Structure of vacuum states and mechanisms of charge screening in two-dimensional massless electrodynamics

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Charge screening and confinement of zero-mass fermions (quarks) in two-dimensional electrodynamics (QED_2) are due to the transition of the local charges into the vacuum of the system under the action of a field that changes the topological number. An exact solution of the quark structure of vacuum for two QED_2 variants shows that this problem is consistent with this phenomenon. The structure of the vacuum is therefore directly related to the Adler anomaly and to the character of the change of the topological numbers of the fields in dynamic processes. The obtained solutions make it possible to investigate in explicit form the properties of the chiral condensate whose existence is also a direct consequence of the Adler anomaly.

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1. INTRODUCTION

In the theory with zero-mass fermions, the Adler anomaly[1)] leads to nonconservation of chirality or of the number of right-hand (R) left-hand (L) particles and charges[1)] (Q_R,Q_L). An investigation of two-dimensional quantum electrodynamics (QED_2) (Ref. 2) shows[3] that the physical phenomenon that explains this nonconservation is precisely the one which produces the charges that neutralize and screen any local charge introduced into this system. This mechanism ensures thus the confinement of the charge (and of Q_R and Q_L) in QED_2. The qualitative picture of the processes that occur is more general than its two-dimensional framework.

The gist of the phenomenon is the following.[3] Local colored charges produce a colored gauge field (an electromagnetic field for QED_2), which in turn begins to create quark-antiquark pairs (q,$\bar{q}$). It might seem that such a field can create only $q_R\bar{q}_R$ and $q_L\bar{q}_L$ pairs, without changing the chirality $K = Q_R - Q_L$ and the total charge $Q = Q_R + Q_L$ of the system. What is then the meaning of the Q_R and Q_L nonconservation to which the Adler anomaly leads? An investigation[3] of the corresponding processes in QED_2 has shown that the change of the topological number of the electromagnetic field[4] by an amount Q_T is evidence that in the course of the process the number $|Q_T|$ of the particles q_R and $\bar{q}_L$ ($Q_T > 0$) or of q_L and $\bar{q}_R$ ($Q_T < 0$) (from among the field-produced $q_R\bar{q}_R$ and $q_L\bar{q}_L$ pairs) turns out to be bound in the delocalized complexes that are present in the physical vacuum of the system. In other words, a certain number of R and L quarks with their characteristics go off into the vacuum plasma and are unobservable, meaning nonconservation of the charges Q_R and Q_L. At the same time, the uncompensated right-hand and left-hand charges (again $|Q_T|$) remain in the form of quarks with finite momenta in the local regions of the field action. These particles inevitably produce a screening charge, since they neutralize the field until the change of the topological number stops, and with it the transition of new R and L charges to the vacuum.

Charge screening due to polarization of vacuum could by itself not lead in QED to a model spectrum consisting, as is well known,[5] of only bosons that are fully neutral with respect to all quantum numbers (including chirality). Only the transitions into the vacuum states, introduced by the described "topological effect," ensure confinement here. The physical aspects of the phenomenon in QED_2 were investigated in Ref. 3. In the present paper the quark structure and the properties of the vacuum states for different QED models are investigated from the viewpoint of their role in the confinement mechanism. These are the properties that allow the topological effect to transfer quark characteristics to the vacuum and cause their nonconfinement.

Absorption of particles by delocalized states presupposes degeneracy of the ground state of the system relative to the quark characteristics. The existence of vacuum states with different quantum numbers (Q_R and Q_L or the total charge Q and the chirality K) was proved in QED_2 in Ref. 3. It is precisely these states together with hadrons (neutral massive bosons[2]) which make up the complete system of states of the model.

The considered confinement mechanism requires also that the matrix elements of the transitions between local packets that carry quark characteristics and states with delocalized quantum numbers be finite, i.e., that they be independent of the volume V of the system. It is only then that delocalization of the quantum numbers becomes really possible. The condensate states are most suitable for this purpose.

Indeed, the ground state of QED_2 with one charged quark (the Schwinger model[2]) constitutes a chiral condensate. The vacuum quantum number $\langle\psi_R^+(x)\psi_L(x)\rangle$ ($\psi_{R,L}(x)$ are the operators of the R and L quarks) is not equal to zero here. The chiral vacua absorb the quark chirality and make it unobservable. However, the QED_2 model with several types of charged zero-mass quarks[7] shows that the topological effect can organize delocalization of the quantum numbers also in another way. The model has degenerate chiral vacua, but these have no chirality condensate. The matrix elements

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of the transitions into chiral vacua decrease here with like $V^{\eta_j^2-1}$ as $V\to\infty$, where

$$\eta_j=g_j/\left(\sum_{i=1}^{N}g_i^2\right)^{1/2},$$

g_j is the electromagnetic charge of the quark of type j. But the presence in the model of a neutral zero-mass particle produces an infrared situation, which cancels out precisely the small factors. The system of coherent states, viz., chiral vacua + an infinite number of zero-mass particles with small momenta, replaces here the chiral condensate states of the Schwinger model. The matrix elements of the transition of the chirality from localized packets into coherent states turn out to be finite, and this allows the chirality to be delocalized, thus realizing confinement of the charges Q_R and Q_L.

We shall not consider here the charged vacua of QED_2. Their investigation shows the absence of real charged vacua. The charges in the model are screened but not delocali ·ed. The quark structure of the charged states of the model co. - cides with the quark structure, investigated later, of the uncharged states. The entire charge Q of the state is concentrated only near the external compensating charges $(-Q)$, without the presence of which a consistent definition of charged states in two dimensions is impossible.[6] Therefore the charge of the state has practically no effect on its structure and properties, which are of interest from the confinement point of view.

The absence of local charge in vacuum means its strict local conservation. Therefore what takes place in the models of zero-mass QED_2 is not the Higgs phenomenon, which starting with Ref. 5 was assumed to be the basis of the physics of screening, but an entirely different screening mechanism connected with the topological effect. The latter takes place in locally uncharged vacua, since there is no transition of the total charge into vacuum in any of its stages. For the screening of the charged subsystems it suffices to have the chirality change ensured by the Adler anomaly. The screening of the charge in zero-mass QED_2 is not its statistical discoloring (as in the Higgs phenomenon), but confinement capable of hiding the triality of the confined quarks (see Sec. 5, where the quantum numbers of the observable states ("flavors") do not coincide with the flavors of the quarks.

In the second section of the paper we recall the result of the calculation, in Ref. 6, of the evolution operator $S(T)$ of the QED_2 system in a finite time T, and separate the vacuum part of $S(T)$. In the third section this vacuum part allows us to construct and study the properties of all the chiral vacua of the Schwinger model. Their quark structure points directly to the existence here of a vacuum condensate consisting mainly of bound $q_R\bar{q}_L$ and $q_L\bar{q}_R$ pairs. The loss of R and L quarks via the Adler anomaly, referred to at the beginning of this section, means precisely the formation of such complexes by the electromagnetic field that changes the topological number.

In Sec. 4 we investigate the degeneracy and the quark structure of the vacuum state of a model with several electrons. Here, too, there exist vacua with different quantum numbers—the necessary condition for quark confinement—but their quark structures differ greatly from one another (in contrast to the Schwinger model). The properties of the states and the existence of condensates in them are directly connected with the topological effect described above.

In Sec. 5, the bosonization method is used to study the confinement phenomenon under conditions when there are no vacuum condensates in the model.[7] Such a system makes unobservable to the total charges $Q_{R,L}$ with the aid of the aforementioned mechanism, in which the principal role is played by zero-mass excitations. The charges of the individual quark types turn out in it observable flavors, and this leads to a number of interesting features of the model. The partial charges of the physical states, however, differ from the quark charges. By the same token, the quark characteristics manifest themselves here in hidden form (confinement).

