Anomalous penetration of an electromagnetic field into a superconductor in the presence of an external constant magnetic field

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The effects that appear upon normal incidence of an electromagnetic wave on the surface of a superconductor (at $T \neq 0$) in the presence of a constant magnetic field at the surface are investigated. It is shown that in such a situation there arises in the superconducting material a variable longitudinal electric field that penetrates deep into the material, and owes its origin to Hall-type surface quasiparticle currents. The anomalous penetration effect due to this field can be significantly stronger than the corresponding effect that occurs in the absence of a constant magnetic field.

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Much interest has been shown in recent years in the study of the effects connected with the possible existence of a longitudinal electric field in a superconductor.[1–4] It has, in particular, been shown that in the nonstationary situation the penetration of this field into the superconductor is the result of a specific collective motion due to the two-component structure of the electron system of the superconductor, and has, in a certain frequency range, the character of a slowly decaying mode.[5,6] It was pointed out recently, that such a collective motion makes possible the anomalous penetration of an electromagnetic field through a superconducting plate.[7,8] The mechanism underlying this phenomenon amounts to the following.

In the case of oblique incidence of an appropriately polarized electromagnetic wave on the surface of a superconductor, there exists an electric-vector component that is normal to the surface, and leads, on account of the electrodynamic boundary conditions, to the formation of a time-dependent surface charge. At $T > T_c$ (i.e., in the normal metal), or at $T = 0$, the time variation of this charge is guaranteed by the purely rotational fields and eddy currents localized in the skin or London layer. But in a superconductor at $T \neq 0$ we must take into account, along with the ordinary electromagnetic boundary conditions, the additional conditions determining the connection between the boundary values of the superfluid and quasiparticle components of the total current.[8] All the boundary conditions can be satisfied only when allowance is made for the indicated collective motion of the superfluid and normal components and the related slowly decaying—into the interior of the sample—longitudinal fields, which in turn guarantee the appearance of an electromagnetic field on the opposite side of the plate.

It is, however, not difficult to see that the intensity of the transmitted signal is, in any event, proportional to the coefficient determining the attenuation of the normal—to the surface—electric field component in the conductor in comparison with the component in a vacuum, i.e., to the parameter ω/σ (where σ is the static conductivity of the normal metal), which, under the most favorable conditions, is of the order of 10^{-9}. This is due to the fact that the normal—to the surface—current component, which is, on account of the electrodynamic boundary conditions, equal to the displacement current in the vacuum, is responsible for the effect.

In the present paper we call attention to a factor capable of markedly raising the coefficient of anomalous penetration. We assume normal incidence of the electromagnetic wave on the surface of the sample (i.e., the absence of a normal component of the electric field and the related anomalous-penetration mechanism considered in Refs. 7 and 8). The response to the electromagnetic field is then completely determined by the eddy currents flowing parallel to the surface, which, at $T \neq 0$, have both a superconducting and a quasiparticle component. Let us note that the weakening of the electric field component parallel to the surface at $T \sim T_c$ and in the case of the normal skin effect is determined by the parameter $(\omega/\sigma)^{1/2}$ (and not by ω/σ as for the component normal to the surface), and the quasiparticle eddy currents are then many orders of magnitude higher than the displacement current in the vacuum.

Let us now assume that an external constant magnetic field is produced near the irradiated surface of the superconductor. This field (localized in the London layer) will induce quasiparticle currents in the Hall direction. On the other hand, the electrodynamic boundary conditions require at the same time the vanishing of the total-current component normal to the surface, while the specific supplementary conditions require the vanishing of the quasiparticle current (cf. Refs. 3 and 4). These conditions can be satisfied simultaneously only when allowance is made for the contribution of the longitudinal fields and currents.[3,4] The picture of the anomalous penetration is thus reminiscent of the one discussed earlier for the case of oblique incidence in the absence of a constant magnetic field. But in the present situation the effect stems not from the displacement currents, but from the quasiparticle eddy currents that flow parallel to the surface, which are $(\sigma/\omega)^{1/2}$ times stronger. Naturally, the Hall current is connected with the supplementary small

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parameter[1] $\Omega\tau$ (Ω is the cyclotron frequency and τ is the momentum relaxation time), and, as a result, the estimate for the longitudinal fields in the superconductor as a ratio of the field of the incident wave is proportional to the parameter $\Omega\tau(\omega/\sigma)^{1/2}$. But, as is easy to see, in a real situation the condition $\Omega\tau \gg (\omega/\sigma)^{1/2}$ can be fulfilled even in fairly weak magnetic fields, and, thus, the anomalous-penetration mechanism under consideration can be more effective than the mechanism proposed in Refs. 7 and 8.

