Slowly decaying waves near cyclotron resonance in metallic ferromagnets in an oblique magnetic field

Yu. I. Man'kov, L. M. Gorenko, and R. G. Khlebopros

L. V. Kirenskiĭ Physics Institute, Siberian Division, USSR Academy of Sciences
(Submitted 13 June 1981)
Zh. Eksp. Teor. Fiz. **81**, 2180–2191 (December 1981)

Weakly decaying waves near cyclotron resonance are investigated theoretically in a metallic ferromagnet located in a magnetic field inclined to the surface of the specimen. It is demonstrated that owing to vanishing of magnetic Landau damping and depending on the relative values of the physical parameters, there may exist in the vicinity of the decay minimum one, two, or three waves. The spectrum is shown to depend on the relation between the cyclotron resonance and antiresonance frequencies. The cases when the frequencies are the same or when the frequency of the coupled wave is close to the ferromagnetic resonance frequency are considered. In the latter case the expression for the wave spectrum obtained near the damping minimum is valid throughout the range of permissible values of the wave vector **k**. The dependences of the natural frequency, polarization, and damping on **k**, on the magnetic induction **B**, and on the angle between **B** and **k** are investigated for all waves. The waves near cyclotron resonance or the low-frequency waves are shown to be connected with the classical discrete spectrum.

PACS numbers: 76.40. + b, 76.50. + g

INTRODUCTION

Weakly damped waves in a metallic ferromagnet near cyclotron-resonance frequencies were first investigated by Blank, Kaganov, and Yui Lu[1] (see also the review[2]). These waves are close to cyclotron waves in a normal metal and propagate perpendicular to the magnetic-induction field of the ferromagnet, i.e., they are excited in a magnetic field parallel to the surface of the sample. In contrast to Ref. 1, where the waves investigated were short compared with the cyclotron radius R of the conduction electrons, Bar'yakhtar, Savchenko, and Stepanov[4] considered weakly damped excitations near cyclotron resonance, with wavelength $2\pi k^{-1}$ of the order of R. Silin and Solontsov[5] investigated spin cyclotron waves in a magnetic field parallel to the sample surface. These waves propagate near the frequency of the electron transition between the Landau levels, with reversal of the spin direction.

Weakly damped waves near cyclotron resonance in metallic ferromagnets, propagating at arbitrary angle to the magnetic-induction field, have to our knowledge not been investigated. These waves are excited in a magnetic field inclined to the sample surface. The first to investigate waves in a normal metal, close to cyclotron resonance and propagating at an angle close to $\pi/2$ with the magnetic field, were Blank and Kaner.[6] In view of further progress in the theory of cyclotron resonance in an oblique magnetic field,[7,8] the study of such waves in normal metals has been recently continued.[7,9,10] It is of interest to extend these investigation to include metallic ferromagnets. The appearance of sufficiently pure materials[11,12] uncovers a possibility of performing more and more subtle experiments aimed at observing electron resonances in ferromagnets.

Weakly damped waves propagating at close to a right angle to the magnetic field in ferromagnets, at frequencies far from the cyclotron frequency, were investigated in a number of studies. Thus, Zverev, Silin, and Solontsov[13] investigated the influence of an oblique magnetic field on spin cyclotron waves. The peculiar low-frequency ($\omega \ll \Omega$, Ω is the cyclotron frequency of the conduction electrons in the induction field) excitations having a classical discrete spectrum were investigated in detail[14,15] in ferromagnets in an oblique magnetic field. The possibility of propagation of such waves is due to the vanishing of the Landau damping at a definite relation between k and R.[16] Analogous propagation conditions make waves near cyclotron resonance similar to waves with a discrete spectrum, so that it is also of interest to trace the connection between these excitations.

DISPERSION EQUATION

To describe the propagation of a weakly damped wave in an unbounded isotropic metallic ferromagnetic, we use the system of Maxwell's equations and the Landau-Lifshitz equation

$$i[\mathbf{k}\times\mathbf{h}]=4\pi c^{-1}\mathbf{j},\quad [\mathbf{k}\times\mathbf{E}]=\omega c^{-1}\mathbf{b}, \tag{1}$$

$$-i\omega\mathbf{m}/\gamma=[\mathbf{m}\times\mathbf{H}]+[\mathbf{M}\times\mathbf{h}]+i\lambda\omega[\mathbf{M}\times\mathbf{m}]. \tag{2}$$

Here **E** is the alternating electric field, **m**, **h**, and **b** are, respectively, the high-frequency parts of the magnetization, of the magnetic field, and of the induction ($\mathbf{m}, \mathbf{h}, \mathbf{b}, \mathbf{E} \propto e^{i(\mathbf{k}\cdot\mathbf{r}-t)}$), **H** is the external constant magnetic field, **M** is the saturation magnetization, $\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$, $\mathbf{b}=\mathbf{h}+4\pi\mathbf{m}$, **j** is the current density, **k** is the wave vector, ω is the wave frequency, γ is the gyromagnetic ratio, and λ is the damping in the magnetic subsystem. We neglect in Maxwell's equations the displacement current and it follows then from (1) that

$$\mathbf{kj}=0, \tag{3}$$

which is analogous to the condition that the metal be quasineutral.