2. THE EVOLUTION OPERATOR $S(T)$ IN QED_2

As shown in Sec. 1, the main purpose of this paper is to bring to light the quark structure of the vacuum states of the model, a structure that guarantees in the model quark confinement. To this end we must cite here the result of the calculation[6] of the operator of the evolution of the QED_2 system within a finite time T. The wave functions of the states are obtained from $S(T)$ in accord with the known formula

$$S(T)=\sum_n|\Psi_n\rangle\exp(-iE_nT)\langle\Psi_n|; \tag{1}$$

E_n is the energy of the nth physical state, and its wave vectors $|\Psi_n\rangle$ and $\langle\Psi_n|$ depend respectively only on the creation $(a^+_{R,L},b^+_{R,L})$ and annihilation $(a_{R,L},b_{R,L})$ operators of the R and L quarks and antiquarks. The quark field $\psi(x)$ is expressed in terms of these operators in the form

$$\psi(x)=\psi_R(x)u^{(R)}+\psi_L(x)u^{(L)},\quad (1\pm\gamma_5)u^{(R,L)}=0,$$

$$\psi_{R,L}(x)=\frac{1}{V}\sum_{p_n>0}[\exp(\pm ip_nx)a_{R,L}(p_n)+b^+_{R,L}(p_n)\exp(\mp ip_nx)]$$

$$\equiv a_{R,L}(x)+b^+_{R,L}(x), \tag{2}$$

$$p_n=\frac{2\pi n}{V},\quad n=0,\pm1,\ldots,$$

$$\{a^+(p_n),a(p_n')\}=\{b^+(p_n),b(p_n')\}=V\delta_{p_np_n'}.$$

Calculation of the functional integral for $S(T)$ in a physical (Coulomb) gauge and in a finite volume V yielded in Ref. 6 an expression in the form of a sum of terms with definite number of R and L quarks-antiquarks in the initial and final states. The term ("matrix element") with $\tilde{n}_R,\tilde{n}_L(n_R,n_L)$ quarks and $\tilde{m}_R,\tilde{m}_L(m_R,m_L)$ antiquarks in the initial (final) state is written in the form

$$\left(\prod_{k=1}^{n_R}\int dx_k a_R^+(x_k)\right)\left(\prod_{k=1}^{m_R}\int dx_k' b_R^+(x_k')\right)\left(\prod_{k=1}^{n_L}\int dy_k a_L^+(y_k)\right)$$

$$\times\left(\prod_{k=1}^{m_L}\int dy_k' b_L^+(y_k')\right)|0\rangle$$

$$\langle 0|\left(\prod_{k=1}^{\tilde n_R}\int d\tilde x_k a_R(\tilde x_k)\right)\left(\prod_{k=1}^{\tilde m_R}\int d\tilde x_k' b_R(\tilde x_k')\right)$$

$$\times\left(\prod_{k=1}^{\tilde n_L}\int d\tilde y_k a_L(\tilde y_k)\right)\left(\prod_{k=1}^{\tilde m_L}\int d\tilde y_k' b_L(\tilde y_k')\right)$$

$$\times\ S_0(x_k, x_k', y_k, y_k'; \tilde x_k, \tilde x_k', \tilde y_k, \tilde y_k')$$

$$I(x_k, x_k', y_k, y_k'; \tilde x_k, \tilde x_k', \tilde y_k, \tilde y_k'). \tag{3}$$

The coordinates $x_k(\tilde x_k)$ and $y_k(\tilde y_k)$ represent respectively the R and L quarks in the final (initial) state, and the symbols for the antiquarks are $x_k'(\tilde x_k')$ and $y_k'(\tilde y_k')$. The operators a^+ and b^+ commute with a and b, since they act in different spaces of states (final and initial). $S_0(x_k,..., \tilde y_k')$ is the matrix element of the evolution operator $S_0(T)$ of the system of free fermions:

$$S_0(T)=\exp\left\{\int\frac{dx\,dx'}{2\pi i}[(T-x+x'-i0)^{-1}\right.$$

$$\times[a_R^+(x)a_R(x')+b_R^+(x)b_R(x')$$

$$+a_L^+(x')a_L(x)+b_L^+(x')b_L(x)]$$

$$+(x-x'-i0)^{-1}[a_R^+(x)b_R^+(x')$$

$$\left.+a_R(x)b_R(x')+a_L^+(x')b_L^+(x)+a_L(x')b_L(x)]]\right\}, \tag{4}$$

corresponding to the same particle numbers as (3). The coefficients of the operators in (4) are the Green's functions of the free fermions.

The function $I(x_k,..., \tilde y_k')$ describes the influence of the interaction between the quarks. It is of the form

$$I(x_k,\ldots,\tilde y_k')=Z^{-1}e^{-iE_0T}S_Q\exp\left\{-\frac{\pi}{V}\sum_{p_n\neq 0}[2F_1(p_n)R_f(-p_n)\right.$$

$$\left.\times R_i(p_n)+F_2(p_n)(R_i(p_n)R_i(-p_n)+R_f(p_n)R_f(-p_n))]\right\}, \tag{5}$$

where the functions $F_1(p_n)$ of $F_2(p_n)$ are equal to

$$F_1(p_n)=\frac{1}{|p_n|}\exp(-i|p_n|T)-\frac{4\omega_n\exp(-i\omega_nT)}{(\omega_n+|p_n|)^2(1-\Omega_n^2)},$$

$$F_2(p_n)=\frac{1}{|p_n|}\frac{\omega_n-|p_n|}{\omega_n+|p_n|}\frac{1-\exp(-2i\omega_nT)}{1-\Omega_n^2}, \tag{6}$$

$$\Omega_n=\frac{\omega_n-|p_n|}{\omega_n+|p_n|}\exp(-i\omega_nT),\quad \omega_n=(m^2+p_n^2)^{1/2},$$

$m^2 = g^2/\pi$ is the boson mass in QED$_2$.

The renormalization factor Z, the vacuum energy E_0, and the sources $R_i(p_n)$ and $R_f(p_n)$ which depend on the quark-antiquark coordinates are given by

$$Z=\left(\frac{2\pi}{mV}\right)^{1/2}\exp\left(\frac{4-\pi}{4\pi}mV\right),\quad E_0=\frac{V}{2}\int(\omega_p-|p|)\frac{dp}{2\pi};$$

$$R_i(p_n)=\sum_k\{\theta(p_n)[\exp(-ip_n\tilde x_k)-\exp(-ip_n\tilde x_k')]$$

$$+\theta(-p_n)[\exp(-ip_n\tilde y_k)-\exp(-ip_n\tilde y_k')]\}, \tag{7}$$

$$R_f(p_n)=\sum_k\{\theta(-p_n)[\exp(-ip_nx_k)-\exp(-ip_nx_k')]$$

$$+\theta(p_n)[\exp(-ip_ny_k)-\exp(-ip_ny_k')]\}.$$

Finally, the factor S_Q characterizes the charged states of the model.[6] Since it suffices for our purposes to consider states with $Q = 0$, we have $S_Q = 1$.

Equations (3) and (4) indicate clearly that the operator $S(T)$ in a finite volume conserves the right- and left-hand charges, since the only nonzero matrix elements are $n_R - m_R = \tilde n_R - \tilde m_R$ $\quad n_L - m_L = \tilde n_L - \tilde m_L$, and $(Q_R = n_R - m_R, Q_L = n_L - m_L)$. The nonconservation of the charges Q_R and Q_L, which corresponds to the Adler anomaly, can be tracked only in the limiting transition $V\to\infty$ in terms with $\sim V$ particles.

