Measurement of the Lamb shift in the hydrogen atom ($n=2$)

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(Submitted 24 September 1981)
Zh. Eksp. Teor. Fiz. **83**, 15–32 (July 1982)

A principle is proposed for the observation of the stationary interference pattern of two phase-shifted components of the $2p$ (or $2s$) state of the hydrogen atom [Yu. L. Sokolov, Sov. Phys. JETP **36**, 243 (1973)]; [Proc. 6-th Internat. Conf. on Atomic Phys., Riga, 1978, p. 207]. An atomic interferometer, a device analogous in principle to a two-channel optical (such as Michelson's) interferometer, is used to measure the frequency of the ($2s_{1/2}$, $F=0$)–($2p_{1/2}$, $F=0$) transition in the hydrogen atom, which is found to equal 909.9014 ± 0.0019 MHz. The corresponding Lamb shift is $\delta(\mathrm{H}, n=2) = 1057.8594 \pm 0.0019$ MHz.

PACS numbers: 35.10.Fk, 35.80. + s, 32.70.Jz

1. INTRODUCTION

Measurement of the Lamb shift in a hydrogen atom is known to be one of the important checks on quantum electrodynamics (QED). Even though δ has been measured for more than a quarter of a century,[1)] it must be admitted, however, that the progress in improving the accuracy has been quite modest. The scatter in the experimental values of the Lamb shift measured by the presently existing methods is patently too large to permit a comparison, which is of fundamental interest, with the theory. Furthermore, the scatter of the theoretical values of δ is also quite appreciable, and their accuracy does not exceed 0.01 MHz. Thus, if a discrepancy is observed between δ_{exp} and δ_{theor}, it is not clear whether this should be attributed, at any rate, more readily to measurement and calculation errors than to fader of QED.

Measurements of δ by the atomic interferometer method enabled us to decrease the error of the result to 0.0019 MHz. The data obtained raise hopes of further increase in the accuracy.

2. METHOD

In principle, δ can be measured by a simple interferometer with a longitudinal field consisting of two plane-parallel plates with openings for the passage of a beam of metastable $2s_{1/2}$ atoms.[2,3] The interference pattern observed in this case contains quite complete information on the structure and properties of levels with $n=2$. Extraction of this information, however, for example the determination of δ with accuracy to 1–2 ppm, entails many practically unsurmountable difficulties.

At the same time, a two-electrode interferometer, which can be relatively easily adjusted with respect to a strictly collimated beam of hydrogen atoms and is convenient in operation, permits a study of many specific features of atomic interference and an estimate of their possible influence on the accuracy with which the Lamb shift is measured.

If the field intensity in such an interferometer does not exceed ~300 V/cm, then the influence of the $2p_{3/2}$ level on the interference pattern of the two phase-shifted components of the $2p_{1/2}$ state can be taken into account by introducing small corrections.[2] In this case the theoretical interference curve obtained from an examination of the two-level $2s_{1/2}$–$2p_{1/2}$ system (Fig. 1) is a superposition of three curves corresponding to transitions 1, 2, and 3 (the energies of the transitions 3 and 3′ coincide). Thus, the intensity of the $2p$ component of a beam passing through the interferometer can be represented in the form

$$w_{2p}=c_1w_1+c_2w_2+c_3w_3. \qquad (1)$$

It follows therefore that by aligning the theoretical curve with the experimental one, i.e., by selecting, say by least squares, the values of the coefficients c_1, c_2, and c_3 (given the Lamb shift, the hyperfine splitting frequency, and the time of flight in the interferometer field), we can determine by the same token the population of the sublevels of the hyperfine structure of the $2s_{1/2}$ state with projections F_z of the total angular momentum equal to 1, 0, and −1 (for $F=1$) and $F_z=0$ (for $F=0$).

The contributions w_i of each of the three subsystems are determined by the expression

$$w_i=\frac{x^2}{2F_i^2(x)}\left[1-\alpha^2x^2-\frac{\beta^2}{(1+x^2)^2}\right]\exp\left[-\frac{\gamma T}{2}\left(1+\frac{1}{2}\alpha^2x^2\right)\right]\times\left\{\mathrm{ch}\left[\frac{\gamma T}{2}\frac{G_i(x)}{F_i(x)}\right]-\cos[\Delta_0TF_i(x)]\right\}, \qquad (2)$$

$$F_i(x)=\left\{(1+\xi_i)^2+x^2\left[1-\alpha\frac{1+\xi_i}{1+\eta_i}-\frac{1}{4}\alpha^2(1-3\alpha)x^2-\frac{\beta^2}{(1+\xi_i)^2+x^2(1-\alpha)}\right]\right\}^{1/2},$$

$$G_i(x)=1+\xi_i-\frac{1}{2}\frac{1+\alpha(1+\xi_i)}{1+\eta_i}\alpha x^2,$$

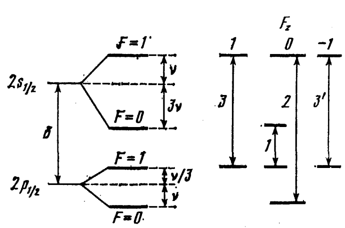

FIG. 1. Scheme of $2s_{1/2}$ and $2p_{1/2}$ levels of the hydrogen atom.

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where $\alpha = \Delta_0/\Delta_1$; $\beta = \gamma/2\Delta_0$; $\Delta_0 = 2\pi\delta$; δ is the Lamb shift; $\Delta_1 = 2\pi\nu_1$; ν_1 is the fine-splitting frequency; $x = 2dE_0/\hbar\Delta_0 = E_0/238.7$; E_0 is the field strength in the interferometer (in V/cm); $\gamma = 1/\tau$ is the decay constant of the $2p$ state; T is the time of flight through the interferometer; the parameters ξ_i and η_i take into account the hyperfine splitting Δ_2: $\xi_i = \Delta_2/\Delta_0 \approx 10^{-1}$, $\eta_i = \Delta_2/\Delta_1 \approx 10^{-2}$.

Equation (2), which determines the yield of the $2p_{1/2}$ atoms for the transitions shown in Fig. 1, was calculated with accuracy not less than 1% (i.e., up to terms 10^{-2} inclusive). Owing to the large phase of the cosine (the phase is $F\Delta_0 T \sim 10\pi$ for $E_0 \sim 300$ V/cm and flight times $T \sim 5\times 10^{-9}$ sec), the energy eigenvalues in the electric field were determined up to terms 10^{-3} inclusive. Equation (2) was obtained assuming abrupt termination of the field at the boundaries.

As already stated, the resultant interference pattern constitutes a superposition of individual curves corresponding to the transitions 1, 2, and 3. The term containing the hyperbolic cosine determines the central line of the curve; periodic changes of the intensity of the flux of the $2p$ atoms is described by the term with the trigonometric cosine.[2,3]

We registered in the experiment the total radiation from the states $2p_{1/2}$ and $2p_{3/2}$. For a comparison with the experimental results it is therefore necessary to add to the expression for $2p_{1/2}$ the probability of the yield of atoms in the state $2p_{1/2}$, which is described by an equation similar to (2). The contribution of the $2p_{3/2}$ also undergoes periodic changes, which, however, have at $E_0 \sim 300$ V/cm a frequency ~10 times higher and an amplitude ~100 times lower, i.e., they constitute minute rapid oscillations relative to the first cosine term. As a result, in the calculation of the contribution of $w_{2p_{1/2}}$ we averaged over the rapid oscillations whose frequency is close to that of the fine splitting.

