

# Superconductivity induced by impurities

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The effect of impurities on the properties of a system with competing superconductive and dielectric transitions of the Peierls type is studied. The case is considered when in the absence of impurities the system would be a dielectric. It is shown that under three-dimensional conditions a sufficiently high impurity level induces a transition to the superconductive state. At a certain concentration the superconductive parameter is maximal. Further increase of the concentration destroys the superconductivity and restores the dielectric properties. Some arguments are presented which indicate that in one or two dimensions impurities do not induce superconductivity. The possibility of applying the present model to superconductivity in the organic superconductor  $(\text{TMTSF})_2\text{ClO}_4$  is discussed.

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## 1. INTRODUCTION

In recent experiments<sup>1</sup> superconductivity was observed at atmospheric pressure in the organic conductor  $(\text{TMTSF})_2\text{ClO}_4$ . This conductor belongs to the family of organic salts  $(\text{TMTSF})_2\text{X}$ , where X is an anion of high symmetry. Superconductivity had already been observed<sup>2</sup> in one of the compounds of this family, namely  $(\text{TMTSF})_2\text{PF}_6$ ; however, in this case a pressure of 6–10 kbar was necessary. The difference between these two isostructural compounds is very slight; in particular, the volume of the unit cell is  $V = 694.3 \text{ \AA}^3$  and  $V = 714.3 \text{ \AA}^3$  for  $\text{X} = \text{ClO}_4^-$  and  $\text{PF}_6^-$  respectively. Nevertheless, there is a basic difference in that the orientation of the  $\text{ClO}_4^-$  anions is random, whereas that of the  $\text{PF}_6^-$  anions is regular; thus, the compound  $(\text{TMTSF})_2\text{ClO}_4$  is characterized by internal disorder. This disorder may be the main reason for the existence of superconductivity at atmospheric pressure, if there is a competition in the material between a superconducting state and a dielectric state of the Peierls type.

The question of the effect of impurities on the competition between the superconducting and Peierls transitions has been investigated many times.<sup>3–5</sup> This question is of interest because in the self-consistent-field approximation impurities suppress the dielectric state and do not affect the superconducting one.<sup>4</sup> However, Larkin and Mel'nikov showed<sup>5</sup> that taking fluctuations into account in a one-dimensional metal leads to suppression of both transitions by impurities; this suppression occurs when the electron mean free path  $l$  becomes less than the radius  $\xi_0$  of electron pairs.

Below a model is propounded in which the transition temperatures to the dielectric and superconducting states are close to one another. Such a model was used by the author<sup>6</sup> to describe the properties of  $(\text{TMTSF})_2\text{PF}_6$ . Small additional interactions will then strongly affect the choice between the dielectric and superconducting states. It was shown<sup>6</sup> that many properties of  $(\text{TMTSF})_2\text{PF}_6$  may be understood if we assume that the effects of commensurability, which may facilitate the occurrence of superconductivity, are important. It turns out that in a model with competing tran-

sitions impurities may also lead to superconductivity, even though in ion-disordered samples and dielectric state is favored. As a result of the closeness of the transition temperature to the superconducting and dielectric states, the concentration of impurities can be very small, so that  $l \gg \xi_0$ . The situation considered here differs from that of Larkin and Mel'nikov,<sup>5</sup> where the impurity concentration was assumed fairly large.

## 2. CHOICE OF MODEL

We consider a system of conducting filaments. An attraction between electrons in the same and different filaments leads to the possibility of formation of both superconducting and Peierls states. In the case of strong one-vertex repulsion a situation may arise where a  $4k_F$  rather than a  $2k_F$  anomaly occurs. In both these cases a charge density wave (CDW) occurs in the dielectric state. We assume that the transition temperatures are close together. A random potential will lead to fluctuations of the CDW period (forward scattering) and to pinning of the CDW (backward scattering).

