Superconductivity stimulation by a microwave field in superconductor—normal metal—superconductor junctions

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It is shown that, in accord with experiment, the critical current of a superconductor—normal metal—superconductor junction can increase substantially in a microwave field. The effect is attributed to the disequilibrium of electrons with energies on the order of the reciprocal time of diffusion in the normal-metal neck, whose contribution to the superconducting current decreases slowly with increasing neck length.

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1. INTRODUCTION

Superconductivity stimulation is usually observed in superconducting bridges and point junctions; a microwave field can raise considerably their critical parameters.[1] This effect is attributed to the nonequilibrium electron-energy distribution, which corresponds to an effective cooling of the junction.[2] A similar phenomen was recently observed in S-N-S (superconductor–normal metal–superconductor) junctions.[3] The critical current of such junctions increases substantially in a microwave field, and in some cases the induced superconductivity appears also under conditions when the critical current of the non-irradiated junction is zero.

This behavior of S-N-S junctions in a microwave field can also be attributed to the disequilibrium of the electron-energy distribution. It is shown in the present paper that the nonequilibrium contribution to the current, which is proportional to the irradiation power, decreases in power-law fashion with increasing length d of the normal-metal neck; at the same time, if there is no irradiation, the critical current decreases exponentially with increasing d (Ref. 4). The critical current in sufficiently long junctions, starting with a certain irradiation power, is determined therefore mainly by the nonequilibrium electrons and it value can increase appreciably. Thus, the critical current in a microwave field can be measured even in the case when its non-irradiated value is exponentially small.

The analysis is based on the microscopic equations for the Green's functions integrated with respect to the energy variable.[5,6] It turns out that the main contribution to the nonequilibrium current in the S-N-S junction is made by electrons having energies ε on the order of the reciprocal time $\hbar D/d^2$ of diffusion along the junction, where D is the diffusion coefficient. Superconductivity is stimulated when this characteristic energy is low compared with the external-field frequency ω, as is the case in experiment. The energy relaxation of the electrons, as usual, is slow enough so that the relaxation time τ is large compared with all the characteristic parameters of the problem.

2. GREEN'S FUNCTIONS OF ELECTRONS IN JUNCTION

If the electron mean free path is small, the current density is given by[6]

$$\mathbf{j}(\tau)=\frac{\pi\sigma}{4e}\mathrm{Sp}\,\{\tau_z\,[G^R\mathbf{d}\,(G^R\hat{f}-\hat{f}G^A)+(G^R\hat{f}-\hat{f}G^A)\,\mathbf{d}G^A]\}. \tag{1}$$

Here $G^{R,A}$ are the retarded and advanced matrix Green's functions made up of the ordinary Green's functions g and the Gor'kov functions F:

$$G^{R,A}(\tau,\tau')=\begin{pmatrix} g_1^{R,A} & F_1^{R,A} \\ -F_2^{R,A} & g_2^{R,A}\end{pmatrix}; \tag{2}$$

τ_z is a Pauli matrix; $\mathbf{d}=\partial/\partial\mathbf{r}-ie\mathbf{A}(\tau)\tau_z$, $\mathbf{A}$ is the vector potential; the matrix $\hat{f}=f+f_1\tau_z$ is made up of two distribution functions; σ is the conductivity of the metal in the normal state. A product of the type $G^R\hat{f}$ means convolution with respect to the internal variable.

We assume that electron-phonon interaction in the normal metal is weak and that the order parameter $\Delta=0$. In this case we have for the Green's functions the equation[6]

$$-D\frac{\partial}{\partial\mathbf{r}}\,[G^{R,A}\mathbf{d}G^{R,A}]+\tau_z\frac{\partial G^{R,A}}{\partial\tau}+\frac{\partial G^{R,A}}{\partial\tau'}\tau_z+ieD\,[\mathbf{A}(\tau)\,\tau_z G^{R,A}\,\mathbf{d}G^{R,A} - G^{R,A}\mathbf{d}G^{R,A}\mathbf{A}(\tau')\,\tau_z]=0, \tag{3}$$