Determination of the coefficients c_1, c_2, and c_3 at various instants of time, e.g., every 20 minutes, permits monitoring the possible changes of the populations of the sublevels of the hyperfine structure of the $2s_{1/2}$ state (the time necessary to obtain the interference curve from which the coefficients are determined is approximately 4 min at $T \sim 2\times 10^{-9}$ sec).

Starting from general considerations, one might assume that the populations of the sublevels of the $2s_{1/2}$ state with projections F_z equal to 1, 0, and −1 (for $F=1$) and $F_z=0$ (for $F=0$) would be equally probable, i.e., $c_1 = c_2 = 0.25$ and $c_3 = 0.50$. An analysis of the interference curves has shown, however, that the populations of the levels with $F_z = 1, 0$, and −1 change significantly with time, i.e., for some reason the beam becomes polarized. Cases were observed when the population of the sublevel $F=1$, $F_z=0$ (i.e., the coefficient c_2) was first determined (15–20 min after turning on the setup) to be close to zero, while the population of the sublevels $F=1$, $F_z=\pm 1$, characterized by the coefficient c_3, was correspondingly increased. Then, for 1–1.5 hr, the coefficients c_1 and c_2 changed gradually and finally became stabilized and in most cases the population of the sublevel $F=1$, $F_z=0$ turned out to be less than the populations with $F_z=1$ and $F_z=-1$. The population of the level $F=0$, $F_z=0$ changed insignificantly — the coefficient c_1 fluctuated in the range 0.21–0.25. The described process had a random character: the dependence of the coefficients on the instant of observation was not reproducible in repeated startups of the setup.

This effect may be caused by variable field fluctuations produced by the proton component and acting on the metastable $2s_{1/2}$ atoms in that part of the beam where both components (H^+ and H^0) have a common trajectory.

The polarization of the beam of $2s_{1/2}$ atoms was investigated at different times of flight in the interferometer field. Experiments of this type revealed an unexpected phenomenon, namely that at fields of the order of 500–700 V/cm and at long times of flight the central line of the theoretical curve cannot be aligned with the central line of the experimental curve.[3] One of the possible causes of this discrepancy is that the error of the approximate theoretical formula (2) is larger when the interference is observed at a field strength ~600 V/cm and at $T \sim 8\times 10^{-9}$ sec. It should be noted, however, that the same formula leads to sufficiently good agreement with the experimental curve for fields of the order of 600 V/cm, but at shorter flight times.[3]

The observed disparity between the theoretical and experimental data will be studied in detail with a special interferometer intended to operate with strong fields and long T.

To determine the Lamb shift at an error of the order of several ppm, the accuracy given by Eq. (2) is obviously insufficient. A computer calculation can likewise ensure the required accuracy, mainly because of the complicated behavior of the atom in the interferometer and the uncertainty in the field characteristics at the boundaries, i.e., near the entrance and exit openings in the electrodes.

These difficulties can be eliminated by using an interferometer consisting of two independent systems, I and II, separated by a variable gap l (Fig. 2). The systems are parallel-plate capacitors with slits for the passage of the beam.

An atom traveling with velocity v through such a double interferometer is acted upon by nonadiabatic fields in each capacitor, and thus leads to a mixing of the states $2s$ and $2p$. In the gap between the capacitors, i.e., in the region where there is no field, $2s$ and $2p$ are eigenstates, and their evolution can be determined

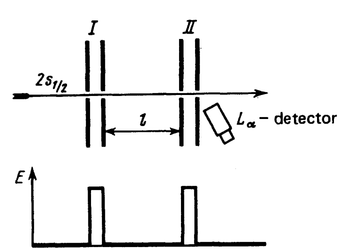

FIG. 2. Scheme of "double" interferometer.

accurately. It follows thus, that an exact expression containing several parameters determined by the action of the fields E_1 and E_2 in the capacitors can be written for the probability $w(l)_{E_1,E_2}$ of the emission of $2p$ atoms following the flight through the double interferometer, as a function of the length l (or of the time of flight $T=l/v$). If the conditions in the capacitors are maintained constant while the length l is varied, these parameters are fixed and need not be calculated (as shown below, the number of these parameters can be decreased to one by suitable reduction of the $w(l)$ curve).

It is important that when the function $w(l)$ is determined in experiment the variable l can be chosen to be not the absolute value of the flight length, but its increment reckoned from a certain arbitrary null point.

Let us illustrate the considered scheme using as an example a three-level system, in which the states $|1\rangle = |2s_{1/2}, F=0|$, $|2\rangle = |2p_{1/2}, F=1, F_z=0|$ and $|3\rangle = |2p_{3/2}, F=1, F_z=0|$ are mixed (to realize this state in experiment it is necessary first to remove from the atomic beam the components of the $2s_{1/2}$ state with $F=1$).

In its own coordinate system, the atom is acted upon by two successive electric-field pulses separated by a time interval $T'=T(1-v^2/c^2)^{1\ 2}$. In this case the expression for the probability of emission of atoms in the $2p$ state takes the form

$$w=w(T')_{E_1,E_2}=|S_{2k}(E_2)a_k(T')_{E_1}|^2+|S_{3k}(E_2)a_k(T')_{E_1}|^2,$$
$$a_1(T')_{E_1}=a_1(E_1), \quad a_2(T')_{E_1}=a_2(E_1)e^{-i\omega T'-\gamma T'/2}, \qquad (3)$$
$$a_3(T')E_1=a_3(E_1)e^{-i\omega T'-\gamma T'/2}.$$

Here $a_k(E_1)$ are the amplitudes of the atomic states $|k\rangle$ following the action of the pulse E_1 from the first capacitor on the initial state of the atom, and $a_k(T')_{E1}$ are the amplitudes by the instant of time when the second field pulse begins to act. The dependence on T' is determined by the atomic-transition frequencies $\omega=2\pi\nu$ and $\omega_1=2\pi\nu_1$, with allowance for the spontaneous damping (Fig. 3). The matrix $S_{ik}(E_2)$ describes the amplitudes of the probabilities of the transitions $|k\rangle \to |i\rangle$ under the influence of the pulse E_2 from the second capacitor.

The reversal $E_2 \to -E_2$ causes the signs of the elements S_{21} and S_{31} to be reversed. Therefore if the sign of the field in the second capacitor is reversed, other conditions being equal, we obtain for the probability

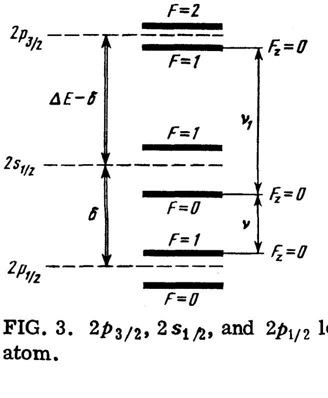

FIG. 3. $2p_{3/2}$, $2s_{1/2}$, and $2p_{1/2}$ level scheme of the hydrogen atom.

