Nonlinear Langmuir perturbations in an inhomogeneous plasma

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The high-frequency nonlinear potential perturbations excited by a beam in an inhomogeneous plasma with initial particle power-law density and velocity distributions are investigated theoretically. Neglecting the thermal motion of the plasma particles, the exact solution of the nonlinear equations was obtained for spherically symmetrical electron-density, velocity, and electric-field perturbations having a frequency close to the local electron Langmuir frequency, and the amplitude of these perturbations is determined. The results are used to calculate, under the conditions of the corona of a laser plasma, the energy density of the Langmuir oscillations generated in the critical-density region.

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One of the important problems connected with the realization of controlled thermonuclear fusion is the investigation of the various physical processes that occur in the "corona" of a laser plasma. Most of these processes, e.g., generation of laser harmonics, excitation of quasistationary magnetic fields, the appearance of fast particles, and others are due to intense fluctuations of the internal field in the inhomogeneous plasma of the corona. The plasma gasdynamics of a target heated by powerful laser radiation was investigated experimentally and theoretically in a number of studies (see the review[1]). This has made it possible to present a sufficiently complete picture of the distributions of the density, velocity, and temperature in the laser-plasma corona. It is therefore necessary to investigate the structure of the intense high-frequency self-consistent field in an inhomogeneous plasma whose density and velocity profiles are specified and are governed by its gasdynamic motions. This is the subject of the present paper, where we report the evolution of nonlinear potential perturbations excited in an inhomogeneous plasma by the passage of a beam of charged particles. This problem could be solved because the initial state of the plasma was defined as having particle power-law density and velocity profiles. Such an initial distribution of the density and velocity of the particles is typical of the laser-plasma corona. The stationary spherically symmetrical flow of this plasma is characterized by a practically constant radial velocity of the particles and by a density n_0 that varies in inverse proportion to the square of the radius.[1] In addition, beams of accelerated particles have been registered in such a plasma.[2-4]

It is shown in the present paper that in a cold electron plasma the spherically symmetrical perturbations of the self-consistent electric field manifest themselves as potential oscillations at a frequency close to the local electron Langmuir frequency. The essentially nonlinear Langmuir oscillations excited by a beam having a density n_b and propagating from the region of the high-density plasma constitute a series of peaks that move with definite velocity towards the lower plasma density. The maximum energy density of the electric field of these oscillations is smaller by a factor n_b/n_0 than the energy density of the electron beam.

In a laser plasma, the ratio of the energy density of the obtained Langmuir oscillations to the density of the thermal energy of the plasma particles, at typical values of the parameters in the vicinity of the critical density, amounts to $\sim 0.5\times 10^{-3}$.

1. We consider a plasma consisting of electrons and of one species of ions. Neglecting the thermal motion of the particles, the one-dimensional spherically symmetrical perturbations of the plasma are described by the system of equations

$$\begin{gathered}\frac{\partial}{\partial t}\omega_{L\alpha}^2(r,t)+r^{-2}\frac{\partial}{\partial r}(r^2\omega_{L\alpha}^2(r,t)v_\alpha(r,t))=0,\\ \frac{\partial}{\partial t}v_\alpha(r,t)+v_\alpha(r,t)\frac{\partial}{\partial r}v_\alpha(r,t)=-a_\alpha\frac{\partial}{\partial r}\Psi(r,t),\\ r^{-2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}\Psi(r,t)=-\omega_{Le}^2(r,t)-a_i^{-1}\omega_{Li}^2(r,t).\end{gathered}\tag{1.1}$$

Here $\Psi=(e_e/m_e)\varphi(r,t)$, $\varphi(r,t)$ is the potential of the self-consistent electric field in the plasma; $\omega_{L\alpha}=(4\pi n_\alpha e_\alpha^2/m_\alpha)^{1/2}$ is the local Langmuir frequency of the particles of species α with mass m_α and charge e_α; $n_\alpha(r,t)$ and $v_\alpha(r,t)$ are the density and velocity of the electrons ($\alpha=e$) and ions ($\alpha=i$) having a coordinate r at the instant of time t; $a_\alpha=1$ at $\alpha=e$ and $a_\alpha=e_i/m_e/e_e m_i$ at $\alpha=i$.