We consider below the case of small concentrations. In this limit only backward scattering is important. To describe a system having competing superconducting and dielectric states, the author in a previous paper<sup>6</sup> wrote the free energy, from phenomenological considerations, in a Ginzburg-Landau form. Taking into account interactions with impurities, this energy is given by

$$F = \int \left[ -(A_1|\Delta|^2 + A_2|\kappa|^2) + \frac{1}{2}B(|\Delta|^2 + |\kappa|^2)^2 + \sum_{i=1}^3 C_i (|\nabla_i \Delta|^2 + |\nabla_i \kappa|^2) - (A_i/B)^{1/2} (V(\mathbf{r})\kappa^*(\mathbf{r}) + V^*(\mathbf{r})\kappa(\mathbf{r})) \right] d\mathbf{r}. \quad (1)$$

The free energy (1) contains two complex order parameters,  $\Delta$  and  $\kappa$ , which determine the magnitudes of the superconducting and dielectric gaps. In accordance with the assumption that the transition temperatures are close, we assume that the inequality

$$|A_1 - A_2|/A_1 \ll 1 \quad (2)$$

is satisfied. In the following we assume that all the

difference in ease of formation of the dielectric and superconducting states is included in the difference between  $A_1$  and  $A_2$ . Strictly speaking, of course, the coefficients of the gradients of  $\Delta$  and  $\kappa$  in (1) are different, and the coefficients of  $\Delta$  and  $\kappa$  in the fourth-order term may also be different; however, to simplify the formula we shall take them equal. The appropriate generalizations of the calculations carried out below are straightforward.

The third term in (1) contains a sum over the three directions in the crystal. A quasi-one-dimensional system satisfies the inequality

$$C_1 \gg C_2, C_3.$$

The presence of impurities in the metal leads to the appearance of a random potential  $V(r)$ . The potential  $V(r)$  in (1) describes backward scattering and is given by

$$V(r) = \left(\frac{B}{A}\right)^{1/2} \sum_{\mathbf{Q}} \frac{V_0}{g} e^{i\mathbf{Q}\cdot\mathbf{r}} \delta(\mathbf{r}-\mathbf{r}_0) = V_0 \sum_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\mathbf{r}} \delta(\mathbf{r}-\mathbf{r}_0), \quad (3)$$

where  $V_0$  is the scattering amplitude at the CDW wave vector  $\mathbf{Q}$  at a single impurity and  $g$  is the magnitude of interaction of the electrons. The sum (3) is taken over all the impurities.

A more detailed description of the form of the term linear in  $\kappa$  has been given elsewhere.<sup>7,8</sup> We do not take into account the destruction of dielectric ordering connected with breaking of pairs, which is analogous to that considered by Bulaevskii.<sup>4</sup>

For a random distribution of impurities and a weak interaction with them the distribution of random fields is Gaussian with correlators

$$\langle V(\mathbf{r}) V^*(\mathbf{r}') \rangle = 2\gamma \delta(\mathbf{r}-\mathbf{r}'), \quad (4)$$

$$\langle V(\mathbf{r}) \rangle = \langle V^*(\mathbf{r}) \rangle = \langle V(\mathbf{r}) V(\mathbf{r}') \rangle = \langle V^*(\mathbf{r}) V^*(\mathbf{r}') \rangle = 0,$$

where  $\gamma$  describes the impurity concentration and is given by

$$\gamma \sim \frac{V_0^2 n}{g^2} \frac{1}{T_c(T_c - T)}. \quad (5)$$

The order of magnitude of the coefficients in (1)–(3) is given by the formula

$$A_{1,2} \sim \frac{T_c - T}{T_c} N(0) \equiv \tau N(0), \quad B \sim \frac{1}{T_c^2} N(0), \quad C \sim \frac{v^2}{T_c^2} N(0). \quad (6)$$

In (5) we have

$$T_c \sim \varepsilon_0 \exp(-1/gN(0)),$$

where  $N(0)$  is the density of states, and  $v$  the velocity, at the Fermi surface.

The coefficients  $C_2$  and  $C_3$  depend in a complicated way on the hopping of electrons from chain to chain and on the interaction of different chains. If the coefficients  $C_2$  and  $C_3$  are not too small and the temperature  $T$  is not too close to the critical temperature, all physical quantities are determined by the condition that the free energy be a minimum, and the contribution of thermodynamic fluctuations be small. The condition for such fluctuations to be small can be written in the form

$$\tau \gg \frac{C_1^2}{C_2 C_3} \left( \frac{T_c^2}{v^2 N(0)} \right)^2. \quad (7)$$

The inequality (7) shows that over a wide range of the parameters the contribution of thermodynamic fluctuations is small. A fairly large amount of hopping does not necessarily destroy dielectric pairing, since the latter requires only the presence of parts of the Fermi surface which coincide under parallel translation, and these need not be planes.