$\omega'=w(T')_{E_1,-E_2}$ of the emission of $2p$ atoms expression (3) with the substitutions[2)] $S_{21} \to -S_{21}$ and $S_{31} \to -S_{31}$.

Then

$$F(T')=w-w'=e^{-\gamma T'/2}[A\cos(\omega T'+\alpha)+B\cos(\omega_1 T'+\beta)]. \qquad (4)$$

The real parameters A, B, α, and β are determined by the influence of the fields in the first and second capacitors on the atom. For fields $E_1, E_2 \sim 300$ V/cm, the amplitudes turn out to be $A \sim 1$ and $B \leqslant 10^{-2}$. The second term in (4) yields therefore minute oscillations with high frequency relative to the first cosine. The amplitude A is a general scale factor; as a result, expression (4) contains in fact three independent parameters, namely B/A, α, and β, and their number, as already stated, can be reduced to a single one. To this end the time of flight in the function $F(T')e^{\gamma T'/2}$ is reckoned from a certain value t_0 in such a way that the new variable t can take on both positive and negative values. A similar operation leads to a shift of the phases α and β (e.g., $\alpha \to \alpha' = \alpha + \omega t_0$). Therefore, adding the values of the function $F(T')e^{\gamma T/2}$ for opposite values of t, we obtain an expression that does not contain unknown phases in the arguments of the cosines:

$$\Phi(t)=\cos\omega t+C\cos\omega_1 t, \quad t=|t|>0. \qquad (5)$$

We have left out here a common scale factor, so that only one unknown parameter C remains. In this expression, t is not the absolute time of flight but its change, which is connected in obvious fashion with the change δl of the transit distance: $t=\delta l(1-v^2/c^2)^{1/2}/v$. Equation (5) then takes the form

$$\Phi(\delta l)=\cos\left(\frac{\omega}{v}(1-v^2/c^2)^{1/2}\delta l\right)+C\cos\left(\frac{\omega_1}{v}(1-v^2/c^2)^{1/2}\delta l\right). \qquad (5a)$$

In accordance with the described scheme, the following procedure was used to determine the function $w(l)$:

1) at given values of the field intensity in the systems I and II and at an arbitrary initial distance between them (corresponding to $l_1=0$), the detector of the interferometer was used to determine the number of $L\alpha$ quanta from the decay of the $2p$ state after passage through the systems I and II, i.e., the quantity $I_{2p}(T')_{E_1,E_2}=I(l_1)$ is determined.

2) The direction of the field in the system II is reversed, other conditions remaining unchanged. The measurement yields the quantity $I_{2p}(T')_{E_1,-E_2}=I'(l_1)$;

3) The difference $I-I'$ is determined.

4) The field E_2 is returned to its initial value. The distance l is then changed by an amount Δl, and two measurements are then again made at opposite directions of the field E_2, i.e., the quantities $I(l_2)$ and $I'(l_2)$ are determined.

5) The procedure is repeated for the chosen discrete series of values of the increment Δl. The function $F(l)=I(l)-I'(l)$ obtained in this manner is described by the theoretical formula (4).

6) The data obtained are used to plot the experimental curve (5a). The frequency ν, of interest to us, of the transition $2s_{1/2}(F=0, F_z=0) \to 2p_{1/2}(F=1, F_z=0)$ is determined by fitting the theoretical curve to the experi-

mental points by least squares, i.e., by fitting the values of ω/v and C. Equation (4) can also be used to determine ω/v.

To determine $I_{2p}(l)_{E_1,E_2}$, the measurements were made for $0 \leq l \leq 2$ cm at 41 points, i.e., at $\Delta l = 0.05$ cm. At $l = 2$ cm the intensity of the $L\alpha$-quantum flux decreases by a factor ~20, thereby substantially lowering the accuracy of ω/v. As a result, the counting time was increased in proportion to $\exp(\gamma T/2)$ with increasing l.

In the reduction of the experimental data we determined the difference $I - I'$, i.e., the difference between the coordinates of the curves $I_{2p}(l)_{E_1,E_2}$ and $I_{2p}(l)_{E_1,-E_2}$. At points close to the intersection of these curves, $I - I'$ turns out to be a small difference between two large numbers, i.e., subject to a considerable error. Since discarding these points, i.e., in essence decreasing the number of the measurements, influences adversely the accuracy of the result, the following measurement procedure was used. The quantities I and I' were determined each 10 times at each point, i.e., the quantities $I_i(n)$ and $I_i'(n)$ were determined, with $n = 1, 2, \ldots, 10$. The differences $I_i(1) - I_i'(1, 2, \ldots, 10), \ldots, I_i(10) - I_i'(1, 2, \ldots, 10)$ were then calculated. The 100 values of $I - I'$ obtained in this manner for each i make it possible to obtain the mean value $\langle I - I' \rangle$, determine its error, and exclude the stray points.

In our case, the errors of the differences $\delta(n)_i = I_i(n) - I_i'(n)$ were found to be, generally speaking, correlated. To estimate the influence of this circumstance on the error of the mean value $\langle I_i - I_i' \rangle$ we performed control measurements wherein we obtained 100 values each of I_i and I_i' at the given point i. The mean value determined from this and its error did not differ, within the limits of the statistical scatter, from the analogous values obtained by the method described above.

The values of ω/v and C in (5a) were fitted by a least-squares program. In the case when the velocity remained constant, reduction of the initial data obtained with the aid of the procedure described above yields the values of ω/v and C with accuracy not lower than 5 ppm. If the velocity drifts, however, the interference curve is deformed and does not agree with relation (4), so that its reduction becomes impossible.

It follows from (5a) that to determine the Lamb frequency ν it is necessary to measure by an independent method the velocity of the 2s atoms. The stabilization and measurement of the velocity are found to be the most complicated part of the experiment and are the main limitations of the method.

The velocity of the 2s atoms can in principle be determined by passing them through a quenching field and observing the resultant $L\alpha$ emission at a small angle to the beam trajectory. The Doppler shift of the $L\alpha$ line can then yield the atom velocity v.

This method was tested with the "Pamir" facility (a vacuum spectrometer was used with a l-meter diffraction grating having 1200 lines/mm. The results have shown, however, that this procedure does not make it possible to determine the velocity at the required accuracy, mainly because of the small transmission of the instrument. In addition, the measurement of this method is increased by the contribution of the $L\alpha$ emission produced as a result of cascade transitions of the highly excited H atoms, whose velocity differs from the velocity of the atoms in the 2s state.

We have also tested a method of determining the velocity by counting the number of interference peaks registered per unit time with the field intensity in the two-electrode interferometer varied linearly.[3] This method is quite promising, since it can substantially decrease the measurement time. It calls for further improvement, however, since its existing variant is not accurate enough.