The condition for neglecting the effects connected with the thermal motion of the plasma particles, i.e., the condition for the applicability of the system of equations (1.1), is smallness of the thermal velocity of the plasma particles $V_{T\alpha}$ compared with the ratio of the characteristic spatial scale l of the inhomogeneity of the electric field of the plasma perturbations $E(r,t)=\partial\varphi/\partial r$ to the characteristic time τ of the change of this field: $l/\tau\gg V_{T\alpha}$. We confine our analysis to the zeroth order in the small ratio $V_{T\alpha}\tau/l\to 0$.

The dimensionalities of the quantities in the system (1.1) are determined only by the scales of the *length* L and time T:

$$[\omega_{L\alpha}^2]=T^{-2};\quad [v_\alpha]=LT^{-1};\quad [\Psi]=L^2T^{-2}.$$

Therefore the solution of the system of (1.1) will depend on one (self-similar) variable $\lambda=r/\gamma t^\delta$ (cf. Refs. 5 and

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6) and it can be sought in the form

$$|a_\alpha|^{-1}\omega_{L\alpha}^2=\frac{1}{t^2}N_\alpha(\lambda),\quad v_\alpha=\frac{r}{t}W_\alpha(\lambda),\quad \Psi=\frac{r^2}{t^2}\Phi(\lambda). \tag{1.2}$$

The constants γ and δ contained in the variable λ are determined by the specific formulation of the problem. Assume that a density profile $|a_\alpha|^{-1}\omega_{L\alpha}^2(r,0)=C_\alpha r^{-p}$ is specified at the initial instant of time $t=0$. The evolution of this profile at $t>0$ is determined by the three dimensional parameters r, t, and C with dimensionalities $[r]=L$, $[t]=T$ and $[C]=L^pT^{-2}$. It is possible to combine these parameters into a single variable $\lambda=r/C^{1/p}t^{2/p}$; in this case $\gamma=C^{1/p}$ and $\delta=2/p$. If the initial conditions of the problem contain also a nonzero velocity of the plasma particles $v_\alpha(r,0)=B_\alpha r^{-q}$, then self-similar solutions of the system (1.1) are possible only under the condition $p=2(1+q)$. In particular, a constant initial plasma-particle velocity $v_\alpha(r,0)=v_\alpha^*(q=0)$ corresponds to $p=2$ $(\delta=1)$.

Substituting (1.2) in (1.1) we obtain a system of ordinary differential equations

$$Q_\alpha(\lambda)'_\lambda=R_\alpha(\lambda)(2-3\delta), \tag{1.3}$$

$$V_\alpha(\lambda)(1-\delta)-V_\alpha(\lambda)_\lambda'(V_\alpha(\lambda)-\delta\lambda)=a_\alpha E(\lambda), \tag{1.4}$$

$$(\lambda^2E(\lambda))_\lambda'=R_i-R_e, \tag{1.5}$$

where we have introduced the new functions: the "flux" $Q_\alpha(\lambda)=R_\alpha(V_\alpha-\delta\lambda)$, the "density" $R_\alpha(\lambda)=\lambda^2N_\alpha(\lambda)$ and the "velocity" $V_\alpha(\lambda)=\lambda W_\alpha(\lambda)$ of the particles of species α, as well as the "electric field" $E(\lambda)=[\lambda^2\Phi(\lambda)]'_\lambda$. Using (1.3) and (1.5) in relations (2.4), we write down two equations for the fluxes $Q_e(\lambda)$ and $Q_i(\lambda)$:

$$(2-3\delta)\lambda^2\left[(2-3\delta)^2\frac{Q_e^2Q''_{e\lambda^2}}{Q'^3_{e\lambda}}+(2-3\delta)(\delta-1)\frac{Q_e}{Q'_{e\lambda}}+\lambda\delta(1-\delta)\right]+Q_e-Q_i+G=0, \tag{1.6}$$

