

# Photomagnetization of the Landau-Lifshitz domain structure by circularly polarized light, and the velocity of domain-wall motion under the influence of light

G. M. Genkin and I. D. Tokman

*Institute of Applied Physics, Academy of Sciences, USSR*  
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It is shown that in absorption of circularly polarized light by multidomain magnets possessing circular dichroism, photomagnetization occurs. The effect is caused by the fact that the values of the exchange-interaction parameter in different domains become different. In magnetic semiconductors, this is due to the different concentrations of the photocarriers in different domains. For the Landau-Lifshitz structure, the equilibrium values of its period and of the domain widths are found, and also the relative value of the photomagnetization. The velocity of an interdomain wall under the action of light is considered. According to estimates for  $\text{CdCr}_2\text{Se}_4$ , the photomagnetization at light intensities of the order of  $10 \text{ W/cm}^2$  reaches values of  $\Delta M/M_0$  of the order of  $10^{-2}$ .

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1. The interaction of magnetic materials with optical radiation leads to a change of their magnetic properties. Thus under the influence of light, the magnetic permeability and the hysteresis loop change (experiments on chromium spinels  $\text{CdCr}_2\text{Se}_4$ , Refs. 1 and 2; the photoferromagnetic effect). There are experimental results<sup>3</sup> on the effect of circularly polarized light on the magnetic semiconductor  $\text{EuS}$ .

The present paper carries out a theoretical treatment of the magnetization that occurs under the influence of circularly polarized optical radiation in a ferromagnet with a domain structure in zero magnetic field. The effect occurs because the values of the exchange-interaction parameter  $J$  in different domains may become different. Thus in magnetic crystals there is a circular dichroism, characterized by different coefficients of absorption of light,  $K_+$  and  $K_-$ , for different circular polarizations. Such circular dichroism exists in magnetic semiconductors [chromium spinel,  $\text{CdCr}_2\text{Se}_4$  (Ref. 4), the europium chalcogenides  $\text{EuO}$  and  $\text{EuS}$  (Ref. 5)] and in ferroelectrics (the garnet  $\text{YIG}$ , Ref. 6). On illumination of the magnet by light with a definite circular polarization, the coefficient of absorption of light in neighboring domains will be different; here, for simplicity, we choose the direction of propagation of the light along the direction of magnetization of the main domains of the Landau-Lifshitz structure. As a result, there are different numbers of photoelectrons in neighboring domains. And since the photoelectrons take part in the indirect exchange in magnetic semiconductors (see, for example, Ref. 7), the effective exchange-interaction constants are different.<sup>1)</sup> In magnetodielectrics a similar situation occurs; here, as a result of the circular dichroism, there are different numbers of ions (in  $\text{YIG}$  garnets,  $\text{Fe}^{3+}$  ions<sup>6</sup>) in the excited state in neighboring domains, in which the exchange-interaction parameter will in general be different. As a result, changes occur in the equilibrium domain structure, and these lead to photomagnetization of a multidomain crystal, in the absence of an external magnetic field, by light with a definite circular polarization. Earlier,<sup>8</sup> we studied the photomagnetization of a stripe domain

structure for small light intensities. The present paper considers photomagnetization of the Landau-Lifshitz domain structure. This structure is realized in cubic crystals with a positive anisotropy constant<sup>9</sup> and is energetically advantageous in a plate of finite dimensions.<sup>10</sup> The present paper treats photomagnetization by light sufficiently intense so that the resulting photomagnetization is in general not small in comparison with the saturation magnetization  $M_0$ . The paper also considers the velocity of the domain-wall motion that occurs during photomagnetization of a multidomain ferromagnet.

We remark that in all the effects considered in this paper, the fact is taken into account that the energy of uniform exchange is different in different domains. Here, although the relative changes of the effective exchange constant are small (thus for magnetic semiconductors they are proportional to  $\Delta n/N$ , where  $\Delta n$  is the change of the photocarrier concentration and  $N$  is the total number of states in the Brillouin zone), nevertheless, as was shown in our paper Ref. 8, the relative photomagnetization contains, along with this small parameter  $\Delta n/N$ , also a large parameter: the ratio of the energy of exchange interaction to the relativistic magnetostatic energy.

