# Pseudo-Goldstone bosons of technicolor

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A phenomenological approach to the problem of determining the masses of pseudo-Goldstone technicolor particles is developed. The spinless bound states of the techniquarks are interpreted as elementary Higgs fields with a self-action Lagrangian which satisfies the symmetry present in practically all known technicolor models (the symmetry consists in the possibility of independent unitary rotation of all existing Higgs field doublets). An electroweak interaction destroys the postulated symmetry of the skeleton Higgs Lagrangian; a consequence is the appearance of the same 10-GeV mass for all charged pseudo-Goldstone particles. Possible mechanisms of splitting of the particle masses are considered, particularly their interaction with fermions. The characteristic splitting scale is of the order of (0.1–1) GeV. The possibility that neutral pseudo-Goldstone particles acquire mass at the expense of interaction between the particles and singlet Higgs field is discussed. Such an interaction was introduced earlier in the theory of the "phantom" axion. It is shown that the existence of a phantom axion should involve the existence of a "resurrected" axion, i.e., of a particle with the same Yukawa coupling to quarks and leptons as the standard Weinberg-Wilczek axion, but with an arbitrary mass.

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## 1. INTRODUCTION

The minimum standard model of electric-weak interactions requires, as is well known, the existence of only one doublet of elementary scalar fields. There are, however, no *a priori* grounds for the sector of the Higgs bosons (elementary or composite) to be so impoverished. At the existing variety of fermions (quarks and leptons) the presence of only one (neutral) observable Higgs boson does not seem to be a most natural possibility. The fact that not a single Higgs boson has been experimentally observed so far cannot serve in any way as proof that only one such boson exists.

There are also some theoretical grounds for expecting a certain variety of scalar Higgs particles in theories that operate with both elementary and composite scalar bosons. In theories based on supersymmetry, there naturally arises a large number of elementary spinless particles. If on the other hand, a situation of the "technicolor" type is realized, then the simple parallelism in the multiplication of the number of techniques by flavor, in analogy with ordinary quarks, leads immediately to the appearance of a large number of composite scalar particles.

Questions connected with including $CP$ nonconservation in the theory can require an increase in the number of Higgs bosons. Thus, for example, the Weinberg $CP$ nonconservation model,[1] in which the Higgs bosons are responsible for the $CP$ nonconservation, calls for the existence of at least three Higgs doublets. The problem of the natural conservation of $P$ and the $CP$ parity in strong interaction can be solved by introducing an additional $U(1)$ symmetry, realized on account of the increase in the number of Higgs bosons.[2]

Of particular interest are, of course, light Higgs particles, both neutral and charged. Such particles appear when the interaction has an exact or an approximate global symmetry that is spontaneously broken. A classical example of a Goldstone particle connected with spontaneous breaking of the chiral Peccei-Quinn $U(1)$ symmetry[2] is the axion.[3] In Ref. 4 we have considered all the exact global symmetries of gauge interactions of the standard model and attempted to extend them to the Yukawa interactions (of fermions with scalar particles) and to self-action of Higgs particles. We have reached the conclusion that a second (strictly massless) axion can exist, namely a Goldstone particle connected with spontaneous breaking of the $U(1)$ symmetry of the chiral rotation of leptons.

In the present paper we investigate the possible approximate symmetries connected with the existence of several multiplets of Higgs bosons. Assume that we have $n$ doublets of Higgs fields $\varphi_i$, whose neutral components develop nonzero vacuum mean values: $\varphi_i^{(0)} = v_i/\sqrt{2}$. (It is reasonable to confine ourselves only to doublets, since the relation $M_w = M_z \cos\theta_w$, which follows from the doublet structure of the Higgs fields is well satisfied in experiment.) It can be assumed that in the tree approximation the self-action of the Higgs fields satisfies the $[U(2)]^n$ symmetry relative to global $U(2)$ transformations of each of the Higgs doublets separately.

The basis for the existence of such a rather high symmetry can be the assumption that the Higgs scalars are actually strongly coupled states of certain fundamental fermions—techniquarks.[5] In attempts to construct realistic models of technicolor there appear many possibilities, in each of which there can be realized a certain global symmetry of the effective Lagrangian of the self-action of the bound scalar states. Numerous examples of such symmetries are given in Ref. 6. It is typical, however, that in all the considered cases a $[U(2)]^n$ symmetry is certainly postulated. If the weak doublets $\varphi_i$ are made up of weak doublets of techniquarks $(U_i, D_i)$, $i = 1, \ldots, n$, then such a symmetry corresponds to $[SU(2)_R \times SU(2)_L]^n$ invariance of the technicolor interaction relative to independent rotations $(U_1, D_1)$, $(U_2, D_2)$ etc. The successive $(U_1, D_1)$, $(U_2, D_2)$, ... techniquark doublets themselves can differ in principle from one another, so that the higher symmetries considered in Ref. 6 and connected mainly with the mixing of the techniquarks from different doublets can generally speaking be absent. Using only $[U(2)]^n$ symmetry, we hope to obtain for the masses of certain pseudo-Goldstone bosons predictions not connected with a concrete technicolor scheme.

In contrast to Ref. 6, we do not use for the determination of the masses of the pseudo-Goldstone bosons the rather complicated, in second order, technique of chiral perturbation theory. We consider instead from the very beginning the doublets $\varphi_i$ as elementary Higgs fields; technicolor leads only to a cutoff of the logarithmically diverging integrals at $\Lambda \approx r_{TC}^{-1} \approx 1$ TeV, where $r_{TC}$ is the radius of the technicolor interaction. By virtue of the foregoing, one can even hope that some of the results obtained below are of comparatively general character, not connected directly with the technicolor hypothesis.