$$(2-3\delta)\frac{\lambda^2}{a_i}\left[(2-3\delta)^2\frac{Q_i^2Q''_{i\lambda^2}}{Q'^3_{i\lambda}}+(2-3\delta)(\delta-1)\frac{Q_i}{Q'_{i\lambda}}+\lambda\delta(1-\delta)\right]+Q_e-Q_i+G=0, \tag{1.7}$$

$$G=-(2-3\delta)\lambda_0^2E(\lambda_0)+Q_e(\lambda_0)-Q_i(\lambda_0).$$

Eliminating with the aid of (1.6) the electron flux Q_e from (1.7), we can obtain one differential equation of fourth order for the ion flux Q_i. The constant λ_0 is defined below.

2. We consider now spherically symmetrical perturbations in the plasma at $\delta=1$. This investigation is of interest for the analysis of the evolution of the plasma perturbations in the corona of a laser plasma, since the system of self-similar equations (1.3)–(1.5) in the absence of a potential $\Phi=0$ corresponds to a spherically symmetrical distribution of the density and velocity of the plasma particles: $|a_\alpha|^{-1}\omega_{L\alpha}^2=C_\alpha r^{-2}$ and $v_\alpha=v_\alpha^*$.

To describe the perturbations in the plasma at $\delta=1$, we use Eqs. (1.6) and (1.7). In addition, we take into account the presence in the plasma of an external beam of charged particles with velocity u_b. This beam corresponds in Maxwell's equation

$$\frac{\partial E(r,t)}{\partial t}+4\pi j+4\pi j_0=0$$

to an extraneous current with density $j_0=m_e\alpha_1u_b/4\pi e_er^2$ (the influence of the electric field of the perturbations in the plasma on the density and velocity of the beam is neglected). Using the self-similar variables we obtain (here and elsewhere $\lambda=r/t$):

$$\lambda^2(\lambda^2E)_\lambda'=R_iV_i-R_eV_e-\alpha_1u_b. \tag{2.1}$$

It follows from Eq. (2.1) that the potential electric field in the plasma is due not only to the difference between the particle fluxes, but also to the presence of a beam with a density proportional to α_1:

$$\lambda^2E=Q_e-Q_i-\alpha_1\lambda+\alpha_1u_b. \tag{2.2}$$

The integration constant is determined by comparing the integral of Eq. (2.1) with the integral of the Poisson equation (1.5), in the right-hand side of which is taken into account an extraneous charge with density

$$\rho_0=m_e\alpha_1r^{-2}/4\pi e_e,$$

determined by the beam of the charged particles.

We confine ourselves below to an investigation of the perturbations of only the electronic component of the plasma, assuming that the influence of the electric field on the ion motion is so weak that it can be neglected. This condition corresponds to taking the limit as $|a_i|\to0$ in Eq. (1.7), which takes in this case the form $Q_{i\lambda^2}=0$. From this we obtain the ion flux $Q_i(\lambda)$ connected with the initial value of the density and velocity of the ions by the relation $Q_i=C_i(v_i^*/-\lambda)$.

Eliminating from (2.2) the quantity λ^2E with the aid of Eq. (1.4) and using the obtained value of the ion flux Q_i, we obtain the following equation for the quantity $\lambda(Q_e)\equiv\lambda(Q)$:

$$Q^2\lambda^2\lambda_{Q^2}''+Q+C(\lambda-v_i^*)+\alpha=0, \tag{2.3}$$

$$C=C_i-\alpha_1,\quad \alpha=\alpha_1(u_b-v_i^*).$$

The solution of Eq. (2.3) describes completely the perturbation of the electronic component of the plasma with the aid of the functions $R_e(\lambda)$, $V_e(\lambda)$, and $E(\lambda)$:

$$R_e\equiv R(\lambda)=-\lambda_Q',\quad V_e\equiv V(\lambda)=\lambda-Q\lambda_Q',\quad E(\lambda)=-Q^2\lambda_{Q^2}'', \tag{2.4}$$

which are connected with the real values of the density, velocity, and electric field by the following relations:

$$n_e(r,t)=\frac{m_e}{4\pi e_e^2}\frac{R(\lambda)}{r^2},\quad v_e(r,t)=V(\lambda),\quad E(r,t)=-\frac{m_e}{e_er}\lambda E(\lambda). \tag{2.5}$$