The time dependence $\exp(-i\omega T)$ in (6) corresponds to a particle with mass. It is well known[2,5] that this is a neutral vector boson ($Q = K = 0$). Its properties and parton wave function that stems from (3) were investigated by us earlier. The operation of the confinement mechanism is determined by the properties of the states without massive particle. We omit therefore $\exp(-i\omega T)$ from all the formulas. The time dependence $\exp(-ipT)$ in (6) is typical of zero-mass particles. For the Schwinger model, however, the time-dependent factors that arise in various parts of expressions (4) and (5) cancel each other completely.[6] In the Schwinger model we have only massive excitations.

Transformation of expressions (4) and (5) with the aid of the formula

$$\frac{2\pi}{V}\sum_{p_n>0}\frac{\exp(ip_nx)}{p_n}=-\ln[1-\exp(2\pi ix/V)]=\ln\frac{V}{2\pi i(x+i0)} \tag{8}$$

leads to the following form of the time-independent part of the operator $S(T)$, which can be designated S_{vac}:

$$S_{vac}=AZ^{-1}\left\{\left(\prod_k^{\tilde n_R}\frac{1/\tilde n_R!}{2\pi i(\tilde x_k'-\tilde x_k-i0)}\right)\right.$$

$$\times\left(\prod_j^{\tilde n_L}\frac{1/\tilde n_L!}{2\pi i(\tilde y_j-\tilde y_j'-i0)}\right)$$

$$\times\left.\frac{\Phi^*(\tilde x_k'-\tilde y_j')\Phi^*(\tilde x_k-\tilde y_j)}{\Phi^*(\tilde x_k-\tilde y_j')\Phi^*(\tilde x_k'-\tilde y_j)}\right\}\left\{\left(\prod_k^{n_R}\frac{1/n_R!}{2\pi i(x_k-x_k'-i0)}\right)\right.$$

$$\times\left.\left(\prod_j^{n_L}\frac{1/n_L!}{2\pi i(y_j'-y_j-i0)}\right)\frac{\Phi(x_k-y_j)\Phi(x_k'-y_j')}{\Phi(x_k-y_j')\Phi(x_k'-y_j)}\right\}$$

$$\times\delta_{n_R-m_R,\tilde n_R-\tilde m_R}\delta_{n_L-m_L,\tilde n_L-\tilde m_L}. \tag{9}$$

We have not written out in (9) the operator part of Eq. (3) and consider only the uncharged states: $Q = Q_R + Q_L = 0$ (A = const).

The function $\Phi(x)$ characterizes the interaction and is equal to

$$\Phi(x)=\exp\left\{\frac{2\pi}{V}\sum_{p_n}\frac{1-\exp(ip_nx)}{p_n}\,\frac{\omega_n-p_n}{\omega_n+p_n}\right\}. \quad (10)$$

The expression under the summation signs in (10) has no singularity as $p\to 0$. This is the result of a subtraction carried out in the exponential factor of (6) when (3) was transformed into (9). All the numerical factors that arise in this case are gathered into the constant factor A that does not depend on the volume (at $Q = 0$).

Thus, besides the states of massive bosons, the Schwinger model has only vacuum states with different quantum numbers. These states are of zero momentum in any Lorentz system.

3. CHIRAL CONDENSATE OF MODEL WITH ONE ELECTRON

We begin with a construction of the simplest vacuum state of the Schwinger model, viz., a vacuum with $Q=K=0$. By observing in it a chiral-condensate quark structure, we obtain infinite degeneracy[5] of the model with respect to the chirality K.

In expression (9), the variables of the initial and final states are explicitly factored out. Therefore, according to Eq. (1), the $Q = K = 0$ sector of (3) is represented by the following vacuum wave function:

$$\Omega_0=Z^{-1/2}\left\{1+\sum_{n_R,n_L=1}^{\infty}\frac{1}{n_R!n_L!}\left[\prod_{k=1}^{n_R}\int\frac{dx_k\,dx_k'}{2\pi i}\right.\right.$$
$$\times\frac{a_R^+(x_k)\,b_R^+(x_k')}{x_k-x_k'-i0}\prod_{j=1}^{n_L}\int\frac{dy_j\,dy_j'}{2\pi i}\,\frac{a_L^+(y_j)\,b_L^+(y_j')}{y_j'-y_j-i0}$$
$$\left.\left.\times\frac{\Phi(x_k-y_j)\,\Phi(x_k'-y_j')}{\Phi(x_k-y_j')\,\Phi(x_k'-y_j)}\right]\right\}|0\rangle. \quad (11)$$

The normalization factor Z defined by (7) increases exponentially with volume as $V\to\infty$. Consequently the main configuration in (11) is one with a finite quark density ($n_q = \bar{n}_q \sim V$).

We write down Ω_0 in a form analogous to the representation that holds for n-particle Green's functions of field theory, namely as an exponential function of the sum of connective vacuum complexes $\varphi_n(x,x'; y, y')$:

$$\Omega_0=Z^{-1/2}\hat{P}(K=0)\exp\left\{\int\frac{dx\,dy}{2\pi i}\varphi_2(x,y)\,a_R^+(x)\,b_L^+(y)\right.$$
$$+\int\frac{dx\,dy}{2\pi i}\varphi_2(x,y)\,a_L^+(y)\,b_R^+(x)+\int\frac{dx\,dx'}{2\pi i}$$
$$\left.\times\int\frac{dy\,dy'}{2\pi i}\varphi_4(x,x';y,y')\,a_R^+(x)\,b_R^+(x')\,a_L^+(y)\,b_L^+(y')+\ldots\right\}|0\rangle. \quad (12)$$

By connective complexes we mean those contributions to the sum (11), in which the integrand φ_n decreases when any group of variables x,x', y, y' is separated from the remaining variables of the given term. Owing to the condition $K = 0$ and to the existence of chiral quark complexes (with $K \neq 0$) in the vacuum of the model (this will be proved below), the vacuum Ω_0 is not simply an exponential function of such complexes, but is a projection of the exponential on the state $K = 0$. The formal projection operator $\hat{P}$ in (12) emphasizes this fact. The proof of (12) is a repetition of the analogous proofs for Green's functions. The arbitrary term of the sum in (11) is represented by a sum of products of all the possible connective parts. The combinatorics customarily used for such calculations leads then to Eq. (12).

We now ascertain which vacuum complexes exist in Ω_0. We consider the simplest four-particle complex in (11):

$$f_4=\int\frac{dx\,dx'}{2\pi i}\,\frac{a_R^+(x)\,b_R^+(x')}{x-x'-i0}\int\frac{dy\,dy'}{2\pi i}\,\frac{a_L^+(y)\,b_L^+(y')}{y'-y-i0}$$
$$\times\frac{\Phi(x-y)\,\Phi(x'-y')}{\Phi(x'-y)\,\Phi(x-y')} \quad (13)$$

The function $\Phi(r)$ of (10) takes as $r\to\infty$ the asymptotic form

$$\Phi(r)=\frac{imr}{2}e^{C-1/2}+\text{const},\qquad r\gg 1/m, \quad (14)$$

which stems from the contribution of small p in the integral (10) (C is the Euler constant). Thus, if we consider in (13) the variable regions $x'\sim y$ and $x\sim y'$, whereas the distance between these two regions of r is large ($r\gg m^{-1}$), then the functions $\Phi(x-y)\sim\Phi(x'-y')\sim r$. They cancel out the large denominators $|x-x'|\sim|y-y'|\sim r$ of the integrand in (13). The expression for f_4 breaks up into two independent parts with helicities $K = \pm 2$:

$$f_4\approx\left[\int\frac{dx\,dy'}{4\pi}\,\frac{me^{C-1/2}}{\Phi(x-y')}a_R^+(x)\,b_L^+(y')\right]$$
$$\times\left[\int\frac{dx'\,dy}{4\pi}\,\frac{me^{C-1/2}}{\Phi(x'-y)}a_L^+(y)\,b_R^+(x')\right]+\varphi_4. \quad (15)$$