In all the experiments, the velocity was measured by observing the emission of the 2p atoms produced from the 2s atoms under the action of a nonadiabatic field.

When the detector was moved along the beam trajectory, the intensity of the $L\alpha$ radiation, as a function of the distance x measured from an arbitrary reference point, varies like $I = I_0 \exp(-x/l_0)$, where $l_0 = v\tau$ (v and τ are the velocity and lifetime of the 2p atom). Thus, the velocity v (more accurately, the quantity $1/l_0$) can be determined from the slope of the straight line $\ln I = \mathrm{const} - x/l_0$. The corresponding measurement system should include two detectors, one of which (the monitor) is immobile and the other moves along the beam trajectory).

If a collimator that separates an element of length Δx on the beam is placed ahead of the moving detector, the registered $L\alpha$-quantum flux is proportional to

$$I_{x,\Delta x} = \frac{1}{\Delta x}\int_x^{x+\Delta x} I_0 e^{-x/l_0} dx = \frac{l_0}{\Delta x}\left(1 - e^{\Delta x/l_0}\right) I_0 e^{-x/l_0} = kI. \qquad (6)$$

The coefficient $k = (l_0/\Delta x)(1 - e^{\Delta x\ l_0})$ is independent of x and thus remains constant for all the trajectory sections on which the measurements are made.

It may seem at first glance that the quantity $I_{x,\Delta x}$ can be determined by the considered method by measuring the intensity at two beam-trajectory points separated by a distance x_0, as is done when the absorption coefficient is determined. This measurement scheme, however, is applicable only when it is known beforehand that the intensity of the registered emission decreases exponentially with increasing x and, second, the experimental parameters do not change with time. Consequently, the "two-point" method turns out in principle to be unsuitable for the measurement of the beam-atom velocity, since it is assumed *a priori* that the function $I_{2p}(x)$ is exponential, and this assumption can generally speaking be made untenable by some deficiencies in the experiment (e.g., if the velocity oscillates). Thus, the necessary element in the analysis of the experimental data is a check on the hypothesis that the investigated function is linear. In addition, when the intensity is registered at only two points of the trajectory, the error in the measurement of the distance becomes systematic. If the measurements are made at several points, however, errors of this kind becomes random, so that the results can be statistically reduced by least squares.

In accordance with the considered formulation of the problem, the following procedure was developed. The intensity of the $L\alpha$ emission was measured at several points of the beam trajectory, i.e., for a number of discrete values x_i ($i = 1, 2, \ldots k$), with n measurements at each i-th point made at equal (or gradually increasing) time intervals. An empirical relation (regression line) $Y = a - bx$ was next obtained for the set of values $y_{il} = \ln I_{il}$ ($l = 1, 2, \ldots n$), corresponding to the theoretical relation $\Psi = \alpha + \beta x$, where $\Psi = \ln I$, $\alpha = \ln I_0$ and $\beta = -1/l_0$. Calculation of the regression-line parameters, i.e., of the coefficients a and b, makes it possible to find l_0 and to calculate its random error.

The actual regression-analysis scheme depends on whether the variances

$$s^2(y_i) = \frac{1}{n-1} \sum_l (y_{il} - \bar{y}_i)^2$$

of the reproducibility of the coordinates y_i of the investigated function, with mean values

$$\bar{y}_i = \frac{1}{n_i} \sum_l y_{il}.$$

are homogeneous.

If n_i measurements of the intensity are made at the i-th point of the trajectory at equal time intervals, the obtained values (the numbers I_{il} of the counted quanta) will have a Poisson distribution. In this case the intensity variance measured at the i-th point will equal the mean value $\bar{I}_i$.

It must be noted, however, that in our case the distribution of the intensities I_{il} does not, strictly speaking, obey the Poisson law. The feature that distinguishes the statistics of the emission of the $2p$ component is that the number of statistical trials is limited, since the number of H_{2p} atoms in the beam element situated in the field of view of the detector is limited. Thus, one of the conditions for the realization of the Poisson distribution is not satisfied, namely, in each interval t_{il} there can occur, with a definite probability, an arbitrary number of events. Such a deviation from the Poisson law, however, does not play any role whatever at high intensities of the $L\alpha$-quantum flux, when this distribution goes over into a normal one whose variance is likewise equal to the mean value. At the same time, this difference can manifest itself in the study of weak effects, e.g., in the investigation of the behavior of a superposition of states at a large distance from the point of its occurrence.[4]

Since the dependence of the intensity on x is determined by an exponential law, the general variances $\sigma^2(I_i)$, which are equal to the mean values $\bar{I}_i$, will differ in magnitude. It follows therefore that the sampling variances $s^2(I_i)$ are certainly inhomogeneous. The variances $s^2(y_i)$ are likewise inhomogeneous, as is clearly observed when the intensity is measured on a trajectory section longer than ~0.5 cm.

In the case of inhomogeneous variances $s^2(y_i)$, to take into account the fact that the experimental data differ in accuracy, it is possible to introduce a weighting function ω_i and minimize by least squares the weighted sum $\sum_i \omega_i(\bar{y}_i - Y)$, and not $\sum_i n_i(\bar{y}_i - Y)$. It should be noted, however, that such a transformation of the variances is an artificial mathematical device that permits the performance of the regression analysis, but makes "valueless" the measurements at large x. Therefore equalization of the variances is best carried out in principle not by introducing a weighting function, but by increasing the counting time with increasing x.

To determine l_0, we used the regression-analysis variant described in Ref. 4.

In the experiment, the intensity of the flux of the $L\alpha$ quanta was measured on a trajectory section not longer than 1 cm, inasmuch as the effect/background ratio becomes much worse for larger segments. The measurements were performed in 26 points at $\Delta x = 0.02$–0.04 cm; 20 measurements were made at each point. Comparison of different measurement schemes shows that, if the counting rate is the same at all points, acceptable results are obtained if the length of the trajectory segment is 0.5–0.7 cm and $\Delta x = 0.02$ cm. In this case the error Δb in the coefficient $b \approx 3$ can be reduced to $\sim 5 \times 10^{-6}$ and $\Delta b/b = \Delta v/v \sim 2 \cdot 10^{-6}$. A somewhat higher accuracy can be obtained at $0 \leqslant x \leqslant 1$ cm and $\Delta x = 0.04$ by increasing the counting rate in proportion to $\exp(x/l_0)$. The first of the measurement schemes, however, is simpler and consumes less time.

The gas in which the proton charge exchange took place was molecular hydrogen. It might seem that this is not the best choice, since a proton can interact with an H_2 molecule via several channels, as a result of which $2s$ atoms with different velocities appear in the beam. In view of the stringent collimation conditions, however, only a fraction of the beam, with a divergence not more than 10^{-4}, entered the interferometer. Such a selection by trajectories ensured high homogeneity of the neutral-atom velocities.

To determine the velocity from the experimentally obtained decay length $l_0 = v\tau$ we must know τ, i.e., the lifetime of the $2p$ state.