The solution of Eq. (2.3) is determined by the two constants, C_e and v_e^*:

$$C_e=-Q'_{\lambda|\lambda=\lambda_0},\quad v_e^*=\lambda_0-Q(\lambda_0)/Q'_{\lambda|\lambda=\lambda_0}.$$

One of these constants can be determined from the condition that there is no electric field in the plasma, $\lambda_0E(\lambda_0)=0$ at $\lambda=\lambda_0$:

$$C_e=C(v_i^*-\lambda_0)(v_e^*-\lambda_0)^{-1}-\alpha(v_e^*-\lambda_0)^{-1}. \tag{2.6}$$

The constant λ_0 is determined by the initial conditions. The choice of the value $\lambda_0=\infty$ corresponds to specifying at the instant of time $t=0$ the initial densities and velocities of the plasma and beam particles $(\alpha_1\neq0)$ in all of space $0<r<\infty$. Another choice of the value $\lambda_0=u_b$ is also possible. In this case the self-similar perturbations in the plasma occur at $t\geq t^*(r)$ [the time $t^*(r)=ru_b^{-1}$ determines the instant when a beam injected into the plasma at a point $r=0$ and at $t=0$ arrives at the point with coordinate r); at shorter times $t<t^*$ we have the unperturbed state of the plasma ($R_e=R_i=C_e=C_i=C$, $v_\alpha=v_e^*=v_i^*=v^*$, $\alpha_1=0$).

We investigate now the solutions of Eq. (2.3) as functions of the relations between the parameters α, C, and v^*_i. Equation (2.6) is simplest in form at $\alpha=Cv_i^*$:

$$(Z-1)^2Z_\xi''+Z=0,$$

$$Z=1+C\lambda/Q,\quad \xi=C^{3/2}/Q. \tag{2.7}$$

Equation (2.7) has a first integral

$$\mathscr{E}={}^1/{}_2Z_\xi'^2+U(Z),\quad U(Z)={}^1/{}_2\ln(Z-1)^2-\frac{Z}{Z-1} \tag{2.8}$$

the value of which for the initial conditions $Z(-C^{1/2}/\lambda_0)=0$ and $Z_\xi'(-C^{1/2}/\lambda_0)=v_e^*/C^{1/2}$ is $\mathscr{E}=v_e^{*2}/2C$. At finite values of $\mathscr{E}$, Z is bounded, $Z_1\leqslant Z\leqslant Z_2$; the turning points Z_1 and Z_2 are determined by the equation $\mathscr{E}=U(Z_{1,2})$. This restricted variation of $Z(\xi)$ corresponds to a nonlinear periodic wave defined, with account taken of (2.8), by the following relation:

$$\xi=\int\limits_0^Z dZ\left(\frac{v_e^{*2}}{C}+\frac{2Z}{Z-1}-\ln(Z-1)^2\right)^{-1/2}-C^{1/2}/\lambda_0.$$

The period of the linear oscillations is then given by

$$\Lambda=\int\limits_{Z_1}^{Z_2} dZ\left(\frac{v_e^{*2}}{C}+\frac{2Z}{Z-1}-\ln(Z-1)^2\right)^{-1/2}.$$

For small values $|Z(\xi)|\ll1$, when the potential $U(Z)\propto Z^2$, the solution of (2.7) can be written in explicit form

$$Z(\xi)=\frac{v_e^*}{C^{1/2}}\sin\left(\xi+\frac{C^{1/2}}{\lambda_0}\right)=\frac{v_e^*}{C^{1/2}}\sin\left(\frac{C^{3/2}}{Q}+\frac{C^{1/2}}{\lambda_0}\right),\quad \frac{v_e^*}{C^{1/2}}\ll1.$$

