Determination of baryon and baryon-resonance masses from quantum-chromodynamics sum rules. Nonstrange baryons

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The masses of the nucleon, the Δ isobar, and the $N^*(J^P = \frac{3}{2}^-, T = \frac{1}{2})$ resonance as well as the amplitudes of virtual transitions of these states into quark currents are determined on the basis of the QCD sum rules with allowance for the higher-power corrections. The consistency of results obtained from the analysis of different sum rules is demonstrated.

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I. INTRODUCTION

Quantum chromodynamics (QCD) is now becoming a universally accepted strong-interaction theory. Therefore one of the most interesting and most important problems of QCD is a model-free description of hadron mass spectra. The QCD sum-rule method, first proposed in Ref. 1, has made possible much progress in the description of meson properties. This method was extended in Ref. 2 to include baryons, and the first results were obtained there on the masses of both strange and ordinary baryons (see also Ref. 3). The present paper continues the investigation of the sum rules for baryons. We pursue simultaneously several goals. First, refinement of the sum rules obtained in Ref. 2, by taking higher-power corrections into account, and demonstration, by the same token, of the reliability of the values calculated in Ref. 2 for the nucleon mass m_N and the isobar mass m_Δ as well as of residues in the quark currents β_N and λ_Δ. Second, formulation of a new independent set of sum rules from which m_N and β_N can also be determined, and to check the consistency of these two methods of determining m_N and β_N. Third, calculations more definite than in Ref. 2, on the basis of the new sum rules, of the mass of the baryon resonance $N^*(J^P = \frac{3}{2}^-, T = \frac{1}{2})$. Fourth, demonstration of the self-consistency of the results, on the basis of a large overdefined system of sum rules. Fifth, refinement of the amplitudes of the transition of a nucleon into quark currents, which determine the matrix element of the proton decay in the grand-unification theory and the asymptotic form of the nucleon electromagnetic form factor. Sixth and last, finding the magnitude of the quark-gluon condensate $g_s\langle 0|\bar{u}\sigma_{\mu\nu}(\lambda^k/2)G^k_{\mu\nu}u|0\rangle$ from the conditions of best agreement of the sum rules.

2. THE METHOD

We describe briefly in this section the sum-rule method for baryons, define the notation, consider the properties of the baryon currents, and describe the sum-rule saturation procedure to be used subsequently.

We consider the polarization operator

$$\Pi_{\mu\nu,\alpha\beta}(q) = i\int e^{iqx}\langle 0|T\{\eta_{\mu,\alpha}(x), \bar{\eta}_{\nu,\beta}(0)\}|0\rangle d^4x, \tag{1}$$

where $\eta_{\mu,\alpha}$ is a colorless spin vector made up of quark operators and having the quantum numbers of the baryons (baryon charge, isotopic spin, strangeness); μ and ν are the Lorentz indices; α and β are the spinor indices and will be omitted hereafter; $\bar{\eta}_\mu = \eta_\mu\gamma^0$.

The polarization operator (1) is calculated in the region $q^2 \sim -1\ \text{GeV}^2$ where, on the one hand, perturbation theory is still applicable, $\alpha_s \approx 0.3$, but on the other the contribution of the nonperturbative effects connected with the structure of the QCD vacuum is no longer small. Nonperturbative corrections will be taken into account when the operator expansion is used. The operator $\Pi_{\mu\nu}(q)$ contains various structures ($g_{\mu\nu}$, $g_{\mu\nu}\hat{q}$, $\gamma_\mu\gamma_\nu$, $q_\mu q_\nu$, etc). For the function at each structure in the polarization operator we write a dispersion relation

$$f(Q^2) = \frac{1}{\pi}\int_0^\infty \frac{\operatorname{Im} f(s)\,ds}{s+Q^2}, \qquad Q^2 = -q^2. \tag{2}$$

Expression (2) is written in simplified form, since it would actually contain a sufficient number of the subtractions necessitated by the divergence of the dispersion integral. If however, we apply to (2) the Borel transformation proposed in Ref. 1,

$$f_B(M^2) = \hat{B}f = \frac{(Q^2)^{n+1}}{n!}\left(-\frac{d}{dQ^2}\right)^n f(Q^2);$$
$$\frac{Q^2}{n} = M^2, \qquad n\to\infty, \tag{3}$$

all the subtraction terms vanish and we get the equality

$$f_B(M^2) = \frac{1}{\pi}\int_0^\infty e^{-s/M^2}\operatorname{Im} f(s)\,ds. \tag{4}$$

Relations (4), written for each of the structure functions of the polarization operator, are sum rules whose left-hand sides are calculated on the basis of quantum chromodynamics, and the right-hand sides phenomenologically. Expressions (4) offer the following advantages over (2): 1) absence of subtraction terms; 2) stronger suppression, due to the factor $\exp(-s/M^2)$, of the contribution of the heavy intermediate states, so that (4) can be used to study the lightest resonances. We consider the following currents η_μ with baryon quantum numbers:

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$$\eta_{1\mu}=\varepsilon^{abc}[(u^aCd^b)\gamma_\mu u^c-(u^aC\gamma_5d^b)\gamma_\mu\gamma_5u^c]=\gamma_\mu\eta/2,$$
$$\eta_{2\mu}=\varepsilon^{abc}[(u^aC\sigma_{\rho\lambda}d^b)\sigma_{\rho\lambda}\gamma_\mu u^c-(u^aC\sigma_{\rho\lambda}u^b)\sigma_{\rho\lambda}\gamma_\mu d^c],$$
$$\eta_\mu{}^\Delta=\varepsilon^{abc}(u^aC\gamma_\mu u^b)u^c, \quad (5)$$

where $a, b, c, = 1, 2, 3$ are the color indices, $C = -C^T$ is the charge-conjugation matrix, and $\sigma_{\mu\nu} = \frac{1}{2}i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$. The currents η and η_μ^Δ coincide with the currents used in Ref. 2 to obtain sum rules for the case of the nucleon and the Δ isobar. The current $\eta_{2\mu}$ corresponds to isotopic spin $T=\frac{1}{2}$ and to total angular momenta $J=\frac{3}{2}$ and $\frac{1}{2}$, so that in the sum over the states, in the right-hand side of (4),

$$\mathrm{Im}\,\Pi_{\mu\nu}(s)=\pi\sum_n\delta(s-M_n^2)\langle 0|\eta_\mu|n\rangle\langle n|\bar\eta_\nu|0\rangle, \quad (6)$$

it is necessary in the case of the current $\eta_{2\mu}$ to take into account among the single-particle states the baryons with $T=\frac{1}{2}$ and $J^P=\frac{3}{2}^\pm$, $\frac{1}{2}^\pm$ (nucleon, N^* resonance with $J^P=\frac{3}{2}^\pm$, etc.).

We consider first the calculation of the left-hand sides of the sum rules. As already mentioned, the strong-inte·‹c-tion constant is quite small in the region $|q|^2\sim 1$ GeV^2 an‹, neglecting α_s, the principal diagram corresponding to the zeroth term of the operator expansion is the simple quark loop of Fig. 1a. The first term of the operator expansion (dimensionality $d=3$) is proportional to the quark condensate $\langle 0|\bar\psi\psi|0\rangle$ with $\psi = u$ or d (diagram e of Fig. 1). In the case of baryon sum rules, it is the most significant (see Ref. 2), since it causes the appearance of structures that violate the chiral invariance and, in particular, determines the values of the baryon masses. From among the operators with larger dimensionality, next in significance are the four-quark operators $\bar\psi\Gamma\psi\bar\psi\Gamma\psi$ ($d=6$), whose contribution to the sum rules is described by diagram c of Fig. 1 and, as discussed in Ref. 2, its importance is due to the presence of the large numerical coefficient $\sim(2\pi)^4$ compared with diagram a of Fig. 1. In the vacuum mean values of the four-quark operators we shall use the factorization hypothesis (see Ref. 1), i.e., in the sum over the intermediate states

$$\sum_n\langle 0|\bar\psi\Gamma\psi|n\rangle\langle n|\bar\psi\Gamma\psi|0\rangle$$