The invariance of the Lagrangian $\mathscr{L}_\varphi$ of the self-action of the Higgs fields $\varphi_i \ldots \varphi_n$ relative to their individual $SU(2)$ rotations presupposes a dependence of $\mathscr{L}_\varphi$ only on invariance of the type $\varphi_1^+\varphi_1, \varphi_2^+\varphi_2 \ldots$, but not on, for example, $(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1)$. This restriction on the form of the skeleton Lagrangian is a direct generalization of the Peccei-Quinn symmetry[2] or of symmetry with respect to chiral transformation of leptons.[4] The latter symmetries require that the Lagrangian contain no terms of the type $(\varphi_1^+\varphi_2)^2$. The principal difference between these cases consists, however, in the fact that terms of the type $(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1)$ inevitably arise in the higher orders in the electric-weak interaction. As a result, the pseudo-Goldstone bosons corresponding to breaking of these approximate symmetries acquire a small mass. Our aim is precisely to determine the spectrum of these bosons. Naturally, the masses obtained from radiative corrections for these bosons regarded as elementary diverge in proportion to $\ln \Lambda$. It is obvious that in the technicolor interpretation of the standard model we have $\Lambda \approx r_{TC}^{-1} \approx 1$ TeV.

The paper is organized in the following manner. In the second section we consider charged pseudo-Goldstone bosons and calculate their mass with account taken of the standard electric-weak interaction. In the base of $n$ doublets, all $n-1$ charged bosons are degenerate and have a mass ~10 GeV. (The expression for the mass coincides with that obtained in the first paper of Ref. 6 by the methods of chiral perturbation theory for the case of two doublets made up of different techniquarks.) Next, in the second section we discuss the possible splitting of the masses of the considered charge bosons, particularly because of their Yukawa interaction with the quarks. At a $t$-quark mass of the order of tens of GeV, the splitting can be of the order of hundreds of MeV.

In the third section we proceed to a discussion of neutral bosons. Neutral particles turn out to be Goldstones of spontaneously broken $U(1)$ symmetries of the Peccei-Quinn symmetry type[2] or of the symmetry considered in Ref. 4. These symmetries are not broken by the electric-weak interactions, and therefore the corresponding Goldstone particles remain massless (accurate to allowance for the anomaly). Since the experimental situation with the axion seems unclear at present, we investigate the possibility connected with the existence of an "invisible" ("harmless," "phantom") axion.[7] To this end we introduce an additional scalar field that interacts with the scalar doublet and develops a large vacuum mean value. We call attention to the fact that besides the usually discussed phantom axion, such a construction contains a Higgs particle that interacts with fermions in exactly the same manner as the standard axion interacts with them in the simplest theory. However, the mass of this Higgs boson ("resurrected axion") can be arbitrary. We shall show that within the framework of the concepts of expanded technicolor, a value on the order of hundreds of MeV would not be unnatural for its mass. As for the charged bosons considered in the preceeding section, in this case their masses remain as before ~10 GeV.

At the end of the article we again summarize our expectations concerning the spectrum and the simplest properties of a pseudo-Goldstone technicolor.

## 2. CHARGED BOSONS

We consider first for simplicity the case of two doublets of Higgs bosons, $\varphi_1$ and $\varphi_2$. The Lagrangian of the theory, which has in the Higgs sector the $[U(2)]^2$ symmetry described above, takes in the presence of Yukawa couplings with fermions the form

$$\mathscr{L} = -\frac{1}{4}G_{\mu\nu}{}^a G_{\mu\nu}{}^a - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \sum_{i=1,2}\left| i\partial_\mu\varphi_i + g\frac{\tau^a W_\mu{}^a}{2}\varphi_i + \frac{g'}{2}B_\mu\varphi_i\right|^2$$
$$-\mu_1{}^2\varphi_1{}^+\varphi_1 - \mu_2{}^2\varphi_2{}^+\varphi_2 - \lambda_1(\varphi_1{}^+\varphi_1)^2 - \lambda_2(\varphi_2{}^+\varphi_2)^2 - \lambda_{12}(\varphi_1{}^+\varphi_1)(\varphi_2{}^+\varphi_2). \quad (1)$$

It is easy to note that at $g=0$ the total Lagrangian $\mathscr{L}$ is invariant to independent $U(2)$ rotations of the doublets $\varphi_1$ and $\varphi_2$. This, of course, causes the effective potential $V(\varphi_1, \varphi_2)$, calculated in any order in $g'$ at $g=0$, not to contain terms of the type $(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1)$, $(\varphi_1^+\varphi_2)^2$, or $(\varphi_2^+\varphi_1)^2$, which violate the $[U(2)]^2$ invariance. Less trivial is the fact that at $g \neq 0$ but $g'=0$ the theory also has an exact additional $U(2)$ invariance compared with the gauge-transformation group corresponding to the common unitary rotation of the doublets $\varphi_1$ and $\varphi_2$. To verify this, it is convenient to change to a gauge in which one of the doublets, for example $\varphi_1$, takes the form

$$\varphi_1(x) = \begin{pmatrix} 0 \\ \sigma_1(x)/\sqrt{2} \end{pmatrix}.$$

In this gauge the interaction of $\varphi_1$ and $\varphi_2$ with vector fields takes the form

$$\frac{1}{2}(\partial_\mu\sigma_1)^2 + \frac{g^2}{8}(W_\mu^{(1)2} + W_\mu^{(2)2})\sigma_1{}^2 + \frac{1}{8}(gW_\mu^{(3)} - g'B_\mu)^2\sigma_1{}^2$$
$$+\left| i\partial_\mu\varphi_2 + g\frac{\tau^a W_\mu^{(a)}}{2}\varphi_2 + \frac{g' B_\mu\varphi_2}{2}\right|^2. \quad (2)$$

It is seen therefore that at $g'=0$ expression (2) is invariant to global isotopic rotations of $\varphi_2$ and of $W_\mu^a$ without any transformation whatever of the first doublet. Thus, in the case $g=0$, $g \neq 0$ the effective potential $V(\varphi_1, \varphi_2)$ preserves the property of the exact invariance to independent global rotations of both doublets.