Returning to the variables λ and Q, we obtain the following equations:

$$\lambda=-\frac{Q}{C}\left[1-\frac{v_e^*}{C^{1/2}}\sin\left(\frac{C^{3/2}}{Q}+\frac{C^{1/2}}{\lambda_0}\right)\right],\quad V=v_e^*\cos\left(\frac{C^{3/2}}{Q}+\frac{C^{1/2}}{\lambda_0}\right),$$

$$R=C\left[1-\frac{v_e^*}{C^{1/2}}\left(\sin\left(\frac{C^{3/2}}{Q}+\frac{C^{1/2}}{\lambda_0}\right)-\frac{C^{3/2}}{Q}\cos\left(\frac{C^{3/2}}{Q}+\frac{C^{1/2}}{\lambda_0}\right)\right)\right]^{-1}, \tag{2.9}$$

$$E(\lambda)=v_e^*\frac{C^{3/2}}{Q}\sin\left(\frac{C^{3/2}}{Q}+\frac{C^{1/2}}{\lambda_0}\right).$$

Plots of λ, R, V, and E against the parameter $\varkappa=-\xi$ at $\lambda_0=\infty$ and $v_e^*/C^{1/2}=0.03$, based on Eqs. (2.9), are shown in Fig. 1.

We investigate the solution of Eq. (2.3) at $\alpha\neq Cv_i^*$. Using the substitution

$$u=\frac{C\lambda}{Cv_i^*-\alpha-Q},\quad \eta=-\ln\left|1-\frac{Cv_i^*-\alpha}{Q}\right| \tag{2.10}$$

we reduce (2.3) to the following system of first-order equations:

$$\frac{du}{d\eta}=\dot u,\quad \frac{d\dot u}{d\eta}=\dot u-\frac{C^3}{(Cv_i^*-\alpha)^2}\frac{u-1}{u^2}. \tag{2.11}$$

The qualitative behavior of the solution of Eqs. (2.11) is analyzed on the phase plane $(u,\dot u)$ (Fig. 2–4). The system (2.11) has a singular point $u=1$, $\dot u=0$. Depending on the ratio of the parameters C^3 and $(Cv_i^*-\alpha)^2$, this point can be a focus, a node, or a degenerate node. In the vicinity of the singular point[1)] the solution of Eqs. (2.11) can be obtained analytically. Thus, if the singular point is a focus $[4C^3>(Cv_i^*-\alpha)^2]$, then

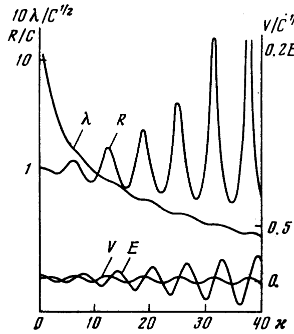

FIG. 1. Dependences of λ, R, V, and E on the parameter $\varkappa=-C^{3/2}/Q$ at $C_i^*/C^{1/2}=0.03$ and $\lambda_0=\infty$, plotted in accord with Eqs. (2.9).

$$u(\eta)=1+\frac{2}{a}\frac{C(v_i^*-v_e^*)-\alpha}{Cv_i^*-\alpha}\exp\left[\frac{\eta-\eta_0}{2}\right]\sin\left[\frac{a}{2}(\eta-\eta_0)\right],$$

$$a=\left[\frac{4C^3}{(Cv_i^*-\alpha)^2}-1\right]^{1/2},\quad \left|\frac{2}{a}\frac{C(v_i^*-v_e^*)-\alpha}{Cv_i^*-\alpha}\right|\ll1. \tag{2.12}$$

A solution of the system (2.11) in the case of a degenerate node $[4C^3=(Cv_i^*-\alpha)^2]$ is of the form $(Cv_i^*>\alpha)$

$$u(\eta)=1+\left(1-\frac{v_e^*}{2C^{1/2}}\right)(\eta-\eta_0)\exp\left[\frac{\eta-\eta_0}{2}\right],\quad \left|1-\frac{v_e^*}{2C^{1/2}}\right|\ll1. \tag{2.13}$$