It is convenient to introduce a four-component order parameter  $\Sigma$  with the following components

$$\Sigma_1 = \kappa', \quad \Sigma_2 = \kappa'', \quad \Sigma_3 = \Delta', \quad \Sigma_4 = \Delta'', \quad (8)$$

where  $\kappa', \Delta'$  and  $\kappa'', \Delta''$  are the real and imaginary parts of the dielectric and superconducting order parameters. When the inequalities (2) and (7) are fulfilled and there is a fairly small impurity concentration, the modulus of the vector  $\Sigma$  is rigidly fixed by the condition that the free energy (1), without taking account of the impurities, be a minimum, and is described by the formula

$$\Sigma^2 = A_1/B. \quad (9)$$

Assuming that Eq. (9) is fulfilled, we write the vector  $\Sigma$  in the form

$$\Sigma = S(A_1/B)^{1/2}, \quad (10)$$

where  $S$  is a four-component unit vector.

Going over to new coordinates

$$x = u_1(C_1)^{1/2}, \quad y = u_2(C_2)^{1/2}, \quad z = u_3(C_3)^{1/2} \quad (11)$$

and renormalizing the constant  $\gamma$  in the rule that determines the distribution of random potentials (3) by the formula

$$\gamma \rightarrow \bar{\gamma} = \gamma / (C_1 C_2 C_3)^{1/2}, \quad (12)$$

we represent the free energy functional (1) in the form

$$F = -\frac{A_1^2}{2B} \Omega + \frac{A_1(C_1 C_2 C_3)^{1/2}}{B} \int \left[ \frac{1}{2} (\nabla S)^2 - \mathbf{V} S_{\perp} + \frac{1}{2} a S_{\parallel}^2 \right] dx, \quad (13)$$

where  $\Omega$  is the volume of the sample.

In expression (13) the distribution of random fields  $\mathbf{V}$  is described by formula (3), with the substitution (12). The vector  $S_{\perp}$  describes the dielectric order parameter and the vector  $S_{\parallel}$  the superconducting one. The vectors  $S_{\perp}$  and  $S_{\parallel}$  are mutually perpendicular and satisfy the condition

$$S_{\perp}^2 + S_{\parallel}^2 = 1. \quad (14)$$

The vector  $\mathbf{V}$  has components

$$V_1 = \text{Re } V, \quad V_2 = \text{Im } V.$$

The anisotropy parameter  $a$  characterizes the degree of proximity of the superconducting and dielectric transitions and is given by

$$a = A_1 - A_2. \quad (15)$$

The minimum of the energy (13) determines the behavior of the system for a given impurity concentration; the existence of a nonzero vector  $S_{\parallel}$  indicates the appearance of superconducting properties in the system. The condition of a minimum of the free energy (13) will of course determine only the modulus of the

vector  $S_{||}$ , not its direction. The minimum corresponds to a constant direction of the vector  $S_{||}$  in space; therefore in what follows we shall take  $S_{||}$  to be a scalar quantity which defines the modulus of the superconducting order parameter. In subsequent sections we shall investigate the solution which realizes the minimum of the functional (13) for various impurity concentrations.

### 3. INSTABILITY OF THE DIELECTRIC STATE

So far we have made no assumption about the sign of  $a$  in (13). We shall restrict consideration to the more interesting case  $a > 0$ , when in the absence of impurity the dielectric state with  $S_{||} = 0$  is favored. The presence of a random potential leads to the appearance at some impurity concentration of a nonzero value of  $\langle S_{||} \rangle$ , where the angular brackets indicate an average over the impurities. We may verify this by examining the stability of the solution with  $S_{||} = 0$ . Expanding the free energy (13) up to terms quadratic in  $S_{||}$  and using the relation (14), we obtain

$$F = F_0 + \frac{A_1}{2B} (C_1 C_2 C_3)^{1/2} \int [(\nabla S_{||})^2 + f(\mathbf{r}) S_{||}^2] d\mathbf{r}, \quad (16)$$

$$f(\mathbf{r}) = -(\nabla \chi)^2 + V_1 \cos \chi + V_2 \sin \chi + a, \quad (16a)$$

$$F_0 = -\frac{A_1^2}{2B} \Omega + \frac{A_1 (C_1 C_2 C_3)^{1/2}}{2B} \int \left[ \frac{1}{2} (\nabla \chi)^2 - V_1 \cos \chi - V_2 \sin \chi \right] du. \quad (17)$$