Upon development of the vacuum mean values $\langle\varphi_i^{(0)}\rangle = \langle\sigma_i\rangle/\sqrt{2} = v_i$, spontaneous violation of the exact gauge $SU(2)_L\times U(1)_Y$ invariance to $U(1)_{em}$ takes place as well as violation of the remaining (approximate at $g$, $g'\neq 0$) global $U(2)$ invariance to the group $U(1)$ (inasmuch as without allowance for the weak interactions the strong electric charges of $\varphi_1$ and $\varphi_2$ remain separately conserved). As a result, besides the usual Goldstone bosons, which make $W_\mu^{(\pm)}$ and $Z_\mu$ heavier, we expect the appearance of one neutral Goldstone boson $a^{(0)}$ and of one charged pseudo-Goldstone boson $a^{(\pm)}$, whose mass vanishes at $g=0$ or $g'=0$.

In the single-loop approximation, the contribution of the gauge bosons to the effective potential is[8]

$$\delta V=\frac{3}{64\pi^2}\operatorname{Tr}[M^4(\varphi_i)\ln M^2(\varphi_i)/\Lambda^2], \tag{3}$$

where $M^2(\varphi_i)$ is the mass matrix of the gauge bosons in an external field. The contribution of the scalar-particle loops can be left out, since they obviously do not lead to the noninvariant term $(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1)$.

Diagonalization of $M^2(\varphi_i)$ yields the following eigenvalues:

$$M_{1,2}^2=A,\quad M_{3,4}^2=\frac{A+A'}{2}\pm\left[\frac{(A-A')^2}{4}+\mathbf{B}^2\right]^{1/2},$$
$$A=\frac{g^2}{2}\sum_{i=1,2}(\varphi_i^+\varphi_i),\quad A'=\frac{g'^2}{2}\sum_{i=1,2}(\varphi_i^+\varphi_i),\quad \mathbf{B}=\frac{gg'}{2}\sum_{i=1,2}(\varphi_i^+\boldsymbol{\tau}\varphi_i). \tag{4}$$

Retaining in expression (3) for $\delta V$ only the invariance-violating term, we easily obtain

$$\delta V=-\frac{3}{32\pi^2}g^2g'^2(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1)\ln\frac{\Lambda^2}{M_Z^2}=-k_{12}(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1). \tag{5}$$

Equation (5) is valid with logarithmic accuracy; a more accurate calculation with allowance for the dependence of the logarithm in (5) on $\varphi_1$ and $\varphi_2$ in the final expression for the mass leads only to a slight redefinition of $\Lambda$.

The mass of the scalar particles are determined by the second derivatives of $V_0+\delta V$, calculated at $\langle\varphi_i^{(0)}\rangle = v_i/\sqrt{2}$. It is easy to write out the equations for $v_i$:

$$\mu_1^2+\lambda_1v_1^2+{}^1/_2(\lambda_{12}-k_{12})v_2^2=0,\quad \mu_2^2+\lambda_2v_2^2+{}^1/_2(\lambda_{12}-k_{12})v_1^2=0. \tag{6}$$

Using (6), it is a simple matter to find the mass spectrum of the scalar particles. Besides ordinary Goldstones, which are not observable because of the Higgs mechanism, we obtain, as expected, the states

$$a^{(+)}=\frac{1}{v_{12}}[v_2\varphi_1^{(+)}-v_1\varphi_2^{(+)}],\quad a^{(0)}=\frac{\sqrt{2}}{v_{12}}[v_2\operatorname{Im}\varphi_1^{(0)}-v_1\operatorname{Im}\varphi_2^{(0)}]; \tag{7}$$
$$v_{12}=(v_1^2+v_2^2)^{1/2},$$

with masses

$$m^2(a^{(0)})=0,\quad m^2(a^{(+)})=\frac{3}{64\pi^2}g^2g'^2v_{12}^2\ln\frac{\Lambda^2}{M_Z^2}=\frac{3\alpha}{4\pi}M_Z^2\ln\frac{\Lambda^2}{M_Z^2}. \tag{8}$$

The expression for the mass $a^{(+)}$ coincides with that obtained in the first paper of Ref. 6 by methods of chiral perturbation theory within the framework of the technicolor model.

We consider now the case of several doublets $\varphi_i$, $i=1,\dots,n$. Assume, as before, that the skeleton Lagrangian does not contain terms of the type $(\varphi_i^+\varphi_j)(\varphi_j^+\varphi_i)$ and $(\varphi_i^+\varphi_j)^2$ at $i\neq j$.

The increment to the effective potential is again determined by vector-boson masses of the type (4), where the summation over $i$ is now over all the doublets. As a result we obtain in place of (5)

$$\delta V=-k\sum_{i,j=1}^{n}(\varphi_i^+\varphi_j)(\varphi_j^+\varphi_i),\quad k=\frac{3}{64\pi^2}g^2g'^2\ln\frac{\Lambda^2}{M_Z^2}. \tag{9}$$

Using Eq. (9) for $\delta V$, we might have proceeded as in the case of two doublets, namely write the equations for the vacuum mean values and substitute the solutions of these equations in expression for the second derivatives of the total potential with respect to $\varphi_i$. This procedure, however, besides being very cumbersome, has strictly speaking no special meaning whatever, since in theories of the technicolor type there is no reason whatever to be restricted to polynomials of the fourth degree in $\varphi$ in the skeleton potential. We use therefore the following simple procedure.