We choose the coordinate system xyz such that the z axis is parallel to the magnetic field **B** and the x axis is perpendicular to the plane containing **B** and **k**. In addition, we introduce also a coordinate system $x\eta\zeta$, in which the ζ axis is parallel to **k** (Fig. 1), and makes an angle $\pi-\varphi$ with the y axis. The connection between the

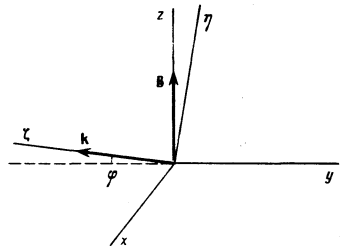

FIG. 1. Coordinate frame. The x axis is perpendicular to the plane passing through the magnetic induction vector **B** and the wave vector **k**. The ζ axis is parallel to **k** and makes an angle $\pi-\varphi$ with the y axis.

current and the electric field, as well as between the high-frequency induction and high-frequency magnetic field, is established with the aid of the conductivity and magnetic permeability, respectively. Then, substituting the second equation of (1) in the first and eliminating with the aid of (3) the electric-field component E_ζ which is longitudinal with respect to k, we reduce Eqs. (1) to a system of linear homogeneous equations for the transverse components E_x and E_η. The vanishing of the determinant of this system yields the dispersion equation of the weakly damped waves in a metallic ferromagnet:

$$16\pi^2\omega^2 \det\hat{\sigma} \det\hat{\mu} + 4\pi i\omega k^2c^2 \operatorname{Sp}\hat{\sigma}\hat{\beta} - k^4c^4 = 0. \tag{4}$$

The first term contains the product of the determinants of the two-dimensional matrices $\hat{\sigma}$ and $\hat{\mu}$, made up of the elements $\tilde{\sigma}_{\alpha\beta}$ and $\tilde{\mu}_{\alpha\beta}$. In the second term the trace is taken of the product of the matrices, where β is obtained from the matrix $\hat{\mu}$ by cyclic permutation of the indices followed by reversal of the signs of the off-diagonal elements

$$\tilde{\sigma}_{\alpha\beta} = \sigma_{\alpha\beta} - \sigma_{\alpha\zeta}\sigma_{\zeta\beta}/\sigma_{\zeta\zeta}, \tag{5}$$

where $\sigma_{ij} \equiv \sigma_{ij}(\mathbf{k}, H, \omega)$ are the Fourier components of the conductivity tensor. The indices α, β and i, j take on the respective values x, η and x, η, ζ:

$$\tilde{\mu}_{\alpha\beta} = \mu_{\alpha\beta} - \mu_{\alpha\zeta}\mu_{\zeta\beta}/\mu_{\zeta\zeta}, \tag{6}$$

where $\mu_{ij} \equiv \mu_{ij}(\mathbf{k}, \mathrm{H}, \omega)$ are the Fourier components of the magnetic permeability.

In the case of an isotropic ferromagnet we obtain from the Landau-Lifshitz equation (2) for the renormalized components of the magnetic-permeability tensor $\tilde{\mu}_{\alpha\beta}$ (6), neglecting the terms $\propto\varphi^2$,

$$\tilde{\mu}_{xx} = \frac{\omega_{\rm I}^2-\omega^2}{\omega_{\rm II}^2-\omega^2}, \quad \tilde{\mu}_{x\eta} = -i\varphi\frac{\omega\omega_M}{\omega_{\rm II}^2-\omega^2}, \quad \tilde{\mu}_{\eta\eta}=1. \quad \tilde{\mu}_{\eta x} = -\tilde{\mu}_{x\eta};$$
$$\omega_{\rm I} = \omega_H + \omega_M, \quad \omega_{\rm II}^2 = \omega_{\rm I}\omega_H, \tag{7}$$
$$\omega_H = \gamma H - i\beta\omega, \quad \omega_M = 4\pi\gamma M, \quad \beta = \gamma M\lambda.$$

If we now go in the dispersion equation (4) to the ferromagnetic-dielectric limit ($\tilde{\sigma}_{\alpha\beta}\to 0$), we obtain at $k\neq 0$ the dispersion law of the magnetostatic wave that is transverse to **B** (see, e.g., Ref. 17):

$$\omega_s = \gamma(HB)^{1/2}. \tag{8}$$

In the case $M=0$ ($\mu_{\alpha\beta}=\delta_{\alpha\beta}$) we obtain from (4) the known dispersion equations[16] for a normal metal.