In expressions (16) and (17) the angle  $\chi$  determines the vector  $S_{\perp}$  by the formula

$$S_{\perp} = |S_{\perp}| (\cos \chi, \sin \chi). \quad (18)$$

Stability of the dielectric state requires that the quadratic form (16) be positive definite. This condition is equivalent to the positivity of the eigenvalues  $E$  of the corresponding Schrödinger equation. At the critical point  $E$  tends to zero, and the Schrödinger equation can be written in the form

$$-\Delta S_{||}(\mathbf{u}) + f(\mathbf{u}) S_{||} = 0. \quad (19)$$

We represent  $S_{||}$  in the form

$$S_{||} = \langle S_{||} \rangle + \psi, \quad \langle \psi \rangle = 0. \quad (20)$$

Substituting (20) in (19) and averaging, we obtain

$$\langle f(\mathbf{u}) \rangle \langle S_{||} \rangle + \langle f(\mathbf{u}) \psi \rangle = 0. \quad (21)$$

Subtracting (21) from (20), we find in the first approximation

$$\psi(\mathbf{u}) = - \int G(\mathbf{u} - \mathbf{u}_1) (f(\mathbf{u}_1) - \langle f(\mathbf{u}_1) \rangle) \langle S_{||} \rangle d\mathbf{u}_1, \quad (22)$$

where the Green's function  $G(u)$  satisfies the equation

$$-\Delta G(u) = \delta(u). \quad (23)$$

With the help of (22), Eq. (21) takes the form

$$S_{||} \left[ \langle f(\mathbf{u}) \rangle - \int G(\mathbf{u} - \mathbf{u}_1) \langle (f(\mathbf{u}_1) - \langle f(\mathbf{u}_1) \rangle) (f(\mathbf{u}_1) - \langle f(\mathbf{u}_1) \rangle) \rangle d\mathbf{u}_1 \right] = 0. \quad (24)$$

The critical concentration is found from the condition that the expression multiplying  $S_{||}$  in (24) tends to zero. To calculate the averages in (24) we must find the solution  $\chi$  which minimizes the free energy  $F_0$  in (17); this will enable us to find, with the help of (16a), the function  $f(\mathbf{r})$  which enters in (24). The extremal solution  $\chi(\mathbf{r})$  is defined by the equation

$$-\Delta \chi = -V_1(\mathbf{u}) \sin \chi - V_2(\mathbf{u}) \cos \chi. \quad (25)$$

Expanding Eq. (25) in  $\chi$  and solving it by iteration, we verify after substitution in (16a) that

$$\langle f(\mathbf{u}) \rangle = a. \quad (26)$$

For the calculation it is convenient to use the diagram technique developed by Larkin and the author.<sup>7</sup> Consideration of the terms in the perturbation-theory series shows that the equality (26) is satisfied in all orders of perturbation theory. Calculations for the second term in (24) by the same method give the final condition for appearance of a nonzero value of  $\langle S_{||} \rangle$ :

$$a = \bar{\gamma}_c \int G(\mathbf{k}) \frac{d^3 \mathbf{k}}{(2\pi)^3} = \bar{\gamma}_c G(0), \quad (27)$$

where  $G(k)$  is the Fourier component of the function  $C(u)$ .

The integral in (27) over the Green's function  $G(k)$  defined by (23) diverges at large momentum. Imposing an upper cutoff at a momentum  $k_0 \sim C_1^{1/2} / \xi_0$  and going over to dimensional units, we obtain with the help of (4) and (5)

$$l = \frac{l_0}{\tau_1 - \tau_2}, \quad (28)$$

where  $l$  is the mean free path, given by the formula

$$l^{-1} = n V_0^2 N(0) / v, \quad (29)$$

and  $l_0$  is a length of order

$$l_0 \sim \frac{\xi_0}{v \tau_1} \frac{1}{(gN(0))^2} \frac{1}{(C_2 C_3)^{1/2}}, \quad (30)$$

where  $\xi_0$  is the pair radius.