we shall retain only the contribution of the vacuum state, so that all the four-quark operators averaged over the vacuum reduce effectively to $\langle 0|\bar\psi\psi|0\rangle^2$. Besides the operators indicated above, there appear in the sum rules also two operators with dimensionality $d<6$, namely $g_s\bar\psi\sigma_{\mu\nu}(\lambda^n/2)G^n_{\mu\nu}\psi$ ($d=5$) and $G^n_{\mu\nu}G^n_{\mu\nu}$ ($d=4$). The most important among them are the contributions of the vacuum mean vlaue of the operator $g_s\bar\psi\sigma_{\mu\nu}(\lambda^n/2)G^n_{\mu\nu}\psi$ (diagrams f of Fig. 1), which are the corrections of first order in $1/M^2$ to the structures that violate chiral invariance. In the calculation of the polarization operator $\Pi_{\mu\nu}(q)$ (the left-hand side of the sum rules) we shall, using the factorization hypothesis, take into account also the vacuum mean values of the following operators with dimensionality $d>6$: $\langle 0|\bar\psi\Gamma_1\psi\bar\psi\Gamma_2\psi G|0\rangle$ (Fig. 1d), $\langle 0|\bar\psi\Gamma\psi G^2|0\rangle$ (Fig. 1g), and $\langle 0|\bar\psi\Gamma_1\psi\bar\psi\Gamma_2\psi\bar\psi\Gamma_3\psi|0\rangle$ (Fig. 1h). The contribution of the gluon condensate $\langle 0|G^2_{\mu\nu}|0\rangle$ to the baryon sum rules was considered in Ref. 4 and, as already shown there, turned out to be relatively small. In our calculations we shall use the method developed in Ref. 4. The contribution of the operators with dimensionality $d>6$ was not taken into account previously. All the calculations will be carried out in the zero-mass quark approximation. It must be noted (see Erratum to Ref. 2) that consideration of operators with $d\geqslant 5$ in the operator expansions calls also for allowance for the x dependence of the vaccum mean values of the fermion operators $\langle 0|\bar u_\alpha(x)u_\beta(0)|0\rangle$. In our approximation

$$\langle 0|:u_\alpha{}^a(x)\bar u_\beta{}^b(0):|0\rangle=-\frac{\delta_{\alpha\beta}\delta^{ab}}{12}\langle 0|\bar uu|0\rangle$$
$$-\frac{\delta_{\alpha\beta}\delta^{ab}}{3\cdot 2^6}x^2g_s\langle 0|\bar uG_{\mu\nu}{}^n(\lambda^n/2)\sigma_{\mu\nu}u|0\rangle-i\frac{x^2\hat x_{\alpha\beta}\delta^{ab}}{2^3\cdot 3^5}g_s{}^2\langle 0|\bar uu|0\rangle^2$$
$$-\frac{\delta_{\alpha\beta}\delta^{ab}}{3^3\cdot 2^9}x^4\langle 0|g_s{}^2G^2|0\rangle\langle 0|\bar uu|0\rangle, \quad (7)$$

where : : denotes subtraction of the perturbative part of the average over the vacuum. The last two terms of (7) were obtained using the factorization hypothesis. The x dependence of the vacuum mean value is represented in the diagrams as emission of a soft gluon by a quark with a small momentum.

We use in this paper the following notation for the correlators:

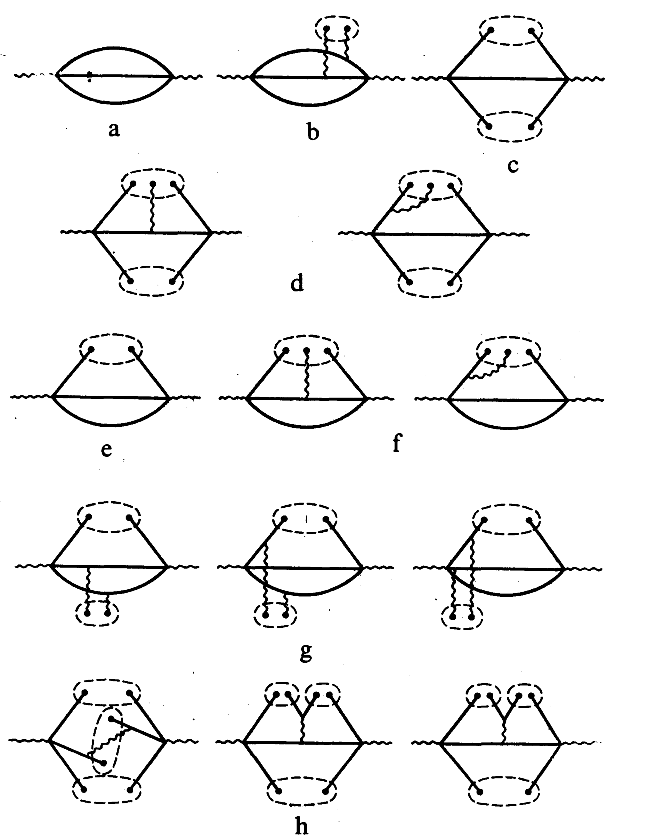

FIG. 1. Feynman diagrams for the polarization operator. The free ends correspond to departure of a soft quark or a gluon to the condensate.

$$\langle 0|\bar{u}u|0\rangle=\langle 0|\bar{d}d|0\rangle=-0{,}24\ \mathrm{GeV}^3;$$
$$a=-(2\pi)^2\langle 0|\bar{u}u|0\rangle=0{,}546\ \mathrm{GeV}^3;$$
$$\left\langle 0\left|\frac{\alpha_s}{\pi}G_{\mu\nu}{}^nG_{\mu\nu}{}^n\right|0\right\rangle=0{,}012\ \mathrm{GeV}^4,$$
$$b=\left\langle 0\left|\frac{\alpha_s}{\pi}G^2\right|0\right\rangle(2\pi)^2, \tag{8}$$
$$g_s\langle 0|\bar{u}G_{\mu\nu}{}^n(\lambda^n/2)\sigma_{\mu\nu}u|0\rangle=g_s\langle 0|\bar{d}G_{\mu\nu}{}^n(\lambda^n/2)\sigma_{\mu\nu}d|0\rangle$$
$$=m_0{}^2\langle 0|\bar{u}u|0\rangle,\qquad m_0{}^2=0{,}8\ \mathrm{GeV}^2.$$

In the calculation of the polarization operator we shall take into account the corrections $\sim\alpha_s^n$ in the principal logarithmic approximation. It can be shown that the currents $\eta_{1\mu}$, $\eta_{2\mu}$ and η_μ^Δ are renormalization-covariant, i.e., each of them is transformed into itself in transformations of the renormalization group, and their anomalous dimensionalities are known.[5] This circumstance allows us to calculate the principal logarithmic corrections in the usual manner.

We denote by γ_{η_i} the anomalous dimensionality of the current η_i, and by γ_n the anomalous dimensionality of the operator $O_n(0)$ in the operator expansion of the T product:

$$T\{\eta_1(x),\bar{\eta}_2(0)\}=\sum_n C_n(x^2)O_n(0). \tag{9}$$

The coefficients of the functions will then contain in the momentum representation the factor

$$(\alpha_s(q^2)/\alpha_s(\mu^2))^{\gamma_{\eta_1}+\gamma_{\eta_2}+\gamma_{\eta_n}}, \tag{10}$$

where $\alpha_s(q^2)$ is the effective QCD charge at the point q^2; μ^2 is the normalization point of the operator expansion. For the currents (5) we have[5]: $\gamma_{\eta i}=-\frac{2}{9}$, $\gamma_{\eta_2}=\gamma_{\eta\Delta}=\frac{2}{27}$

We take into account in the sum rules the anomalous dimensionalities of the following operators: $\bar{\psi}\psi$, $\bar{\psi}\psi\bar{\psi}\psi$, $\alpha_s\pi^{-1}G^2$, and $g_s\bar{\psi}G^n_{\mu\nu}(\lambda^n/2)\sigma_{\mu\nu}\psi$. For operators with dimensionality $d>6$ the factorization hypothesis is not as accurate as, say, for a four-fermion correlator, so that it is meaningless to take into account the anomalous dimensionalities of these operator, according to the results of Ref. 1 the anomalous dimensionalities do not violate the factorization hypothesis to within 10%, and for the operators $\bar{\psi}\psi\bar{\psi}\psi$ we can set $\gamma=8/9$. The anomalous dimensionality of the operator $g_s\bar{\psi}G^n_{\mu\nu}(\lambda^n/2)\sigma_{\mu\nu}\psi$ is small (see Ref. 6), and we shall therefore set it equal to zero hereafter. The anomalous dimensionality of the operator α_sG^2 is equal to zero, since this operator is equal, accurate to terms α_s^2, to the trace of the energy-momentum tensor of the gluon field. In the numerical calculations we shall assume $\alpha_s(q^2)=4\pi/9\ln(-q^2/\Lambda^2)$, $\Lambda=150$ MeV, $\mu=0.5$ GeV.