If the singular point is a node $[4C^3<(Cv_i^*-\alpha)^2]$, then

$$u(\eta)=1+\frac{1}{\tilde a}\frac{C(v_i^*-v_e^*)-\alpha}{Cv_i^*-\alpha}\left[\exp\left(\frac{1+\tilde a}{2}(\eta-\eta_0)\right)-\exp\left(\frac{1-\tilde a}{2}(\eta-\eta_0)\right)\right]$$

$$\tilde a=\left[1-\frac{4C^3}{(Cv_i^*-\alpha)^2}\right]^{1/2},\quad \left|\frac{C(v_i^*-v_e^*)-\alpha}{Cv_i^*-\alpha}\right|\tilde a^{-1}\ll1. \tag{2.14}$$

Equations (2.12)–(2.14) yield the dependences of λ, R, V, and E on $\varkappa$ at an arbitrary ratio of the parameters $\alpha\neq Cv_i^*$. It is easy to show that the oscillatory dependence of λ, R, V, and E on $\varkappa$ at $4C^3>(Cv_i^*-\alpha)^2$ gives way to a monotonic dependence at $4C^3\leqslant(Cv_i^*-\alpha)^2$. In the limit as $\alpha-Cv_i^*\to0$ Eqs. (2.12) lead to the previously obtained relations (2.9).

Equations (2.9) and (2.12)–(2.14) become meaningless at large values of the parameter $\varkappa$, when the function $\lambda=\lambda(Q)$ becomes ambiguous (see, e.g., Fig. 1). This ambiguity can be eliminated by taking into account the thermal motion of the plasma electrons, which was not considered in our paper. However, the limitation connected with the ambiguity of the function $\lambda(Q)$ is not essential, since the ambiguity arises at $\lambda\sim v_e^*$, whereas

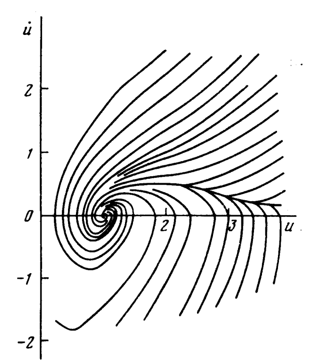

FIG. 2. Integral curves of the system of Eqs. (2.11) on the phase plane at $C^3=2(Cv_i^*-\alpha)^2$. The singular point is a focus.

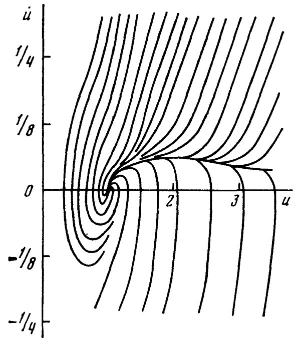

FIG. 3. Integral curves of the system of equations (2.11) on the phase plane at $4C^3 = (Cv_i^* - \alpha)^2$. The singular point is a degenerate node.

Eqs. (2.9) and (2.12)–(2.14) were obtained from the initial system (1.1), which is valid at $l/\tau \sim |Q\lambda_0| \sim |\lambda - V_e| \gg V_{T\alpha}$. For most real plasmas, the thermal velocity of the electrons greatly exceeds the directional velocities of the plasma particles, $v_\alpha \ll V_{Te}$, so that the condition $|\lambda - V_e| \gg V_{Te}$ is the most stringent restriction on Eqs. (2.9) and (2.12)–(2.14).

3. The solutions obtained in Sec. 2 of the present paper for the problem of the potential perturbations of an inhomogeneous plasma depend on the parameters C_i and v_i^*. We obtain now estimates of these parameters as applied to a real physical object, the corona of a laser plasma. Using the results of Ref. 1, we write down equations for the determination of the parameters C_i and v_i^* in the corona of a laser plasma in terms of the laser-radiation flux density q_0[W/cm²] on the target and the initial target radius R_t[cm] (A is the atomic weight and z is the ionization multiplicity):

$$C_i[\mathrm{cm}^2/\mathrm{sec}^2]=1.9\cdot 10^{22} q_0^{4/7}(A/z)^{1/2} R_t^{2},\qquad v_i^*[\mathrm{cm/sec}]=4.7\cdot 10^{5} q_0^{1/7}(A/z)^{-1/2}. \tag{3.1}$$

For values $R_t=10^{-2}$ cm and $q_0=10^{14}$ W/cm², we obtain from these equations the parameter values $C_i=2.7\times 10^{26}$ cm²/sec² and $v^*_i=3.3\times 10^7$ cm/sec.