2. The equilibrium domain structure is found from the condition for a minimum of the total energy  $W$ , which is made up of the magnetostatic energy  $W_{ms}$ , the energy of the interdomain walls  $W_w$ , the energy of the closure domains  $W_{cd}$ , and the exchange energy  $W_{ex}$ . As was shown in Ref. 8, the widths of the domains under illumination by circularly polarized light must be different. As a result, the Landau-Lifshitz structure is reconstructed in such a way (see Fig. 1) that deviations occur from the 45-degree geometry that prevails in the absence of photomagnetization.

The angles  $\beta$ ,  $\gamma$ , and  $\theta$  are connected by the relation

$$2\beta = \pi - 2\gamma = \theta, \quad (1)$$

which follows from the condition of continuity of the normal component of magnetization  $M_n$  at the boundaries

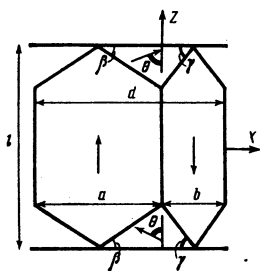


FIG. 1.

between the main and closure domains. We note that the same kind of deviation occurs in the Landau-Lifshitz structure on application of an external magnetic field perpendicular to the surface of the plate.<sup>11</sup> The magnetization  $M_n$  has a discontinuity on the lower and upper faces of the plate: this corresponds to a uniform distribution of "magnetic charges" on the upper and lower faces of the plate, of thickness  $l$ , with densities  $\pm M_0 \cos \theta$ . The magnetostatic energy of such a uniformly magnetized plate, per unit area of the surface, is, from Refs. 8 and 12.

$$W_{ms} = 2\pi l M_0^2 \cos^2 \theta. \quad (2)$$

The effective exchange-interaction parameter  $J$  under illumination by circularly polarized light depends, because of circular dichroism, on the angle  $\psi$  between the magnetization vector  $M_0$  and the direction of propagation of the circularly polarized light, according to the law

$$J = J_0 + J_1 \cos \psi. \quad (3)$$

Thus in the main domains  $J_+ = J_0 + J_1$ ,  $J_- = J_0 - J_1$ , and in the closure domains  $J_c = J_0 + J_1 \cos \theta$ . The exchange energy per unit area of the plate surface is

$$W_{exc} = -\frac{2S^2}{da_0^3} (J_+ C_+ + J_- C_- + J_c C_c), \quad (4)$$

where  $C_+$ ,  $C_+$ , and  $C_-$  are the volumes of the main and closure domains per unit distance along the  $Y$  axis,  $S$  is the value of the spin,  $a_0$  is the lattice constant (the lattice is assumed to be simple cubic), and  $d$  is the period of the structure. Taking account of the relation (1), we get

$$W_{exc} = -\frac{2S^2 J_1}{a_0^3} l \cos \theta. \quad (5)$$

The anisotropy-energy density in the closure domains is

$$\rho_{an} = K \cos^3 \theta \sin^2 \theta. \quad (6)$$

Here  $K$  is the first cubic-anisotropy constant.

The magnetostriction energy of the closure domains can be estimated by assuming that the strain tensor is constant over the whole volume of the body.<sup>9</sup> Following Ref. 9, we get for the density of magnetostrictive energy in the closure domains

$$\rho_{mstr} = \frac{b_1^2 \sin^2 \theta}{2C_2}; \quad (7)$$

here  $b_1$  is the magnetoelastic coupling constant, and  $C_2$  is the elastic modulus. From (6) and (7), the energy of the closure domains per unit area of the plate surface is

$$W_{cd} = -\frac{d}{4} \left( K \cos^3 \theta + \frac{b_1^2}{2C_2} \right) \sin^2 \theta. \quad (8)$$

