On the relativistic semiclassical theory of the Coulomb excitation of atoms

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The processes of Coulomb excitation and ionization of atoms by fast charged particles moving along classical trajectories are considered. The target electrons are described by the Dirac equation and the field of the incoming particle is described by the Lienard-Wiechert potential. The theory is formulated in a form most convenient for the description of all the characteristics of semiclassical atomic collisions. Analytic expressions are obtained in the cases of small and large momentum transfers for K-shell ionization cross sections and for the probabilities and cross sections for the $1S_{1/2}\to 2S_{1/2}$, $1S_{1/2}\to 2P_{1/2}$, and $2S_{1/2}\to 2P_{1/2}$ transitions in a heavy hydrogen-like atom. The stimulated multiphoton emission and absorption processes that occur in inelastic scattering of relativistic electrons by light atoms in an external electromagnetic field are considered. Universal relations are derived in the low-frequency approximation for the total cross section for stimulated absorption in all the inelastic channels.

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The theoretical investigation of the Coulomb excitation and ionization of atomic and nuclear targets in relativistic collisions has very recently become essential owing to the development of particle-acceleration technology and the ever increasingly extensive use of relativistic charged-particle beams.[1–5] At the same time we find, in studying collisions of charged particles with individual atoms,[6–11] collisions of light atoms with multiply charged ions,[12,13] and collisions in crystals under channeling conditions,[14–16] that we not only need to estimate the cross sections, we must also be able to determine the excitation or ionization probabilities as functions of the impact parameter **b**. This is achieved with the aid of a semiclassical approach in which it is assumed that the charged particle moves along a definite trajectory $\mathbf{R}(t)$, that it is structureless, and that it is only a source of a classical alternating electromagnetic field.[17,18]

The most convenient formulation of the semiclassical theory of Coulomb excitation of atoms in relativistic collisions is as follows. The electrons in the atom are described by the Dirac equation and the external field produced by the incoming particle is described by the Lienard-Wiechert potential $(\Phi, \mathbf{A})$. The operator for the interaction between a target electron and a charged particle moving along an arbitrary trajectory with velocity $v(t)$ is

$$\hat{V}(t)=e\Phi(t)-e\hat{\alpha}\mathbf{A}(t)=e\Phi(1-\hat{\alpha}\mathbf{v}/c), \tag{1}$$

where $\hat{\alpha}=\gamma^0\boldsymbol{\gamma}$, the γ^μ being the Dirac matrices. In first order in $\hat{V}(t)$ or $\eta = Ze^2/\hbar v$ (Z is the charge of the incoming particle), the amplitude of the transition between the steady states $|i\rangle$ and $|f\rangle$ $(\neq|i\rangle)$ of the unperturbed Hamiltonian is given by

$$\mathfrak{M}_{fi}=\frac{ie}{\hbar}\int_{-\infty}^{\infty} dt e^{i\Omega t}\langle f|\Phi(1-\hat{\alpha}\mathbf{v}/c)|i\rangle, \quad \Omega=\omega_f-\omega_i. \tag{2}$$

In the case of a straight-line trajectory, i.e., for $\mathbf{R}(t)=\mathbf{b}+\mathbf{v}t$, we obtain for the excitation amplitude and cross section after expanding $\hat{V}(t)$ in a Fourier integral in the coordinates, the expressions (cf. Ref. 10)

$$\mathfrak{M}_{fi}(\mathbf{b})=\frac{i}{\pi}\eta\int\frac{d^3q}{q^2-(\Omega/c)^2}\delta\left(\frac{\mathbf{qv}-\Omega}{v}\right)e^{-i\mathbf{qb}}\mu_{fi}(\mathbf{q}), \tag{3}$$

$$\sigma_{fi}=\int d^2b|\mathfrak{M}_{fi}(\mathbf{b})|^2=4\eta^2\int\frac{d^3q}{[q^2-(\Omega/c)^2]^2}\delta\left(\frac{\mathbf{qv}-\Omega}{v}\right)|\mu_{fi}(\mathbf{q})|^2. \tag{4}$$