We shall consider hereafter waves near cyclotron resonance

$$|\nu - i(\omega - N\Omega)| \ll \Omega,$$

where N is an integer and ν is the electron collision frequency. The wavelength is assumed to be much smaller than the cyclotron radius of the conduction electrons ($kR \gg 1$). We also assume a strong spatial dispersion of the wave in the **B** direction:

$$|\nu - i(\omega - N\Omega)| \ll k_z v,$$

where v is the Fermi velocity of the conduction electrons. The inverse limiting case corresponds to the wave considered by Blank *et al.*[1] We note next that the main contribution to σ_{ij} is made by the resonant electrons that satisfy the phase relation

$$k_z v_z + N\Omega = \omega. \tag{9}$$

We assume that $k_z v \ll \Omega$; then, at a given ω, the relation (9) is satisfied only for one value of N, i.e., we can confine ourselves to only one group of resonant electrons. Gathering together all the inequalities presented above, we can write

$$|\nu - i(\omega - N\Omega)| \ll k_z v \ll \Omega \ll kv. \tag{10}$$

From this follows, in particular, that

$$\varphi \ll 1/kR \quad (k_x = 0, \ k_y \approx -k, \ k_z \approx \varphi k).$$

The asymptotic expressions for the components of the conductivity tensor under conditions (10), in the approximation where the conduction electrons have an isotropic and quadratic dispersion law, were obtained earlier.[9] It was shown that in the real part of σ_{xx}, which is mainly responsible for the damping of the wave, there is an oscillating term

$$1 - (-1)^N \sin(2\alpha + w^2),$$

which describes the magnetic Landau damping[16,18] and vanishes at

$$\alpha + w^2/2 = \alpha_n \equiv \pi[n + (-1)^N/4], \quad n = 1, 2, 3, \ldots. \tag{11}$$

Here $\alpha = kR$, and the parameter w is a measure of the deviation of the section of the Fermi surface of the resonant electrons from the central section. The mechanism of such oscillations of the Landau damping of electromagnetic waves that propagate at an angle to the magnetic field was first described by Kaner and Skobov[18] and is analogous to the geometric resonance that occurs when ultrasound is attenuated in a metal in a magnetic field.[19]

The expressions for the renormalized conductivity-tensor components $\tilde{\sigma}_{\alpha\beta}$ in the vicinity of the points α defined by (11), i.e., near the minimum of the wave damping, are of the form

$$\tilde{\sigma}_{xx} = \sigma_0[\xi + \eta(2\rho Q - a) + i(\rho/\alpha_n^{1/2} - P + Q^2 f)],$$
$$\tilde{\sigma}_{\eta\eta} = \sigma_0(w/\alpha_n^{1/2} - f), \quad \tilde{\sigma}_{x\eta} = -\sigma_0[(a - 2\rho Q)/2\alpha_n^{1/2} + Qf], \quad \tilde{\sigma}_{\eta x} = -\tilde{\sigma}_{x\eta}. \tag{12}$$

Here

$$\sigma_0 = 3n_0e^2/2m\alpha^2\Omega\varphi, \quad \xi = 2\sin^2(\alpha - \alpha_n - \rho^2/2), \quad f = N/2\varphi\alpha_n^2,$$
$$a = 2\pi^{-1/2}\sin(\rho^2 - \pi/4), \quad P = C(\rho) - S(\rho), \quad Q = C(\rho) + S(\rho),$$

where n_0 is the density of the conduction electrons and m is the electron effective mass. C and S denote the Fresnel integrals

$$S(\rho)=\left(\frac{2}{\pi}\right)^{1/2}\int_0^\rho \sin\tau^2 d\tau,\quad C(\rho)=\left(\frac{2}{\pi}\right)^{1/2}\int_0^\rho \cos\tau^2 d\tau.$$

In addition, we have used in (12) the notation

$$\rho=(\omega_0-N\Omega)/\varphi\alpha_n^{1/2}\Omega,\quad \eta=(\nu-\Gamma N\Omega)/\varphi\alpha_n^{1/2}\Omega, \tag{13}$$

where $\omega=\omega_0(1-i\Gamma)$, Γ is the relative damping of the wave, and $w=\eta-i\rho$.