The connective part of φ_4 is defined by the expression

$$\varphi_4=\int\frac{dx\,dx'\,dy\,dy'}{(2\pi i)^2}\,\frac{a_R^+(x)\,b_R^+(x')\,a_L^+(y)\,b_L^+(y')}{\Phi(x-y')\,\Phi(x'-y)}$$
$$\times\left[\frac{\Phi(x-y)\,\Phi(x'-y')}{(x-x'-i0)\,(y'-y-i0)}+\frac{(im)^2}{4}e^{2C-1}\right]. \quad (16)$$

The distances between the particles that make up φ_4 are of the order of the interaction radius m^{-1}.

It is easy to verify that there are no other possibilities of obtaining independent complexes from f_4. Any term of (11) can be investigated in the same manner. It is observed as a result that the vacuum of the model contains arbitrary neutral ($K = Q = 0$) and chiral ($K \neq 0, Q = 0$) complexes and has no charged complexes. The vacua of the Schwinger model comprise a chiral condensate.

The main properties of the condensate can be investigated by considering only the first two terms in the exponential of (12). This is the approximation of noninteracting chiral pairs. With the aid of Eqs. (12) and (15) the vacuum state Ω_0 is then written in the pair approximation as

$$\Omega_0^{(n)}=Z^{-1/2}\hat{P}(K=0)\left\{\exp\left[\int_0^\infty \Phi(p)a_R^+(p)b_L^+(p)\frac{dp}{2\pi}\right]\right.$$

$$\left.\times\exp\left[\int_0^\infty \Phi^*(p)a_L^+(p)b_R^+(p)\frac{dp}{2\pi}\right]\right\}|0\rangle. \qquad (17)$$

We have changed over in (17) to the momentum representation of all the quantities. The function $\Phi(p)$ is equal to

$$\Phi(p)=\int\frac{dx}{4\pi}e^{-ipx}\frac{me^{C-1/2}}{\Phi(x)}. \qquad (18)$$

Expression (17) describes the state of an ideal gas of bound $q_R\bar{q}_L$ and $q_L\bar{q}_R$ pairs with zero total momenta, i.e., a Bose condensate of such pairs. Equation (18) represents the wave function of a bound pair. An equal number of pairs with opposite chirality ensures a zero total chirality of the vacuum Ω_0. But there exist, of course, also states with any other chirality.

The momentum distribution of the vacuum quarks is determined by the function $\Phi(p)$. The density of the R quarks over an interval $dp/2\pi$ is given by the expression well known from Fermi statistics

$$n_R^{(n)}(p)=\frac{1}{V}\langle\Omega_0^{(n)}|a_R^+(p)a_R(p)|\Omega_0^{(n)}\rangle$$
$$=\frac{|\Phi(p)|^2}{1+|\Phi(p)|^2}=n_L^{(n)}(p)=\bar{n}_R^{(n)}(p)=\bar{n}_L^{(n)}(p). \qquad (19)$$

Equation (19) is derived in the Appendix. It is of interest to compare the quark density (19) with the exact expression (12) for the particle density in vacuum, which can be calculated in the Schwinger model (see Ref. 3 and the Appendix):

$$n_R(p)=\int\frac{dz}{2\pi i}\frac{e^{-ipz}}{z-i0}\exp\left\{-\int_0^\infty\frac{dk}{\omega}\left(\frac{\omega-k}{k}\right)^2\sin^2\frac{kz}{2}\right\}. \qquad (20)$$

A numerical comparison of (19) and (20) shows them to be practically equal at small $p\lesssim m$, i.e., in the region where most vacuum particles are concentrated. Only the asymptotic forms (19) and (20) at momenta $p\gg m$ are different:

$$n_R(p)\sim 1/p^4; \qquad n_R^{(n)}(p)\sim 1/p^6.$$

The total densities of the quarks in the vacua (12) and (17) are small and close to each other,

$$\rho_R=\int_0^\infty n_R(p)\frac{dp}{2\pi}=0.0284m, \quad \rho_R^{(n)}=\int_0^\infty n_R^{(n)}(p)\frac{dp}{2\pi}=0.0265m. \qquad (21)$$

The vacuum gas is highly rarefied, and the contribution of multiparticle complexes that describe the interaction of the condensate pairs is small.

However, the representation (17) of the QED_2 vacuum as an ideal gas of condensate chiral pairs has its limitations. Thus, the QED_2 correlation functions decrease exponentially at large distances, whereas in the pair approximation the decrease follows only a power law. This statement in proved in the Appendix for the simplest correlator of the theory, namely the equal-time fermion Green's function. The role of the multiparticle complexes in this phenomenon is obviously large.

The importance of multiparticle complex in the vacuum is emphasized also by the absence of a Goldstone particle in QED_2 (Ref. 9). It is precisely the exponential decrease of the correlators that confirms rigorously this statement. A chiral condensate and spontaneous breaking of the chiral symmetry should have yielded in the theory a Goldstone particle. Its absence denotes rigidity of the vacuum plasma and the long-range correlations between their particles. The proximity of many properties of the vacuum states (12) and (17) suggests the attractive idea that there might exist in field theory a small numerical parameter connected with the density of the vacuum particles. For a low-density gas the normalization factor is connected with the gas density by the equation

$$Z=V^{-1/2}e^{\rho V}. \qquad (22)$$

Comparison with (7) indicates that the analogous parameter for QED_2 is

$$\alpha=(4-\pi)/4\pi=0.07. \qquad (23)$$

The existence of a chiral condensate is a most important property of the model, directly connected with the confinement mechanism. Such a mechanism, as explained in Sec. 1, calls for infinite degeneracy of the vacuum states in the quark characteristics. A chiral condensate makes natural a chirality degeneracy of the Schwinger-model vacuum states. In the approximation of an ideal gas of pairs, the degenerate states differ from one another only by addition of a certain number of pairs. Neither the energy nor the momentum of the state is altered thereby, and we have an infinite number of degenerate states.

The exact wave functions of the chiral vacua differ from (12) only in the projector $\hat{P}(K)$ on a state with given chirality; this projector is applied to a single state Ψ_0 described by the exponential in (12). This obvious statement can be obtained again directly from the operator $S(T)$ by a method similar to the one that leads to Ω_0. It means that the vacuum complexes in all the degenerate vacua are perfectly identical, and the structures of the vacua do not differ. In the next section we shall see that a different situation arises in the model with several electrons.

The total number of chiral objects in any of the vacua is proportional to the volume V. Therefore the matrix elements of the transitions between the local chiral packets and the chiral vacua remain finite. For example, the matrix element

$$\Xi(x-y)=\langle\Omega_1|\psi_R^+(x)\psi_L(y)|\Omega_0\rangle$$

$$=\mathrm{const}\cdot\exp\left\{-m^2\int dp\frac{\sin^2 p(x-y)}{p^2\omega_p}\right\}. \qquad (24)$$

This formula is given in Ref. 3 (see also the Appendix). It is independent of volume precisely because of the condensate nature of the vacua of the model. In this situation, the Bose-Einstein principle adds a factor $\sqrt{N}\sim\sqrt{V}$ (N is the number of condensate particles), which cancels out the factor $1/\sqrt{V}$ typical of transitions between local and nonlocal states.