Unfortunately, to our knowledge there are neither theoretical nor experimental published data of required accuracy on the decay constant of the $2p$ states of the hydrogen atom. Our theoretical calculation yields the following value of γ for the $2p_{1/2}$ state:

$$\gamma = 4\pi c \frac{2^8}{3^8} R_H \alpha^3 \left(1 + \alpha^2 \ln \frac{9}{8}\right) = \frac{2^8}{3^8} \frac{me^4}{\hbar^3} \frac{1}{1 + m/M_p} \alpha^3 \left(1 + \alpha^2 \ln \frac{9}{8}\right), \quad (7)$$

where $\alpha = e^2/\hbar c$ is the fine-structure constant and R_H is the Rydberg constant with allowance for the finite mass of the proton.

Substituting here the presently accepted values for α and R_H (see, e.g., Ref. 5), we obtain

$$\gamma = 6.264936 \cdot 10^8 \text{ sec}^{-1}, \quad \tau = 1.596185 \cdot 10^{-9} \text{ sec}.$$

In the calculation of the decay probability we used exact solutions of the Dirac equation in the Coulomb field of a point nucleus, and in the final expression we expanded in powers of α with allowance for the first relativistic corrections ($\sim\alpha^2$)—the spin-orbit interaction and the photon retardation over the size of the atom. In addition, proton-motion correction necessitated by its finite mass was taken into account.

Further refinement of expression (7) calls for taking into account primarily the radiative corrections ($\sim\alpha^3\ln\alpha$), as well as of the interaction with the magnetic moment of the proton (hyperfine structure), recoil in the radiation, the relativistic corrections of next order, the finite size of the nucleus, and others. None of these corrections were taken into account; however, the error in the obtained value of γ is estimated to be of the order of one ppm, which is acceptable for the determination of the Lamb shift with approximately the same accuracy.

3. EXPERIMENT

The first attempt to determine δ (with the aid of a "single" interferometer) was made in 1971, and the value $\delta = 1058.30 \pm 0.04$ MHz (average of 12 measurements) obtained differed substantially from the values known at that time. This was attributed primarily to the inaccuracy of the formula used to calculate δ (Ref. 2). At the same time, the small error (equal to one standard deviation) pointed to the desirability of improving the method. The problem was to obtain highly accurate initial experimental data to be able to find δ with an error of the order of several ppm.

The greatest difficulties arose when attempts were made to stabilize the velocity of the $2s$ atoms. After trying various systems, the simplest setup was chosen, containing two elements that influenced the proton velocity in such a way that the drifts caused by them were of opposite sign. In this case, the change of the velocity became negligible 30–40 min after the setup was turned on, and the velocity could be measured at least approximately. A gradually diminishing increase of the velocity followed and stopped after approximately 2.5 hr. The velocity could then be regarded as constant within acceptable limits for 2–2.5 hr, unless the operation of the setup was interrupted for some random cause. This was followed again by a slow drift towards decreasing v. When the setup was turned on again, the velocity changed somewhat.

A decisive influence on the attainable accuracy of δ is exerted also by the stability of the parameters of the system used to count the $L\alpha$ quanta, which includes a detector and the pertinent electronics. Over several years we tested a great variety of detectors. The best results were obtained with sealed photomultipliers whose windows were made of MgF_2 (the widely used LiF windows turned out to be of little use because of the gradual change of their transparency, due apparently to the hygroscopicity of lithium fluoride).

Starting with 1975, several variants of the previously described "double" interferometer were constructed. Gradual improvement of this interferometer (as well as of the remaining elements of the setup) enabled us to start accurate measurements of the Lamb shift.

Figure 4 shows the diagram of the latest variant of

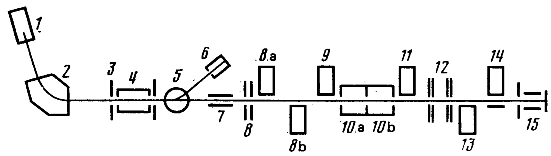

FIG. 4. Diagram of the "Pamir" facility.

the "Pamir" facility, used to measure δ and also for a number of experiments on atomic interference.

Protons of energy ~20 keV from ion source 1 were passed through a velocity analyzer consisting of magnet 2 and slit diaphragm 3 of width 0.02 cm. The dispersion of the magnet, measured in the plane of this diaphragm, is 163 eV/cm; thus, the energy scatter of the protons passing through the diaphragm does not exceed 3.25 eV, i.e., $\sim 1.6\times10^{-4}$ of their energy.

The neutral hydrogen atoms were produced in charge-exchange chamber 4. The mixed beam was passed through a weak magnetic field 5, which deflected the proton component by ~0.2° (the proton current was measured with Faraday cylinder 6). The purpose of the parallel-plate capacitor 7 was to "quench" the $2s$ atoms, a procedure necessary to determine the background. The atom velocity was measured with the aid of system 8 which constituted in essence a two-electrode interferometer with a longitudinal field[2,3] and with a fixed distance between the electrodes (i.e., with a fixed time of flight). The $2p$ atoms produced under the influence of the nonadiabatic field were registered with immobile detector 8a and with mobile detector 8b, which could be moved in a range 0–1.5 cm. In the non-working state, the electrodes of this system 8 were removed from the beam trajectory.

To monitor the flux of the $2s$ atoms, part of the beam was passed through sensor 9, which was a parallel-plate capacitor in which the $2s$-atoms were "quenched," and an $L\alpha$ detector. To remove the component of the $2s_{1/2}$ state with total angular momentum $F=1$, the beam was passed through RF fields 10a and 10b with frequencies 1147 and 1087 MHz. Thus, only one component of the $2s$ state remained in the beam, with total angular momentum $F=0$, and was monitored with the aid of sensor 11, which was similar to sensor 9. The beam passed next through interferometer 12 with detector 13. The sensor 14 was equipped with a quenching field and its purpose was to observe the interference of the $2s$-state components produced under the influence of the nonadiabatic field, as well as to adjust the interferometer. The total beam current was measured with an end-window sensor 15.

In the described experiments, the length of the beam of the neutral H atoms, measured from the charge-exchange chamber 4 to the entrance slit of the interferometer 12, was 300 cm. This length was chosen to give the states with $n = 3-7$, which are quite densely populated and are at the same time relatively long-lived, enough time to radiate, since the cascade transitions from them to the ground state $1s$ produce a background of $L\alpha$ quanta.

The conditions that must be satisfied by a beam of hydrogen atoms in experiments with an atomic interferometer are considered in Ref. 2. Naturally, these conditions become more stringent when it is proposed to measure the Lamb shift with accuracy of the order of several ppm.

The main requirement imposed on the ion source was the production of a thin (with cross section 3–4 mm) weakly diverging proton beam that is stable in time. The last variant of the source developed by us produces a two-component beam whose total current is 2.5–3 mA. This beam contains a relatively weak central component with angle divergence of the order of 10^{-3} at a current 70–100 μA. The diverging part of the beam was confined by a diaphragm, and the "jet" component was directed without any focusing whatever to the entrance slit 3 of the charge-exchange chamber. The ion source and the analyzing magnet were fed from highly stabilized current and voltage sources developed by Grodzinskiĭ and Chashchin.[6] In some cases, an astigmatic lens placed between the magnet 2 and the diaphragm 3 contracted the beam in the vertical direction, (i.e., in the plane of the slit diaphragm 3). The use of such a lens, however, did not increase significantly the beam intensity and at the same time served as an additional source of instability of the flux and of the velocity of the neutral atoms. It should be noted in general that the improvement of the "Pamir" facility followed the path of maximum simplification.