where $D=v_F l/3$ is the diffusion coefficient, and we use a gauge in which the scalar potential is zero; the right-hand side of (3) can be set equal to zero because of the large value of τ_ε. From (3) we find that the relation

$$G^A(\tau,\tau')=-\tau_z G^{R+}(\tau',\tau)\tau_z, \tag{4}$$

which is valid in an equilibrium superconductor, is valid also in a normal metal. In addition, the Green's functions satisfy the normalization condition

$$\int G^{R,A}(\tau,\tau_1)G^{R,A}(\tau_1,\tau')d\tau_1=\delta(\tau-\tau'). \tag{5}$$

To investigate the behavior of the junction in a microwave field we shall need expressions for the unperturbed Green's functions. We obtain them by using a junction model in the form of a bridge of variable thickness, in which the transverse dimensions (thickness and width) of the normal-metal neck are small compared with the thickness of the superconducting "shores," with the length d of the bridge, and with the other characteristic dimensions of the problem. This allows us to assume that all the quantities in the bridge depend only on the longitudinal coordinate x. We change over in Eq. (3) to the Fourier representation in terms of the time difference $\tau-\tau'$ with the corresponding energy ε:

$$-D\frac{\partial}{\partial x}\left(G^R\frac{\partial}{\partial x}G^R\right)+i\varepsilon\,(G^R\tau_z-\tau_z G^R)=0. \tag{6}$$

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On the edges of the junction, at $x=\pm d/2$, the Green's functions should be matched to the equilibrium functions in the shores. When calculating the nonequilibrium current it is necessary, as will be shown later, to find a solution of Eq. (6) for real energies ε that are small compared with the order parameter Δ in the shores. For such energies, the boundary condition for Eq. (6) is of the form

$$G^R\left(\pm\frac{d}{2}\right)=i\begin{pmatrix}0 & -e^{\pm i\varphi/2}\\ e^{\mp i\varphi/2} & 0\end{pmatrix}, \quad (7)$$

where φ is the phase difference of the order-parameter at the shores.

In the solution of the system (6) for the Green's functions g and for the Gor'kov function F it suffices to solve one equation

$$\frac{\partial}{\partial x}\left(g_1^R\frac{\partial}{\partial x}F_1^R-F_1^R\frac{\partial}{\partial x}g_1^R\right)+iyF_1^R=0, \quad (8)$$

since, in addition, the following relations hold:

$$g_1^R=-g_2^R,\quad (g_1^R)^2-F_1^RF_2^R=1, \quad (9)$$

$$\frac{\partial}{\partial x}\left(F_2^R\frac{\partial}{\partial x}F_1^R-F_1^R\frac{\partial}{\partial x}F_2^R\right)=0. \quad (10)$$

We have introduced here the dimensionless energy $y=2\varepsilon d^2/D$, and the coordinate x is measured in units of the bridge length d. To solve Eq. (8) we make the substitution

$$g_1^R=\eta/(\eta^2-1)^{1/2},\quad F_1^R=e^{i\chi}/(\eta^2-1)^{1/2} \quad (11)$$

(the branches of the root are chosen such that the functions g_1^R and F_1^R have the necessary analytic properties) and introduce the complex parameter

$$\psi=\frac{i}{2}\left(F_2^R\frac{\partial}{\partial x}F_1^R-F_1^R\frac{\partial}{\partial x}F_2^R\right), \quad (12)$$

which, as follows from (10), is independent of the coordinates. In this case we obtain for the functions η and χ the equations

$$\left(\frac{\partial\eta}{\partial x}\right)^2-(\eta^2-1)^2\left[\psi^2(\eta_0^2-\eta^2)+2iy\left(\frac{\eta_0}{(\eta_0^2-1)^{1/2}}-\frac{\eta}{(\eta^2-1)^{1/2}}\right)\right]=0, \quad (13)$$