The discussed property of the matrix element is extremely important. The confinement mechanism in QED_2

(see Sec. 1 and the discussion in Ref. 3) operates precisely because of its existence.

4. VACUUM STATES IN THE MODEL WITH SEVERAL ELECTRONS (QUARKS)

The presence of several types of zero-mass quarks with charges g_i makes for a number of interesting features of the QED_2 model. We have here excitations of two types, massive bosons with mass

$$m^2=\frac{1}{\pi}\sum_{j=1}^{N}g_j^2 \tag{25}$$

and zero-mass particles.[5] The confinement phenomenon (confinement of total Q_R and Q_L charges) proceeds here differently than in the Schwinger model, since the vacuum states of the model (at arbitrary g_j) have no chiral condensate. But before we proceed to this question we must study the physical states of the model.

The $S(T)$ operator matrix element analogous to the product $S_0(x,...)I(x,...)$ in Eq. (3) is equal here to the expression (again, we consider only states with $Q=0$)

$$S(T)=Z^{-1}\exp(-iE_0T)\prod_{j=1}^{N}S_0^{(j)}(T)$$
$$\times\exp\left\{-\frac{\pi}{V}\sum_{n\neq0}\sum_{j,l=1}^{N}\eta_j\eta_l[2F_1(p_n)R_f^{(j)}(-p_n)\right.$$
$$\left.\times R_i^{(l)}(p_n)+F_2(p_n)(R_f^{(j)}(p_n)R_f^{(l)}(-p_n)+R_i^{(j)}(p_n)R_i^{(l)}(-p_n))]\right\}. \tag{26}$$

Equations (4)–(7) define all the factors of (26) for each type of quark $j=1,2,...,N$. In place of the charges g_j we use the dimensionless constants

$$\eta_j=g_j\Big/\left(\sum_{j=1}^{N}g_j^2\right)^{1/2}. \tag{27}$$

Expression (26) proves the presence of zero-mass excitations. They appear because the factor $F_1(p_n)$ is multiplied here by $|\eta_j\eta_l|<1$, so that the time dependence $\exp(-i|p_n|T)$ is not completely offset by the dependence on T in $S_0^{(j)}(T)$, as was the case in the Schwinger model.

Thus, in the considered case the evolution operator has the following structure:

$$S(T)=S_{z.m.}(T)S_{vac}+\text{terms}\,(e^{-i\omega T}). \tag{28}$$

The zero-mass part of $S(T)$ is equal to

$$S_{z.m.}(T)=\exp\left\{-\frac{2\pi}{V}\sum_{n\neq0}\sum_{j,l=1}^{N}\frac{\exp(-i|p_n|T)}{|p_n|}\right.$$
$$\left.\times[R_f^{(j)}(-p_n)R_i^{(l)}(p_n)\eta_j\eta_l-R_f^{(j)}(-p_n)R_i^{(j)}(p_n)]\right\}, \tag{29}$$

and the expression for the matrix element S_{vac} takes the form

$$S_{vac}=\exp\left[\left(Q_R^2+Q_L^2-\sum_j Q_j^2\right)\ln V\right]Z^{-1}\exp(-iE_0T)$$
$$\times\prod_{j,l=1}^{N}\left\{\frac{1}{\tilde n_R!\,\tilde n_L!}\prod_{k=1}^{\tilde n_R}\prod_{m=1}^{\tilde n_L}\frac{1/2\pi i}{\tilde x_k'-\tilde x_k-i0}\,\frac{1/2\pi i}{\tilde y_m-\tilde y_m'-i0}\right.$$
$$\left.\times\left[\frac{\Phi^*(\tilde x_k-\tilde y_m)\Phi^*(\tilde x_k'-\tilde y_m')}{\Phi^*(\tilde x_k-\tilde y_m')\Phi^*(\tilde x_k'-\tilde y_m)}\right]^{\eta_j\eta_l}\right\}$$
$$\times\left\{\frac{1}{n_R!\,n_L!}\prod_{k=1}^{n_R}\prod_{m=1}^{n_L}\frac{1/2\pi i}{x_k-x_k'-i0}\right.$$
$$\left.\times\frac{1/2\pi i}{y_m'-y_m-i0}\left[\frac{\Phi(x_k-y_m)\Phi(x_k'-y_m')}{\Phi(x_k-y_m')\Phi(x_k'-y_m)}\right]^{\eta_j\eta_l}\right\}. \tag{30}$$

To simplify the notation we have omitted here the particle-species labels j and l from all the coordinates. Therefore in Eq. (30) the coordinates of the R particles $x,\tilde x,x',\tilde x'$ are $x_k=x_k^{(j)}$ etc., the coordinates of the L particles $y,\tilde y,y',\tilde y'$ are $y_m=y_m^{(l)}$, and so on. All the constant factors that arise when (26) is tranformed into (28) are included in (30). The total R and L charges of the states are

$$Q_{R,L}=\sum_{j=1}^{N}\eta_jQ_{R,L}^{(j)}, \tag{31}$$

and $Q_{R,L}^{(j)}$ are defined as the differences of the number of quarks and antiquarks of type j.

Inasmuch as in the two-dimensional case any number of zero-mass particles all moving in the same direction does not differ in energy and momentum from a single particle with the same total momentum, it follows that an infinite number of different systems of states can be constructed here for zero-mass excitations. From (29), by expanding the exponential in a series in $\exp(-ip_nT)$ and identifying each p_n with the momentum of one particle, we obtain a system of states of neutral zero-mass bosons. The sum over all such excitation does indeed yield $S_{z.m.}(T)$, i.e., they form a complete system of states. We shall see later that other choices, greatly differing from the one just made by us, are also possible.

Vacuum states of the model exist for arbitrary $Q_R^{(j)},Q_L^{(j)}$ (we confine ourselves as before to the case $Q_R+Q_L=Q=0$). Let us compare the simplest completely neutral state $Q_R^{(j)}=Q_L^{(j)}=0, j=1,2,...,N$:

$$\Omega_0=Z^{-1/2}\left\{1+\sum_{n_R,n_L}\left\{\frac{1}{n_R!n_L!}\prod_{k=1}^{n_R}\prod_{m=1}^{n_L}\int\frac{dx_k\,dx_k'}{2\pi i}\right.\right.$$
$$\times\frac{a_R^+(x_k)b_R^+(x_k')}{x_k-x_k'-i0}\int\frac{dy_m\,dy_m'}{2\pi i}\frac{a_L^+(y_m)b_L^+(y_m')}{y_m'-y_m-i0}$$
$$\left.\left.\times\left[\frac{\Phi(x_k-y_m)\Phi(x_k'-y_m')}{\Phi(x_k-y_m')\Phi(x_k'-y_m)}\right]^{\eta_j\eta_l}\right\}\right\}|0\rangle \tag{32}$$

with the vacuum function of the sector in which of the charges differs from zero, $Q_R^{(j)}=-Q_L^{(j)}=1$:

$$\Omega_j=V^{\eta_j^2-1}\left\{\int\frac{dx\,dy}{2\pi i}\frac{a_R^+(x)b_L^+(y)}{[\Phi(x-y)]^{\eta_j^2}}+\sum_{l=1}^{N}\int\frac{dx\,dy}{2\pi i}\int\frac{dx_1\,dy_1'}{2\pi i}\right.$$
$$\left.\times\frac{a_R^+(x)b_L^+(y)a_R^+(x_1)b_R^+(x_1')[\Phi(x_1'-y)]^{\eta_j\eta_l}}{(x_1-x_1'-i0)[\Phi(x-y)]^{\eta_j^2}[\Phi(x_1-y)]^{\eta_j\eta_l}}+\dots\right\}|\Omega_0\rangle. \tag{33}$$

In Eqs. (32) and (33) we have again left out the indices j and l of the particle species in all the quantities ($x\equiv x^{(j)}$, $y\equiv y^{(j)}$, $x_1\equiv x_1^{(l)}$, $x_1'\equiv x_1'^{(1)}$; $n_R\equiv n_R^{(j)}$, $n_L\equiv n_L^{(l)}$ etc.) and in the creation and annihilation operators.