We proceed now to consider the right-hand (phenomenological) part of the sum rules. By virtue of the exponential suppression of the large masses, the dominant contribution in the right-hand side of (4) is that of the lowest hadron state in the channel with the given quantum numbers. The right-hand side of (4) will therefore take accurate account of the lowest baryon state (in the approximation of infinitely narrow resonances), and all the remaining heavier states will be taken into account via a model. Namely, following Refs. 1 and 2, we approximate the entire remainder by the contribution of a continuum that starts at a certain threshold $s_0=W^2$ and is determined by the simplest quark loop corresponding to the given structure function (i.e., by the diagrams a, b, and e, f of Fig. 1 for structures with odd and even numbers of γ matrices, respectively).

The amplitude product $\langle 0|\eta_\mu|R\rangle\langle R|\bar{\eta}_\nu|0\rangle$ in (6) takes in the case of resonances with spin and parity $J^P=\frac{3}{2}^\pm$ and $\frac{1}{2}^\pm$ the form

$$\langle 0|\eta_{i\mu}|{}^1/{}_2{}^\pm\rangle\langle{}^1/{}_2{}^\pm|\bar{\eta}_{j\mu}|0\rangle=-\beta_i\beta_j\gamma_\mu\gamma_\nu\hat{q}+\frac{\alpha_i\beta_j+\alpha_j\beta_i}{2}(\gamma_\mu q_\nu-\gamma_\nu q_\mu)q$$
$$+\frac{\alpha_j\beta_i-\alpha_i\beta_j}{2}(\gamma_\mu q_\nu+\gamma_\nu q_\mu)\hat{q}+\alpha_i\alpha_jq_\mu q_\nu\hat{q}\mp M_R\beta_i\beta_j\gamma_\mu\gamma_\nu$$
$$+\left(\beta_i\beta_j\mp M_R\frac{\alpha_i\beta_j+\alpha_j\beta_i}{2}\right)(q_\nu\gamma_\mu+q_\mu\gamma_\nu)+\left(\beta_i\beta_j\mp M_R\frac{\alpha_j\beta_i-\alpha_i\beta_j)}{2}\right)$$
$$\times(q_\nu\gamma_\mu-q_\mu\gamma_\nu)+(2\alpha_i\beta_j\mp M_R\alpha_i\alpha_j)q_\mu q_\nu, \tag{11}$$

$$\langle 0|\eta_{i\mu}|{}^3/{}_2{}^\pm\rangle\langle{}^3/{}_2{}^\pm|\bar{\eta}_{j\nu}|0\rangle=-\lambda_i\lambda_j\{g_{\mu\nu}\hat{q}-{}^1/{}_3\gamma_\mu\gamma_\nu\hat{q}$$
$$+{}^1/{}_3(q_\nu\gamma_\mu-q_\mu\gamma_\nu)-{}^2/{}_3q_\mu q_\nu\hat{q}/M_R{}^2\pm M_R[g_{\mu\nu}-{}^1/{}_3\gamma_\mu\gamma_\nu$$
$$+{}^1/{}_3(q_\nu\gamma_\mu-q_\mu\gamma_\nu)\hat{q}/M_R{}^2-{}^2/{}_3q_\mu q_\mu/M_R{}^2]\},\quad i,j=1,2,\Delta, \tag{12}$$

where M_R is the resonance mass; λ, α, and β are constants defined by the formulas

$$\langle 0|\eta_{i\mu}|{}^1/{}_2{}^+\rangle=(\alpha_iq_\mu+\beta_i\gamma_\mu)\gamma_5v(q),$$
$$\langle 0|\eta_{i\mu}|{}^1/{}_2{}^-\rangle=(\alpha_iq_\mu+\beta_i\gamma_\mu)v(q), \tag{13}$$
$$\langle 0|\eta_{i\mu}|{}^3/{}_2{}^+\rangle=\lambda_iv_\mu(q),\quad\langle 0|\eta_{i\mu}|{}^3/{}_2{}^-\rangle=\lambda_i\gamma_5v_\mu(q).$$

Here $v(q)$ is a spinor $((\hat{q}-M_R)v(q)=0,\ \bar{v}v=2M_R)$; $v_\mu(q)$ is a spin-vector $((\hat{q}-M_R)v_\mu(q)=0,\ \bar{v}_\mu v_\mu=-2M_R,\ \gamma_\mu v_\mu(q)=q_\mu v_\mu(q)=0)$. Since the current $\eta_{1\mu}$ actually corresponds by virtue of (5) to $J=\frac{1}{2}$, we have $\lambda_1=\alpha_1=0$.

The sum rules should be considered at values of the parameter M^2 such that, on the one hand the contribution of the higher-power corrections to the rules is small, and on the other, the contribution of the model continuum is small. These two conditions determine the interval of the allowed values of M^2. (This interval will hereafter be designated by the symbol Ω.) In the present paper we impose specifically the following restriction on the contribution made to the sum rules by the continuum and by the higher-power corrections that contain operators with dimensionality $d>6$:1) The contribution of the continuum is $<30\%$; 2) the contribution of the higher-power corrections is $<30\%$.

We introduce the measure δ of the agreement of the sum rules. To this end we transfer to contribution of the continuum to the left-hand side and define the quantity δ as follows:

$$\delta=\max\left\{\left|\ln\frac{F(M^2)}{f_i(M^2)}\right|,\left|\ln\frac{f_i(M^2)}{f_j(M^2)}\right|\right\}_{M\in\Omega}, \tag{14}$$

where $F(M^2)=\lambda^2\exp(-M_R^2/M^2)$ is the contribution of the resonance (the right-hand side of the sum rule), and $f_i(M^2)$ and $f_j(M^2)$ are the left-hand sides of those sum rules to which the given resonance contributes.

It is easily seen that the relative disparity Δ of the sum rules in the region Ω is connected with δ by the relation

$$\Delta \approx 1 - e^{-\delta}, \tag{15}$$

$$\Delta = \max \left\{ \left| \frac{F(M^2) - f_i(M^2)}{F(M^2)} \right|, \quad \left| \frac{f_i(M^2) - f_j(M^2)}{F(M^2)} \right| \right\}_{M \in \Omega} . \tag{16}$$

We minimize with respect to the parameter δ the disparities of the sum rules, and thus obtain the masses and residues of the baryons.[1)] The fact that several sum rules must be reconciled allows us to improve the accuracy with which the masses and residues of the resonances are determined.

The threshold W of the continuum will not be specified beforehand, but will be determined from the condition of the best agreement of the sum rule in the region Ω. For each polarization operator we can, generally speaking, introduce two different continuums at structures with even and odd numbers of γ matrices. The reason is that the contributions of resonance with different parities to a sum rule with an odd number of γ matrices are of the same sign, whereas the sign of their contributions to sum rules with even numbers of γ matrices are different. By the same token, the threshol of the model continuum can generally speaking be different these two cases. Since we are restricting the size of the contribution of the continuum to the sum rules, it exerts no significant influence on the calculated masses and residues of the baryons. Even though we require that the sum rules be in agreement only in the region Ω, the fact that the sum rules are satisfied also in the region where the continuum dominates indicates that, despite the crude model of the higher states, the continuum approximates them well enough, meaning that the correction to the baryon mass to account for the continuum is of the correct sign and order. The accuracy of our results is thereby improved.