Consideration of higher orders of perturbation theory than those used in the derivation of (24) shows that formula (28) is valid for sufficiently long mean free paths  $l \gg l_0$ , for which the inequality

$$\bar{\gamma}_c \ll G(0) \quad (31)$$

is satisfied.

For  $\bar{\gamma} < \bar{\gamma}_c$  there is no superconducting order. The formula which describe the dielectric state can be obtained from the solution of Eq. (25). An investigation of Eq. (25) was carried out in previous work,<sup>7</sup> where in particular the absence of long-range order was proved to all orders of perturbation theory and the structure factor

$$\Pi(\mathbf{r}) = \langle \kappa(0) \kappa^*(\mathbf{r}) \rangle \quad (32)$$

was calculated. In dimensional units  $\Pi(\mathbf{r})$  has the form

$$\Pi(\mathbf{r}) = \frac{A_1}{B} \exp \left\{ -\frac{\gamma}{8\pi} \left( \frac{x^2}{C_1} + \frac{y^2}{C_2} + \frac{z^2}{C_3} \right)^{1/2} \right\}. \quad (33)$$

When the condition (31) is fulfilled the function  $\Pi(\mathbf{r})$  changes slowly over a range  $\xi_0$ , which indicates the applicability of expression (33) under the assumptions made above. If  $\bar{\gamma} > \bar{\gamma}_c$ , the superconducting order parameter is nonzero. This region is investigated in the next section.

### 4. THE SUPERCONDUCTING REGION

To investigate the region in which the superconducting order parameter is nonzero we write the equations

which determine the minimum of the free energy (13) in general form. Varying (13) with respect to  $S_{\perp}$  and using (14), we obtain

$$S_{\perp}(1-S_{\perp}^2)^{-1/2}\Delta(1-S_{\perp}^2)^{1/2}-\Delta S_{\perp}-V-aS_{\perp}=0. \quad (34)$$

In accordance with the assumption of a transition to the superconducting state for  $\bar{\gamma} > \bar{\gamma}_c$ , we assume that

$$\langle S_{\perp}^2 \rangle \ll 1. \quad (35)$$

We shall show below that this condition is fulfilled for a wide range of concentrations. Keeping only the linear and cubic terms in  $S_{\perp}$ , we write Eq. (34) in the form

$$-\Delta S_{\perp} - S_{\perp}(\Delta S_{\perp}) - S_{\perp}(\nabla S_{\perp})^2 - aS_{\perp} = V. \quad (36)$$

The nonlinear terms in (36) can be treated by the method of self-consistent fields. Carrying out a decoupling according to the formula

$$S_{\perp i} (S_{\perp k} \Delta S_{\perp k}) \rightarrow S_{\perp i} \langle S_{\perp k} \Delta S_{\perp k} \rangle + \langle S_{\perp i} \Delta S_{\perp k} \rangle S_{\perp k}, \\ S_{\perp i} \nabla S_{\perp k} \nabla S_{\perp k} \rightarrow S_{\perp i} \langle \nabla S_{\perp k} \nabla S_{\perp k} \rangle$$

and calculating the superconducting order parameter  $\langle S_{\parallel} \rangle$ , we obtain

$$\langle S_{\parallel} \rangle = 1 - \frac{1}{2} \langle S_{\perp}^2 \rangle = 1 - \frac{\bar{\gamma}}{(2\pi)^3} \int \frac{d^3 k}{(k^2 + b - a)^2}, \quad (37) \\ b = \frac{\bar{\gamma}}{(2\pi)^3} \int \frac{k^2 d^3 k}{(k^2 + b - a)^2} \approx \bar{\gamma} G(0).$$

An estimate of the corrections to the self-consistent field approximation shows that they may be neglected if condition (35) is satisfied.