Only neutral complexes with chirality $K=0$ are present in (32). The simplest of them is a four-particle complex of the form $q_R^{(j)}\bar{q}_R^{(j)}q_L^{(l)}\bar{q}_L^{(l)}$. Such a complex cannot be divided into two integral pairs, since the integrand decreases in this case with increasing distance r between pairs like $r^{\eta_j\eta_l-1}$, and $|\eta_j\eta_l|<1$. There is no chiral condensate in Ω_0, nor in Ω_j. But in contrast to the Schwinger model, Ω_j contains complexes missing from Ω_0, as can be seen directly from (33). These complexes constitute a junction of the chiral pair $q_R^{(j)}\bar{q}_L^{(j)}$ with the neutral complexes of (32). The total number of particles in the principal chiral complexes is quite large,[2] $\sim\ln V$. Consequently the Ω_j vacua differ substantially from the Ω_0 vacua and differ from one another (at different j). This is radically different from what we have in the Schwinger model.

As $\eta_1\to 1$ ($\eta_j=0, j\neq 1$) we have a phase transition[10] to the Schwinger model. The density of the chiral complexes jumps from zero to the finite value given in (30). Only in the case of constant charges g_j does a chiral condensate exist also in the model of Ref. 5. For example, for $N=2$ and

$$\eta_1=\eta_2=1/\sqrt{2} \tag{34}$$

eight-particle complexes in the state Ω_0 decay into two four-particle chiral ($K=\pm 4$) complexes of the form

$$\varphi_4=\int\frac{dx_1\,dx_2\,dy_1\,dy_2}{(4\pi)^2},\frac{a_{R1}^+(x_1)\,b_{L1}^+(y_1)\,a_{R2}^+(x_2)\,b_{L2}^+(y_2)}{[\Phi(x_1-y_1)\,\Phi(x_2-y_2)]^{1/2}}\times\frac{(me^{C-1/2})^2}{[\Phi(x_1-y_2)\,\Phi(x_2-y_1)]^{1/2}}. \tag{35}$$

Similarly in the case of arbitrary N and at equal $g_j=g$ there arises in the model a condensate of $2N$-particle complexes, in each of which are represented all N types of chiral quark pairs $q_R^{(j)}\bar{q}_L^{(j)}$ (or $q_L^{(j)}\bar{q}_R^{(j)}$).

A direct connection exists between the properties considered above the vacuum state of the model of Ref. 5 and the topological effects that arises already in the problem of zero-mass free fermions in an external electromagnetic field. 3 It is shown in Ref. 3 that an electromagnetic field that changes the topological number

$$Q_T(t)=-\frac{g}{2\pi}\int_0^t d^2xE(x,t) \tag{36}$$

after a time must inevitably produce, via the Adler anomaly, a system of free fermions with partially delocalized Q_R and Q_L charges. The quantity $Q_T(t)$ is precisely the measure of this delocalization:

$$\Delta Q_R=-\Delta Q_L=Q_T.$$

The explanation of the phenomenon is different in part for integer and noninteger Q_T. The field corresponding to an integer $Q_T(t)$ produces exactly Q_T pairs of particles $q_R\bar{q}_L$ (or $q_L\bar{q}_R$ at $Q_T<0$) with momenta on the order of $1/V$. At the same time we have an infinite coherent distribution dp/p with $\ln V$ particles having small momenta in the case of noninteger Q_T

$$\int_{V^{-1}} dp/p\sim\ln V.$$

A similar situation obtains with the vacua of the model of Ref. 5, and this fact is a direct consequence of the confinement, based on the mechanism of the topological effect (zero-mass quarks). Indeed, as explained in Sec. 1 and in Ref. 3, and electromagnetic field with $Q_T\neq 0$ should produce $|Q_T|$ chiral ($q_R\bar{q}_L$ or $q_L\bar{q}_R$) pairs directly an a delocalized state. In the multielectron model[5] we shall have for each type j of quark a total of $Q_T^{(j)}$ pairs produced in this manner. In the case of equal g_j [when all the $Q_T^{(j)}(t)$ are equal to one another and become integer simultaneously] only the pair combinations that are symmetrical about j should be joined to the vacuum state. And it is precisely such condensate complexes which exist in this case in the vacuum of the model. It suffices simply that the joined particles form such a complex. It is impossible, however, to have equal or integer $Q_T^{(j)}$ for all j simultaneously at arbitrary g_j. The electromagnetic field should produce in this case a distribution of $\ln V$ particles that is not symmetric in j, and the chiral complexes made up of $\ln V$ particles play the principal role, as we have seen in the vacua of the model under these conditions.

We obtain in QED$_2$ a direct connection between the condensate structure of the vacuum states and the Adler anomaly. The vacuum has a condensate structure only when all the partial topological charges $Q_T^{(j)}(t)$ are equal (for all the flavors of the j zero-mass quarks).[3] The Adler anomaly for an individual flavor j does not depend on j in this case. The important consequence is that the topological confinement mechanism can lead only to vacuum condensates that are symmetric in the flavors, and are characterized by symmetric vacuum mean values of the type $\langle\bar{u}u\bar{d}d\bar{s}s...\rangle\neq 0$ (only zero-mass quarks should be considered). All the paired and other vacuum combinations (such as $\langle\bar{u}u\rangle$, $\langle\bar{d}d\rangle$, etc.) are equal to zero. These statements are of interest also from the viewpoint of their possible four-dimensional generalizations.

5. CONFINEMENT MECHANISM IN A MODEL WITH SEVERAL QUARKS[5]

As shown in Sec. 1, the most important feature of the discussed confinement mechanism is the finite value of the matrix elements of the transition between local packets and delocalized vacuum states. It is precisely in such transitions that the locally produced quark characteristics vanish. We readily obtain the desired result if the vacuum is a condensate of such characteristics. Just as in the general case, in the model of Ref. 5 there is no condensate and the confinement mechanics become entirely different.

The simplest way of casting light on the resultant picture is to solve the model by the bosonization method[11,7-9] of solving QED$_2$ models. The method itself and an exact solution of the Schwinger model were formulated also in Ref. 3. The main formula here is the boson representation for fermion operators:

$$\psi_{R,L}(x)=\exp\left\{-\frac{1}{V}\sum_{p_n>0}\exp(\mp ip_nx)\left(\frac{2\pi}{p_n}\right)^{1/2}c^{+}_{R,L}(p_n)\right\}\frac{\sigma_{R,L}}{V^{1/2}}$$

$$\times\exp\left\{\frac{1}{V}\sum_{p_n>0}\exp(\pm ip_nx)\left(\frac{2\pi}{p_n}\right)^{1/2}c_{R,L}(p_n)\right\}. \quad (37)$$

Here $\rho_{R,L}$ are constant operators ($\sigma_R^+\sigma_R=\sigma_L^+\sigma_L=1$, $\{\sigma_R^\pm,\sigma_L^\pm\}=0, \sigma^-\equiv\sigma$), which mark the vacuum state of the model of Ref. 3. The chiral and charged vacua of the Schwinger model can be represented in terms of $\sigma^\pm$ as $(\sigma_R^\pm)^m(\sigma_L^\pm)^n\Omega_0$. The operators $\sigma_{R,L}^\pm$ commute with the creation and annihilation operators $c_{R,L}^\pm(p)$ of zero-mass neutral bosons. The connection between $c_{R,L}^\pm(p)$ and the quark fields (2) is established by the formulas[11,3]

$$c_R^\pm(p)=\left(\frac{2\pi}{p}\right)^{1/2}\int e^{\pm ipx}\rho_R(x)\,dx,$$

$$c_L^\pm(p)=\left(\frac{2\pi}{p}\right)^{1/2}\int e^{\mp ipx}\rho_L(x)\,dx, \quad (38)$$

$\rho_{R,L}(x)$ are the R and L charge density operators.