But if, for example, the actual trajectories of the incoming particles can be represented in the form

$$\mathbf{R}(t)=\mathbf{b}+\mathbf{v}t+\mathbf{a}_1\sin(\omega t+\varphi)-\mathbf{a}_2\cos(\omega t+\varphi), \tag{5}$$

then the excitation amplitude (cf. Ref. 19)

$$\mathfrak{M}_{fi}(\mathbf{b})=\frac{i}{\pi}\eta\int d^3q e^{-i\mathbf{qb}}\mu_{fi}(\mathbf{q}) \times\sum_{\nu=-\infty}^{\infty}J_\nu(\mathcal{N})\delta\left(\frac{\Omega-\omega_\nu}{v}\right)\frac{\exp[i\nu(\alpha-\varphi)]}{q^2-(\Omega/c)^2}; \tag{6}$$

$$\omega_\nu=\mathbf{qv}+\nu\omega,\quad \mathcal{N}=[(\mathbf{qa}_1)^2+(\mathbf{qa}_2)^2]^{1/2},$$
$$\cos\alpha=\mathbf{qa}_1/\mathcal{N},\quad \sin\alpha=\mathbf{qa}_2/N,$$

and the $J_\nu(\mathcal{N})$ are Bessel functions. The φ-averaged cross sections for the inelastic channels are sums of partial cross sections

$$\sigma_{fi}=\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int d^2b|\mathfrak{M}_{fi}(\mathbf{b})|^2=\sum_{\nu=-\infty}^{\infty}\sigma_{fi}^\nu, \tag{7}$$

$$\sigma_{fi}^\nu=4\eta^2\int\frac{d^3q}{[q^2-(\Omega/c)^2]^2}\delta\left(\frac{\Omega-\omega_\nu}{v}\right)J_\nu^2(\mathcal{N})|\mu_{fi}(\mathbf{q})|^2. \tag{8}$$

In the formulas (3), (4), (6), and (8)

$$\mu_{fi}(\mathbf{q})=\langle f|e^{i\mathbf{qr}}(1-\hat{\alpha}\mathbf{v}/c)|i\rangle=\frac{c}{\Omega}\mathbf{Q}\langle f|\hat{\alpha}e^{i\mathbf{qr}}|i\rangle. \tag{9}$$

The vector

$$\mathbf{Q}=\mathbf{q}-\Omega\mathbf{v}/c^2 \tag{10}$$

differs from **q** precisely because of the presence of the magnetic interaction term $\sim\hat{\alpha}\mathbf{A}$ in the operator $\hat{V}(t)$. The transverse components of the vectors **Q** and **q** (in the plane of the impact parameter) coincide, but in the longitudinal direction (i.e., along **v**) $Q_\parallel = q_\parallel/\gamma^2$ ($\gamma=(1-v^2/c^2)^{-1/2}$ is the Lorentz factor).

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The expression for $\mu_{fi}(\mathbf{q})$ is significantly simplified in many of the most important particular cases. For nonrelativistic velocities, i.e., for $v \ll c$, and any αZ_a ($\alpha = e^2/\hbar c$ is the fine-structure constant and Z_a is the effective charge of the nucleus of the target atom for the electron shell in question) we have

$$\mu_{fi}(\mathbf{q}) \simeq M_{fi}(\mathbf{q}) \equiv \langle f | e^{i\mathbf{qr}} | i \rangle. \tag{11}$$

For $\Delta \equiv \Omega a/v \ll \gamma (a = \hbar^2/Z_a me^2)$ the dominant contributions to the amplitude and the cross section are made by $qa \lesssim 1$, for which

$$\mu_{fi}(\mathbf{q}) \approx \begin{Bmatrix} i\mathbf{Q}a\vec{\mathscr{M}}_{fi} & \text{for } qa \ll 1 \\ M_{fi}(\mathbf{q}) & \text{for } qa \sim 1 \end{Bmatrix} \approx M_{fi}(\mathbf{Q}) \equiv \langle f | e^{i\mathbf{Qr}} | i \rangle, \tag{12}$$

$$\vec{\mathscr{M}}_{fi} \equiv \langle f | \mathbf{r}/a | i \rangle. \tag{13}$$

Accordingly, the general expressions for the amplitudes and the cross sections are also significantly simplified in the regions of small and large momentum transfers.