Of particular importance in precision measurements is correct allowance for the $L\alpha$ background registered by the detectors of the setup. We introduce the following notation: M_1 is the "natural" background due to the detector and electronic-circuitry noise, to cosmic radiation, etc.; M_2 is the background produced by emission of highly excited atoms; M_3 is the background due to collisions of neutral atoms with the residual gas; M_4 is the background due to the interaction of the beam atoms with the edges of the exit slit of the interferometer; M_5 is the background due to the entry, into the detector, of $L\alpha$ quanta that leave the interelectrode region through the slit.

The main contribution to the total background registered by the detectors is made by emission of excited H atoms in cascade transitions into the $1s$ state (the M_2 component). It was indicated above that the intensity of this component can be decreased by increasing the distance from the observation point to the charge-exchange chamber. We have calculated the contribution made to the M_2 component by the $(ns-2p)$ and $(nd-2p)$ transitions at $3 \leqslant n \leqslant 25$. It follows from the results that the main component of the $L\alpha$ background is due to the $(4s-2p)$ and $(5s-2p)$ transitions with lifetimes $\tau_{4s-2p} = 3.88 \cdot 10^{-7}$ sec and $\tau_{5s-2p} = 7.76 \cdot 10^{-7}$ sec (the corresponding ranges are $l_{4s} = 78$ cm and $l_{5s} = 155$ cm). The flux of the quanta produced in a beam segment of length $\Delta x = 1$ cm (in the region $l = 300$ cm) reaches 5×10^{-6} of the neutral-atom current.

Control experiments have shown that measurements performed with the quenching field 7 turned on yield identical results when the nonadiabatic field of the interferometer 12 is on or off. It follows therefore that the nonadiabatic field does not exert a noticeable influence on the highly excited atoms.

An estimate of the flux of the $L\alpha$ quanta produced as a result of collisions between $1s$ atoms and particles of the residual gas (in practice, hydrogen leaking in from the charge-exchange chamber) shows that this background component is negligibly small (of the order of 5 quanta/sec at a pressure $\sim 5 \times 10^{-8}$ Torr). Equally small is the contribution of the excited atoms (since the population of the states decreases like $1/n^3$), as well as the contribution of the $2s$ atoms, since their flux is about 2% of the flux of the $1s$ atoms, while the cross section for the quenching of the $2s$ atoms in collisions is approximately 100 times smaller than the cross section of the process $H_{1s} + H_2^0 \rightarrow H_{2p} + R$. The background components M_4 and M_5 are eliminated when the apparatus is adjusted.

The total background was determined in an experiment in which no fields whatever were turned on. In this case comparison of the readings of the end-window sensor 15 and of the detectors 9 and 13 has made it possible to conclude that the registered background is proportional, within the limits of the attainable accuracy, to the neutral-atom current.

From interference-observation experiments we determined the quantity

$$I_{2p} = \frac{N_1 - N_{1b}}{N_2 - N_{2b}}, \tag{8}$$

where N_1 and N_2 are the readings of the detector 13 and of the monitor 11 with the quenching field 7 turned off, while N_{1b} and N_{2b} are the measured values of the background, i.e., the readings of the same detector with the quenching field turned on. In some experiments, to shorten the measurement time, the data registered were the readings of the end-window sensor 15, the readings of the monitor 11, and the readings of the detector 13 with the quenching field turned off, i.e., the quantities N_1 and N_2 were determined. The corresponding values of N_{1b} and N_{2b} were determined from the readings of the previously calibrated end-window sensor 15.

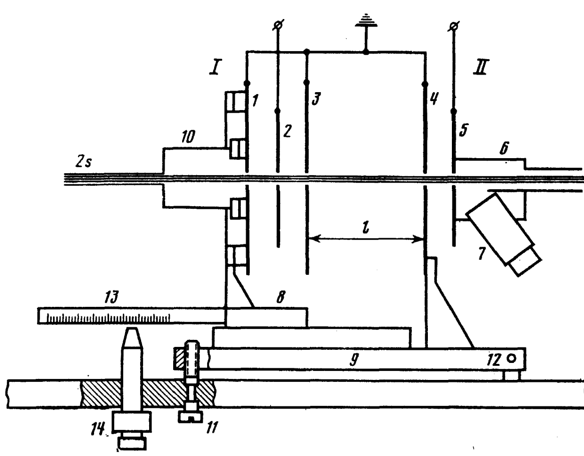

FIG. 5. Construction of the double interferometer.

Figure 5 shows schematically the construction of the described double interferometer.

Since the field over a length l should be strictly equal to zero, the input system I was constructed in the form of a triple electrode whose outer plates 1 and 3 were grounded and the inner plate 2 was under a potential. As a result, the beam passing through such a system is successively subjected to the action of the oppositely directed nonadiabatic fields E_1 and E_2, which are localized between the plate 2 and plates 1 and 3. The output system II consists of two electrodes 4 and 5, the first of which is grounded. The space between electrodes 3 and 4 (i.e., over the length l) is surrounded by an additional removable shield connected to the electrodes. The shield and all the remaining electrodes of the interferometer were gold-coated. The interferometer was placed in a vacuum chamber made of Armco iron, to shield it against magnetic fields.

The requirement that the field be zero over length l might seem to impose definite restrictions on the thicknesses of the electrodes 3 and 4 of systems I and II, since the electric field can "sag" in the interelectrode space near the slit for the passage of the beam.

The expression for the field intensity, in the case of infinitely long electrodes, can be represented in the parametric form[5)]

$$x=\frac{l}{2\pi}\left[r\ln\frac{1+t}{1-t}+2\,\mathrm{arctg}\,\frac{t}{r}\right], \tag{9}$$

$$E=E_0\left(\frac{1-t^2}{1+r^2}\right)^{1/2}, \quad E_0=U/l, \quad r=d/l, \quad -1<t<1.$$

Here l is the distance between the electrodes, d is the width of the slit in the electrode, and U is the applied voltage.

Electrodes 3 and 4 were disks of 10 cm diameter. In the middle of each electrode was a slit for the passage of the beam. In most modifications, the slits measured 0.02×0.4 cm. The electrode thickness (i.e., the length of the slit along the beam trajectory) was taken equal to 0.1 cm, inasmuch as the field attenuates over this length to a value $E/E_0\sim0.0007$. Such a sag does not play any role whatever, the more so since the measurements begin at a distance $l\neq0$ at which the influence of the field vanishes completely.

The electrodes of the output system II are connected to a voltage source that permits reversal of the field direction between the isolated electrode 5 and the grounded electrode 4. A counting chamber (at the same potential), whose internal space is within the field of view of the detector 7, is fastened to electrode 5.