In the model with several quarks[5] we must write down Eqs. (38) for each type of quark j. The Hamiltonian of the model in a Coulomb gauge is diagonalized when Eqs. (37) are substituted in it and the operators $c_{R,L}^{(i)\pm}(p)$ are replaced by the operators ($N=2$)

$$c_m(\pm p)=\frac{1}{2}\left(\frac{p}{\omega_p}\right)^{1/2}\left\{\frac{\omega_p+p}{p}[\eta_1c_{1R,L}(p)+\eta_2c_{2R,L}(p)]\right.$$

$$\left.+\frac{\omega_p-p}{p}[\eta_1c^{+}_{1L,R}(p)+\eta_2c^{+}_{2L,R}(p)]\right\},$$

$$c_{0R}(p)=\eta_2c_{1R}(p)-\eta_1c_{2R}(p);\ c_{0L}(p)=\eta_2c_{1L}(p)-\eta_1c_{2L}(p),$$

$$p>0 \quad (39)$$

and the corresponding $c_m^+(\pm p)$, $c_{0R}^+(p)$, $c_{0L}^+(p)$.

For the Hamiltonian of the model of Ref. 5 this yields the expression

$$H=\frac{1}{V}\sum_{p_n}[\omega_{p_n}c_m^+(p_n)c_m(p_n)+|p_n|c_0^+(p_n)c_0(p_n)]. \quad (40)$$

The spectrum of the model thus consists, as already mentioned, of neutral massive and zero-mass mesons. There is also an infinite set of vacua defined by the operators $\sigma_R^{(i)\pm}$ and $\sigma_L^{(i)\pm}$. The total charges Q_R and Q_L remain confined. The corresponding characteristics should, in accord with the foregoing, to be delocalized in the vacuum states with the same quantum numbers.

But substitution of (37) with use of the conditions $c_m(p)|\Omega_0\rangle=c_0(p)|\Omega_0\rangle=0$ leads to the following result:

$$\langle\Omega_1|\psi_{R1}^+(x)\psi_{L1}(x)|\Omega_0\rangle=\langle\Omega_0|\sigma_{R1}\sigma_{L1}^+\psi_{R1}^+(x)\psi_{L1}(x)|\Omega_0\rangle$$

$$=\frac{1}{V}\exp\left[\frac{2\pi\eta_1^2}{V}\sum_{p_n>0}\frac{\omega_n-p_n}{\omega_np_n}\right]=\frac{\text{const}}{V^{1-\eta_1^2}}. \quad (41)$$

Owing to the absence of chiral condensate in the vacua of the model ($\eta_1^2<1$) no direct transition is possible from local chirality into a delocalized chiral vacuum $\Omega_1=\sigma_{R1}^+\sigma_{L1}|\Omega_0\rangle$.

However, chirality delocalization does take place in the model of Ref. 5 just the same, owing to the existence in it of zero-mass excitations. Indeed, the matrix element of the transition from the local chiral state $\psi_{R1}^+(x)\psi_{L1}(x)|\Omega_0\rangle$ into the coherent state

$$\Psi_{p_0}=\frac{1}{N}$$

$$\times\exp\left\{\frac{\eta_2}{V}\sum_{|p_n|<p_0}c_0^+(p_n)\left(\frac{2\pi}{|p_n|}\right)^{1/2}\varepsilon(p_n)\exp(-ip_nx)\right\}|\Omega_1\rangle, \quad (42)$$

$$\langle\Psi_{p_0}|\Psi_{p_0}\rangle=1,\quad N^2=(p_0V)^{2\eta_2^2},$$

turns out to be finite:

$$\langle\Psi_{p_0}|\psi_{R1}^+(x)\psi_{L1}(x)|\Omega_0\rangle=N^{-1}\exp\left\{\frac{2\pi\eta_2^2}{V}\sum_{|p|<p_0}\frac{1}{|p|}\right\}$$

$$\times\frac{\text{const}}{V^{1-\eta_1^2}}\approx\text{const}(p_0)^{\eta_2^2}. \quad (43)$$

The state Ψ_{p0} contains an infinite number of zero-mass particles with small momenta $|p|<p_0$ over a vacuum Ω_1. It is a state with delocalized Q_R and Q_L, analogous to the chiral vacuum Ω_1 of the preceding section. The last statement follows from the fact that the electromagnetic densities

$$\rho(x)=\rho_R(x)+\rho_L(x) \text{ and } j(x)=\rho_R(x)-\rho_L(x)$$

depend in the model only on the operators of the massive particles $c_m^\pm(p)$, as can be easily verified with the aid of (38) and (39). At the same time there are no heavy bosons in Ψ_{p0}. Therefore the chirality of the state is delocalized in the vacuum Ω_1.

Thus, the chirality is delocalized with the aid of the coherent states Ψ_{p0} and is not observable in the model of Ref. 5. But states of this type can be constructed only for the total charges Q_R and Q_L. In the model there are no states capable of ensuring finite transitions between local packets with partial charges $Q^{(j)}\neq0$ (at $Q_{R,L}=0$) and delocalized vacua with $Q^{(j)}\neq0$. The partial charges $Q^{(j)}$ [see (31)] are observable if the total $Q_{R,L}$ charges of the considered local states are equal to zero. We shall now prove this statement with the aid of a rather unexpected and instructive example.

The system of physical states of the (bosons + vacua) model considered by us so far is not the only one possible, owing to the degeneracy of two-dimensional zero-mass excitations. From among the infinite choice of the possible systems of state there is even a variant in which the zero-mass particle is a fermion with nonzero partial charges $Q^{(j)}$. The states of such a particle are produced by creation and annihilation operators $a_{R,L}^{\pm(0)}(p)$ and $b_{R,L}^{\pm(0)}(p)$, defined in terms of the local operator

$$\psi_{0R,L}(x)=\frac{1}{V}\sum_{p_n>0}\{a_{R,L}^{(0)}(p_n)\exp(\pm ip_nx)+b_{R,L}^{+(0)}(p_n)\exp(\mp ip_nx)\}$$

$$=\exp\left\{-\frac{1}{V}\sum_{p_n>0}\exp(\mp ip_nx)\left(\frac{2\pi}{p_n}\right)^{1/2}c^{+}_{0R,L}(p_n)\right\}$$

$$\times\frac{\sigma_{R,L}^{(1)}}{V^{1/2}}\exp\left\{\frac{1}{V}\sum_{p_n>0}\exp(\pm ip_nx)\left(\frac{2\pi}{p_n}\right)^{1/2}c_{0R,L}(p_n)\right\} \quad (44)$$

These operators of the free fermion field anticommute in the usual manner with $\psi^+_{0R,L}(x)$. In the new system of states, the Hamiltonian of the model remains diagonalized:

$$H=\frac{1}{V}\sum_{p_n,i=R,L}|p_n|[a_{i0}^+(p_n)a_{i0}(p_n)+b_{i0}^+(p_n)b_{i0}(p_n)]$$
$$+\frac{1}{V}\sum_{p_n}\omega_{p_n}c_m^+(p_n)c_m(p_n). \qquad (45)$$

Massive bosons and zero-mass fermions are now the excitations of the model. Its vacuum states have charges $Q^{(2)}_{R,L}$.