It follows from these estimates that under laser-plasma corona conditions the following inequalities hold when $|\alpha_1|<C_i$.

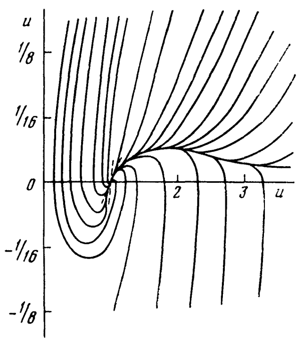

FIG. 4. Integral curves of the system of equations (2.11) on the phase plane at $8C^3 = (Cv_i^* - \alpha)^2$. The singular point is a node.

$$C^{1/2} \gg v_i^*, \quad 4C^3 \gg (Cv_i^* - \alpha)^2. \tag{3.2}$$

Taking into account the inequalities (3.2), we obtain from (2.12) explicit expressions for the density, velocity, and intensity of the electric field. In the limit $t^*(r)=r/\lambda_0 \leqslant t \ll r\min(|v_i^* - \alpha/C|^{-1}, V_{Te}^{-1})$, these expressions take the form

$$\begin{aligned}
\omega_{Le}^2(r,t) &= \frac{C}{r^2}\Big\{1-\frac{C(v_i^*-v_e^*)-\alpha}{C^{1/2}}\Big(1-\frac{Cv_i^*-\alpha}{2C}\Big(\frac{t}{r}+\frac{b}{C^{1/2}}\Big)\Big)\\
&\times\Big[\Big(1+\frac{Cv_i^*-\alpha}{2Cr}t\Big)\sin\Big[\frac{C^{1/2}}{r}t\Big(1+\frac{Cv_i^*-\alpha}{2Cr}t\Big)+b\Big]\\
&-\frac{C^{1/2}}{r}t\Big(1+\frac{Cv_i^*-\alpha}{Cr}t\Big)\cos\Big[\frac{C^{1/2}}{r}t\Big(1+\frac{Cv_i^*-\alpha}{2Cr}t\Big)+b\Big]\Big]\Big\}^{-1},\\
v_e(r,t) &= \Big(v_i^*-\frac{\alpha}{C}\Big)\Big\{1-\frac{C(v_i^*-v_e^*)-\alpha}{2C^{3/2}}\Big(1-\frac{Cv_i^*-\alpha}{2C}\Big(\frac{t}{r}+\frac{b}{C^{1/2}}\Big)\Big)\\
&\times\Big[\sin\Big[\frac{C^{1/2}}{r}t\Big(1+\frac{Cv_i^*-\alpha}{2Cr}t\Big)+b\Big]\\
&+\frac{2C^{1/2}}{Cv_i^*-\alpha}\cos\Big[\frac{C^{1/2}}{r}t\Big(1+\frac{Cv_i^*-\alpha}{2Cr}t\Big)+b\Big]\Big]\Big\},\\
E(r,t) &= \frac{m_e}{e_e}\frac{C}{r}F(r,t)(1-F(r,t)),\qquad b=-\frac{C^{1/2}}{\lambda_0}\Big(1+\frac{Cv_i^*-\alpha}{2C\lambda_0}\Big),\\
F(r,t) &= \frac{C(v_i^*-v_e^*)-\alpha}{C^{3/2}}\Big(1-\frac{Cv_i^*-\alpha}{2C}\Big(\frac{t}{r}+\frac{b}{C^{1/2}}\Big)\Big)\\
&\times\sin\Big[\frac{C^{1/2}}{r}t\Big(1+\frac{Cv_i^*-\alpha}{2Cr}t\Big)+b\Big]\ll 1.
\end{aligned} \tag{3.3}$$