3. THE NUCLEON

The matrix elements between the states of a nucleon and of the vacuum differ from zero for both currents $\eta_{1\mu}$ and $\eta_{2\mu}$. Therefore the nucleon mass and its residues in the quark currents can be determined by considering the polarization operators $\Pi_{\mu\nu}(\eta_1, \bar{\eta}_1)$, $\Pi_{\mu\nu}(\eta_1, \bar{\eta}_2)$ and $\Pi_{\mu\nu}(\eta_2, \bar{\eta}_2)$. It is clear that the agreement between results obtained from independent sum rules for these polarization operators can serve as an important check on the validity of the entire approach.

a) *Sum rules for* $\Pi_{\mu\nu}(\eta_1, \bar{\eta}_1)$. The calculation of $\Pi_{\mu\nu}(\eta_1, \bar{\eta}_1)$ with allowance for the diagrams shown in Fig. 1 leads to the following equations:

$$\Pi_{\mu\nu}{}^a = -\frac{q^4 \ln(-q^2/\Lambda^2)}{16(2\pi)^4} \gamma_\mu \hat{q} \gamma_\nu,$$

$$\Pi_{\mu\nu}{}^b = -\frac{\langle 0 | \alpha_s \pi^{-1} G^2 | 0 \rangle}{32(2\pi)^2} \ln\left(-\frac{q^2}{\Lambda^2}\right) \gamma_\mu \hat{q} \gamma_\nu,$$

$$\Pi_{\mu\nu}{}^c = -\frac{\langle 0 | \bar{u}u | 0 \rangle^2}{6q^2} \gamma_\mu \hat{q} \gamma_\nu, \qquad \Pi_{\mu\nu}{}^d = -\frac{m_0{}^2 \langle 0 | \bar{u}u | 0 \rangle^2}{24q^4} \gamma_\mu \hat{q} \gamma_\nu,$$

$$\Pi_{\mu\nu}{}^e = -\frac{\langle 0 | \bar{u}u | 0 \rangle}{4(2\pi)^2} q^2 \ln\left(-\frac{q^2}{\Lambda^2}\right) \gamma_\mu \gamma_\nu, \qquad \Pi_{\mu\nu}{}^f = 0, \tag{17}$$

$$\Pi_{\mu\nu}{}^g = +\frac{\langle 0 | \alpha_s \pi^{-1} G^2 | 0 \rangle \langle 0 | \bar{u}u | 0 \rangle}{3^2 \cdot 2^3 q^2} \gamma_\mu \gamma_\nu,$$

$$\Pi_{\mu\nu}{}^h = \frac{17}{81} \frac{\alpha_s}{\pi} (2\pi)^2 \frac{\langle 0 | \bar{u}u | 0 \rangle^3}{q^4} \gamma_\mu \gamma_\nu.$$

The letters labeling the polarization operators indicate the diagrams of Fig. 1 which were used in their calculation.

Using the method expounded in Sec. 2, we write down the sum rules for the structures $\gamma_\mu \hat{q} \gamma_\nu$ and $\gamma_\mu \gamma_\nu$ (the contribution of the continuum has been transferred to the left-hand part):

$$(\gamma_\mu \hat{q} \gamma_\nu): \quad \frac{1}{8} L^{4/9} M^6 \left(1 - e^{-W_1^2/M^2} \left(\frac{W_1^4}{2M^4} + \frac{W_1^2}{M^2} + 1\right)\right)$$

$$+ \frac{b}{32} L^{4/9} M^2 (1 - e^{-W_1^2/M^2}) + \frac{1}{6} a^2 L^{4/3} - \frac{1}{24} a^2 \frac{m_0{}^2}{M^2} = \tilde{\beta}_1{}^2 e^{-m^2/M^2},$$

$$(\gamma_\mu \gamma_\nu): \quad \frac{1}{4} a L^{8/9} M^4 \left(1 - e^{-W_2^2/M^2} \left(\frac{W_2^2}{M^2} + 1\right)\right) - \frac{ab}{3^2 \cdot 2^3}$$

$$+ \frac{17}{81} \frac{\alpha_s}{\pi} \frac{a^3}{M^2} = \tilde{\beta}_1{}^2 m e^{-m^2/M^2},$$

$$L = \frac{\ln(M/\Lambda)}{\ln(\mu/\Lambda)}, \tag{18}$$

where on the left of the sum rules are indicated the structures for which they have been written, m is the nucleon mass, $\tilde{\beta}_1 = (2\pi)^2 \beta_1$, W_1 and W_2 are the thresholds of the continuum.

The reconciliation of sum rules in the region Ω yields the following values for the mass and the residue of the nucleon:

$$m = 1.02 \pm 0.12 \, \text{GeV}, \; \tilde{\beta}_1{}^2 = 0.45 \pm 0.15 \, \text{GeV}^6;$$

$$\tilde{\beta}_1 = 0.66 \pm 0.11 \, \text{GeV}^3. \tag{19}$$

The scatter of the values of the mass and of the residue are due to the uncertainty of the continuum threshold and to the fact that at a fixed value of the threshold it is possible to choose different m and β_1 without substantially increasing disparity of the sum rules. Figure 2 shows the M dependence of the left and right parts of these sum rules. The values of m and β_1 [Eq. (19)] agree well with the corresponding value obtained in Ref. 2. The residue β_1 determines the amplitude of the proton decay in the SU(5) grand unification theory. Its value (17) confirms the proton lifetime calculations of Ref. 7.

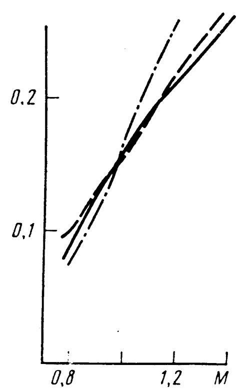

FIG. 2. Plots of the sum rules of the polarization operator $\Pi_{\mu\nu}(\eta_1, \bar{\eta}_1)$. In this and succeeding diagrams we use the following designations; solid line—right-hand part of sum rules, the dashed line corresponds to the left-hand part of the sum rules for structure with odd number of γ matrices, and the dash-dot line—for structures with an even number of γ matrices. The following values were chosen for the mass and residues of the nucleon and for the thresholds of the continuum: $m = 1$ GeV, $\tilde{\beta}_1^2 = 0.43$ GeV^6, and $W_1 = W_2 = 1.5$ GeV.

b) *Sum rules for* $\Pi_{\mu\nu}(\eta_1,\bar\eta_2)$. A contribution to $\mathrm{Im}\Pi_{\mu\nu}(\eta_1,\bar\eta_2)$ is made only by states with spin $\frac{1}{2}$ (nucleon), with $\alpha_1=0$. The values of the residues α_2 and β_2 can also be related to each other. We note to this end that the current $\eta_{2\mu}$ has the following property:

$$\gamma_\mu\eta_{2\mu}=0. \tag{20}$$

Using (13), (20), an the Dirac equation, we obtain

$$\alpha_2=4\beta_2/m. \tag{21}$$

In the case of resonance with negative parity, we must replace m by $-m$ in (21).