We write the energy of the interdomain walls, per unit area of the plate surface, in the form

$$W_w = W_{w,180} + W_{w,c}, \quad (9)$$

where  $W_{w,180}$  is the energy of the 180-degree walls and  $W_{w,c}$  is the energy of the boundaries of closure domains:

$$W_{w,180} = \frac{2\Delta_{180}}{d} \left( l - \frac{d}{2} \sin \theta \right), \quad (10)$$

$$W_{w,c} = 2\Delta_a \cos \frac{\theta}{2} + 2\Delta_b \sin \frac{\theta}{2}. \quad (11)$$

Here  $\Delta_{180}$ ,  $\Delta_a$ , and  $\Delta_b$  are the surface energy densities of 180-degree boundaries and of boundaries of the main domains  $a$  and  $b$  with closure domains.<sup>2)</sup>

We consider the reconstruction of the domain structure. The equilibrium values of the angle  $\theta_0$  and of the period of the structure  $d_0$  are determined by the conditions for a minimum of the energy:

$$\frac{\partial W}{\partial \theta} = -4\pi l M_0^2 \cos \theta_0 \sin \theta_0 + \frac{2S^2 J_1}{a_0^3} l \sin \theta_0 + \frac{3}{4} d_0 \sin^2 \theta_0 \cos \theta_0 \left( K \cos^2 \theta_0 + \frac{b_1^2}{2C_2} \right) - \frac{d_0}{2} K \sin^4 \theta_0 \cos \theta_0 = 0, \quad (12)$$

$$\frac{\partial W}{\partial d} = \frac{1}{4} \left( K \cos^3 \theta_0 + \frac{b_1^2}{2C_2} \right) \sin^3 \theta_0 - \frac{2\Delta_{180} l}{d_0^2} = 0. \quad (13)$$

The system of equations (12)–(13) has in general two roots,  $\theta_{01}$  and  $\theta_{02}$ , with  $\theta_{01} < \theta_{02}$ . It can be shown that the larger root  $\theta_{02}$  corresponds to a minimum of  $W$ , the lower  $\theta_{01}$  to a maximum of  $W$ . With increase of the pumping (the light intensity), the values of  $\theta_{01}$  and of  $\theta_{02}$  approach each other; and on attainment of a critical value of the pumping and correspondingly of  $J_{1cr}$ , there is a single root  $\theta_{0cr} = \theta_{01} = \theta_{02}$ . But this solution does not correspond to a stable state of the system, since the state with  $\theta_0 = 0$  (the whole specimen uniformly magnetized) has a lower energy than the state with  $\theta_{0cr}$ . Here

$$\theta_{0cr} \approx \arcsin \frac{(8K\Delta_{180}l)^{1/2}}{(4\pi l M_0^2)^{1/2}}, \quad (14)$$

$$J_{1cr} \approx \frac{2\pi a_0^3 M_0^2}{S^2} \left[ 1 - \frac{(8K\Delta_{180}l)^{1/2}}{(4\pi l M_0^2)^{1/2}} \right]^2. \quad (15)$$

But on the other hand, it is known that a structure with branched domains gives an energy that is smaller than the energy of a uniformly magnetized plate.<sup>10</sup> In our case also it turns out that branching begins before  $J_{1cr}$  is attained; namely, on attainment of  $J_1' \sim 0.5 J_{1cr}$ .<sup>3)</sup>

For  $J_1 < J_1'$ ,

$$\cos \theta_0 \approx J_1 S^2 / 2\pi a_0^3 M_0^2. \quad (16)$$

We note that this result corresponds to the fact that in the range  $J_1 < J_1'$ , the contribution from the last two terms in formula (12) is small in comparison with the first two. And it is only in the range  $\theta_0 \ll 1$  (near  $\theta_{0cr}$ ), when the period  $d_0$  of the equilibrium structure becomes sufficiently large, that they must be taken into account and the expression for  $\cos \theta_0$  becomes different. But our whole treatment is limited, as has already been indicated, to angles  $\theta_0$  larger than  $\theta_{0cr}$ .