It follows from (10) that $|\rho|\ll\alpha^{1/2}$. In the expression for η we put $\omega_0=N\Omega$. We consider the case $f\ll 1$, and therefore omit from (12) terms of higher order in f. From the condition on f and from the inequalities (10) we have $N/2\alpha^2\ll\varphi\ll\alpha^{-1}$.

SPECTRUM, POLARIZATION, DAMPING

We turn now to the dispersion equation (4). It is known that the modification of the spectrum of the interacting excitations is most substantial at the intersection of their spectral curves. The intersection of ω_s and $N\Omega$ as functions of B in an external magnetic field is possible only at $\gamma>Ne/mc$. Expressing this inequality in the form of the ratio of the characteristic frequencies, we obtain $\omega_a>N\Omega$, where $\omega_a=\mathrm{Re}\,\omega_1=\gamma B$ is the antiresonance frequency. The magnetic field corresponding to the equation $\omega_s=N\Omega$ is equal to

$$H'=4\pi M/[(\omega_a/N\Omega)^2-1].$$

The excitation spectrum near cyclotron resonance in a magnetic field parallel to the sample surface also depends on the ratio $\omega_a/\Omega=gm/2m_0$ (m_0 is the mass of the free electron and g is the Landé factor), a fact shown by Blank *et al.*[1]

Changing over to the solution of the dispersion equation, we note that since we have fixed kR, it is convenient to change in (4) from k to α_n. In this case

$$k^2c^2/4\pi\omega_0\sigma_0=b^2/q\equiv X,\quad b^2=B^2/3\pi^{1/2}n_0mv^2,$$
$$q=2\pi^{1/2}(\rho+N/\varphi\alpha_n^{1/2})\alpha_n^{-1/2},$$

where b is the normalized magnetic-induction field ($b\sim\Omega c/\omega_p v$, and ω_p is the plasma frequency of the condition electrons). Substituting the expressions for the elements of the conductivity tensor $\tilde\sigma_{\alpha\beta}$ (12) and the magnetic permeability tensor $\tilde\mu_{\alpha\beta}$ (7) in the dispersion equation (4), we obtain for its real part

$$b^4-b_sb^3-A_1b^2-A_2b_sb-A_3=0, \tag{14}$$

where

$$A_1=q(\sigma_2-\sigma_1),\quad A_2=q[\sigma_1-2\varphi\delta^{-1}(N/\varphi\alpha_n^{1/2}+\rho)\sigma_3],\quad \delta=\omega_a/\varphi\alpha_n^{1/2}\Omega,$$
$$A_3=q^2(\sigma_3^2+\sigma_1\sigma_2),\quad \sigma_1=\mathrm{Im}\,\tilde\sigma_{xx}/\sigma_0,\quad \sigma_2=-\mathrm{Im}\,\tilde\sigma_{\eta\eta}/\sigma_0,\quad \sigma_3=\tilde\sigma_{x\eta}/\sigma_0.$$

The quantity

$$b_s=\frac{\delta^2\mu}{\delta^2-(N/\varphi\alpha_n^{1/2}+\rho)^2} \tag{8a}$$

is the spectrum of the magnetostatic wave in terms of the variables b and ρ:

$$\mu^2=(4\pi M)^2/3\pi^{1/2}n_0mv^2.$$

For a normal metal ($\mu=0$), the equation analogous to (14) is biquadratic. The previously obtained[9] solution of this equation is

$$b_\pm^2=\tfrac{1}{2}q\{[P-(1+Q^2)f]\pm(L^2+T^2)^{1/2}\},$$
$$L=2\rho\alpha_n^{-1/2}-P+(Q^2+1)f,\quad T=(2\rho Q-a)/\alpha_n^{1/2}-2Qf. \tag{15}$$

Equation (14) can be solved for the entire range of permissible values of ρ ($|\rho|\ll\alpha^{1/2}$) only numerically. We consider first the case when ω_s crosses $N\Omega$, with $|\omega_a-N\Omega|\gg|N\Omega-\omega_0|$, i.e., we consider waves far from antiresonance. We construct the solution in the form of the relation plotted in Fig. 2 by thick solid lines. Account is taken of the fact that $b>\mu$ in an isotropic ferromagnet. In the calculation we assumed $4\pi M=22$ kG, $n_0=10^{23}$ cm^{-3}, $v=1.46\cdot10^8$ cm$\cdot$sec^{-1} (in which case $\mu=6\cdot10^{-3}$), $N=1$, and $\omega_a/\Omega=1.5$. The thin dashed curves of Fig. 2 show the function $b_\pm(\rho)$ that follows from Eq. (15) that describes the spectrum of the electromagnetic waves in a normal metal. The thin solid line is the spectrum of the magnetostatic wave (8a). It is seen from Fig. 2 that, depending on the relation between $b_\pm$ and b_s, the spectrum of the coupled waves in a metallic ferromagnet will consist of different numbers of branches. If $b_s>b_\pm$ and $b_+<\mu$, then the spectrum has only one branch b_1, which lies in the entire considered range of values of ρ. We note immediately that this branch of the spectrum describes a right-polarized wave. On the other hand, $b_+>\mu$ at $\rho<0$, a second branch b_2 close to b_+ appears in the spectrum of the coupled waves (Fig. 2a). However, b_2 describes a left-polarized wave, whereas b_+ in a normal metal is right-polarized. When b_s crosses b_+, the spectral curve of the right-polarized wave b_1 breaks up into two branches (Fig. 2b). In this case b_2 shifts towards lower values of b compared with b_+. Finally, when b_s approaches b_-, the spectral curve b_1 becomes close to b_+, and b_2 becomes close to b_- (Fig. 2c). We note that in all cases at $|\rho|\gtrsim1$ the slope of the spectral curve b_1 of the cou-