$$\partial\chi/\partial x=\psi(1-\eta^2), \quad (14)$$

where $\eta_0=\eta(0)$ is the value of the function η at the center of the junction, and $\chi(0)=0$ (η is an even function of the coordinate, and χ is odd). The parameters η_0 and ψ are expressed in terms of the differences of the order-parameter phases φ at the shores and the relative energy y from the boundary conditions (7)

$$\frac{\varphi}{2}=\int_0^1 P(u)\,du,\quad \frac{\psi}{2}=\int_0^1 P(u)\frac{\zeta\,du}{\zeta-u^2},\quad \zeta=\eta_0^{-2}; \quad (15)$$

$$P(u)=\left[1-u^2+2iy\frac{\zeta}{\psi^2}\left[(1-\zeta)^{-1/2}-u(u^2-\zeta)^{-1/2}\right]\right]^{-1/2},$$

where the integration is carried out in the complex $u=\eta/\eta_0$ plane along a contour that goes around the branch point $u=\pm\zeta^{1/2}$ clockwise.

The asymptotic solutions of the system (15) in the regions of low and high energies (compared with the characteristic energy $\hbar D/d^2$) are of the form

$$\left.\begin{aligned}\zeta=\eta_0^{-2}&=\frac{\varphi^4\cos^2(\varphi/2)}{4y^2\sin^4(\varphi/4)}\\ \psi&=\varphi,\end{aligned}\right\}\quad |y|\ll 1; \quad (16)$$

$$\left.\begin{aligned}\zeta&=-2c\cos^2(\varphi/2)\,e^{-Z(y)},\\ \psi&=c\sin\varphi Z(y)\,e^{-Z(y)},\end{aligned}\right\}\quad |y|\gg 1; \quad (17)$$

$$Z(y)=(-iy)^{1/2}, \quad (18)$$

$$c=2\exp\left\{2\int_0^\infty\frac{dv}{1+v^2}\left[2^{-1/2}\left(1-\frac{v}{(1+v^2)^{1/2}}\right)^{-1/2}-v\right]\right\}=32(\sqrt{2}-1)/(\sqrt{2}+1)\approx 9.3.$$

The asymptotic equations were written for the case when the phase difference φ is not too close to π, for otherwise there appears one more intermediate region of values of y.

These asymptotic relations allow us to track the energy dependence of the Green's functions of the electrons in the junction. At energies $\varepsilon\gg\hbar D/d^2$, as follows from (11) and (17), the Gor'kov function at the center of the junction, $F^R\sim\zeta^{1/2}$, is exponentially small, and the function g^R is close to unity, i.e., to its value in the normal metal. At $\varepsilon\sim\hbar D/d^2$, however, both functions are of the order of unity. Thus, the presence of superconducting shores influences strongly the electrons in the normal metal only at energies of the order of the reciprocal time of diffusion along the junction.

3. CRITICAL CURRENT OF JUNCTION

The critical current of the junction is obtained from Eq. (1). After calculating the average value of the current, we can substitute in this formula the distribution function $f(\varepsilon)$ averaged over the time and the coordinates (the average nonequilibrium increment to the distribution function is proportional to the large quantity τ_ε, therefore the terms that contain the alternating parts of the distribution function are relatively small). Introducing the parameter $\psi(\varepsilon)$ defined by Eq. (12), we obtain from (1)

$$J_s=\frac{1}{2eR}\int_{-\infty}^{+\infty}\operatorname{Im}\psi(\varepsilon)f(\varepsilon)\,d\varepsilon, \quad (19)$$

where R is the normal resistance of the junction, and when finding the dependence of the parameter ψ on the energy ε one can use the unperturbed Green's functions [the corresponding asymptotic forms of $\psi(\varepsilon)$ are given by Eqs. (16) and (17)].

In the equilibrium case (without an external field) $f(\varepsilon)=\tanh(\varepsilon/2T)$ and the current is determined by the value of ψ in the poles of the tangent $\varepsilon=(2n+1)\pi Ti$, i.e., on the Matsubara energies.[4] As can be seen from (17), in this case the current is exponentially small ($\hbar D/d^2\ll T$). In the nonequilibrium case, however, $f(\varepsilon)$ becomes nonanalytic at low energies $\varepsilon\sim\hbar D/d^2$, at which $|\psi|\sim 1$, and the nonequilibrium current turns out to be relatively large. Thus, to calculate the nonequilibrium current we must find the distribution function in the microwave field at low energies.