To estimate the effect, let us use the kinetic equation[9] for the quasiparticle-distribution function n:

$$\frac{\partial n}{\partial t}+\frac{\partial \tilde{\varepsilon}}{\partial \mathbf{p}}\frac{\partial n}{\partial \mathbf{r}}-\frac{\partial \tilde{\varepsilon}}{\partial \mathbf{r}}\frac{\partial n}{\partial \mathbf{p}}+\hat{I}n=0, \tag{1}$$

where $\hat{I}$ is the phonon-impurity collision integral,

$$\tilde{\varepsilon}=(\xi_p^2+\Delta^2)^{1/2}+\mathbf{p}\mathbf{v}_s,\quad \xi_p=\frac{p^2}{2m}-\mu-\Phi,\quad \mathbf{v}_s=\frac{\hbar}{2m}\left(\nabla\chi-\frac{e}{\hbar c}\mathbf{A}\right),$$
$$\Phi=\frac{1}{2}\dot{\chi}+e\varphi.$$

Here v_s contains both the contribution corresponding to the response to the electromagnetic field and the contribution describing the screening of the constant magnetic field.

We shall solve the problem by iteration assuming that, in first order in the field intensities, we can independently compute the responses to the constant magnetic and electromagnetic fields, i.e., independently solve the problems of their screening. Separating out the local-equilibrium part n_0 of the distribution function [i.e., setting $n=n_0(\tilde{\varepsilon})+n^1$], we obtain

$$\frac{\partial n^1}{\partial t}+\frac{\partial \tilde{\varepsilon}}{\partial \mathbf{p}}\frac{\partial n^1}{\partial \mathbf{r}}-\frac{\partial \tilde{\varepsilon}}{\partial \mathbf{r}}\frac{\partial n^1}{\partial \mathbf{p}}+\hat{I}n_1-\frac{e}{c}[\mathbf{V}\times\mathbf{H}_0]_z\frac{\partial n^1}{\partial p_z}=-\frac{e}{m}(\mathbf{p}\mathbf{E}_1)\frac{\partial n_0}{\partial \varepsilon}-[n_0]_t^1, \tag{2}$$

where $\mathbf{H}_0$ is a constant magnetic field, $\mathbf{E}_1$ is the eddy electric field (we neglect the magnetic field of the electromagnetic wave, assuming that it is, at any rate, weaker than H_0), z is a coordinate measured along the normal to the surface into the superconductor, and $[n_0]_t^1$ is the result of the differentiation of n_0 with respect to t with allowance for Φ and the terms of higher order in H_0 and E_1.

Let us discuss the limits of applicability of our approach. The classical description requires the fulfillment of the condition

$$\hbar\omega\ll\Delta,\quad \lambda\gg\hbar v_F/\bar{\xi}.$$

As will be shown below, the following condition should also be satisfied:

$$\bar{\xi}\gg(\hbar\omega\Delta)^{1/2}$$

(here $\bar{\xi}$ is the characteristic value of ξ_p; λ is the characteristic E_1-attenuation length: $\lambda=\min(\lambda_L,\lambda_s)$, where λ_L and λ_s are the London and skin depths respectively). We shall, for simplicity, consider the case of the normal skin effect: $v_F\tau\ll\lambda_s$. The applicability of the iterative procedure requires, as can be shown, the fulfillment of the condition

$$\Omega\tau\mu\varepsilon/\bar{\xi}^2\ll 1.$$

We shall, however, require the fulfillment of the following more rigid condition, which will allow us to neglect the "capture" of the quasiparticles by the field H_0 near the surface:

$$pv_s^0\varepsilon_p/\bar{\xi}^2=(\Omega\lambda/v_F)\mu\varepsilon/\bar{\xi}^2\ll 1.$$

When these conditions are fulfilled, the response to the electromagnetic field has, in the lowest order in E_1 and with H_0 neglected, the form

$$n_e^1=-\hat{I}^{-1}\frac{e}{m}(\mathbf{p}\mathbf{E}_1)\frac{\partial n_0}{\partial \varepsilon}. \tag{3}$$