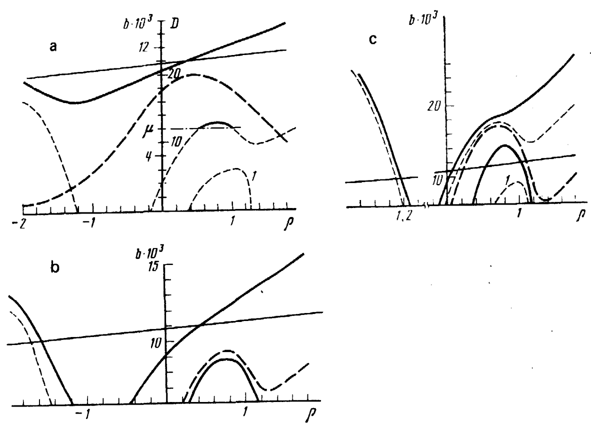

FIG. 2. The thick solid curves show the spectrum of the coupled waves, $N-1$, $\omega_a/\Omega=1.5$. The thin dashed curves and the thin solid lines show the spectrum of the electromagnetic waves in the normal metal (15) and of the magnetostatic wave in the ferromagnet (8a); curve 1—spectrum of left-polarized wave $b_-(\rho)$; a) $n-17$, $\varphi-4\cdot10^{-3}$, the thick dashed line shows the ρ-dependence of the polarization coefficient D, b) $n=14$, $\varphi=4\times10^{-4}$; c) $n=8$, $\varphi=10^{-2}$, the thick dashed line shows the spectrum of the coupled wave near the antiresonance ($N=\omega_a/\Omega=1$). In Figs. b and c, the induction b is reckoned from the value of the normalized magnetization μ.

pled wave in the ferromagnet coincides with the slope of b_+.

If ω_s does not cross $N\Omega$, but $|\omega_a - N\Omega| \gg |N\Omega - \omega|$ as before (for example, $N=2$, $\omega_a/\Omega = 1.5$), the spectrum of the coupled waves in the ferromagnet will be close to the spectrum of the waves in the normal metal, and the polarizations of the spectrum branches are also preserved; $b_1 \approx b_+$ and $b_2 \approx b_-$. An interaction between b_+ and b_- now appears, so that at $\rho > \rho_2$ (at the upper limit of the region of existence of b_-), b_1 will lie somewhat lower than b_+. At lower values of the induction, b_2 is likewise shifted in comparison with b_-.

Particular interest attaches to the case when the cyclotron resonance and antiresonance frequencies coincide ($|\omega_a - N\Omega| \ll |N\Omega - \omega|$). Then $b_s \gg b_+$ and Eq. (14) has a solution $b_1' \approx b_s$ that coincides with the spectrum of the magnetostatic wave. In addition, there are solutions that are close to the spectrum of the electromagnetic waves in a normal metal. Thus, at $\rho < 0$ the branch $b_2' \approx b_+$ and is left-polarized (the magnetostatic mode is right-polarized in this region of ρ). At $\rho > 0$, there exist a right-polarized branch b_3' and a left-polarized b_4'. In the region $\rho < \rho_2$ the branch $b_3' \approx b_+$. At $\rho \gtrsim \rho_2$ an interaction appears between b_+ and b_-, but a stronger one than in the preceding case, so that b_3' lies much lower than b_+ (Fig. 2c, thick dashed line). The branch b_4' is likewise shifted towards lower values of the induction compared with b_-. Thus, at certain values of the physical parameters, by virtue of the condition $b > \mu$, an interval $\rho > \rho_2$ in which there are no waves can exist.