Calculation of the polarization operator leads to the equations

$$\Pi_{\mu\nu}{}^a=\Pi_{\mu\nu}{}^b=0,$$

$$\Pi_{\mu\nu}{}^c=\frac{\langle 0|\bar uu|0\rangle^2}{q^2}\{\gamma_\mu\gamma_\nu\hat q+2q_\nu\gamma_\mu\},$$

$$\Pi_{\mu\nu}{}^d=\frac{5}{12}m_0{}^2\frac{\langle 0|\bar uu|0\rangle^2}{q^4}\{\gamma_\mu\gamma_\nu\hat q+2q_\nu\gamma_\mu\},$$

$$\Pi_{\mu\nu}{}^e=-\frac{\langle 0|\bar uu|0\rangle}{2(2\pi)^2}q^2\ln\left(-\frac{q^2}{\Lambda^2}\right)\left\{\gamma_\mu\gamma_\nu-4\frac{q_\nu\gamma_\mu\hat q}{q^2}\right\}, \tag{22}$$

$$\Pi_{\mu\nu}{}^f=-\frac{m_0{}^2\langle 0|\bar uu|0\rangle}{4(2\pi)^2}4\frac{q_\nu\gamma_\mu\hat q}{q^2},$$

$$\Pi_{\mu\nu}{}^g=\frac{1}{24}\frac{\langle 0|\alpha_s\pi^{-1}G^2|0\rangle\langle 0|\bar uu|0\rangle}{q^2}\left\{\gamma_\mu\gamma_\nu-4\frac{q_\nu\gamma_\mu\hat q}{q^2}\right\},$$

$$\Pi_{\mu\nu}{}^h=-\frac{2}{9}\frac{\alpha_s}{\pi}(2\pi)^2\frac{\langle 0|\bar uu|0\rangle^3}{q^4}\left\{\gamma_\mu\gamma_\nu-4\frac{q_\nu\gamma_\mu\hat q}{q^2}\right\}.$$

These lead to three independent sum rules:

$$(\gamma_\mu\gamma_\nu q):\ a^2L^{28/27}-\frac{5}{12}m_0{}^2\frac{a^2}{M^2}=\tilde\beta_1\beta_2e^{-m^2/M^2},$$

$$(\gamma_\mu\gamma_\nu):\ \frac{1}{2}aL^{16/27}M^4\left(1-e^{-W_2{}^2/M^2}\left(\frac{W_2{}^2}{M^2}+1\right)\right)+\frac{ab}{24}$$

$$-\frac{2}{9}\frac{\alpha_s}{\pi}\frac{a^3}{M^2}=\tilde\beta_1\beta_2me^{-m^2/M^2}, \tag{23}$$

$$(q_\nu\gamma_\mu\hat q):\ \frac{1}{2}aL^{16/27}M^2(1-e^{-W_2{}^2/M^2})-\frac{1}{4}am_0{}^2L^{4/27}-\frac{ab}{24M^2}$$

$$+\frac{1}{9}\frac{\alpha_s}{\pi}\frac{a^3}{M^4}=\frac{\tilde\beta_1\beta_2}{m}e^{-m^2/M^2}.$$

From the sum rules (23) we obtain for the mass and residue of the nucleon

$$m=1.05\pm0.15\ \mathrm{GeV},\ \tilde\beta_1\tilde\beta_2=0.65\pm0.15\ \mathrm{GeV}^6. \tag{24}$$

Figure 3 shows the behavior of the sum rules as a function of M at the chosen values of the mass, residue, and continuum.

The residue β_2, which can be determined from (19) and (24),

$$\beta_2=(2.5\pm0.8)\cdot10^{-2}\ \mathrm{Gev}^3, \tag{25}$$

is of interest because it is connected with the constant f_0 in the asymptotic expression for the neutron electric form factor. Namely (see Ref. 8), as $Q^2\to\infty$ the electric form factor of the neutron is given by

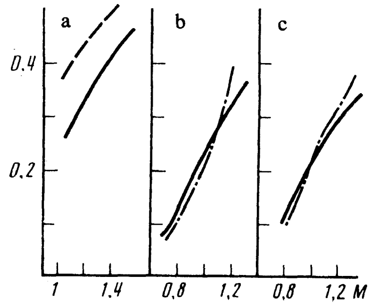

FIG. 3. Plot a corresponds to the sum rules for the $\gamma_\mu\gamma_\nu\hat q$ structure; b—for the $\gamma_\mu\gamma_\nu$ structure; c—for the $q_\nu\gamma_\mu\hat q$ structure. The mass, residue and continuum threshold are: $m=1.05$ GeV, $\tilde\beta_1\tilde\beta_2=0.67$ GeV^6, and $W_2=1.5$ GeV.

$$F_n(Q^2)=(4\pi\alpha_s(Q^2))^2\left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right)^{4/27}\times\frac{100}{3}|f_0|^2\frac{1}{Q^4}, \tag{26}$$

where the constant f_0 is determined by the part of the matrix element

$$\varepsilon^{abc}\langle 0|(u^aC\gamma_\mu u^b)d^c|N\rangle=(q_\mu f_0+\gamma_\mu\beta)\gamma_5v(q) \tag{27}$$

that is proportional to q_μ. Separating from the current in the matrix element (27) the current corresponding to the isospin $\frac{1}{2}$, we can easily transform the left-hand side of (27) into

$$\varepsilon^{abc}\langle 0|(u^aC\gamma_\mu u^b)d^c|N\rangle=\frac{1}{12}\langle 0|\eta_{2\mu}|N\rangle=\frac{1}{3}\frac{\beta_2}{m}q_\mu v(q), \tag{28}$$

where $\eta_{2\mu}$ is defined in (5) and we have used Eq. (21) (the terms proportional to γ_μ are left out). Hence

$$f_0={}^1/_3\beta_2m^{-1}=0.8\cdot10^{-2}\ \mathrm{GeV}^2. \tag{29}$$

Substituting this value in (25) we obtain, say at $Q^2=25$ GeV^2,

$$F_n(Q^2)_{\mathrm{asymp}}\approx1.2\cdot10^{-2}/Q^4 \tag{30}$$

as against the experimental value

$$F_n(Q^2)_{\exp}\approx-1/Q^4,$$

i.e., besides the difference in sign, the form factor calculated from the asymptotic formula is smaller than the experimental value by almost two orders of magnitude even at $Q^2=25$ GeV^2.

c) Sum rules for $\Pi_{\mu\nu}(\eta_2,\bar\eta_2)$. In contrast to the already considered sum rules, contributions to $\mathrm{Im}\Pi_{\mu\nu}(\eta_2,\bar\eta_2)$ are made not only by particles with spin $\frac{1}{2}$, but also by particles with spin $\frac{3}{2}$. It is therefore necessary to choose structures to which only fermions with spin $\frac{1}{2}$ contribute. Such structures can be chosen by using (11) and (12). They take the form

$$q_\nu\gamma_\mu+q_\mu\gamma_\nu,\qquad \gamma_\mu\gamma_\nu+{}^1/_3g_{\mu\nu}. \tag{31}$$

The sum rules for these structures will be used in the calculation of the nucleon mass.

The calculated polarization operator $\Pi_{\mu\nu}(\eta_2,\bar\eta_2)$ is

$$\Pi_{\mu\nu}{}^{a}=\frac{12}{5}\frac{q^4}{(2\pi)^4}\ln\left(-\frac{q^2}{\Lambda^2}\right)\left\{g_{\mu\nu}\hat{q}-\frac{5}{16}\gamma_\mu\gamma_\nu\hat{q}+\frac{5}{16}(q_\nu\gamma_\mu-q_\mu\gamma_\nu)+\frac{1}{16}(q_\nu\gamma_\mu+q_\mu\gamma_\nu)-\frac{q_\mu q_\nu}{q^2}\hat{q}\right\},$$

$$\Pi_{\mu\nu}{}^{b}=-\frac{2}{3}\frac{\langle 0|\alpha_s\pi^{-1}G^2|0\rangle}{(2\pi)^2}\left\{\ln\left(-\frac{q^2}{\Lambda^2}\right)\left[g_{\mu\nu}\hat{q}-\frac{3}{8}\gamma_\mu\gamma_\nu\hat{q}+\frac{3}{8}(q_\nu\gamma_\mu-q_\mu\gamma_\nu)-\frac{1}{8}(q_\nu\gamma_\mu+q_\mu\gamma_\nu)\right]-\frac{1}{2}\frac{q_\mu q_\nu}{q^2}\hat{q}\right\},$$

$$\Pi_{\mu\nu}{}^{c}=-16\frac{\langle 0|\bar{u}u|0\rangle^2}{q^2}\left\{g_{\mu\nu}\hat{q}-\frac{3}{8}\gamma_\mu\gamma_\nu\hat{q}+\frac{3}{8}(q_\nu\gamma_\mu-q_\mu\gamma_\nu)-\frac{1}{8}(q_\nu\gamma_\mu+q_\mu\gamma_\nu)\right\}, \qquad (32)$$