Substituting (3) into the term $\propto H_0$, we obtain in the equation in first order in the parameter $\Omega\tau$ a current term corresponding to the high-frequency Hall current. The direction of this current is given by the vector $[\mathbf{E}_1\times\mathbf{H}_0]$, i.e., it is normal to the surface, and its curl is equal to zero. We should, on the basis of this circumstance, expect the appearance of longitudinal fields normal to the surface. Iterating (2) in the appropriate order in E_1 and H_0, we obtain for the determination of these longitudinal fields the equation

$$\frac{\partial \tilde{n}}{\partial t}+\frac{\xi}{\varepsilon}v_z\frac{\partial \tilde{n}}{\partial z}+\hat{I}\tilde{n}=F^{-}-[n_0]_t^1, \tag{4}$$
$$F^{-}=\frac{e}{c}[\mathbf{v}\times\mathbf{H}_0]_z\frac{e}{m}(\mathbf{p}\mathbf{E}_1)v_z\frac{\partial}{\partial\xi}\left(-\tau\frac{\varepsilon}{|\xi|}\frac{\partial n_0}{\partial\varepsilon}\right).$$

Here we have used the estimate[2] $\hat{I}^{-1}\sim|\xi|/\tau\varepsilon$.

Using a procedure similar to the one used in Refs. 6 and 8, and assuming that $\omega\tau\ll 1$, we can derive from (4) the following hydrodynamic equations for the quantities $\tilde{v}_s$ and Φ corresponding to the longitudinal fields:

$$j=j^H+eN\left[\frac{N_s}{N}+i\omega\left(i\omega-\frac{1}{\tau_b}\right)\tau\left(-\frac{1}{\hat{D}}+\frac{\Delta}{2T}\frac{\ln\Lambda}{(i\omega-1/\tau_b)}\right)\right]\tilde{v}_s^0$$
$$-i\omega\tau\frac{Ne}{m}\frac{1}{\hat{D}}\nabla\Phi^0=0, \tag{5a}$$
$$\delta N=\frac{3N}{2\mu}\left(-1-\frac{\pi\Delta}{4T}+i\omega\frac{1}{\hat{D}}\right)\Phi^0-\frac{3}{2}i\omega m\frac{N}{\mu}D\frac{1}{\hat{D}}\nabla\tilde{v}_s^0$$
$$+\tau\sum_{\mathbf{p}}\frac{|\xi|}{\varepsilon}\nabla\overline{(vF^{-})}\hat{D}_{q_0}^{-1}=0. \tag{5b}$$

Here we have set

$$\tilde{v}_s=\tilde{v}_s^0e^{-i\omega t},\quad \Phi=\Phi^0e^{-i\omega t},\quad \hat{D}=(i\omega-1/\tau_b)+D\nabla^2,$$
$$\hat{D}_q=e^{-qz}\hat{D}e^{qz},\quad D=\tau v_F^2/3,\quad q_0=\lambda^{-1},$$

τ_b is the branch-population-imbalance relaxation time,

$$j^H=\left(-i\omega+\frac{1}{\tau_b}\right)\tau\sum_{\mathbf{p}}\frac{\varepsilon}{|\xi|}\mathbf{v}F^{-}\hat{D}_{q_0}^{-1}$$

and is the Hall current.

The appearance of the logarithmic term is due to the presence of a divergence in the corresponding integral with respect to ξ_p, i.e., to the fact that the important contribution to the conductivity is made by the particles with small ξ_p. Estimates with the aid of the kinetic equation lead to a cutoff energy $\xi_p\sim\omega\tau\Delta$ (cf. Ref. 9) and the value $\Lambda=(\omega\tau)^2$. But the important role played by the low-energy region requires in the general case an assessment of the applicability of the classical approach.[9] The question of the expression for the current is discussed from first principles, in particular, in Ref. 10, where the logarithmic term is computed within the framework of a rigorously quantum-mechanical approach (see also Ref. 11). It can be seen in this case that the characteristic quantum energy, which is connected with the density-of-states singularity and determines the limits of applicability of the classical approach in the low-energy region, is given by the estimate $\varepsilon-\Delta\sim\hbar\omega$, and, thus, $\xi_p\sim(\hbar\omega\Delta)^{1/2}$. Therefore, in the case $\omega\tau\Delta\gg(\hbar\omega\Delta)^{1/2}$ or $\hbar\omega>\Delta(\Delta\tau)^{-2}$, the expression for the conductivity can be obtained within the framework of the classical approach. In the opposite case to estimate the logarithmic term we must go outside the limits of applicability of the kinetic equation; in this case the cutoff energy is $\xi_p\sim(\hbar\omega\Delta)^{1/2}$, and the quantity Λ is determined by the quantum parameter $\hbar\omega/\Delta$. Finally, for Λ we have the following interpolation formula:

$$\Lambda\approx\max((\omega\tau)^2,\ \hbar\omega/\Delta).$$

It can be verified that the quantum-mechanical calculation[10] also yields an estimate of this type. Notice that the foregoing interpolation formula ensures a connection between the expressions obtained for the conductivity in the various frequency regions, since, on the one hand, at $\omega\tau\gg1$ the expression for the conductivity does not contain an ξ_p singularity, and, on the other, the quantum parameter $\hbar\omega/\Delta$ does not depend on the quantity $\omega\tau$.

The expression for the Hall current j^H also contains a singularity in the region of small ξ_p. Taking the foregoing into account, we can estimate j^H strictly within the framework of the classical approach in the case $\hbar\omega>\Delta/(\Delta\tau)^2$. In the opposite case we shall limit ourselves to an order-of-magnitude estimate, assuming that the cutoff point for the integration is determined by the quantum energy $\xi\sim(\hbar\omega\Delta)^{1/2}$. Finally, we have the estimate

$$j^H\sim\left(1+\frac{D}{\lambda^2}\frac{1}{i\omega-1/\tau_b}\right)^{-1}\Omega\tau\sigma E_1\left(1+\frac{\Delta c}{T\Lambda}\right);\quad c\sim1. \tag{6}$$

The homogeneous equations corresponding to (5a) and (5b) describe the collective motion of the condensate and the normal component; the dispersion law for this motion can be obtained by assuming that $v_s^0,\ \Phi^0\propto e^{iqz}$, and computing the determinant of the system (cf., for example, Refs. 6 and 8). We then arrive at the following results: for $\omega\tau_b\Delta/T\ll1$ (purely diffusional quasistatic case) we have

$$q^2=i/D\tau_b; \tag{7a}$$

for $(\tau_b\Delta/T)^{-1}\ll\omega\ll\tau^{-1}(\Delta/T)^2$ we have

$$q^2\approx(i\omega/D)\Delta/T; \tag{7b}$$

for $(\Delta/T)^2\ll\omega\tau\ll\Delta/T\ln\Lambda$ (acoustic-type mode[5,6]) we have

$$q^2=\frac{\omega^2}{D}\left(\frac{\pi\tau/4}{\Delta/T+i\omega\tau\ln\Lambda}+\frac{i\Delta}{\omega T}\right); \tag{7c}$$

for $\omega\tau\gg\Delta/T\ln\Lambda$ we have

$$q^2\approx-(i\omega/D)\pi/2\ln\Lambda. \tag{7d}$$

Let us now turn to the particular solution of the inhomogeneous system (5). Since $j^H\propto e^{-z/\lambda}$, it is not difficult to see that, if $\omega\ll v_F/\lambda$, then the terms $\propto iw$ in the first equation can be neglected; we then obtain

$$j=j^H+eN_sv_s^H=0.$$

Thus, in this case the quasiparticle current corresponding to the particular solution to the inhomogeneous system is simply j^H.

The boundary conditions for the present problem reduces[3,4,8] to the form[3)]

$$eN_sv_{sz}|_{z=0}=0;\quad j_{nz}|_{z=0}=0, \tag{8}$$

where $\mathbf{j}_n$ is the total quasiparticle current. It can be seen that it can be satisfied only when allowance is made for the general solution to the homogeneous equation, i.e., for the longitudinal fields that penetrate deep into the sample.

Using (7), and determining the quasiparticle component corresponding to the solution to the homogeneous equation, we obtain for $\nabla\Phi$ the estimate:

$$\nabla\Phi|_{z=0}\approx ej^H|_{z=0}A(\omega)/\sigma\sim e(\Omega\tau)E_1|_{z=0}B(\omega);$$

$$A(\omega)=\frac{-i\omega}{D_q}\left[1+\frac{\hat{D}_q}{Dq^2}\left(1+\frac{i}{\omega\tau_b}+\frac{i\hat{D}_q}{\omega}\frac{\Delta}{2T}\ln\Lambda\right)\left(1+\frac{\pi\Delta}{4T}\frac{i\omega}{D_q}\right)\right]. \tag{9}$$

$$B(\omega)\approx(\lambda^2/D)(i\omega-1/\tau_b)A(\omega).$$

An estimate shows that in the case of the normal skin effect E_1 is connected with the field intensity E_0 in the incident wave by the relation $E_1\sim(\omega/\sigma)^{1/2}E_0$.