We now represent the spectrum of the wave in the form of the function $\omega_0(k)$ at a fixed value of the magnetic field H. From the definition (13) of ρ we obtain

$$\omega_0 = N\Omega[1 + \varphi\alpha_n^{1/2} N^{-1}\rho(B, \alpha_n, \varphi)], \tag{16}$$

where ρ is no longer a function of B, α_n, and φ, and must be determined from (14). At fixed H and φ, the function $\rho(\alpha_n)$ can be obtained by using the numerical solutions $b(\rho)$ of Eq. (14), after which it is easy to construct the dispersion curve $\omega_0(k)$, a plot of which at $B = 4.25 \cdot 10^4$ G and $\varphi = 10^{-2}$ is shown in Fig. 3 in the coordinates Δ/Ω and kR, where $\Delta = \omega_0 - \Omega$. The dashed lines join points belonging to one and the same branch of the spectrum in terms of the coordinates b and ρ. It is seen

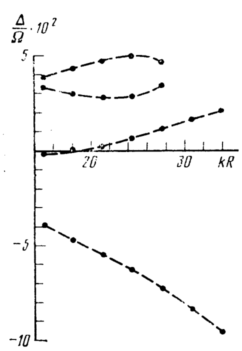

FIG. 3. Spectrum of coupled waves. Plot of $\omega_0(k)$ at $N=1$ and at constant H. The points mark the values of ω_0 at $kR = \alpha_n$. The dashes join points of the same branch of the spectrum; $\Delta = \omega - \Omega$.

that the branch that exists in the region $\omega < \Omega$ has anomalous dispersion.

We note that the spectral curves of the coupled and magnetostatic waves intersect. It can be found from (14) that the value of ρ at the intersection point is given by the equation

$$b_s^2 = -q(\sigma_3^2 + \sigma_1\sigma_2)/\sigma_2. \tag{17}$$

The right-hand side of (17) is small ($\sim b_+^2$) everywhere with the exception of the vicinity of the point where σ_2 goes through zero. Therefore at $b_s \gg b_+$ the intersection point of ω_0 and ω_s is determined by the vicinity of the zero of σ_2. Determining ρ from the condition $\sigma_2 = 0$ and using (16), we obtain approximately the spectrum of the coupled wave at frequencies close to ω_s:

$$\omega_0 = N\Omega[1 + (2\alpha_n)^{-1}]. \tag{18}$$

In limiting cases, analytic solutions of the dispersion equation can be obtained. At $|\rho| \gg 1$ the coefficients of Eq. (14) are given by

$$A_1 \approx 0, \quad A_2 \approx q\rho\alpha_n^{-1/2}, \quad A_3 \approx 2q^2\rho^2\alpha_n^{-1}$$

($P \approx 0$, $|Q| \approx 1$). Solving (14) for ρ, we obtain

$$\rho = -b^2\alpha_n^{1/2} U_\pm/2q,$$
$$U_\pm = s \pm (s^2 - 2s + 2)^{1/2}, \qquad s = \omega_a\omega_M/(\omega_a^2 - \omega_0^2) \equiv b_s/b. \tag{19}$$

Using (16), we get

$$\omega_0 = N\Omega[1 - {}^1/_3\alpha_n^5 K^2 U_\pm], \tag{20}$$

where $K = \varphi\Omega c/N\omega_p v$. Letting M tend to zero, we obtain from this an expression for the wave spectrum in the normal metal.[9] In the opposite limit $|\rho| \ll 1$ ($P \approx Q \approx (2/\pi)^{1/2}\rho$) we have

$$A_1 \approx q[(2/\pi)^{1/2}\rho - f], \quad A_2 \approx -q(2/\pi)^{1/2}\rho, \quad A_3 \approx q^2(2\pi\alpha_n)^{-1}.$$

We have neglected here terms of higher order in ρ and α_n^{-1}.

Solving now Eq. (14) for ρ and using (16), we get under the condition $s \neq 1$

$$\omega_0 = N\Omega\{1 + \tilde{K}^2\alpha_n^{9/2} + [(\pi/8)^{1/2}\alpha_n^{-3/2} - {}^1/_4\tilde{K}^{-2}(\varphi/N)^2\alpha_n^{-9/2}]/(1-s)\}, \tag{21}$$

where $\tilde{K} = (2\pi/9)^{1/2}K$. At $M=0$ this yields the spectrum obtained by Blank and Kaner[6] for electromagnetic waves in a normal metal.