It is obvious that even though the masses of charged pseudo-Goldstone bosons result only from the increment (9) to the effective potential, these masses cannot be calculated simply by differentiating (9) with respect to $\varphi_i$ at $\varphi_i=\langle\varphi_i\rangle$. Proceeding in this manner we would, for example, obtain a nonzero mass for ordinary Goldstone bosons that increase the weights of $W^{(\pm)}$ and $V^0$. The reason is that addition of $\delta V$ to the skeleton potential leads to replacement of equations of type (6) by vacuum mean value, and this must be taken into account. It is possible, however, to proceed in the following manner. We replace the increment (9) to the potential by the quantity

$$\delta\tilde{V}=-k\sum_{i,j}(\varphi_i^+\varphi_j)(\varphi_j^+\varphi_i)+k\sum_{i,j}(\varphi_i^+\varphi_i)(\varphi_j^+\varphi_j). \tag{10}$$

The second term in (10) does not violate the considered invariance, so that its addition reduces to a redefinition of the skeleton-Lagrangian parameters. On the other hand, $\delta\tilde{V}$ no longer alters the equations for the vacuum mean values $v_i$. The mass matrix of the sought scalar particles is made up of the derivatives of the skeleton potential with respect to $\varphi_i$ and of $\delta\tilde{V}$. All the second derivatives of the skeleton potential that have a bearing on the matter should be equal to zero at equilibrium, because the masses of all the (pseudo-) Goldstone particles vanish at $\delta\tilde{V}=0$. Since, as already mentioned, $\delta\tilde{V}$ does not alter the equations for the vacuum mean value, we cannot confine ourselves to differentiation of $\delta\tilde{V}$ only.

Direct differentiation of (10) leads to the following mass matrix of the charged scalars:

$$(m^2)_{ik}=\left.\frac{\partial^2\delta\tilde{V}}{\partial\varphi_i^{(+)}\partial\varphi_k^{(-)}}\right|_{\varphi_i^{(0)}=\langle\varphi_i^{(0)}\rangle}=k[v^2\delta_{ik}-v_iv_k],\quad v^2=\sum_{i=1}^{n}v_i^2. \tag{11}$$

Diagonalizing the matrix (11), we obtain one massless Goldstone boson that makes $W^{(+)}$ heavier, and $n-1$ particles that have equal masses (8). It can be shown that the obtained mass degeneracy is not lifted in a more accurate calculation of the single-loop potential—when account is taken of the logarithmic dependence of

(5) on $\varphi_i$.

If the technicolor concept is assumed, then $\Lambda \sim 1$ TeV. In this case $m(a_i^{(+)}) \approx 10$ GeV.

What can cause the mass difference of these charged particles? One of the possible causes is the interaction of Higgs bosons with fermions.

Consider a scheme in which fermions with specified electric charge receive mass from only one Higgs field (one doublet). It is known[9] that this is the most general type of interaction compatible with the requirement of "natural" flavor conservation in exchange with neutral scalar bosons:

$$-\mathscr{L}_Y=\sum_{i,j} h_1^{ij}[\bar d_R^i d_L^j \varphi_1^{(0)*}+\bar d_R^i u_L^j \varphi_1^{(+)*}]+\sum_{i,j} h_2^{ij}[\bar u_R^i u_L^j \varphi_2^{(0)}-\bar u_R^i d_L^j \varphi_2^{(+)}]$$
$$+\sum_{i,j} h_3^{ij}[\bar l_R^i l_L^j \varphi_3^{(0)*}+\bar l_R^i \nu_L^j \varphi_3^{(+)*}]. \qquad (12)$$

The summing is carried out here over the generation, so that $d^i \equiv (d, s, b \ldots)$, $u^i \equiv (u, c, t \ldots)$, and $l^i \equiv (e, \mu, \tau \ldots)$.

If the total number of doublets $n > 3$, then the remaining $n-3$ doublets do not interact with the fermions.

The interaction (12) makes an additional contribution to the effective potential, which takes in the single-loop approximation the form[8]

$$V_f=-\frac{1}{64\pi^2}\mathrm{Tr}(mm^+)^2\ln(mm^+), \qquad (13)$$

where $m$ is the mass matrix of the fermions in an external field. It is easy to show that the matrix $m$ in (13) can be replaced by the matrix $m_{RL}$, which connects in the Lagrangian fields of given helicity:

$$\mathscr{L}\sim\bar\Psi_R m_{RL}\Psi_L+\text{H.c.}$$

(Obviously,

$$m=(m_{RL}+m_{RL}^+)/2+(m_{RL}-m_{RL}^+)\gamma_5/2.)$$

Since interaction with scalar fields (12) does not mix quarks with leptons, the trace in (13) breaks up into a sum of contributions of quarks and leptons. For the corresponding mass matrices we obtain

$$m_{RL}^{(q)}=\begin{pmatrix} h_1\varphi_1^{(0)*}, & h_1\varphi_1^{(+)*} \\ -h_2\varphi_2^{(+)}, & h_2\varphi_2^{(0)}\end{pmatrix}, \quad m_{RL}^{(l)}=\begin{pmatrix} h_3\varphi_3^{(0)*}, & h_3\varphi_3^{(+)*} \\ 0, & 0\end{pmatrix}, \qquad (14)$$

where $h_1 = h_1^{ij}$, $h_2 = h_2^{ij}, \ldots$ are matrices over the indices that number the generations. The contribution of the leptons to $V_f$ leads only to terms of the type $(\varphi_3^+\varphi_3)^2\ln(\varphi_3^+\varphi_3)$, which do not violate the considered invariances. The quark contribution contains terms $\sim(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1)\ln \ldots$. Direct calculation yields

$$\delta V_f=-\frac{3}{8\pi^2}\mathrm{Tr}(h_1^+h_1h_2^+h_2)[(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1)-(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2)]\ln\frac{\Lambda^2}{m_q^2}. \qquad (15)$$

Here, as earlier, we have confined ourselves to logarithmic accuracy. The trace in (3) can be easily calculated by diagonalizing the matrices $h_1^{ij}$ and $h_2^{ij}$. As a result we obtain

$$\mathrm{Tr}(h_1^+h_1h_2^+h_2)=\frac{4}{v_1^2v_2^2}\mathrm{Tr}(M_d^2C^+M_u^2C), \qquad (16)$$

where $M_d$ and $M_u$ are diagonal mass matrices of the lower and upper quarks, and $C$ is a generalized Kabibbo matrix.