The kinetic equation for the distribution function in the case of a small electron mean free path takes the form[6]

$$\operatorname{Sp}\left\{-D\frac{\partial}{\partial x}(G^R\partial G+G\partial G^A)+\tau_\varepsilon\left(\frac{\partial G}{\partial\tau}+\frac{\partial G}{\partial\tau'}\right)+ieD\tau_\varepsilon[A(\tau)-A(\tau')](G^R\partial D+G\partial G^A)\right\}=-4I^{ph}, \quad (20)$$

$$G=G^R f-f G^A. \qquad (21)$$

In a linear alternating field, the Green's functions depend not only on the time differences, but also on their arithmetic mean $t=(\tau+\tau')/2$. In a weak field the dependence on t can be assumed harmonic:

$$G(\tau,\tau')=G_0(\tau-\tau')+G_\omega(\tau-\tau')e^{-i\omega t}+G_{-\omega}(\tau-\tau')e^{i\omega t}, \qquad (22)$$

where ω is the frequency of the external field, G_0 is the unperturbed function (in the Fourier representation

$$G_0=(G_0^R-G_0^A)\,\mathrm{th}(\varepsilon/2T),$$

where G_0^R and G_0^A are determined by Eqs. (2) and (11)), while the amplitudes $G_{\pm\omega}$ are proportional to the vector potential of the field.

Using (22), we go over in (20) to the Fourier representation in terms of the time difference, with the corresponding energy ε, and average it over the time t and over the coordinate x (the latter will be designated by the angle brackets $\langle\ldots\rangle$). The first two terms in the left-hand side vanish in this case; for the collision integral in the right-hand side we use the expression[6]

$$I^{ph}=\frac{1}{4\tau_\varepsilon}\langle \mathrm{Sp}\{\tau_z(G_0^R-G_0^A)\}\rangle\left[f(\varepsilon)-\mathrm{th}\frac{\varepsilon}{2T}\right],$$

when the frequency ω of the external field is low compared with the temperature T (in the collision integral, the energies of importance in the term describing the arrival of the particles are of the order T, whereas a significant disequilibrium sets in at energies $\sim\omega$).

We shall assume at the same time that the frequency ω is high compared with the characteristic energies D/d^2. This simplifies the solution of Eq. (20), for in this case the derivatives $\partial G_0^{R,A}(\varepsilon\pm\omega)/\partial x$ are exponentially small in practically the entire region of the junction [except for edges of size $\sim(D/\omega)^{1/2}$],[1)] and the matrix $G_0^R(\varepsilon\pm\omega)=\tau_z$ [see Eqs. (2), (11), and (17)]. Using this circumstance, we obtain

$$f(\varepsilon)-\mathrm{th}\frac{\varepsilon}{2T}=ieD\tau_\varepsilon\langle 2\,\mathrm{Sp}\{\tau_z\,\mathrm{Re}[G_0^R(\varepsilon)]\}\rangle^{-1}$$

$$\times\mathrm{Sp}\Big\{\tau_z\Big\langle A_\omega\Big[G_0^R(\varepsilon)\frac{\partial}{\partial x}G_{-\omega}\Big(\varepsilon+\frac{\omega}{2}\Big)-G_{-\omega}\Big(\varepsilon-\frac{\omega}{2}\Big)\frac{\partial}{\partial x}G_0^A(\varepsilon)\Big] \qquad (23)$$

$$+A_{-\omega}\Big[G_0^R(\varepsilon)\frac{\partial}{\partial x}G_\omega\Big(\varepsilon-\frac{\omega}{2}\Big)-G_\omega\Big(\varepsilon+\frac{\omega}{2}\Big)\frac{\partial}{\partial x}G_0^A(\varepsilon)\Big]\Big\rangle\Big\},$$

where $A_{\pm\omega}$ is the amplitude of the vector potential of the external field. Thus, there remain in the right-hand side of (23) only the terms that contain the amplitudes of the alternating part of the function G. This means that the main cause of the disequilibrium of the electrons in an S-N-S junction at sufficiently high frequencies is not the direct acceleration in the microwave field (the terms $\sim A_\omega A_{-\omega}G_0G_0^R$), but the "jitter" of the Green's functions and accordingly of the state density of the quasiparticles.