For large impact parameters (i.e., for $\rho \equiv b/a \gg 1$) and small momentum transfers ($\Delta \ll 1$) we have

$$\mathfrak{M}_{fi}(\mathbf{b}) \approx 2i\eta \frac{\Delta}{\gamma^2} \vec{\mathscr{M}}_{fi} \left[\gamma \frac{\mathbf{b}}{b} K_1\left(\frac{\Delta}{\gamma}\rho\right) + i\frac{\mathbf{v}}{v} K_0\left(\frac{\Delta}{\gamma}\rho\right) \right], \tag{14}$$

the $K_\nu(z)$ are modified Bessel functions. The differential probability for ionization is given in this approximation by the expression

$$\frac{dW(\mathbf{b})}{d(ka)\,do_e} \approx (2\eta\Delta/\gamma^2)^2 \frac{(ka)^2}{(2\pi a)^3} \vec{\mathscr{M}}_{fi} \vec{\mathscr{M}}_{if}$$
$$\times \left[\gamma^2 \left(\frac{\mathbf{kb}}{kb}\right)^2 K_1^2\left(\frac{\Delta}{\gamma}\rho\right) + \left(\frac{\mathbf{kv}}{kv}\right)^2 K_0^2\left(\frac{\Delta}{\gamma}\rho\right) \right]. \tag{15}$$

It is assumed here that the continuous-spectrum wave function entering into $\vec{\mathscr{M}}_{fi}$ describes the state of an electron with definite momentum and polarization at infinity; do_e is the solid-angle element in which the momentum $\hbar\mathbf{k}$ of the outgoing electron lies; and the scalar product $\vec{\mathscr{M}}_{fi}\vec{\mathscr{M}}_{if}$ does not depend on the direction of $\mathbf{k}$. The condition $\Delta \ll 1$ of applicability of (14) and (15) is rather weak for transitions between highly excited states or fine-structure components. This condition reduces in other bound-bound channels and in the case of ionization with $ka \lesssim 1$ to the inequality $\xi \equiv \hbar/mav \ll 1$, and is replaced in the case of ionization channels in which fast electrons are ejected (i.e., for which $ka \gg 1$) by the weaker condition $\Delta \ll ka$. The latter case is due to the fact that the dominant contribution to the ionization amplitude is made in this case by $r \lesssim 1/k$.

The cross sections for the dipole-allowed bound-bound and bound-free transitions are given to within logarithmic accuracy (i.e., under the condition that $\ln(\gamma/\Delta) \gg 1$) by the expressions

$$\sigma_{fi} \approx 8(\pi a^2)\eta^2 \ln(\gamma/\Delta) |\vec{\mathscr{M}}_{fi}|^2, \tag{16}$$

$$\frac{d\sigma(\mathbf{k})}{d(ka)\,do_e} \approx 4(\pi a^2)\eta^2 \frac{(ka)^2}{(2\pi a)^3} \left[1 - \left(\frac{\mathbf{kv}}{kv}\right)^2 \right] \ln(\gamma/\Delta) \vec{\mathscr{M}}_{fi}\vec{\mathscr{M}}_{if}. \tag{17}$$