$$\Pi_{\mu\nu}{}^{d}=-\frac{28}{3}m_0^2\frac{\langle 0|\bar{u}u|0\rangle^2}{q^4}\left\{g_{\mu\nu}\hat{q}-\frac{3}{8}\gamma_\mu\gamma_\nu\hat{q}+\frac{3}{8}(q_\nu\gamma_\mu-q_\mu\gamma_\nu)\cdot\frac{1}{8}(q_\nu\gamma_\mu+q_\mu\gamma_\nu)\right\},$$

$$\Pi_{\mu\nu}{}^{e}=16\frac{\langle 0|\bar{u}u|0\rangle}{(2\pi)^2}q^2\ln\left(-\frac{q^2}{\Lambda^2}\right)\left\{g_{\mu\nu}-\frac{5}{16}\gamma_\mu\gamma_\nu+\frac{1}{4}\frac{q_\nu\gamma_\mu-q_\mu\gamma_\nu}{q^2}\hat{q}-\frac{1}{2}\frac{q_\mu q_\nu}{q^2}\right\},$$

$$\Pi_{\mu\nu}{}^{f}=-8m_0^2\frac{\langle 0|\bar{u}u|0\rangle}{(2\pi)^2}\left\{\ln\left(-\frac{q^2}{\Lambda^2}\right)\left[g_{\mu\nu}-\frac{1}{4}\gamma_\mu\gamma_\nu\right]+\frac{1}{4}\frac{q_\nu\gamma_\mu-q_\mu\gamma_\nu}{q^2}\hat{q}-\frac{1}{2}\frac{q_\mu q_\nu}{q^2}\right\},$$

$$\Pi_{\mu\nu}{}^{g}=-\frac{2}{3}\frac{\langle 0|\alpha_s\pi^{-1}G^2|0\rangle\langle 0|\bar{u}u|0\rangle}{q^2}\left\{g_{\mu\nu}-\frac{1}{2}\gamma_\mu\gamma_\nu+\frac{q_\nu\gamma_\mu-q_\mu\gamma_\nu}{q^2}\hat{q}-2\frac{q_\mu q_\nu}{q^2}\right\},$$

$$\Pi_{\mu\nu}{}^{h}=\frac{11\cdot 2^5}{27}\frac{\alpha}{\pi}(2\pi)^2\frac{\langle 0|\bar{u}u|0\rangle^3}{q^4}\left\{g_{\mu\nu}-\frac{19}{88}\gamma_\mu\gamma_\nu+\frac{3}{22}\frac{q_\nu\gamma_\mu-q_\mu\gamma_\nu}{q^2}\hat{q}-\frac{3}{11}\frac{q_\mu q_\nu}{q^2}\right\}.$$

Comparing the relations between the different structure functions in the right-hand side with Eq. (12), we can readily seen that these relations are very close to those obtainable, for example, if the right-hand sides of the sum rules were to contain only a resonance with spin $\frac{3}{2}$. This means that the contribution of spin-$\frac{1}{2}$ states to the sum rules for the polarization operator $\Pi_{\mu\nu}(\eta_2,\bar{\eta}_2)$ is strongly suppressed (by an order of magnitude or even more). Therefore the different corrections not taken into account by us, particularly the corrections $\sim\alpha_s$, may turn out to be important for the calculation of this contribution.

For the structures (25) we obtain from (26) the following sum rules:

$$\frac{1}{10}L^{-4/27}M^6\left(1-e^{-W_1^2/M^2}\left(\frac{W_1^4}{2M^4}+\frac{W_1^2}{M^2}+1\right)\right)+\frac{1}{36}bM^2L^{-4/27}(1-e^{-W_1^2/M^2})+\frac{2}{3}a^2L^{20/27}-\frac{7}{18}a^2\frac{m_0^2}{M^2}=\tilde{\beta}_2^2e^{-m^2/M^2}, \qquad (33a)$$

$$-\frac{1}{3}aL^{8/27}M^4\left(1-e^{-W_2^2/M^2}\left(\frac{W_2^2}{M^2}+1\right)\right)+\frac{2}{3}aL^{-4/27}m_0^2M^2(1-e^{-W_2^2/M^2})+\frac{ab}{9}+\frac{124}{81}\frac{\alpha_s}{\pi}\frac{a^3}{M^2}=\tilde{\beta}_2^2me^{-m^2/M^2}. \qquad (33b)$$

The right-hand side of the sum rule (33b) is positive, whereas the first (principal) term in the left-hand side is negative. Several ways out of the resultant contradiction can be considered. The first, obvious and most likely, is that the sum rules (33) are of no great significance, since the contribution of the unaccounted-for corrections $\sim\alpha_s$ can exceed that of the accounted-for terms. We assume, however, that this is not the case, and attempt to satisfy the sum rules (33) in their present form. It is then clear that the second term in the left-hand side of (33b), which is proportional to am_0^2, should be large enough, i.e., parameter $m_0^2>0.5\ \mathrm{GeV}^2$ is large. As will be shown below, this condition does indeed hold, so that we can attempt to satisfy the sum rule (33). The best fit leads to the following values of the parameters:

$$m=0.9\pm0.2\ \mathrm{GeV},\ \tilde{\beta}_2^2=0.45\pm0.25\ \mathrm{GeV}^6,\ \tilde{\beta}_2=0.65\pm0.15\ \mathrm{GeV}^3. \qquad (34)$$

However, as seen from Fig. 4a, the left-hand side of the second of the sum rules, as a function M^2, is in quite poor agreement with the right-hand side, as well as with the left-hand side of the first sum rule. The second sum rule can be brought into agreement only by assuming that besides the nucleon contribution an important role is played in the right-hand side of (33) also by the contribution of the negative-parity resonance which enters in (33b) with a minus sign.

If a negative-parity resonance with mass 1.5 GeV is taken into account explicitly, the agreement between the sum rules improves markedly. This is seen from Fig. 4b. The mass and residue of the nucleon do not change significantly in this case. If we now substitute the mass and residue, which are known from Secs. 3a and 3b, into the sum rules (33), we can obtain the mass of the resonance $N^{1/2}$, since its contribution to the sum rules is not small. The mass and residue of this resonance are found to be

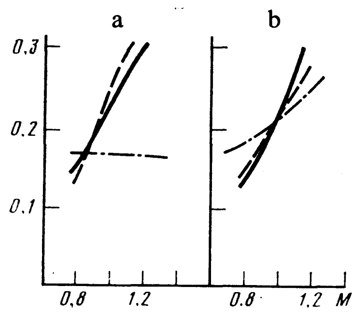

FIG. 4. a) Case of one residue, and thresholds of the continuums are $m=0.9$ GeV, $\tilde{\beta}_2^2=0.52$ GeV^6, $W_1=1.6$ GeV, $W_2=1.4$ GeV; b) case of two resonances. Their parameters are chosen to be $m=1$ GeV, $\tilde{\beta}_2^2=0.6$ GeV^6, $m_-=1.5$ GeV, $\tilde{\beta}^2_-=2.05$ GeV^6, $W_1=1.6$ GeV, $W_2=1.4$ GeV.

$$m_- = 1.5\ \text{GeV}, \quad \tilde{\beta}_-^2 = 0.25\ \text{GeV}^6. \tag{35}$$

The conclusion obtained in this variant should not, of course, be regarded as important. All that matters is that even in this case, which to some extent is extreme, the mass and the residue of the nucleon remain practically the same as before, and the mass of the resonance with $J^P = \frac{1}{2}^-$ turns out to be large enough. If we consider seriously the variant with allowance for the resonance R^- with $J^P = \frac{1}{2}^-$, the same resonance would have to be taken into account also in the sum rules for $\Pi_{\mu\nu}(\eta_1, \bar{\eta}_1)$ and $\Pi_{\mu\nu}(\eta_1, \bar{\eta}_2)$. The good agreement between these sum rules without allowance for the resonance R^- shows that its residue in the current η_1 should be small compared with β_1. This is not surprising, since it has a natural explanation within the framework of the constituent quark model, in which R^- consists of quarks situated in a p wave.