As to the general physical picture [determined by the dispersion law (7)] of the penetration of the longitudinal field through a plate and the conversion of this field into an electromagnetic field at the opposite surface, it is similar to the picture considered in Ref. 8. Therefore we discuss it here only briefly. To find the longitudinal-electric-field intensity in the sample,

$$\mathbf{E}_l=-\frac{1}{e}\nabla\Phi+\frac{m\dot{\mathbf{v}}_s}{e},$$

we must also compute $\dot{\mathbf{v}}_s$. We can easily do this using any of the equations of the homogeneous system corresponding to (5), as well as the solution to the dispersion equation (7). Finally, we obtain

$$E_l\sim-\frac{1}{e}\nabla\Phi,\quad \omega\tau<\frac{\Delta}{T};$$

$$E_l\sim-\frac{1}{e}\frac{\Delta}{T\omega\tau}\nabla\Phi,\quad \omega\tau\gg\Delta/T.$$

The field strength in the vacuum in the vicinity of the opposite surface of the plate, i.e., the electromagnetic field that penetrates through the plate, can be computed knowing $\mathbf{E}_l$ in the sample near this surface and using the standard boundary conditions. With allowance for these conditions, the electric field in the vacuum is determined by the $\mathbf{E}_l$ component parallel to the corresponding plate surface, and it is this component which is matched with the field in the vacuum (cf. Ref. 8).

Thus, in our situation, when the plate is perfectly plane-parallel, i.e., when the opposite face is perpendicular to the axis $z\parallel\mathbf{E}_l$, the field does not penetrate into the vacuum. If, on the other hand, the normal to this face is inclined at some angle α to the z axis (i.e., if the plate is wedge-shaped or notched), the field intensity in the vacuum near the sample surface is $E_v \sim \alpha E_l$ (let us note that, in the situation considered in Refs. 7 and 8, the effect also increases significantly with increasing α, although it is nonzero at $\alpha = 0$). Finally, we have for the penetration coefficient $K \sim E_v/E_0$ the order-of-magnitude estimate

$$K \sim \alpha(\Omega\tau)\left(\frac{\omega}{\sigma}\right)^{1/2} B(\omega)\left(1+\omega\tau\frac{\Delta}{T}\right)^{-1} \exp(-\operatorname{Im} qL), \qquad (10)$$

where L is the plate thickness and q is determined from (7). In particular, for $\omega/\sigma \sim 10^{-9}$, $\Omega\tau \sim 10^{-4}$, and $(\Delta/T, \omega\tau, \alpha) \sim 1$, the pre-exponential factor is of the order of 10^{-9}–10^{-8}.

As already indicated, the connection with the magnitude of the effect considered in Refs. 7 and 8 is determined by the parameter $\Omega\tau(\omega/\sigma)^{-1/2}$. In this connection, let us point out two important consequences. First, since $\omega/\sigma \sim 10^{-9}$ in the optimal situation for the observation of the latter effect, whereas $\Omega\tau$ can have a value $\sim 10^{-5}$ even in a field ~1 Oe, we must, in particular, take into account the possible effect of the background magnetic fields in the observation of this effect. Second, the conditions for the observation of the anomalous penetration and, in particular, the excitation of the slowly decaying mode[5,6] are eased significantly.

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[1)] Everywhere below we shall, for simplicity, limit ourselves to the case of the normal skin effect. The smallness of $\Omega\tau$ is due, on the one hand, to the fact that the field strength should not be higher than H_{c1} and, on the other, with the fact that τ is limited by the requirement $v_F\tau < \lambda_s$, where λ_s is the skin depth.

[2)] It can be seen that F^- contains parts both even and odd in ξ_p. The charge transport is, however, connected with only the even part, which is the part that we shall consider below; the odd part corresponds to currents that ensure the variation of the distribution-function correction even in ξ_p and p, which is negligible in comparison with n_0.

[3)] The fulfillment of this condition is guaranteed by the absence of surface recombination of the quasiparticles.

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