The input system I is mounted on guides 8 fastened to a base 9, on which is also mounted the output system II. The system I can be displaced relative to system II in a range 0–2 cm by a precision mechanism. In front of system I is located a slit collimator 10, mounted on mutually perpendicular guides, so that the collimator can be adjusted relative to the slits in the triple electrode. This electrode (i.e., the system I) is likewise mounted, integral with collimator 10, on crossed guides to permit its adjustment relative to the system II. The interferometer assembled on the base 9 is adjusted relative to the atomic beam with the aid of micrometer screw 11, which rotates the entire setup around an axis 12 perpendicular to the beam direction. In the course of adjustment it is necessary to reach a position such that the beam separated by the collimator 10 does not strike the electrodes of the systems I and II at any value of l from 0 to 2 cm. This condition is verified with the aid of detectors 13 and 14 (Fig. 3), whose readings should remain constant both in the measurements of the $2p$-atom flux and in the measurement of the $L\alpha$ background of the beam.

As already stated, the difference interference curve (obtained as a result of replacing E_2 by $-E_2$ and the operation $I-I'$) is described by formula (4). The parameters A and B in this formula are given by

$$A=F(\mathrm{I})F(\mathrm{II})=F(E_1,T_1)F(E_2,T_2), \quad B=G(\mathrm{I})G(\mathrm{II})=G(E_1,T_1)G(E_2,T_2),$$

where E_1, E_2, T_1, and T_2 are the field intensities and the flight times in the systems I and II.

Since the parameter B, which determines the contribution of the $2p_{1/2}$ level, is small compared with A ($B\sim0.01A$), the yield of the $2p$ atoms after passage of the beam through the interferometer is determined in practice by the parameter A. Consequently, when constructing the interferometer and choosing the optimal conditions for its operation (i.e., to obtain an $F(T')$ curve with a maximum modulation depth), it is necessary to find values of E_1, E_2, T_1, and T_2 such that A is a maximum. These values are chosen from the family of the theoretical plots of $A(E_1)_{E_2,T_1,T_2}$ and $A(E_2)_{E_1,T_1,T_2}$. For fields E_1 and $E_2\sim250-300$ V/cm the distances between the electrodes of systems I and II are found to be 0.17–0.20 cm.

When determining the velocity, the relative error of a single measurement of the flux of the $L\alpha$ quanta at the first point ($i=1$), located at a distance 0.03 cm from the exit electrode 8 of the system (Fig. 4), was of the order of 3×10^{-3}. For an exponential variation of the $L\alpha$ intensity along the beam trajectory, this error corresponds to a length $\Delta x=10$ μm. In order for the error in the determination of the number of pulses, due to the inaccuracy Δx of the setting of the detector 8b (Fig. 4) as well as of the detector 7 of the interferometer (Fig. 5) not to exceed with certainty the statistical error of the reading, the permissible values of Δx and Δl were assumed equal to 1 μm. For calibration and for certain control measurements (e.g., to determine the influence of the temperature), however, it was necessary to measure the detector displacements with even greater precision. We used therefore the optical system employed in linear Abbe comparators, which permits measurements to be made accurate to ~0.1–0.2 μm.

In the interferometer shown in Fig. 5, this system consists of a scale 13 rigidly secured to electrode 1, and a reading microscope 14 fastened to the cover of the vacuum chamber. The microscope is provided with an eyepiece micrometer with a double-spiral reading line. The scale and the microscope were a product of Carl Zeiss in Jena.

The optical systems were calibrated at 20° C; where necessary, corrections for the reading error were introduced in accordance with the test certificates (the absolute value of the corrections did not exceed 0.8 μm). If the measurements were performed at a temperature other than 20°, the results were corrected for the change in the length of the scale. In those cases when the temperature did not remain constant during the measurement time, the scale was shifted relative to the microscope. For this purpose, provision was made for temperature compensation both in the velocity meter and in the interferometer. It was impossible, however, to attain sufficient compensation, owing to the change of the effective length of the scale and of the guides upon displacement of the moving elements. The temperature was not allowed to change by more than 0.8° during the measurement run. Under real conditions, its fluctuations did not exceed 0.2°. It should be noted that direct allowance for the temperature effects was necessary primarily in the control measurements.

We can name the following sources of systematic errors of ω/v:

1) errors connected with the inaccuracy of the optical systems;

2) errors connected with the inaccurate counting of the $L\alpha$ quanta;

3) error due to the influence of random electric and magnetic fields in the space between electrodes 3 and 4;

4) errors due to variation of the fields E_1 and E_2;

5) errors due to deviation from parallelism of the electric RF field in the resonators 10a and 10b (Fig. 4) and the atom-velocity direction;

6) errors due to inaccuracy of Eqs. (4) and (7).

7) Errors of unknown origin.

The experiments were organized in such a way that the corrections were made only for the scale (when displacements were measured) and for the dead time of the $L\alpha$-quantum counting system. The remaining elements of the facility were so arranged that the influence of all other sources of the systematic errors was negligibly small. This was the only correct way, inasmuch as in such a complicated experiment it was impossible to observe, and all the more visualize, all the sources of systematic errors and to estimate numerically their influence on the accuracy of the result.

The method of eliminating the systematic errors consisted of repeatedly modifying the setup and comparing the resultant interference curves. This method, while extremely laborious, has justified itself fully and made it possible to investigate in detail the properties of each element of the system.

According to estimates, the following random fields can be tolerated in the space between electrodes 3 and 4 (i.e., over the length l): an electric field $E_r \leq 10^{-4}$ V/cm and a magnetic field $H_r \leq 10^{-2}$ Oe. To reduce these fields to a minimum value that would have no effect on the shape of the interference curves (at a given measurement accuracy), comparative experiments were performed with electrodes 3 and 4 made of different materials. Poor results were obtained with stainless steel, for which it is known that potential reliefs are easily formed on its surface. The results were indistinguishable in the case of electrodes with galvanic coatings of platinum, gold, and tin. We used therefore gold-coated electrodes. The interferometers were then disassembled and reworked, and the electrodes were degreased, treated with an ethylene-diamine-tetra-acetic acid solution, and washed with distilled water.

To shield against magnetic fields, the interferometer was placed in a vacuum chamber made of Armco iron. Comparative experiments were performed with additional multilayer Permalloy shields placed both outside and inside the chamber. It was found that one Armco-iron chamber is sufficient to protect against magnetic fields (the thicknesses of the cylindrical sidewall, the bottom, and the cover were 0.8, 1.5, and 2.5 cm, respectively).

The fields E_1 and E_2 were produced with the aid of highly stabilized sources ("Takeda-Riken" model TR6120 or domestic model V1-7). The voltage error of these instruments is $\sim 2\times 10^{-5}$ of the set value (in our case, 50–55 V); their stability, however, is higher by approximately one order of magnitude, i.e., $\sim 2\times 10^{-6}$. At this constancy fields E_1 and E_2 have no influence whatever on the accuracy of the result.