Equations (3.3) describe the electron Langmuir oscillations of the plasma: at each point r the electron density varies with a frequency close to the local Langmuir frequency $\omega_{Le}(r, 0)$. At any fixed instant of time T_0 the profile of the electron density in the interval $r_0 \leqslant r \leqslant \lambda_0 T_0$ constitutes a set of N peaks, where

$$N=\Big[\frac{C^{1/2}}{2\pi}\Big(\frac{T_0}{r_0}-\frac{1}{\lambda_0}\Big)-\frac{1}{2}\Big].$$

Each of these peaks moves over an immobile ion profile towards decreasing density with constant velocity v_n:

$$v_n=\frac{C^{1/2}\lambda_0}{\lambda_0(2n+1)\pi+C^{1/2}},\qquad n=0,1,2\ldots .$$

The distance between the neighboring peaks $\Delta r_{n,n+1}$ increases linearly with time:

$$\Delta r_{n,n+1}=2\pi\lambda_0^2 C^{1/2} t\,[(C^{1/2}+\pi\lambda_0(2n+1))(C^{1/2}+\pi\lambda_0(2n+3))]^{-1}.$$

For small $n \ll C^{1/2}/2\pi\lambda_0$ the distance between peaks is practically independent of the number n, namely $\Delta r_{n,n+1} \approx 2\pi\lambda_0^2 t C^{-1/2}$. The quantity $\Delta r_{n,n+1}$ can be interpreted as the length l of the nonlinear Langmuir wave.

Using these results, we estimate now the energy density of the plasma Langmuir oscillations excited in the laser-plasma corona by beams of fast electrons. These electrons are constantly recorded in experiments (see, e.g., Ref. 2–4). Assuming equal electron and ion velocities in (3.3), $v_e^*=v_i^*$, and putting $\lambda_0=u_b$, we write down the maximum energy density of the Langmuir waves in the form $[n_0 \equiv n_e(r, 0)]$

$$\frac{E^2}{8\pi}=\frac{1}{2}m_e n_b u_b^2\Big(\frac{n_b}{n_0}\Big). \tag{3.4}$$

For a laser plasma with an electron temperature $T_e = 1$ keV the energy density of the plasma waves excited by a beam of electrons with energy $\varepsilon=10$ keV and with density $n_b/n_0=\alpha_1/C=10^{-2}$ is of the order of $E^2/8\pi n_0\varkappa_B T_e$

$=0.5\times10^{-3}$ ($\varkappa_B$ is Boltzmann's constant).

The wavelength l and the phase velocity v_{ph} of the Langmuir oscillations in the region of the critical density are determined by the relation

$$l=2\pi u_b/\omega_0(1+2\pi n u_b/C^{1/2}),\ v_{ph}=\omega_0 l/2\pi=u_b/(1+2\pi n u_b/C^{1/2}) \quad (3.5)$$

and for the plasma of a target with $R_t=10^{-2}$ cm, heated by a flux $q_0=10^{14}$ W/cm^2 of neodymium-laser radiation of frequency $\omega_0=1.78\times10^{15}$ sec^{-1} they amount at the initial instant of time to $l=0.15$ μm and $v_{ph}=u_b=4.2\cdot10^9$ cm/sec. Using (3.5), we estimate the time in which the phase velocity decreases to a value comparable with the thermal velocity of the plasma electrodes: $\Delta\tau=2\pi n\omega_0^{-1}$, For the parameters given above, this estimate of $V_{Te}\sim v_{ph}\sim\lambda$ sec shows when Eqs. (3.3) cease to be valid as a result of the increased role of the kinetic effect and the need for taking into account in the region of small phase velocities $v_{ph}\lesssim V_{Te}$.

[1)]The required proximity to the singular point (0, 1) leads to inequalities that limit the values of the parameters α, C, and v_i^* as well as the range of variation of η. In the case of the conditions imposed on the parameters C, v_i^* and α in the derivation of Eqs. (2.12)–(2.14) the range of variation of η is restricted by the inequality

$$\eta<2-\ln\ 1-\left|\frac{Cv_{i^*}-\alpha}{Cv_{i^*}-\alpha-C\lambda_0}\right|\equiv2+\eta_0\ .$$

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