To determine the amplitudes $G_{\pm\omega}$ we solve Eq. (20) by perturbation theory, substituting the unperturbed Green's function and the equilibrium function $f(\varepsilon)=\tanh(\varepsilon/2T)$ in the terms proportional to the vector potential. Since the frequency $\omega\gg D/d^2$, we have at substantial energies

$$G_0(\varepsilon\pm\omega)=\pm\frac{\omega}{T}\tau_z, \qquad (24)$$

and the terms containing $G_0(\varepsilon)$ are relatively small in the parameter ε/ω. The collision integral in the right-hand side of (20) can be neglected because of the large value of τ_ε. As a result we have

$$G_{\pm\omega}\Big(\varepsilon\mp\frac{\omega}{2}\Big)\tau_z=\tau_z G_{\mp\omega}^{+}\Big(\varepsilon\mp\frac{\omega}{2}\Big)=-\frac{eDA_{\pm\omega}}{T}\frac{\partial G_0^R(\varepsilon)}{\partial x}. \qquad (25)$$

It follows from (24) that the amplitudes of the alternating parts of the Green's functions and of the distribution functions at energies $\varepsilon\pm(\omega/2)$ are expressed in terms of the unperturbed Green's functions at energies $\varepsilon\sim\hbar D/d^2$, which already differ substantially from the Green's functions of the normal metal even at the center of the junction [all their components at these energies are of the order of unity, see Eqs. (11), (16), and (17)].

Equation (25) ceases to hold in a region of size $\sim(D/\omega)^{1/2}$ near the edges of the junction, for in this case Eq. (24) for the unperturbed Green's function G_0 can no longer be used and the derivatives $\partial G_0^{R,A}(\varepsilon\pm\omega)/\partial x$ are no longer small. For similar reasons, the contribution from the edges of the function is not accounted for in Eq. (23), too. Therefore, substituting (25) in (23) we obtain the nonequilibrium increment, apart from a numerical coefficient. With the same accuracy we can replace the averaging over the entire region of the junction by calculating the right-hand side of (23) at the center of the junction.

Changing over from the matrix Green's function to the ordinary functions g and F [with the aid of Eq. (2)], we have

$$f(\varepsilon)-\mathrm{th}\frac{\varepsilon}{2T}=\frac{\alpha D\tau_\varepsilon}{d^2T}k_1,\quad \alpha=e^2DA_\omega A_{-\omega}, \qquad (26)$$

$$k_1=2\,\mathrm{Im}\left\{\frac{\partial F_1^R}{\partial x}\frac{\partial F_2^R}{\partial x}-\left(\frac{\partial g_1^R}{\partial x}\right)^2\right\}\Big|_{x=0}=2\,\mathrm{Im}(\zeta^{-1}\psi^2),$$

where the number k_1 is expressed in terms of the parameters ζ and ψ with the aid of (11) and Eqs. (13) and (14) in the asymptotic limit (17).

With the aid of Eq. (26) for the nonequilibrium distribution function, we calculate now the critical current of the junction using Eq. (19). We use here the asymptotic equations (17) for the parameters ζ and ψ. As a result we have for the maximum value of the nonequilibrium current $J_c^{(ne)}$

$$J_c^{(ne)}=\frac{3^{3/2}}{8}c^2\frac{\alpha\tau_\varepsilon D^2}{eRd^4T}k_2, \qquad (27)$$

$$k_2=\int_{-\infty}^{+\infty}\mathrm{Re}\{\exp[-Z(y)]\}\,\mathrm{Im}\{-Z(y)\exp[-Z(y)]\}\,y\,dy,$$

where $Z(y)$ and the constant c are defined by Eqs. (18).