Since, generally speaking, the relativistic wave functions, which are adequate for the formulation of the scattering problem, and describe the states with definite momentum and polarization at infinity, cannot be written in closed form even in the case of the Coulomb field, it is more convenient to use somewhat different relations to compute the ionization cross sections $d\sigma/dE_k$ (E_k is the energy of the secondary electron). In the case when electrons are knocked out from, for example, the S state, the square, averaged over the initial and summed over the final degenerate states, of the modulus of the form factor is

$$\langle |M_{S\to k}(\mathbf{q})|^2 \rangle = \sum_{j,l} |\varkappa| \left\{ \int dr r^2 j_l(qr) S_\varkappa(r) \right\}^2, \tag{18}$$

$$S_\varkappa(r) = G_S(r) G_k^*(r) + F_S(r) F_k^*(r), \tag{19}$$

$\{j, l, \varkappa\}$ are the quantum numbers of the continuous-spectrum wave functions with definite energy, momentum, and parity, $G(r)$ and $F(r)$ are respectively the "large" and "small" components of the Dirac radial wave functions, and the $j_l(x)$ are spherical Bessel functions. Then if the $G_k(r)$ and $F_k(r)$ are normalized to the delta function in energy, we find in the region of small momentum transfers that

$$\frac{d\sigma_S}{dE_k} \approx \frac{8\pi}{9} \eta^2 \ln(\gamma/\Delta) \sum_{\varkappa=+1,-2} |\varkappa| \left\{ \int dr r^3 S_\varkappa(r) \right\}^2. \tag{20}$$

For light targets (i.e., for $\xi \ll 1$) the relativistic effects due to the closeness of v to c practically do not manifest themselves in close ($\rho \lesssim 1$) encounters, and in the case of large impact parameters the magnetic interaction of the electron with the incoming particle almost completely cancels out the increase in the role of the retardation in the Coulomb interaction. The effective strength of the Coulomb interaction increases rapidly with increasing γ in view of the relativistic contraction of the range of the field in the direction of $\mathbf{v}$ (as a consequence the impact-parameter range where the excitation of the atom occurs nonadiabatically broadens rapidly). But the weakening of the total interaction as a result of the presence of the magnetic term leads to a situation in which the excitation cross section depends weakly (logarithmically) on the Lorentz factor.

The main problem in the case of large momentum transfers is the determination of the relative contributions of the various terms in the expansion in inverse powers of q of the form factor:

$$M_{fi}(\mathbf{q}) = \sum_{n=0}^{\infty} A_{fi}^{\ n}/q^{\nu+n} \tag{21}$$

or of the properly averaged square of its modulus, i.e., of an expression of the type (18), (19):

$$\langle |M_{fi}(\mathbf{q})|^2 \rangle = \sum_{n=0}^{\infty} B_{fi}^{\ n}/q^{\mu+n}. \tag{22}$$

Estimates for nonrelativistic atoms[20] show that, as a rule, $\nu = 4 + l_i + l_f$ and $\mu = \nu^2$ (l_i and l_f are the orbital angular momenta of the initial and final states), and, consequently, the expansions are in integral powers. In this case the transition probabilities and cross sections can be approximately computed by replacing the series (21) and (22) by their first terms. As will be seen from what follows, the case of relativistic targets differs from the nonrelativistic case in two important respects. First, the exponents ν and μ are now not integers and depend on the quantity αZ_a. Second, and more important, the coefficients A_{fi}^n and B_{fi}^n depend also on αZ_a. Therefore, generally speaking, for any αZ_a and different momentum transfers we must take several first terms in the expansions (21) and (22) into consideration, and thus make the behavior of the transition probabilities and cross sections depend essentially on the specific values of αZ_a and Δ.

In accordance with an expansion of the type (21), the amplitude $\mathfrak{M}_{fi}(\mathbf{b})$ is a sum, the n-th term of which can be calculated analytically under the assumption that A_{fi}^n does not depend on the direction of $\mathbf{q}$ and that $v \ll c$:

$$\mathfrak{M}_{fi}^n(\mathbf{b}) \approx 2i\eta A_{fi}^n \left(\frac{\rho}{2\Delta}\right)^{(\nu+n)/2} K_{(\nu+n)/2}(\Delta\rho) \Big/ \Gamma\left(\frac{2+\nu+n}{2}\right). \tag{23}$$