Calculating the integral in (37) and using relation (27), we have

$$\langle S_{\parallel} \rangle = 1 - \frac{\bar{\gamma}}{8\pi} \frac{1}{(\bar{\gamma} - \bar{\gamma}_c)^{1/2} (G(0))^{1/2}}. \quad (38)$$

An analysis of the expression (38) shows that as a function of  $\bar{\gamma}$  the quantity  $\langle S_{\parallel} \rangle$  has a maximum which is attained for  $\bar{\gamma} = 2\bar{\gamma}_c$ . At this point the value of the superconducting order parameter  $\langle S_{\parallel} \rangle_{\max}$  is equal to

$$\langle S_{\parallel} \rangle_{\max} = 1 - \frac{1}{4\pi} \left( \frac{\bar{\gamma}_c}{G(0)} \right)^{1/2}. \quad (39)$$

When condition (31) is fulfilled the quantity  $\langle S_{\parallel} \rangle_{\max}$  is close to unity. Moving away from  $2\bar{\gamma}_c$  leads to a decrease of  $\langle S_{\parallel} \rangle$  and an increase of  $\langle S_{\perp}^2 \rangle$ . The condition (36) allows us to write down the extent of applicability of (38). This condition may be written in the form

$$\bar{\gamma}/\bar{\gamma}_c - 1 \gg \bar{\gamma}_c/G(0), \quad \bar{\gamma} \ll G(0). \quad (40)$$

At the edges of the region defined by the inequalities (40),  $\langle S_{\perp}^2 \rangle$  is of order unity. In the previous section it was shown that for small concentrations this leads to the disappearance of superconductivity. In the region of large concentration,  $\bar{\gamma} \gg G(0)$ , vanishing of superconductivity is also to be expected. This region is characterized by strong pinning of CDW's to impurities, such that the vector  $S$  follows the variation of the vector  $V$  in space and accordingly has a component  $S_{\parallel}$  equal to zero.

In the superconducting region the existence of a finite  $\langle S_{\parallel} \rangle$  is accompanied by quite strong correlations of the dielectric order parameter. For example, the correlation function  $\Pi(r)$ , defined by expression (32), has the

form

$$\Pi(r) = \frac{A_1}{B} \frac{\bar{\gamma}}{\pi (G(0))^{1/2} (\bar{\gamma} - \bar{\gamma}_c)^{1/2}} \exp\left(-G^{1/2}(0) (\bar{\gamma} - \bar{\gamma}_c)^{1/2}\right) \\ \times \left( \frac{x^2}{C_1} + \frac{y^2}{C_2} + \frac{z^2}{C_3} \right)^{1/2}. \quad (41)$$

This formula is also applicable when the inequality (40) is satisfied. On the boundary between the superconducting and dielectric regions the correlation lengths and pre-exponential factors in expressions (33) and (41) are of the same order of magnitude. At the same time it remains unclear whether the correlation length in  $\Pi(r)$  has a singularity at the point  $\gamma_c$ .

For sufficiently large impurity concentration or sufficiently weak interaction between different filaments the inequality (31) is no longer satisfied and the theory developed in the previous section becomes inapplicable. In this case we would guess that the superconducting order parameter  $\langle S_{\parallel} \rangle$  would not appear for any concentration, if  $a > 0$ . A weak interaction between filaments can result in a one- or two-dimensional situation. In the next section we shall give arguments which suggest that in such a low-dimensional case superconductivity cannot occur for  $a > 0$ .

## 5. LOW DIMENSIONALITY

If the spatial dimensionality  $d$  is less than three, the formulae obtained in the two preceding sections are inapplicable, since for any impurity concentration the discarded terms are of the same order of magnitude as those kept. This is a result of the large contribution from the small-momentum region. The simplest case to consider is that of small concentrations or sufficiently strong impurities in a one-dimensional system, which corresponds to strong pinning.<sup>8,9</sup> In the notation used above, strong pinning sets in if the inequality

$$V_0 r_c = \left( \frac{B}{A} \right)^{1/2} \frac{V_0}{g} r_c \gg C_1, \quad (42)$$

is satisfied, where  $r_c$  is the mean distance between impurities. In this case the representation (3) in terms of individual impurities is essential; the transition to the Gaussian distribution (4), which was made in the previous sections for the three-dimensional case, is now invalid.