It is easy to see from (15) and (16) that the mass matrix $m^2$ [see (11)] of the charged pseudo-Golstone bosons takes the form

$$\widetilde{m}^2=\varkappa\begin{vmatrix} v_2^2, & -v_1v_2, & 0,\ldots \\ -v_2v_1, & v_1^2, & 0,\ldots \\ \cdots & \cdots & \cdots \\ 0, & 0, & 0,\ldots\end{vmatrix}, \qquad (17)$$

$$\varkappa\approx\frac{3}{4\pi^2}\frac{m_t^2m_b^2}{v_1^2v_2^2}\ln\frac{\Lambda^2}{m_t^2}. \qquad (18)$$

We have retained in (18) only the most essential contribution of the heavy $t$- and $b$-quarks, and have replaced $C \approx 1$.

Using (11) and (17), we can find the eigenstates and eigenvalues of the total mass matrix $m^2 + \tilde m^2$. The true Goldstone state

$$g^{(+)}=\frac{1}{v}\sum_{i=1}^{n} v_i\varphi_i^{(+)} \qquad (19)$$

remains, of course, strictly massless (and is absorbed by the $W$ boson). The state of the "charged axion" type

$$a^{(+)}=\frac{1}{v_{12}}[v_2\varphi_1^{(+)}-v_1\varphi_2^{(+)}], \quad v_{12}=[v_1^2+v_2^2]^{1/2} \qquad (20)$$

now has a mass

$$m^2(a^{(+)})=m_0^2+\varkappa(v_1^2+v_2^2), \qquad (21)$$

where $m_0^2$ is the mass of the charged bosons without allowance for the interaction with the fermions, and is defined by relation (8). The remaining $n-2$ charged scalar particles have as before a mass $m_0^2$.

We see that the mass amounts to

$$\Delta m^2=\left(\frac{m_t^2m_b^2}{v_{12}^2}\right)\frac{3}{4\pi^2}\left(x+\frac{1}{x}\right)^2\ln\frac{\Lambda^2}{m_t^2}, \quad x=\frac{v_1}{v_2}. \qquad (22)$$

At $m_t \approx 50$ GeV, $v_{12}^2 \sim v^2 = (G_F\sqrt{2})^{-1}$, $x \sim 1$, $\Lambda \sim 1$ TeV, and $m_0 \approx 10$ GeV we have

$$\Delta m\approx\Delta m^2/2m_0\approx 60\ \text{MeV}. \qquad (23)$$

This figure should be more readily understood as the lower bound of $\Delta m$, since $v_{12}^2 < v^2$, $x + 1/x < 2$, and possibily $\Lambda > 1$ TeV.

What can be said concerning the value of $\Lambda$ in (22) from the point of view of the technicolor concepts? It may seem at first glance that, just as in (8), $\Lambda \sim r_{TC}^{-1}$. This, however, is incorrect, since in models of the technicolor type the Yukawa interactions are the low-energy limits of the interactions connected with exchange of heavy gauge bosons of the "expanded technicolor" ($ETC$) group, which transform ordinary fermions into technifermions. Bearing this in mind, we can easily verify that the real cutoff of the integration momenta in the fermion loop is $q^2 \sim M_E^2$, where $M_E$ is the mass of the $ETC$ boson. From the known relation that connects the masses of ordinary fermions with the techniquark condensate and with $M_E$ (Ref. 5)

$$m_q=\frac{g_{ETC}^2}{M_E^2}\langle\bar Q_{TC}Q_{TC}\rangle, \quad \langle\bar Q_{TC}Q_{TC}\rangle\sim\Lambda_{TC}^3\sim r_{TC}^{-3}, \qquad (24)$$

we easily find that ($g_{ETC} \sim 1$)

$$\ln(\Lambda^2/m_q^2) \approx \ln(\Lambda_{TC}^3/m_q^3). \qquad (25)$$

Thus, the logarithms in (8) and (22) differ in the technicolor model by a factor $\frac{3}{2}$. For the ratio $\Delta m/m_0$ we obtain a final value

$$\frac{\Delta m}{m_0} = \left(\frac{3}{\sin^2 2\theta_W}\right)\left(\frac{m_t^2 m_b^2}{M_Z^4}\right)\left(\frac{v^2}{v_{12}^2}\right)\left(x+\frac{1}{x}\right)^2, \qquad (26)$$

where $\theta_W$ is the Weinberg angle.

Only one of the $n-1$ charged pseudo-Goldstone bosons is split off on account of interaction with the fermions. If $\varphi_i$ are effectively regarded as elementary fields, the only difference in the interaction between different $\varphi_i$, besides the Yukawa couplings, are self-actions of the type $\lambda(\varphi^+\varphi)^2$. In the higher orders of perturbation theory, when account is taken of the interactions $\mu^2\varphi^2$ and $\lambda\varphi^4$ together with the electric-weak or Yukawa interaction (without which the mass of the pseudo-Goldstone remains strictly equal to zero!), the charged scalar particles certainly differ in mass. The value of this difference depends, of course, on the concrete values of the different $\lambda$, but should at any rate be small compared with $m_0 \approx 10$ GeV and perhaps also compared with the difference on account of the fermion contribution (22).