4. THE RESONANCE $N^{3/2-}$

As seen from (11) and (12), only resonances with spins $\frac{3}{2}$ contribute to the sum rules at the structures $g_{\mu\nu}\hat{q}$ and $g_{\mu\nu}$. From (32) and (12) we obtain the following sum rules for a resonance with spin $\frac{3}{2}$:

$$(g_{\mu\nu}\hat{q}):\ \frac{24}{5} M^6 L^{-4/27}\left(1 - e^{-W_1^2/M^2}\left(\frac{W_1^4}{2M^4} + \frac{W_1^2}{M^2} + 1\right)\right)$$

$$-\frac{2}{3} b M^2 L^{-4/27}\cdot(1 - e^{W_1^2/M^2}) - 16 a^2 L^{28/27}$$

$$+\frac{28}{3} a^2 \frac{m_0^2}{M^2} = \tilde{\lambda}_R^2 e^{-M_R^2/M^2}, \tag{36}$$

$$(g_{\mu\nu}):\ 16 a M^4 L^{8/27}\left(1 - e^{-W_2^2/M^2}\left(\frac{W_2^2}{M^2} + 1\right)\right)$$

$$-8 a M^2 m_0^2 L^{-4/27}(1 - e^{-W_2^2/M^2}))$$

$$-\frac{2}{3} ab - \frac{352}{27}\frac{\alpha_s}{\pi}\frac{a^3}{M^2} = \tilde{\lambda}_R^2 M_R e^{-M_R^2/M^2},$$

where M_R and $\tilde{\lambda}_R = (2\pi)^2 \lambda_R$ are the mass and residue of the resonance $N^{\frac{1}{2}-}$.

The fact that the sum rule (36) should be saturated with a resonance with negative parity follows from the sign of the second sum rule. The mass and residue of the resonance from (36) are found to be

$$M_R = 1.75 \pm 0.25\ \text{GeV}, \quad \tilde{\lambda}_R^2 = 20 - 100\ \text{GeV}^6. \tag{37}$$

Figure 5 shows the behavior of the sum rules for this resonance. The large scatter of the resonance $N^{3/2-}$ in mass and in resonance is due to the fact that the region Ω is very small.

5. THE Δ ISOBAR

As shown in Ref. 2, the current η_μ^Δ with isospin $T = \frac{3}{2}$ is the most suitable for the calculation of the Δ-isobar mass. The polarization operator $\Pi_{\mu\nu}(\eta^\Delta, \bar{\eta}^\Delta)$ is of the form

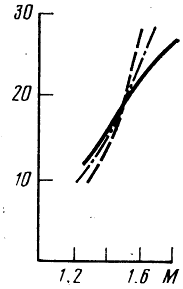

FIG. 5. The mass and residue of the resonance $N^{3/2-}$ are chosen to be $M_R = 1.6$ GeV, $\tilde{\lambda}_R^2 60 =$ GeV^6; $W_1 = 1.6$ GeV, $W_2 = 2.2$ GeV are the thresholds of the continuums.

$$\Pi_{\mu\nu}^a = \frac{q^4}{10(2\pi)^4}\ln\left(-\frac{q^2}{\Lambda^2}\right)\left\{g_{\mu\nu}\hat{q} - \frac{5}{16}\gamma_\mu\gamma_\nu\hat{q} + \frac{5}{16}(q_\nu\gamma_\mu - q_\mu\gamma_\nu) + \frac{1}{16}(q_\nu\gamma_\mu + q_\mu\gamma_\nu) - \frac{q_\mu q_\nu}{q^2}\hat{q}\right\}$$

$$\Pi_{\mu\nu}^b = -\frac{5}{72}\frac{\langle 0|\alpha_s\pi^{-1}G^2|0\rangle}{(2\pi)^2}\left\{\ln\left(-\frac{q^2}{\Lambda^2}\right)\left[\hat{q}g_{\mu\nu} - \frac{3}{8}\gamma_\mu\gamma_\nu\hat{q} + \frac{3}{8}(q_\nu\gamma_\mu - q_\mu\gamma_\nu) - \frac{1}{8}(q_\nu\gamma_\mu + q_\mu\gamma_\nu)\right] + \frac{1}{5}\frac{q_\mu q_\nu}{q^2}\hat{q}\right\},$$

$$\Pi_{\mu\nu}^c = \frac{4}{3}\frac{\langle 0|\bar{u}u|0\rangle^2}{q^2}\left\{g_{\mu\nu}\hat{q} - \frac{3}{8}\gamma_\mu\gamma_\nu\hat{q} + \frac{3}{8}(q_\nu\gamma_\mu - q_\mu\gamma_\nu) - \frac{1}{8}(q_\nu\gamma_\mu + q_\mu\gamma_\nu)\right\},$$

$$\Pi_{\mu\nu}^d = \frac{7}{9} m_0^2 \frac{\langle 0|\bar{u}u|0\rangle^2}{q^4}\left\{g_{\mu\nu}\hat{q} - \frac{3}{8}\gamma_\mu\gamma_\nu\hat{q} + \frac{3}{8}(q_\nu\gamma_\mu - q_\mu\gamma_\nu) - \frac{1}{8}(q_\nu\gamma_\mu + q_\mu\gamma_\nu)\right\},$$

$$\Pi_{\mu\nu}^e = -\frac{4}{3}\frac{\langle 0|\bar{u}u|0\rangle}{(2\pi)^2} q^2 \ln\left(-\frac{q^2}{\Lambda^2}\right)\left\{g_{\mu\nu} - \frac{5}{16}\gamma_\mu\gamma_\nu + \frac{1}{4}\frac{q_\nu\gamma_\mu - q_\mu\gamma_\nu}{q^2}\hat{q} - \frac{1}{2}\frac{q_\mu q_\nu}{q^2}\right\},$$

$$\Pi_{\mu\nu}^f = \frac{2}{3} m_0^2 \frac{\langle 0|\bar{u}u|0\rangle}{(2\pi)^2}\left\{\ln\left(-\frac{q^2}{\Lambda^2}\right)\left[g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu\right] + \frac{1}{2}\frac{q_\nu\gamma_\mu - q_\mu\gamma_\nu}{q^2}\hat{q} - \frac{q_\mu q_\nu}{q^2}\right\}, \tag{38}$$

$$\Pi_{\mu\nu}^g = \frac{1}{18}\frac{\langle 0|\alpha_s\pi^{-1}G^2|0\rangle\langle 0|\bar{u}u|0\rangle}{q^2}\left\{g_{\mu\nu} - \frac{7}{18}\gamma_\mu\gamma_\nu + \frac{5}{2}\frac{q_\nu\gamma_\mu - q_\mu\gamma_\nu}{q^2}\hat{q} - 5\frac{q_\mu q_\nu}{q^2}\right\},$$

$$\Pi_{\mu\nu}^h = -\frac{4}{27}\frac{\alpha_s}{\pi}(2\pi)^2\frac{\langle 0|\bar{u}u|0\rangle^3}{q^4} \times\left\{g_{\mu\nu} - \frac{3}{4}\gamma_\mu\gamma_\nu + 2\frac{q_\nu\gamma_\mu - q_\mu\gamma_\nu}{q^2}\hat{q} - 4\frac{q_\mu q_\nu}{q^2}\right\}.$$

From the signs of the polarization operator at the structure $g_{\mu\nu}$ it follows that a resonance with positive parity, i.e., a Δ isobar, will predominate in the $g_{\mu\nu}\hat{q}$ and $g_{\mu\nu}$ sum rules. The sum rules for Δ are

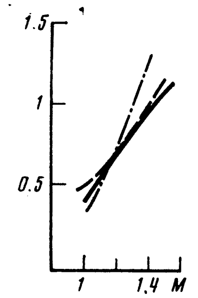

FIG. 6. Δ isobar. $M_\Delta = 1.35$ GeV, $\tilde{\lambda}_\Delta^2 = 2.5$ GeV^6, $W_1 = 2.1$ GeV, $W_2 = 2.2$ GeV.