Substituting (3) in (16), (16a) and (17), we find that the dielectric state is stable if the quadratic form  $F_2$  is positive definite, where

$$F_2 = \int \left[ \left( \frac{d\psi}{du} \right)^2 + \left( - \left( \frac{d\chi}{du} \right)^2 + \sum_i V_i \cos(\chi + \bar{Q}u_i + a) \right) \psi^2 \right] du, \quad (43)$$

where the function  $\chi(u)$  minimizes the functional  $F_0$ :

$$F_0 = \int \left[ \frac{1}{2} \left( \frac{d\chi}{du} \right)^2 - \sum_i V_i \cos(\chi + \bar{Q}u_i) \right] du, \quad \bar{Q} = (C_1)^{1/2} Q. \quad (44)$$

It is easy to see that when the inequality (42) is satisfied the minimum of (44) is obtained for

$$\chi + \bar{Q}u_i \approx 2m\pi, \quad (45)$$

where  $m$  is an integer. The term in  $(d\chi/du)^2$  is small and its contribution may be neglected. Substituting the solution (45) in (43), we can convince ourselves that the

quadratic form (43) is everywhere positive definite. The impurities only make the "dielectricization" of the state easier.

In the opposite limit of concentrated and weak impurities the analysis is much more complicated. In both the one- and two-dimensional cases slow spatial variations are essential. For a qualitative investigation we use the idea of partitioning the system in its ground-state into domains<sup>9,10</sup> with dimension  $L$ , over which the change of  $\chi$  is of order unity. The formation of domains leads to a loss of elastic energy in (44), but in return there is a gain in the energy of interaction with impurities because of the phase matching. The energy per unit volume can be written in the form

$$F' = \frac{1}{2}L^{-2} - \nabla_0 L^{-d/2}. \quad (46)$$

Minimization of  $F'$  [Eq. (46)] with respect to the domain size  $L$  gives the characteristic domain size  $L_0$ :

$$L_0^{2-d/2} = 2/d\nabla_0. \quad (47)$$

The characteristic energy  $E$  multiplying  $\psi^2$  in (43) can be written in the form

$$E = -L^{-2} + \nabla_0 L^{-d/2} + a. \quad (48)$$

Substitution of (47) in (48) gives

$$E = L_0^{-2}(2/d-1) + a. \quad (49)$$

It is clear from this formula that for spatial dimension  $d \leq 2$  the characteristic energy in (43) is everywhere positive for  $a > 0$ , which indicates the stability of the dielectric state for any impurity concentration.

Expressions (45)–(49) have of course only a qualitative significance and cannot serve as a rigorous proof of the stability of the dielectric. For  $d \geq 3$  large momenta are important for the occurrence of superconductivity, and therefore it would be insufficient to take into account only the contribution of slow variations to the characteristic energy  $E$ .

## 6. CONCLUSION

It has been shown above that impurities may lead to the onset of superconductivity if there is a competition in the system between transitions of the superconducting and Peierls types. A further increase of impurity concentration destroys superconductivity and evidently leads again to a transition to the dielectric state. The

sequence just described, dielectric–superconductor–dielectric, is possible in a space with dimensionality  $d \geq 3$ . In the lower-dimensional system with  $d \leq 2$  no such sequence takes place and the dielectric state always occurs. A characteristic feature of the transition to the superconducting state is the sensitivity of the superconducting order parameter, and hence of the transition temperature of the superconductor, to impurity concentration.

The model considered above may shed light on the simultaneous existence of superconductivity in the organic conductor  $(\text{TMTSF})_2\text{ClO}_4$  (Ref. 1), which is characterized by internal disorder in the distribution of the  $\text{ClO}_4^-$  anions, and its absence in the ordered crystal  $(\text{TMTSF})_2\text{PF}_6$ , which has the same structure and closely similar dimensions of the unit cell. The quite strong variations of critical temperature from sample to sample in  $(\text{TMTSF})_2\text{ClO}_4$ , which are evidently connected with sample purity, are also evidence in favor of the validity of the above model for this material. It would be very interesting to investigate experimentally the properties of the organic salts  $(\text{TMTSF})_2\text{X}$  as a function of crystal purity by varying the impurity concentration in a controlled way.

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