In a ferromagnet, just as in a normal metal, the spectrum of the wave can cross $N\Omega$. In the expression for s at $|\omega_a - N\Omega| \gg |N\Omega - \omega_0|$ we can put $\omega_0 \approx N\Omega$. In the opposite limit, i.e., if the cyclotron resonance and antiresonance frequencies coincide, replacing ω_a by $N\Omega$ in the expression for s and recognizing that in this case $s = -\delta\mu/2\rho b$ and $|s| \gg 1$ ($U_+ = 2s$, $U_- = 1$), we obtain from (19) and (20)

$$\omega_0 = N\Omega[1 - {}^1/_3\alpha_n^5 K^2], \tag{22}$$

$$\omega_0 = N\Omega[1 \pm 3^{-1/2}\alpha_n^{5/2} K(\omega_M/N\Omega)^{1/2}]. \tag{23}$$

Expression (22) describes a wave close to an electromagnetic wave in a normal metal ($b_2' = 2^{1/4}b_+$). The dispersion law (23) is typical only of a wave in a ferromagnet and does not go over at $M=0$ into any expression for the spectrum of the wave in a normal metal. By virtue of the condition $b>\mu$, the waves (22) and (23) exist respectively at

$$|\rho|>\varphi\alpha_n^{5/2}\mu/4\pi^{1/2}, \quad |\rho|>\mu\alpha_n^2/2\pi^{1/2}.$$

In the limit $|\rho|\ll 1$ at $\omega_a\approx N\Omega$ we find, since $|s|\gg 1$, that the spectrum of the wave is described by the first two terms in the curly brackets of (21). In this case the wave propagates only at a frequency higher than $N\Omega$. The dispersion law of this wave coincides with the spectrum obtained by Blank and Kaner[6] for an electromagnetic wave in a normal metal.

The ratio of the longitudinal and transverse components of the electric field vector **E** of a weakly damped wave in a ferromagnet is determined by the same expression as in the case of a normal metal[9]:

$$E_\zeta=-(iQE_x+E_\eta)/2\varphi\alpha,$$

whence $E_\zeta\gg E_x$, E_η, i.e., the field **E** in the coupled wave wave is directed mainly along **k**. The transverse components of the electric field are connected by the relation

$$E_\eta=-iDE_x,$$

where D is the polarization coefficient,

$$D=\frac{k^2c^2/4\pi\omega-i(\tilde{\sigma}_{xx}-\tilde{\mu}_{x\eta}\tilde{\sigma}_{x\eta})}{\tilde{\sigma}_{x\eta}+\tilde{\mu}_{x\eta}\tilde{\sigma}_{\eta\eta}}. \tag{24}$$

Changing, just as the dispersion equation, from k to α we have near the damping minimum, i.e., at α determined by (11),

$$D=\left[X+\sigma_1+\varphi\frac{s}{1-s}\frac{N\Omega}{\omega_a}\sigma_3\right]\Big/\left[\sigma_3-\varphi\frac{s}{1-s}\frac{N\Omega}{\omega_a}\sigma_2\right]. \tag{25}$$

When we considered above the spectrum of the coupled waves, we discussed also the sign of D (the wave is right-polarized at $D>0$ and left-polarized at $D<0$). As for the $D(\rho)$ dependence, it differs in a ferromagnet from the form obtained for a normal metal[9] primarily because of the terms $\sim\mu_{x\eta}$ in (24), which are small everywhere except in the vicinity of the point of intersection of ω_0 and ω_s. Near this point $s\approx 1$ and according to (25) $D\approx-\sigma_3/\sigma_2$. At $b_s\gg b_+$, as noted above, the intersection of ω_s and ω_0 takes place in the vicinity of a zero of σ_2, and it is this which causes the existence and position of the maximum of the function $D(\rho)$. Far from the point of the intersection of ω_0 and ω_s, those terms in D which are proportional to $\tilde{\mu}_{x\eta}$ can be neglected, and the difference between D of the coupled wave and its value in the normal metal is determined by the difference between the spectra of the coupled and electromagnetic waves (Fig. 2a).

In the limit $|\rho|\gg 1$, substituting in (25) the expression for b from (19) and expanding $\sigma_{1,2,3}$ accurate to $\rho\alpha^n$, we obtain

$$D=1-2U_\pm^{-1}.$$

At $|s|\gg 1$ it follows from this that $D=\pm 1$, i.e., in this case the wave polarization is circular and, in particular, the wave (22) is left-polarized and the wave (23) right-polarized. If $|\rho|\ll 1$, then in the case $|s|\gg 1$ we have

$$D=(2\pi)^{1/2}\rho-\varphi N\Omega/\omega_a.$$

At $|s|\ll 1$ the polarization, just as the wave spectrum, is close to its value in the normal metal.