$$(g_{\mu\nu}\hat{q}):\ \frac{1}{5} M^6 L^{-4/27} \left(1-e^{-W_1^2/M^2}\left(\frac{W_1^4}{2M^4}+\frac{W_1^2}{M^2}+1\right)\right) - \frac{5}{72} bM^2L^{-4/27}(1-e^{-W_1^2/M^2}) + \frac{4}{3}a^2L^{20/27} - \frac{7}{9}a^2\frac{m_0^2}{M^2} = \tilde{\lambda}_\Delta^2 e^{-M_\Delta^2/M^2},$$

$$(g_{\mu\nu}):\ \frac{4}{3}aM^4L^{8/27}\left(1-e^{-W_2^2/M^2}\left(\frac{W_2^2}{M^2}+1\right)\right) - \frac{2}{3}aM^2m_0^2L^{-4/27}(1-e^{-W_2^2/M^2}) - \frac{ab}{18} - \frac{4}{27}\frac{\alpha_s}{\pi}\frac{a^3}{M^2} = \tilde{\lambda}_\Delta^2 M_\Delta e^{-M_\Delta^2/M^2}, \qquad (39)$$

where $\tilde{\lambda}_\Delta = (2\pi)^2\lambda_\Delta$, and M_Δ is the Δ-isobar mass.

From the sum rules (39) we obtain the following mass and residue:

$$M_\Delta = 1.37 \pm 0.12 \text{ GeV}, \quad \tilde{\lambda}_\Delta^2 = 2.3 \pm 0.6 \text{ GeV}^6. \qquad (40)$$

These values are also in accord with those obtained in Ref. 2 (to be sure, the residue λ_Δ^2 turns out to be smaller by a factor 1.5). Figure 6 shows the behavior of the sum rules for the Δ isobar.

6. RECONCILIATION OF THE SUM RULES

In the preceding section we investigated the sum rules for different baryons. The most interesting case is that of the nucleon. It is not trivial at all that by using different sum rules of different polarization operators for the calculation we obtain the same value of the nucleon mass. The residues obtained from these sum rules also agree with one another and with the results obtained earlier in Ref. 2.

Thus, even if no account is taken of the sum rules (33) which have low accuracy, in the case of a nucleon with four free parameters ($m, \beta_1, \beta_2, m_0^2$) we have five sum rules that in good agreement as functions of M^2.

Another check on the sum rules is also possible.

We write the sume rules for $\Pi_{\mu\nu}(\eta_2, \bar{\eta}_2)$ at the structures $q_\mu q_\nu \hat{q}$ and $q_\mu q_\nu$, saturating them with resonances having already known residues and masses obtained in the preceding sections:

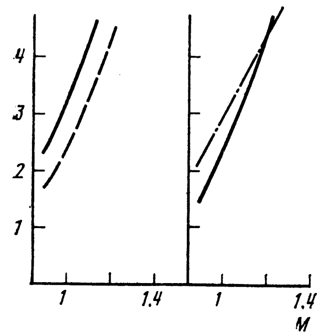

FIG. 7. The masses and residues of the resonances are the following: $m = 1$ GeV, $\tilde{\beta}_2^2 = 0.4$ GeV^6, $m_R = 1.7$ GeV, $\tilde{\lambda}_R^2 = 60$ GeV^6, $m_- = 1.5$ GeV, $\tilde{\beta}_-^2 = 0.25$ GeV.^{6} The thresholds chosen for the continuums are $W_1 = 2.1$ GeV and $W_2 = 1.6$ GeV.

$$(q_\mu q_\nu \hat{q}):\ \frac{12}{5}M^4L^{-4/27}\left(1-e^{-W_1^2/M^2}\left(\frac{W_1}{M^2}+1\right)\right) + \frac{1}{3}bL^{-4/27} = \frac{2}{3}\frac{\tilde{\lambda}_R^2}{M_R^2}e^{-M_R^2/M^2} + \frac{16\tilde{\beta}_2^2}{m^2}e^{-m^2/M^2} + \frac{16\tilde{\beta}_-^2}{m_-^2}e^{-m_-^2/M^2},$$

$$(q_\mu q_\nu):\ 8aM^2L^{8/27}(1-e^{-W_2^2/M^2}) - 4am_0^2L^{-4/27} + \frac{4}{3}\frac{ab}{M^2} - \frac{16}{9}\frac{\alpha_s}{\pi}\frac{a^3}{M^4} = \frac{2}{3}\frac{\tilde{\lambda}_R^2}{M_R}e^{-M_R^2/M^2} + \frac{8\tilde{\beta}_2^2}{m}e^{-m^2/M^2} - \frac{8\tilde{\beta}_-^2}{m^-}e^{-m_-^2/M^2}. \qquad (41)$$

If we represent graphically the agreement with the sum rules (see Fig. 7), it can be seen that accurate to 15–30% the right-hand side agrees with the left. We note here that the substituted masses and residues are already those obtained, and are not adjusted for a best fit. We have thus shown that a large number of sum rules can be transformed into one another at a relatively small number of free parameters, thereby verifying convincingly the validity of our analysis.

7. ESTIMATE OF m_0^2

Even earlier, in Sec. 3c, we advanced argument favoring the assumption that m_0^2 is relatively large and positive. In this section we show that the value of m_0^2 can be obtained from other sum rules. To this end we shall use Eqs. (23) and (39), since these sum rules contain m_0^2 with a relatively large weight, and the region Ω is not small.

We shall vary the quantity m_0^2 in the sum rules (23) and (39) and reconcile these sum rules in the best fashion at each fixed value of m_0^2. The mismatch parameter δ will vary in this case. If we plot $\delta(m_0^2)$ we obtain the picture shown in Fig. 8, from which it is seen that $m_0^2 = 0.6$–1.1 GeV^2. In all the

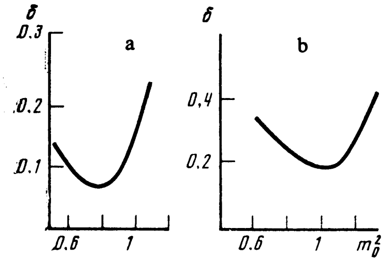

FIG. 8. Plots obtained from the sum rules (23) (a) and from the sum rules (39) (b).

preceding calculations we used the value $m_0^2 = 0.8\ \text{GeV}^2$, a good choice in accordance with the result of this section.

8. CONCLUSION

The present investigation of the QCD sum rules for baryons has shown that it of these sum rules, the masses of the nucleon, of the Δ isobar, and of the resonance with $J^P = \frac{3}{2}^-$, $T = \frac{1}{2}$ as well as their residues in the quark currents. Allowance for higher-power corrections hardly changes the results, which agree with those obtained earlier in Ref. 2 and, within the limits of the calculation accuracy, with experiment. At the same time, the calculated masses were found to be systematically 10–15% higher than the observed ones, possibly indicating that the quark condensate $\langle 0|\bar{\psi}\psi|0\rangle = -(0.24\ \text{GeV})^3$ assumed in the calculation is somewhat ($\sim 20\%$) overestimated. It was shown further that a large number of independent sum rules are in good agreement with one another and lead to the same results for the masses and the residues. On the basis of the sum rules, we determined the value of the quark-gluon condensate

$$g_s\langle 0|\bar{\psi}\sigma_{\mu\nu}(\lambda^n/2)G_{\mu\nu}^n\psi|0\rangle = m_0^2\langle 0|\bar{\psi}\psi|0\rangle,$$
$$m_0^2 = 0.8\pm 0.2\ \text{GeV}^2$$

and obtained the residues that determine the proton decay amplitude in the SU(5) grand unification theory, and the asymptotic value of the electric form factor of the neutron.

[1)]The mass and residue errors cited below correspond to the condition $\Delta \sim 2\Delta_{\min}$ ($\Delta_{\min}$ is obtained with account taken of the uncertainty of the contribution of the continuum; as a rule $\Delta_{\min} \sim 10\%$).

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