Similarly, the excitation cross section, averaged or summed over the degenerate final states, can be found:

$$\langle\sigma_{fi}\rangle \approx 8\pi\eta^2 \sum_{n=0}^{\infty} \frac{\langle B_{fi}^n\rangle}{2+\mu+n} (a/\Delta)^{2+\mu+n}. \tag{24}$$

Thus far, in the semiclassical theory of Coulomb excitation there has been considered, more or less in detail, retardation by light atoms (see Refs. 21 and 22 for reviews), and the probabilities and cross sections for ionization of heavy atoms by slow particles[11,23,24] and the probabilities for certain light-atom excitation channels in nonrelativistic collisions[25,27] have been studied. Of great interest at present is the investigation of not only the ionization processes, but also the bound-bound transitions that occur in multiply charged ions of medium-weight and heavy atoms, which is due, for example, to the prospects of using them in x-ray lasers and to their important role in thermonuclear plasmas.

Below we consider the general case, i.e., the case in which $\alpha Z_a \lesssim 1$ and $v \lesssim c$ at the same time, within the framework of the hydrogen-like atom model, analytically calculating not only the cross sections, but also the probabilities for the individual transitions. Since the relativistic effects in the heavy atoms are most important for the $1S_{1/2}$, $2S_{1/2}$, and $2P_{1/2}$ states, the main attention will be given to the transitions between them and the knocking out of electrons from the K shell. We can neglect in the computations in the region of large momentum transfers the finite nuclear dimensions and the nuclear structure if $qr_0A^{1/3} \ll 1$ (A is the atomic weight and $r_0 \approx 10^{-13}$ cm). In the case of the inner atomic shells allowance for the screening and its effect on the normalization of the wave functions of the continuous spectrum can be made within a broad range of collision parameters (where the knocked-out electron velocities $v_{el} \gtrsim e^2/\hbar$) approximately with the aid of the Slater rules, i.e., by simply redefining Z_a (for a discussion of this question, see, for example, Ref. 1).

In the dipole region the probabilities for excitation involving $1S_{1/2} \to 2P_{1/2}$ and $2S_{1/2} \to 2P_{1/2}$ transitions are, in the case of small momentum transfers and any γ, given by

$$W_{1S_{1/2}\to 2P_{1/2}}(\mathbf{b}) \approx \frac{2^{4s+3}}{9} N^{2s+6} \frac{(N-1)^2(N+2)}{(N+1)^{4s+5}} \left(\frac{\Delta}{\gamma^2}\eta\right)^2 \times\left[\gamma^2 K_1^2\left(\frac{\Delta}{\gamma}\rho\right) + K_0^2\left(\frac{\Delta}{\gamma}\rho\right)\right], \tag{25}$$

$$W_{2S_{1/2}\to 2P_{1/2}}(\mathbf{b}) \approx (2s+1)\left[\frac{\Delta(1+s)}{\gamma^2}\eta\right]^2 \times\left[\gamma^2 K_1^2\left(\frac{\Delta}{\gamma}\rho\right) + K_0^2\left(\frac{\Delta}{\gamma}\rho\right)\right]. \tag{26}$$

The cross sections for these transitions are given with logarithmic accuracy by

$$\sigma_{1S_{1/2}\to 2P_{1/2}} \approx \frac{2^{4s+4}}{9}(\pi a^2)\eta^2 \ln(\gamma/\Delta) N^{2s+6} \frac{(N-1)^2(N+2)}{(N+1)^{4s+5}}, \tag{27}$$

$$\sigma_{2S_{1/2}\to 2P_{1/2}} \approx \frac{\pi a^2}{2}\eta^2 N^4 (N^2-1)\ln(\gamma/\Delta). \tag{28}$$