The magnetic moment of unit volume of the ferromagnet caused by photomagnetization,  $\Delta M$ , is  $M_0 \cos \theta_0$ ; thus the relative magnetization of unit volume is

$$\Delta M / M_0 \approx J_1 S^2 / 2\pi a_0^3 M_0^2, \quad J_1 < J_1'. \quad (17)$$

We note that photomagnetization occurs both in the main

domains and in the closure domains; by virtue of condition (1), which results from the continuity of  $M_n$  on the boundary of a main and a closure domain, it is the same in both types of domains. That is, all domains, both main and closure, thus "perform." Hence it follows that photomagnetization will occur, in general, for an arbitrary direction of propagation of the light with respect to the axes of the domain structure, although the direct analysis in the present paper relates to the case when  $\mathbf{k}$  is parallel to the direction of the magnetization in the main domains of the Landau-Lifshitz structure.

While the relative magnetization over the whole permissible interval of variation of the effective exchange parameter,  $0 < J_j < J'_j$ , and correspondingly of the intensity of illumination, is determined to a good approximation by the single expression (17), the period of the structure and the domain widths  $a$  and  $b$  depend differently on  $J_1$  for the cases of small and of large reconstructions. In the case of small reconstructions ( $\theta = \pi/2 + \varphi$ ,  $\varphi \ll 1$ ) of the domain structure, we have the following expression for the period:

$$d_0 \approx d'_0 \left(1 - \frac{KC_2}{b^2} B_1\right). \quad (18)$$

The domain widths are

$$a \approx d_0(1+B)/2, \quad (19)$$

$$b \approx d_0(1-B)/2, \quad (20)$$

where  $d'_0 = 4(\Delta_{180} l C_2 / b^2)^{1/2}$  is the equilibrium period of the structure in the absence of pumping,<sup>9</sup> and where we have introduced the notation

$$B = S^2 J_1 / 2\pi a_0^3 M_0^2.$$

In the case of large reconstructions, when<sup>4)</sup>

$$K \cos^2 \theta \gg b^2 / 2C_2,$$

the period of the structure is determined by the following expression:

$$d_0 \approx \frac{(8\Delta_{180} l)^{1/2}}{K^{1/2} B (1-B^2)^{1/4}}. \quad (21)$$

The domain widths are

$$a \approx \frac{(8\Delta_{180} l)^{1/2} (1+B)}{2K^{1/2} B (1-B^2)^{3/4}}, \quad (22)$$

$$b \approx \frac{(8\Delta_{180} l)^{1/2} (1-B)}{2K^{1/2} B (1-B^2)^{3/4}}. \quad (23)$$

For magnetic semiconductors, where the carriers participate in the indirect exchange, we have, following Ref. 7, for the case of relatively small concentrations of the carriers ( $n/N \ll 1$ )

$$\frac{\Delta J}{J} \approx \frac{\Delta T_c}{T_c} \approx \frac{(\pi)^{3/2} (AS)^{1/2} \hbar n}{6^{1/2} 2^{1/2} a_0 m^{1/2} k_B T_c N}, \quad (24)$$

where  $N$  is the total number of states in the Brillouin zone,  $A$  is the  $s$ - $d$  exchange constant, and  $m$  is the electron mass. Using formula (24) and the expression for the difference  $\Delta n$  of the stationary concentrations of the photocarriers in the different domains,

$$\Delta n = P \Delta K \tau / \hbar \omega, \quad (25)$$

we get as a result of the following formula:

$$\frac{\Delta M}{M_0} \approx 0.1 \frac{P \Delta K \tau (AS)^{1/2}}{\omega a_0 m^{1/2} M_0^2}, \quad (26)$$

where  $P$  is the flow of light power of frequency  $\omega$ ,  $\tau$  is the lifetime of a photocarrier, and  $\Delta K = K_+ - K_-$ . For the magnetic semiconductor  $\text{CdCr}_2\text{Se}_4$ , where according to Ref. 4  $\Delta K \sim 10^2 \text{ cm}^{-1}$ , for light power flow  $P \sim 10 \text{ W/cm}^2$  of frequency  $\omega \sim 10^{15} \text{ s}^{-1}$ , setting  $\tau \sim 10^{-6} \text{ s}$ , we have  $\Delta M/M_0 \sim 10^{-2}$ .