An important part of the experiment was the determination of the resolution of the facility, regarded either as an instrument for the measurement of the Lamb shift, or as an instrument for the determination of the velocity of the 2*s* atoms.[2] In the latter case, we measured the difference between the velocities of the H_{1s} and H_{2s} atoms produced in the charge-exchange chamber. The obtained difference between the values of l_0 corresponds to an energy difference of the order of 10 eV. This is evidence, first, of the sufficiently high resolution of the method, and second, that the beam contains one monokinetic group of 2*s* atoms.

In the last experiments, the resolution, both with respect to velocity and with respect to the Lamb shift, was determined by very slightly varying the velocity of the protons passing through the diaphragm 3 (Fig. 4). The sensitivity threshold of the described apparatus, obtained by this method, is lower than the random errors in the measurement of ν.

4. RESULTS

During the last two years we have made more than 200 measurements of the function $I_{2p}(l)$ using three different interferometer modifications. It followed from the results, however, that only in 42 cases could the velocity of the 2*s* atoms be regarded as constant (it was measured at the beginning and at the end of the run[6)]). These 42 cases are clearly grouped in a region for which the difference between the first and second values

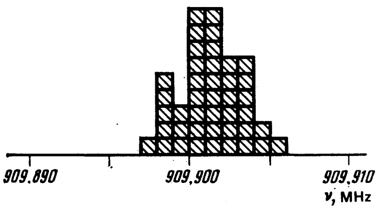

FIG. 6. Histogram of the values of ν.

of v does not exceed ~5 ppm. For the remaining experiments this quantity was distributed in a much larger interval, depending on the drift dimensions.

The values of ν obtained by reducing the data corresponding to the constant-velocity selection criterion constitute a set of 42 values determined accurate to within several ppm. The equalization of the accuracy is due to the fact that all the elements of the considered investigation were so constructed, that the initial experimental and theoretical data, from which ν was calculated, (i.e., ω/v, v, and γ) were determined with practically the same relative error.

Figure 6 shows a histogram of the values of ν. They form a compact group with a mean value

$$\nu=909.9014\pm0.0019 \text{ MHz}$$

(the error is assumed to equal one standard deviation).

The corresponding value of the Lamb shift is

$$\delta=1057.8594\pm0.0019 \text{ MHz}.$$

The value $\nu=909.9003\pm0.0022$ MHz obtained by us earlier[7] differs from the value given here by 0.0011 MHz. This is explained, first, by the increase of the number of measurements (42 in place of 34) and, second, by the fact that all the experimental data were reduced anew, and this also altered somewhat the value of ν. However, the difference between the old and new values does not exceed the error limit.

It should be noted that there is no absolute criterion of the reliability of the result. An exceedingly important part of the investigation is the determination and allowance for the systematic errors, to the elimination of which we pay particular attention. The fact that the values of ν obtained with the aid of interferometers of three different designs turned out to be indistinguishable give grounds for hoping that the influence of errors of this kind has been reduced to a minimum.

At the present time we know the following theoretical and experimental values of δ:

$$\delta_{\text{theor}}=1057.912\pm0.011 \text{ [8]},\ \delta_{\text{theor}}=1057.864\pm0.014 \text{ [9]},$$
$$\delta_{\text{exp}}=1057.862\pm0.020 \text{ [10]},\quad \delta_{\text{exp}}=1057.845\pm0.009 \text{ [11]}.$$

We see that the scatter of the obtained quantities exceeds the range of random errors. It is apparently impossible therefore to speak of the existence of some discrepancy (or, conversely, agreement) between the theoretical and experimental values. The only conclusion that can be drawn in our opinion on the basis of the presented data is that the accuracy of the measurement of δ must be increased.

The general impression gained by us as a result of many years work with atomic interferometers indicates that further increase of the accuracy is perfectly feasible. One of the ways of improving the procedure is to replace optical reading systems by precision converters of linear displacement of moving elements into electric signals expressed in a numerical code. A more radical approach is an experimental variant in which the quantity measured (or specified) is not the velocity v but its increment Δv. In addition, it is necessary to increase substantially the statistics, i.e., increase the number of determinations of ν from which its mean value and standard deviation are calculated.

It is also obvious that a more accurate calculation of the lifetime of the $2p$ atom is necessary, inasmuch as the quantity determined in experiment is in fact $\nu\tau$.

An analysis of the method considered shows that physically it necessitates no corrections to the obtained data. In fact, the observed effect, namely the time evolution of a superposition of $2s$ and $2p$ states, takes place in a space free of any fields, as a result there are, in principle, no factors that distort the function $I_{2p}(l)$. The possible experimental errors are purely instrumental effects whose influence is eliminated by suitable calibration and control experiments.

It must be noted that at the present time the theoretical value of δ can apparently be refined.[5,12,13] As a result, an increase in the accuracy of the measurement of δ is extremely important, since a comparison with the theory would lead to a number of fundamental conclusions concerning, in particular, the size of the proton.

The authors are grateful to N. G. Kokina for developing the $L\alpha$ detectors, I. L. Beĭgman for a discussion of the calculation of the decay constant of the $2p$ state, V. P. Spiridonov for a discussion of problems connected with the statistical reduction of the experimental data, D. N. Lin for taking part in the measurements, and E. V. Grodzinskiĭ and V. V. Chashchin for developing and constructing the current and voltage stabilizers.

[1)] The classical investigation by Triebwasser, Dayhoff, and Lamb[1] was performed in 1953.

[2)] This is due to the simple fact that the electric field mixes states of opposite parity. Therefore, if the atom entering the capacitor is in a state with definite parity (e.g., in the $2s$ state), the probability of emerging in the $2s$ or $2p$ state does not depend on the sign of the field. On the other hand, if the initial wave function is a superposition of states of opposite parity ($2s$ and $2p$), the emergence probabilities for opposite field values differ by an amount proportional to the product of the amplitudes of the atomic states $2s$ and $2p$ in the initial wave function. A measurement method with fields of two signs can be used, in particular, to determine a small admixture of the $2p$ state in the initial state of the atom.

[3)] The "passage of peaks" in the field of view of the detector is an analog of the effect known in optics: when one of the mirrors of a Michelson interferometer is moved at a velocity v in the beam direction, the intensity of the interference pattern becomes a function of the time: $I=I_0(1+\cos 2\pi ft)$, where $f=\nu v/c$ is the modulation frequency.

[4)] It must be noted that in experiments of the type considered there is no simple relation between the flux intensity of the

$L\alpha$ quanta entering the window of the detector and the corresponding reading of the counting system. However, the group of problems connected with optimization of the $L\alpha$-quantum counting, the introduction of the necessary corrections, etc., is far beyond the scope of the present article.

[5)]Equations (9) were obtained by V. M. Galitskiĭ.

[6)]The second measurement of the velocity is, generally speaking, not obligatory, since, as already mentioned, in the course of its drift during the measurement run the curve $\Phi(\delta l)$ becomes deformed and does not agree with the theoretical relation (5a). The reduction of such data is impossible and the measurement run was discarded.

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Translated by J. G. Adashko