In the technicolor interpretation there appear new interactions that distinguish different composite doublets $\varphi_i$ from one another. For example, the doublets can be bound states of techniquarks, having or not having the usual color interaction. In this case one can expect splittings of the order

$$\Delta m^2 \sim [\alpha_s(\Lambda_{TC})/\pi]\, m_0^2$$

($\Delta m \sim 150$ MeV at $\alpha_s \sim 0.1$).

Summing the content of the present section, we can note that if the postulated approximate $[U(2)]^n$ symmetry does indeed hold, we expect a spectrum of charged scalar Higgs boson with masses in the 10-GeV region, which differ in mass by an amount $\Delta m \sim 0.1-1$ GeV. The characteristic decays of these bosons should be the channels $a^{(+)} \to (c+\bar{b})$, $(c+\bar{s})$ with widths on the order of several keV.[1)]

## 3. NEUTRAL BOSONS

Whereas in the model considered with $[U(2)]^n$ symmetry of the Higgs sector the charged boson acquire mass because of the electric-weak interaction, neutral Goldstone particles remain massless, since they correspond to spontaneous breaking of symmetry that is not broken by the electric-weak interactions. An exception is, of course, the axion,[3] which acquires a small mass on account of the anomaly of the axial current. The question of the possible existence of a massless axion was considered in Ref. 4. If the number of doublets is $n>3$, there may even be several such axions. Although at the present time we do not exclude the possible existence of the massless axion,[4] the possibility of existence of a massive axion that decays into $2\gamma$ is quite doubtful from the experimental point of view. In order not to enter into a contradiction with the existing constraints on its existence, it was proposed[7] to expand the Higgs sector by introducing one more singlet complex Higgs field interacting with doublet scalar fields and developing a large vacuum mean value. In this section we examine how the described situation with charges bosons changes in this case, and discuss neutral Higgs particles. Since we do not adhere consistantly in this paper to the point of view of a certain "effective technicolor," we shall not assume that the vacuum mean value of the singlet field $V = \langle\Phi\rangle\sqrt{2}$ is excessively large (for example, $V \sim 10^{14}$ GeV, just as when the phantom axion is introduced into the grand unification model). On the contrary, we consider the situation in which, say, $V \sim 10 v_{12} \approx 2.5$ TeV.

We take any pair of the Higgs doublets $\varphi_1$ and $\varphi_2$ considered above, for example the one that interacts with quarks in accordance with (12), and assume that a direct interaction takes place between these fields and the complex singlet field $\Phi$ (Ref. 7):

$$\mathcal{L}' = \gamma[(\varphi_1^+\varphi_2)\Phi^2 + (\varphi_2^+\varphi_1)\Phi^{*2}]. \qquad (27)$$

The interaction (27) obviously violates the invariance of the theory with respect to the independent $SU(2)$ rotations of $\varphi_1$ and $\varphi_2$. It is therefore clear that the states of the charge-axion type considered in the preceeding section,

$$a^{(+)} = \frac{1}{v_{12}}[v_2\varphi_1^{(+)} - v_1\varphi_2^{(+)}], \qquad v_{12} = (v_1^2+v_2^2)^{1/2} \qquad (28)$$

acquire a finite mass that does not depend on the electric-weak interactions. The value of this mass is easy to calculate. It turns out to be

$$m^2(a^+) = \frac{\gamma V^2}{2}\left(x+\frac{1}{x}\right), \quad x = \frac{v_1}{v_2} = \frac{\langle\varphi_1^{(0)}\rangle}{\langle\varphi_2^{(0)}\rangle}, \quad V = \langle\Phi\rangle\sqrt{2}. \qquad (29)$$

Although by assumption $V$ is a large quantity, we shall see below that within the framework of the technicolor interpretation one can expect a very small value of $\gamma$, so that $m(a^{(+)})$ (29) may turn out to be only a negligible correction to the electric-weak part of the mass $a^{(+)}$ (8).

As for neutral bosons, which are combinations of $\mathrm{Im}\,\varphi_1^{(0)}$, $\mathrm{Im}\,\varphi_2^{(0)}$, and $\mathrm{Im}\,\Phi$, the situation here is the following. The state

$$g^{(0)} = \sqrt{2}\, v_{12}^{-1}[v_1\,\mathrm{Im}\,\varphi_1^{(0)} + v_2\,\mathrm{Im}\,\varphi_2^{(0)}]$$

is a true massless Goldstone that vanishes from the spectrum of the real state because of the Higgs mechanism. The state

$$a_F^{(0)} = \frac{\sqrt{2}}{N_F}\left[v_2\,\mathrm{Im}\,\varphi_1^{(0)} - v_1\,\mathrm{Im}\,\varphi_2^{(0)} + \frac{V}{2}\left(x+\frac{1}{x}\right)\mathrm{Im}\,\Phi\right], \qquad N_F^2 = v_1^2+v_2^2+\frac{V^2}{4}\left(x+\frac{1}{x}\right)^2, \qquad (30)$$

is a "phantom" axion[7] and its mass differs from zero only because of the anomaly and equals

$$m(a_F^{(0)}) = \frac{2f_\pi m_\pi}{V} N \frac{z^{1/2}}{1+z}\left[1+\frac{4v_{12}^2}{V^2(x+1/x)^2}\right]^{-1/2} \approx 150\left(\frac{v_{12}}{V}\right)\ [\mathrm{keV}]. \qquad (31)$$

Here $N$ is the number of quark doublets, $z = m_u/m_d$ [for numerical estimates we put in (31) $f_\pi = 90$ MeV, $N=3$, $z=0.56$, $v_{12} = (G_F\sqrt{2})^{-1/2} = 250$ GeV, and $V \gg v_{12}$].