3. We turn to consideration of the velocity of a domain wall under the action of circularly polarized light. We shall for simplicity consider the photomagnetization of a stripe domain structure. Let there be an external magnetic field  $H$ , directed along the  $Z$  axis, and circularly polarized light also directed along the  $Z$  axis. If  $\psi$  is the angle between the direction of the magnetization and the direction of propagation of the circularly polarized light, then according to (3),  $J = J_0 + J_1 \cos \psi$ , and  $\psi = 0$  for  $x = -\infty$ ,  $\psi = \pi$  for  $x = +\infty$ ; the  $X$  axis is chosen perpendicular to the plane of the interdomain wall, and the value  $x = 0$  corresponds to the middle of the wall. The volume density of exchange energy  $\omega_{\text{ex}}$  can be obtained in the usual way from the expression for the exchange energy of two spins,  $-2JS_1S_2$ , going over to the approximation of a continuous medium, and taking account of the variation of  $J$  with the coordinates. As a result we have

$$\omega_{\text{ex}} = -\frac{2J_0 M_0^2 a_0^3}{g^2 \mu_B^2} - \frac{2J_1 M_0^2 a_0^3 M_z}{g^2 \mu_B^2 M_0} + \left( \frac{J_0 a_0^3}{g^2 \mu_B^2} + \frac{J_1 a_0^3 M_z}{g^2 \mu_B^2 M_0} \right) \left( \frac{\partial M_0}{\partial x} \right)^2, \quad (27)$$

where  $g$  is the Landé factor.

It is necessary to point out the difference of the expression (27) from the usual expression for the exchange-energy density. Namely, terms proportional to  $J_1$  have appeared in the uniform and in the nonuniform parts of the expression  $\omega_{\text{ex}}$ . Usually the uniform part of the exchange has not been taken into account at all, because it corresponds to the first term in formula (27), constant over the volume of the body; now, however, the uniform exchange corresponds not only to the first term, whose contribution to all the effects is still zero, but also to the second term in the expression (27). It is the contribution of the second term that was also taken into account in the expressions (4) and (5) for the exchange energy  $W_{\text{ex}}$ ; the contribution of the third term of (27) could be disregarded in the previous part of the paper, because in the main and closure domains  $\partial M_0 / \partial x = 0$ . The term proportional to  $J_1$  in the nonuniform part of  $\omega_{\text{ex}}$  that is proportional to  $(\partial M_0 / \partial x)^2$  exerts an influence on the dynamics of the wall motion; but as will be shown below, this contribution to the velocity of motion of the wall is also relatively small.<sup>5)</sup>

The volume density  $\omega_{\text{tot}}$  of the total energy of the crystal, with allowance for the energy of uniaxial anisotropy and for the energy of interaction with an external field  $H$ , can be written in the form

$$\omega_{\text{tot}} = -FJ_1 \frac{M_0^2 M_z}{a_0^2 M_0} + F \left( \frac{1}{2} J_0 + \frac{1}{2} J_1 \frac{M_z}{M_0} \right) \left( \frac{\partial M_0}{\partial x} \right)^2 - \frac{1}{2} KM_z - HM_z; \quad (28)$$

here and below, we introduce the notation  $F = 2a_0^2 / g^2 \mu_B^2$ .