The analytic properties of the integrand in (27) are such that the integral differs from zero ($k_2=6$) and the significant values of y are of the order of unity. Therefore our initial assumption that the main contribution to the nonequilibrium current is made by electrons with energies $\varepsilon\sim\hbar D/d^2$ is justified. Thus, apart from a numerical factor, we obtain from (27)

$$J_c^{(ne)}=3\cdot10^2 e\frac{\tau_\varepsilon D^3}{d^4RT}|A_\omega|^2. \qquad (28)$$

The critical current is found to have a power-law rather than exponential dependence on the junction length d, unlike in the case of an equilibrium current. Recognizing[6] that $\tau_\varepsilon \sim T^{-3}$, we obtain for the temperature dependence of the critical current $J_c^{(ne)} \sim T^{-4}$. The determination of the numerical coefficient in (28) calls for an exact solution of Eqs. (3) and (20), something possible only with the aid of very complicated computer calculations.

Equation (28) is valid at not too high irradiation powers, since the kinetic equation (20) was solved by perturbation theory. All that was important there was that the distribution function $f(\omega)$ differed little from the equilibrium value $(\omega/2T)$. The corresponding restriction on the value of the parameter α which is proportional to the irradiation power can be obtained from Eq. (26):

$$\alpha \ll \hbar\tau_\varepsilon^{-1}(\omega d^2/D)^{1/2}, \tag{29}$$

where we used the fact that at energies $\varepsilon \sim \omega$ the right-hand side of (26) increases by a factor $(\omega d^2/D)^{1/2}$ (the main contribution at such energies are made by the edges of the junction). The deviation of the distribution function from equilibrium $[f(\varepsilon) - \tanh(\varepsilon/2T)]$ for energies $\varepsilon \sim \hbar D/d^2$ at values of α satisfying the inequality (29) can already be large.

4. DISCUSSION OF RESULTS

The results show that an external microwave field can stimulate superconductivity in an S-N-S junction: the critical current of the junction increases substantially in a microwave field. The physical picture of the phenomenon consists in the following. In the equilibrium case (without irradiation) electrons with energies $\mathscr{E} \sim T$, diffusing through a normal-metal layer, lose their coherence over distances $\xi \sim (\hbar D/T)^{1/2}$. This leads to an exponential damping of the superconducting current at normal-layer thicknesses $d \gg \xi$. In a microwave field the electron distribution becomes substantially nonequilibrium at energies $\varepsilon \sim \hbar D/d^2$. Electrons with such energies diffuse freely through the junction (d^2/D is the diffusion time) and make the principal contribution to the nonequilibrium current. As a result, the critical current decreases with increasing d only in power-law fashion [Eq. (28)].

The calculations were performed under the condition that the external-field frequency ω satisfies the relation

$$D/d^2 \ll \omega \ll \hbar^{-1}T. \tag{30}$$

The left-hand inequality allows us to neglect the spatial gradients, as well as to expand in powers of the parameter ε/ω (and not of the field frequency, as is customary in the treatment of nonequilibrium phenomena). The right-hand inequality must be satisfied to use the relaxation-time approximation in the collision integral. The value of the order parameter Δ in superconductors does not enter in the final result, but it is assumed that it is large compared with the characteristic energies $\varepsilon \sim \hbar D/d^2$. The restriction on the radiation power is given by condition (29).

Comparison with experiment[3] shows that the results give the correct picture of the phenomenon. The effect is observed when the field frequency $\omega > (D/d^2)$ and the critical current have a power-law dependence on the thickness d of the normal-metal layer. A quantitative comparison, however, is difficult, since the measured quantity was the maximum critical current in a microwave field, whereas Eq. (28) is valid only at sufficiently low irradiation powers.

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[1)] The quantity $\eta_0(2\omega d^2/D)$, as seen from (17) is exponentially large, therefore $g_1^R \approx 1$, whereas $F_1^R \sim \eta_0^{-1}$ is exponentially small; the smallness of the derivatives follows from Eqs. (13) and (14).

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