Finally, the state orthogonal to the Goldstone $g^{(0)}$ and to $a_F^{(0)}$ is the following combination of fields:

$$a_R^{(0)} = \frac{\sqrt{2}}{N_R}\left[v_2 \operatorname{Im}\varphi_1^{(0)} - v_1 \operatorname{Im}\varphi_2^{(0)} - \frac{2v_1v_2}{V}\operatorname{Im}\Phi\right],\qquad N_R^2 = v_1^2+v_2^2+\frac{4v_1^2v_2^2}{V^2}. \tag{32}$$

The mass of $a_R^{(0)}$ contains a normal part and a small anomalous part (the latter coincides, accurate to $v_{12}/V$, with the mass of the standard axion). Neglecting the small anomalous part of the mass, we have

$$m^2(a_R^{(0)}) = \frac{\gamma V^2}{2}(x+1/x)+2\gamma v_1 v_2. \tag{33}$$

If $V \gg v_1, v_2$, then the phantom axion consists almost entirely of $\operatorname{Im}\Phi$, and therefore becomes difficult to observe (it interacts weakly with quarks and leptons). We see, however, that in this case

$$a_R^{(0)} \approx \sqrt{2}\, v_{12}^{-1}[v_2 \operatorname{Im}\varphi_1^{(0)} - v_1 \operatorname{Im}\varphi_2^{(0)}],$$

i.e., it almost coincides with the standard axion (7). Therefore the coupling of $a_R^{(0)}$ with quarks and leptons is entirely the same as for the standard axion. If we call $a_F^{(0)}$ a phantom axion, $a_R^{(0)}$ can be called a resurrected axion.

The main difference between the resurrected axion and an ordinary one is that $a_R^{(0)}$ has an arbitrary mass. To visualize the possible scale of this mass, we attempt again to turn to the technicolor picture.

Assume, as before, that the fields $\varphi_1$ and $\varphi_2$ consist of the techniquarks $Q_1 = (U_1, D_1)$ and $Q_2 = (U_2, D_2)$ and, in addition, there exist the techniquarks $\psi_1$ and $\psi_2$, which are singlet with respect to the Weinberg-Salam group and whose bound states form two singlet complex scalar fields: $\Phi \sim \bar\Psi_{1R}\Psi_{1L}$ and $\Phi_2 \sim \bar\Psi_{2R}\Psi_{2L}$.

The independent phase transformations of the fields $\varphi_1$ and $\varphi_2$

$$\varphi_{1,2} \to e^{i\alpha_{1,2}}\varphi_{1,2}$$

corresponds to independent hypercharge transformations of the techniquarks[2] $Q_{1,2}$:

$$Q_{1,2} \to e^{i\alpha_{1,2}Y_{1,2}}Q_{1,2}.$$

The transformation with $\alpha_1 = \alpha_2 = \alpha$ is the usual Weinberg–Salam gauge transformation of the $U(1)$ group, and at $\alpha_1 = -\alpha_2 = \alpha$ we have the Peccei-Quinn transformation.[2]

The independent chiral rotation groups of the techniquarks

$$\Psi_{1,2} \to e^{i\beta_{1,2}\gamma_5/2}\Psi_{1,2}$$

generate independent phase transformations of the fields $\Phi_1$ and $\Phi_2$:

$$\Phi_{1,2} \to e^{i\beta_{1,2}}\Phi_{1,2}.$$

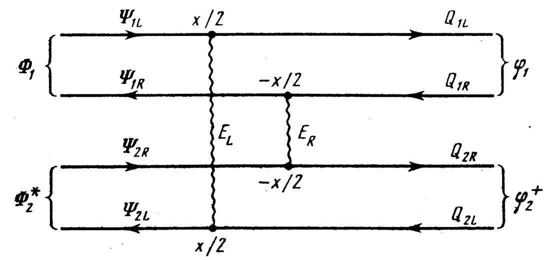
In fact, however, the theory is not invariant to transformations with $\beta_1 = \beta_2$, because of the anomaly connected with the $\Psi_1$ and $\Psi_2$ technicolor interaction, since only the transformations with $\beta_1 = -\beta_2 = \beta$ remain.

In the absence of an interaction that transforms the $Q$ quarks into $\Psi$ quarks, the following independent phase transformations are possible:

$$\varphi_1 \to e^{i\alpha}\varphi_1,\quad \varphi_2 \to e^{-i\alpha}\varphi_2$$

and

$$\Phi_1 \to e^{i\beta}\Phi_1,\quad \Phi_2 \to e^{-i\beta}\Phi_2.$$




The mixing of the $Q$ and $\Psi$ quarks on account of exchange of $E$ bosons of a certain expanded technigroup (see the figure) leads, however, to an effective interaction of the type $\gamma(\Phi_2^*\Phi_1)(\varphi_1^+\varphi_2)$, as a result of which the theory remains invariant only to the one-parameter group

$$\varphi_1 \to e^{i\alpha}\varphi_1,\quad \varphi_2 \to e^{-i\alpha}\varphi_2,$$
$$\Phi_1 \to e^{i\alpha}\Phi_1,\quad \Phi_2 \to e^{-i\alpha}\Phi_2.$$