From (28) we obtain the value of the "effective field"<sup>13</sup>  $f$ :

$$f = \frac{d}{dx} \left[ \partial \omega_{\text{tot}} / \partial \left( \frac{\partial \mathbf{M}_0}{\partial x} \right) \right] - \frac{\partial \omega_{\text{tot}}}{\partial \mathbf{M}_0} = \mathbf{H} + K \mathbf{M}_0 \mathbf{n} + \left[ F \frac{J_1 M_0}{a_0^2} - \frac{1}{2} F \frac{J_1}{M_0} \left( \frac{\partial \mathbf{M}_0}{\partial x} \right)^2 \right] \mathbf{n} + F \left[ J_0 + J_1 \frac{(\mathbf{M}_0 \mathbf{n})}{M_0} \right] \frac{\partial^2 \mathbf{M}_0}{\partial x^2} + F J_1 \left( \frac{\partial \mathbf{M}_0}{\partial x} \frac{\mathbf{n}}{M_0} \right) \frac{\partial \mathbf{M}_0}{\partial x}. \quad (29)$$

Here  $\mathbf{n}$  is the unit vector in the  $Z$  direction.

To find the law of motion of the interdomain wall, the effective field from (29) must be substituted in the Landau-Lifshitz<sup>13</sup> equation:

$$\frac{\mathbf{M}_0}{\mu_B} = [\mathbf{f} \times \mathbf{M}_0] + \lambda \left( \mathbf{f} - \frac{(\mathbf{f} \mathbf{M}_0) \mathbf{M}_0}{M_0^2} \right), \quad (30)$$

where  $\lambda$  is the relaxation parameter.

Carrying out calculations analogous to those made in Ref. 13, we obtain the following expression for the velocity of motion of an interdomain wall:

$$v = \left( \frac{F J_0}{K} \right)^{1/2} \frac{\mu_B (\lambda^2 + M_0^2)}{\lambda M_0} \left\{ H + \frac{F}{a_0^2} J_1 M_0 + \frac{F J_1 M_0}{2(F J_0 / K)} \right\}. \quad (31)$$

Since  $(F J_0 / K)^{1/2}$  is the wall width  $D$  and since  $D \gg a_0$ , the second term in braces in (31) exceeds the third term by a factor  $2(D/a_0)^2$ ; thus the main contribution comes from the second term, which is due to the change of the uniform exchange.<sup>6)</sup>

From (31), the effect of the pumping is similar to the effect of an effective field  $H_{\text{eff}}$ ; here

$$H_{\text{eff}} \approx \frac{2 J_1 M_0 a_0^2}{g^2 \mu_B^2} = \frac{2 J_1 S^2}{a_0^2 M_0}. \quad (32)$$

For magnetic semiconductors we have, in analogy to formula (26),

$$H_{\text{eff}} \approx 0.1 \frac{4\pi P \Delta K \tau (AS)^{1/2}}{\omega a_0 m^{1/2} M_0}. \quad (33)$$

For  $\text{CdCr}_2\text{Se}_4$ , where  $M_0 \sim 4 \cdot 10^2$  G, for light power flux  $P \sim 10$  W/cm<sup>2</sup> of frequency  $\omega \sim 10^{15}$  s<sup>-1</sup> we have  $H_{\text{eff}} \sim 40$  Oe.

<sup>1)</sup>We remark that there is experimental confirmation, indirect to be sure, of the dependence of the effective exchange interaction on the carrier concentration: the dependence of the

Curie temperature of rare-earth chalcogenides on the doping level (see, for example, Ref. 7).

<sup>2)</sup>We remark that  $\Delta_a$  and  $\Delta_b$  in general depend on the angles  $\beta$  and  $\gamma$ , and thus on the angle  $\theta$  by virtue of the relation (1).

<sup>3)</sup>This estimate corresponds to the work of Privorotskiĭ<sup>10</sup>: according to Ref. 10, branching begins when  $H_0 \sim 2\pi M_0$ , while  $J_{1cr}$  corresponds to an effective magnetic field  $H_{\text{eff}} \sim 4\pi M_0$ , whence we have the estimate given.

<sup>4)</sup>Usually the condition  $K \gg b_1^2 / C_2$  holds.

<sup>5)</sup>We remark also that this term leads to a small (since  $J_0 \gg J_1$ ) change of the energy of an interdomain wall.

<sup>6)</sup>In the paper of Merkulov and Samsonidze<sup>14</sup> domain-wall motion under the influence of light was considered; but in this paper the contribution of the change of uniform exchange was disregarded, and thus the largest term was omitted.

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