Here and below we use the following notation

$$\exp(2i\delta) = \frac{-\varkappa + i/ka}{t+iw/ka}, \quad w = \frac{E_k}{mc^2} = \left[1+\left(\frac{\hbar k}{mc}\right)^2\right]^{1/2},$$

$$s=[1-(\alpha Z_a)^2]^{1/2}, \quad N=(2+2s)^{1/2}, \quad t=[|\varkappa|-(\alpha Z_a)^2]^{1/2},$$

$$X=\{[(1+s)(w+1)]^{1/2} - i[(1-s)(w-1)]^{1/2}\}(t+iw/ka)e^{i\delta}.$$

The K-shell ionization cross section for small momentum transfers and any αZ_a and γ is given by

$$\frac{d\sigma_{1s}}{dE_k} \approx \frac{8}{9}(\pi a^2)\eta^2\left(\frac{ma}{\hbar^2 k}\right)\ln\left(\frac{\gamma}{\Delta}\right)\sum_{\varkappa=+1,-2}|\varkappa|\{f_{1s}(\varkappa)\}^2, \tag{29}$$

$$f_{1s}(\varkappa) = \frac{2^s}{[\pi\Gamma(2s+1)]^{1/2}} e^{\pi w/2ka}(2ka)^t \frac{\Gamma(s+t+2)}{\Gamma(2t+1)}\left|\Gamma\left(t+\frac{iw}{ka}\right)\right| \operatorname{Re}\left\{\frac{X}{(1+ika)^{s+t+2}} F\left(s+t+2;\, t+1+\frac{iw}{ka};\, 2t+1;\, \frac{2ika}{1+ika}\right)\right\}. \tag{30}$$

The heavy-atom excitation amplitudes and cross sections can, in the case of large momentum transfers, be written in the form

$$\mathfrak{M}_{fi}(\mathbf{b}) \approx 2i\eta C \sum_{n=n_1}^{n_2} A_n \left(\frac{\rho}{2\Delta}\right)^{(2s+1+n)/2} \times K_{(2s+1+n)/2}(\Delta\rho)/\Gamma\left(\frac{2s+3+n}{2}\right), \tag{31}$$

$$\sigma_{fi} \approx 8(\pi a^2)\eta^2 C^2 \sum_{n=n_3}^{n_4} \frac{B_n}{4s+4+n}\left(\frac{1}{\Delta}\right)^{4s+4+n}. \tag{32}$$

For the $1S_{1/2} \to 2S_{1/2}$ transition, in the formulas (31) and (32) $n_1 = n_3 = 0$, $n_2 = 1$, $n_4 = 2$, $B_0 = A_0^2$, $B_1 = 2A_0A_1$, $B_2 = A_1^2$,

$$A_0 = N\frac{N-1}{2s}\sin(\pi s), \quad A_1 = -N^2\cos(\pi s), \qquad C = \frac{2^{2s+1/2}}{N^{s+2}}\left(\frac{N+2}{N-1}\right)^{1/2}. \tag{33}$$

For the $1S_{1/2} \to 2P_{1/2}$ transition, in the formulas (31) and (32) $n_1 = 1$, $n_2 = n_3 = 2$, $n_4 = 4$, $B_2 = A_1^2$, $B_3 = 2A_1A_2$, $B_4 = A_2^2$,

$$A_1 = -\sin(\pi s),\quad A_2 = 2s\frac{N}{N-1}\cos(\pi s),$$

$$C = \frac{i}{2s}\frac{2^{2s+1/2}}{N^{s+2}}\left(\frac{N+2}{N+1}\right)^{1/2}. \tag{34}$$

The K-shell ionization cross section for large momentum transfers is given by

$$d\sigma_{1s\to k}/dE_k = -(ma/\hbar^2 k)\sigma_{fi}, \tag{35}$$

where σ_{fi} is computed from the formula (32) with $n_3 = 0$ and $n_4 = 2$. In this case in the region $ka \ll 1$ and $\Delta \gg 1$ the coefficients $B_0 = D_0^2$, $B_1 = 2D_0D_1$, and $B_2 = D_1^2$, while

$$D_0 = \sin(\pi s)/s,\quad D_1 = -4[(s+2)/(2s+1)]\cos(\pi s), \tag{36}$$

$$C = \frac{2^{4s-1}}{\pi}\frac{1+s}{\Gamma(2s+1)}(ka)^{2s}e^{\pi w/ka}\left|\Gamma\left(s+\frac{iw}{ka}\right)\right|^2. \tag{37}$$