This situation is perfectly analogous to the interaction (27) considered above. We can therefore attempt to estimate the value of the constant $\gamma$ from the diagram shown in the figure. Within the limits of heavy $E$ bosons, we have

$$\gamma V_1V_2v_1v_2 = \frac{g^2_{ETC,L}\, g^2_{ETC,R}}{M^2_{E_L} M^2_{E_R}} \langle\bar\Psi_1\Psi_1\rangle\langle\bar\Psi_2\Psi_2\rangle\langle\bar Q_1Q_1\rangle\langle\bar Q_2Q_2\rangle r_0^4, \tag{34}$$

where $r_0^4$ stems from integration with respect to $d^4x$, which is cutoff by the smallest of the radii of the composite bosons $\varphi_1, \varphi_2$, and $\Phi_1, \Phi_2$ (after calculating the traces of the matrices $\gamma$, we have put

$$\int d^4x \left\langle \bar\Psi_1\left(\frac{x}{2}\right)\Psi_1\left(-\frac{x}{2}\right)\right\rangle\left\langle \bar\Psi_2\left(-\frac{x}{2}\right)\Psi_2\left(\frac{x}{2}\right)\right\rangle \times \left\langle \bar Q_1\left(-\frac{x}{2}\right)Q_1\left(\frac{x}{2}\right)\right\rangle\left\langle \bar Q_2\left(\frac{x}{2}\right)Q_2\left(-\frac{x}{2}\right)\right\rangle = r_0^4\langle\bar\Psi_1\Psi_1\rangle\langle\bar\Psi_2\Psi_2\rangle\langle\bar Q_1Q_1\rangle\langle\bar Q_2Q_2\rangle,$$

where $\langle\bar\Psi_1\Psi_1\rangle = \langle\Psi_1(0)\Psi_1(0)$, etc.).

For the estimate (34) we assume that $r_0 \sim 1/V (r_0 \ll 1/v$, $V_1 \sim V_2 \sim V$, $v_1 \sim v_2 \sim v)$, and $\langle\bar\Psi\Psi\rangle \sim V^3$, $\langle\bar QQ\rangle \sim v^3$, and $g_{ETC} \sim 1$. Then

$$\gamma \sim \frac{v^4}{M_E^4},\qquad m(a_R^{(0)}) \sim \sqrt{\gamma}\,V \sim \left(\frac{V}{M_E}\right)\left(\frac{v}{M_E}\right)v. \tag{35}$$

Let, for example, $V \sim 10v = 2.5$ TeV and $M_E \sim 10V = 25$ TeV. In this case $m(a_R^{(0)}) \sim 250$ MeV. We see thus that the mass of the resurrected axion could fully amount to hundreds of MeV. At the same relation between $v$ and $V$, the phantom axion has a mass ~15 keV.

We note that from the experimental point of view the masses that are admissible for the resurrected axion are at any rate larger than the difference $m_K - m_\pi$, so as to make the decay $K \to \pi + a_R^{(0)}$ forbidden. It is known (see, e.g., the review[10]), that the failure to observe this decay in experiment may serve as one of the proofs that a standard axion does not exist.

Obviously, interactions similar to (27) can be proposed for any pair of doublets $\varphi_i$ and $\varphi_k$, $i, k = 1, \ldots, n$. In this case, of all the $[U(1)]^n$ symmetries corresponding to independent phase transitions of $\varphi_i$, there can remain, generally speaking, only one—the Peccei-Quinn symmetry.[2] (In the considered example we have $\varphi_1 \to e^{i\alpha}\varphi_1$, $\varphi_2 \to e^{-i\alpha}\varphi_2$, and $\Phi \to e^{i\alpha}\Phi$.) Accordingly, one Goldstone particle should be observed—a phantom axion and $n-2$ neutral Higgs particles with mass of the same order as (35), i.e., say, several hundred MeV.

Of course, the values cited for the masses of the phantom and resurrected axions are absolutely arbitrary. We wished only to illustrate that within the framework of the technicolor ideas all neutral particles can acquire mass and still remain sufficiently light—lighter than the charged pseudo-Goldstone states.

We arrive thus at the following picture of the possible pseudo-Goldstone technicolor particles.

There exists a set of neutral pseudo-scalar particles, which conserve the flavors and parity in the interaction with the fermions. The masses of these particles are known, but are assumed to be less than 10 GeV, the latter being the characteristic mass of charged pseudo-Goldstones. All the considered neutral bosons have a direct interaction of ordinary strength ($\sim m_f/v$) with quarks and leptons, because all these states are orthogonal to the true Goldstone state $\sum v_i \operatorname{Im}\varphi_i^{(0)}$ and therefore, generally speaking, are contained in $\varphi_1$, $\varphi_2$, and $\varphi_3$—the three doublets that interact with fermions.

There is possibly one very light neutral particle (the phantom axion), whose interaction with fermions is strongly suppressed ($\sim m_f/V$, $V \gg v$).

There exists a spectrum of charged Higgs particles with masses grouped in the region of 10 GeV, with a characteristic difference 0.1–1 GeV. The charged Higgs bosons decay predominantly into heavy fermions, and we therefore expect in the main decays into $c+\bar{s}$ or $c+\bar{b}$, with a width of the order of several keV.

The described rich spectrum of pseudo-Goldstone particles is the minimum possible set of light technicolor bosons, since we have used the minimal $[U(2)]^n$ symmetry, which is present in practically all the known technicolor models. In fact, these models usually have broader symmetries, the character of which depends, however, on the details of the considered model.

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[1)] We note that although in the initial Lagrangian only three doublets interact directly with the fermions, all the physical particles with definite mass have a direct connection with quarks and leptons.

[2)] This follows from the identities[5]

$$\begin{pmatrix}\varphi^{(+)}\\ \varphi^{(0)}\end{pmatrix} \sim \begin{pmatrix}-\pi_2 - i\pi_1\\ \sigma + i\pi_3\end{pmatrix}, \quad \pi^a = \frac{i}{2}\bar{Q}\gamma_5\tau^a Q,$$

$$\sigma = \frac{1}{2}\bar{Q}Q, \quad Q = \begin{pmatrix}U\\ D\end{pmatrix},$$

i.e., $$\begin{pmatrix}\varphi^{(+)}\\ \varphi^{(0)}\end{pmatrix} \sim \begin{pmatrix}\bar{D}_R U_L - \bar{D}_L U_R\\ \bar{U}_L U_R + \bar{D}_R D_L\end{pmatrix}.$$

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