The fermion states (44) have zero $Q_{R,L}$, but their partial charges $Q^{(1)}$ are not equal to zero ($Q=\eta_1 Q^{(1)}+\eta_2 Q^{(2)}$):

$$Q_R=Q_L=0;\quad Q^{(1)}=\eta_2;\quad Q^{(2)}=-\eta_1. \qquad (46)$$

To prove (46) we must calculate the commutator

$$\rho^{(i)}(x)=\rho_R^{(i)}(x)+\rho_L^{(i)}(x)$$

with $\psi_{0R,L}(y)$ from (44). To this end we express $\rho^{(i)}(x)$ with the aid of (38) and (39) in terms of $c_0^\pm(p)$ and $c_m^\pm(p)$. Integrating the obtained commutator with respect to the coordinate x, we obtain (46). The charges $Q^{(1)}$ and $Q^{(2)}$ characterize the physical states of the model and can be called flavors. The Q_j for the states (39) and (44), however, obviously differ from the partial charges of the quarks (± 1). Therefore the quark flavor is observable only here in hidden form, meaning confinement.

The quark structure of the fermion (44) is extremely complicated in the case of arbitrary g_j, but it becomes very simple in several particular cases. Thus, for $N=3$ and $\eta_1+\eta_2+\eta_3=0$ the zero-mass fermion is a "nucleon" neutral in all the charges and is made up of three quarks.

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APPENDIX

The density of R quarks in vacuum

$$n_R(p)=\frac{1}{V}\langle\Omega_0|a_R^+(p)a_R(p)|\Omega_0\rangle \qquad (A.1)$$

can be expressed in terms of the Green's function of the fermion:

$$n_R(p)=\int\frac{dx\,dy}{V}e^{ip(x-y)}\langle\Omega_0|\psi_R^+(x)\psi_R(y)|\Omega_0\rangle$$
$$=\int\frac{dx\,dy}{V}e^{ip(x-y)}G(x,y). \qquad (A.2)$$

We replace in (A.2) the operators $\psi_R^\pm(x)$ by the representation (37). Using formula (39) for the case $\eta_2=0$, $\eta_1=1$ (they go over in this case into the corresponding expressions of the Schwinger model), we carry out the calculations in (A.2). We obtain expression (20) for $n_R(p)$ and the following equation for the Green's function:

$$G(x,y)=\frac{1}{2\pi i(y-x-i0)}\exp\left[-\int_0^\infty\frac{dp}{\omega_p}\left(\frac{\omega_p-p}{p}\right)^2\sin^2\frac{p(x-y)}{2}\right]. \qquad (A.3)$$

The asymptotic form of $G(z)$ at $z\gg m^{-1}$ is determined by the contribution of the small p to the integral of (A.3). $G(z)$ decreases exponentially as $z\to\infty$:

$$G(z)=\frac{e^{C+1}}{4\pi i(z-i0)}(mz)\exp\left(-\frac{\pi}{4}m|z|\right). \qquad (A.4)$$

We carry out the same calculations in the interacting-pair approximation [Eqs. (17)–(19)]. We define the state $\Omega_0^{(n)}$ with zero chirality:

$$\Omega_0^{(n)}=\int_0^{2\pi}\frac{d\theta}{2\pi}|\theta\rangle=\int_0^{2\pi}\frac{d\theta}{2\pi}Z^{-1/2}\exp\left\{\int_0^\infty\frac{dp}{2\pi}\right.$$
$$\left.\times[\Phi(p)a_R^+(p)b_L^+(p)e^{i\theta}+\Phi^*(p)a_L^+(p)b_R^+(p)e^{-i\theta}]\right\}|0\rangle. \qquad (A.5)$$

For the operators

$$A_{R,L}(p)=[1+|\Phi(p)|^2]^{-1/2}[a_{R,L}(p)-\Phi(p)e^{\pm i\theta}b^+_{L,R}(p)],$$
$$B_{R,L}(p)=[1+|\Phi(p)|^2]^{-1/2}[b_{R,L}(p)+\Phi(p)e^{\mp i\theta}a^+_{L,R}(p)], \qquad (A.6)$$

the state $|\theta\rangle$ is then a "vacuum" state:

$$A_{R,L}(p)|\theta\rangle=B_{R,L}(p)|\theta\rangle=0. \qquad (A.7)$$

These operators satisfy the usual commutation relations. The operators $a^\pm_{R,L}(p)$ of (A.1) can be expressed in terms of A and B. The quark density in vacuum (A.1) in vacuum is now easily calculated in the considered approximation

$$n_R^{(n)}(p)=\int_0^{2\pi}\frac{d\theta\,d\theta'}{(2\pi)^2}\langle\theta|B_L(p)B_L^+(p)|\theta'\rangle\frac{1}{V}\frac{|\Phi(p)|^2}{1+|\Phi(p)|^2}$$
$$=\frac{|\Phi(p)|^2}{1+|\Phi(p)|^2}. \qquad (A.8)$$

A similar calculation expresses, in the pair approximation, the fermion Green's function in terms of the quark density (A.8):

$$G(x,y)=\langle\Omega_0^{(n)}|(a_R^+(x)+b_R(x))(a_R(y)+b_R^+(y))|\Omega_0^{(n)}\rangle$$
$$=\int_0^\infty\frac{dp}{2\pi}[1+|\Phi(p)|^2]^{-1}[e^{ip(x-y)}+|\Phi(p)|^2e^{-ip(x-y)}]$$
$$=\frac{1}{2\pi i(y-x)}+\int_0^\infty\frac{dp}{i\pi}n_R^{(n)}(p)\sin p(x-y). \qquad (A.9)$$

The asymptotic form as $z=x-y\to\infty$ again depends on the behavior of $n_R^{(n)}(p)$ at small p. From (18) and (A.8) we obtain

$$n^{(n)}(p)=\frac{1}{2}-\frac{p}{m}+\frac{25}{8}\left(\frac{p}{m}\right)^3-\frac{12}{5}\left(\frac{p}{m}\right)^5+\frac{11}{9\cdot 6!}\left(\frac{p}{m}\right)^6. \qquad (A.10)$$

Consequently, as $z\to\infty$, the Green's function is

$$G(z)=-\frac{11}{9}\frac{1}{\pi iz}\frac{1}{(mz)^6}, \qquad (A.11)$$

i.e., it decreases only in power-law fashion. The representation (37) makes it also easy to verify Eq. (24). The calculations are similar to the derivation of Eqs. (A.3) and (20).

[1)] In a two-dimensional world R and L denote directly particles moving to the right and to the left.

[2)]To prove this statement it suffices to calculate the contribution made to the normalization factor Z by the first terms of expansion (33) for Ω_j. We obtain the dependence of the volume $V^{2-4\eta_j^2}$. This is very small compared with the normalization condition $\langle \Omega_j | Q_j \rangle = V^2$, that follows from our choice (2) of the quark operators. Only the terms of the operators a^+ and b^+ with $\ln V$ can compensate for this smallness.

[3)]Strictly speaking, the $Q_T^{(j)}(t)$ can differ from one another by integers. The generalization of the text to include this case is obvious.

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