In the region $ka \gtrsim 1$ the least unwieldy results are obtained when momentum transfers such that $\Delta \gg ka + 1$ are considered. In this case, instead of (36) and (37) we should make the following substitutions in (32):

$$D_0 \to D_0\,\mathrm{Re}\,(X_0),\quad D_1 \to D_1\,\mathrm{Re}\,[X_0(2w+2s+1-ika)]/(2s+4), \tag{38}$$

$$C \to C/N^2,\quad X_0 = X|_{x=-1}. \tag{39}$$

The relative contributions of the individual terms in the amplitude (31) and the cross sections (32) and (35) depend essentially on the relation between Δ and αZ_a; therefore, it makes no sense to raise the question of the universal dependence of the probabilities and cross sections on the momentum transfer in the case of heavy atoms.

Reference 19 reports the investigation of inelastic electron-atom collisions occurring in an external nonresonant laser field, the effect of which amounts largely to a modification of the actual trajectories of the incoming electrons. Unfortunately, in relativistic collisions the representation (5) for the classical trajectory $\mathbf{R}(t)$ is not always, even if often, valid. It is suitable for the description, with a specific degree of accuracy, of the motion of relativistic particles in certain electric and magnetic "undulator," as well as in the case of planar and axial channeling in crystals. In contrast to the case of "undulators," in which the effective electron mass increases and, consequently, the amplitudes $\mathbf{a}_{1,2}$ decrease with increasing γ, the transverse-vibration frequencies depend on γ under channeling conditions: $\omega \sim \gamma^{-1/2}$.

The total cross section for stimulated multiphoton emission-absorption in all the inelastic channels,

$$\sigma_{\text{inel}}^{\text{obs}} = \sum_{f\neq i}\sum_{\nu=1}^{\infty}\nu(\sigma_{fi}^{\nu} - \sigma_{fi}^{-\nu}), \tag{40}$$

can be computed for arbitrary targets in the low-frequency ($\omega \ll \Omega$) approximation in the same way as is done in Ref. 19. All the transformations necessary for this purpose are most simply performed in the approximation (12) if

$$\gamma^2\frac{\omega}{\Omega}\left(\frac{\tilde{v}}{v}\frac{\Omega}{\omega}+1\right) \ll 1; \tag{41}$$

$\hbar\Omega$ is the mean excitation-energy value, equal approximately to the ionization potential, and $\tilde{v}$ is the additional variable velocity acquired by the incoming electron in the external field. As a result, we find that (cf. Ref. 19)

$$\sigma_{\text{inel}}^{\text{obs}} \approx Z^*\pi a_0^2\left(\frac{e^2}{\hbar v}\right)^4\frac{\hbar\omega}{Ry}\frac{\omega\varkappa_0}{2\hbar v^2}(3a_0^2 - a_1^2 - a_2^2),$$
$$a_0^2 = (\mathbf{a}_1\mathbf{v}/v)^2 + (\mathbf{a}_2\mathbf{v}/v)^2, \tag{42}$$

Z^* is the number of effectively excited target electrons and $\varkappa_0$ is the effective cross section for the stopping of an electron by an atom in the absence of an external field ($\varkappa_0 \sim Z^*$). It follows from the general expression (42) that there always occurs in the case of transverse (with respect to $\mathbf{v}$) motion of the incoming electrons in the external field stimulated "absorption of the quanta" of this field. Stimulated "emission of quanta," i.e., stimulated amplification of the external field, is also possible. The cross section for this process increases as the angle between $\mathbf{v}$ and the direction of the linear oscillations $\mathbf{a}_1$ ($\mathbf{a}_2 \equiv 0$) decreases. As γ increases, the angle range in which $\sigma_{\text{inel}}^{\text{obs}} < 0$ broadens, though insignificantly, since $\varkappa_0$ depends on γ weakly, i.e., logarithmically.

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