The damping of the coupled waves in a ferromagnet is determined from the condition that the imaginary part of the dispersion equation (4) vanish, and has an electron contribution due to the dissipative currents and the Landau damping, as well as a magnetic contribution due to the absorption of energy of the wave by the magnetic subsystem of the ferromagnet. The relative damping of the coupled wave is

$$\Gamma=\frac{\nu}{N\Omega}\frac{l}{r}+\xi\frac{\psi}{r}+\beta\frac{L_0\omega_0(2\omega_a-\omega_M)}{r}, \tag{26}$$

where

$$\psi=\sigma_2(\omega_0^2-\omega_a^2)-X(\omega_0^2-\omega_s^2), \quad r=l+2\omega_0^2X^2,$$

$$l=-\frac{N}{\varphi\alpha^{1/2}}[(a-2\rho Q)\psi+(\omega_0^2-\omega_a^2)X\alpha_n^{-1/2}].$$

Here L_0 determines the dispersion equation of an electromagnetic wave in a normal metal; this equation can be obtained from (14) by multiplying by q^{-2} at $M=0$.

If the coupled and electromagnetic waves are close, $X\ll 1$ and the last term of r can be neglected; then

$$\Gamma=\nu/N\Omega+\xi\varphi\alpha_n^{1/2}/N(2\rho Q-a). \tag{27}$$

The magnetic part of the damping is small in this case, since L_0 is close to zero, i.e., the damping of the coupled wave, is the same as in a normal metal.[9] We note that the damping is close to (27) wherever $X\ll N/\varphi\alpha_n^{1/2}$; an exception is the situation $\omega_0\approx\omega_s$. In this case $\psi\sim\sigma_2$ and, since the point of intersection of ω_0 and ω_s is determined at $b_s\gg b_+$ by the vicinity of the zero of σ_2, the value of ψ will tend to zero, while l is determined by the last term. In this case the Landau damping of the coupling wave is small, and the magnetic damping is increased by a factor $\alpha^{1/2}$. If, furthermore, $X\gg N/\alpha_n\varphi$, the magnetic part of the damping is

$$\Gamma_2=\beta(2\omega_a-\omega_M)/2\omega_0$$

($L_0\approx X^2$ at $b\gg b_+$), which coincides with the damping of the damping of the magnetostatic wave. At $X\gg N/\varphi\alpha_n^{1/2}$, the same expression describes the total damping of the coupled wave.

The dispersion equation (14) describes also a low-frequency ($\omega\ll\Omega$) wave with a discrete spectrum. At $N=0$ Eq. (14) leads to an equation similar to that obtained earlier.[15] The difference between the coefficients is due to the fact that in Ref. 15 the integral in σ_{xx} was calculated approximately, but this leads only to a numerical difference between the results; all the wave features noted earlier[15] remain in force here, too.

CONCLUSION

Thus, the dispersion equation obtained for weakly damped waves propagating in a ferromagnet at an angle to the magnetic field describes both waves near cyclotron resonance, as well as low-frequency ($\omega\ll\Omega$) waves with a classical discrete spectrum. The number of branches of the spectrum of the coupled waves and their polarization depend on the relation between the resonant value of the induction field and the magnetization of the ferromagnet, as well as on the magnitude and the sign of the frequency difference between the cyclotron resonance and antiresonance. A distinguishing feature of the spectrum of the coupled waves is its crossing the spectrum of the magnetostatic wave. In the vicinity of this point, the dispersion law of the wave $\omega_0(k)$ has the simple form (18), and the polarization coefficient D reaches a maximum. This behavior of D explains the obtained decrease of the Landau damping of the wave at $\omega_0 \approx \omega_s$. Indeed, the increase of D corresponds to a decrease of the component of the electric field E_x, and since the magnetic Landau damping is described by $\tilde{\sigma}_{xx}$, this leads to a decrease of the Landau damping. The latter is evidence, particularly, that (18) is valid also when α is far from $\tilde{\alpha}_n$. This conclusion follows also from an analysis of the dispersion equation (4).

The cyclotron radius of the resonant electrons is $R_\perp = R(1=\rho^2/2\alpha)$, so that the data obtained by observing the coupled waves near the cyclotron resonance can be used to obtain information on the non-extremal sections of the Fermi surface of the conduction electrons in the ferromagnet. We note, however, that the section of the resonant electrons can be distinguished from the central section if the difference of their radii $R\rho^2/2\alpha$ is at least larger than the straggling $\Delta R = R(\Delta\theta)^2/2$ of the radii of the resonant electrons, which is determined by the width $\Delta\theta = \nu/k_z v$ of the resonance curve

$$[\nu - i(\omega - N\Omega - k_z v\cos\theta)]^{-1},$$

i.e., if $\nu/k_z v < \rho/\alpha^{1/2}$; $v_z = v\cos\theta$. From this inequality it follows that $\nu < |\omega_0 - N\Omega|$. At $B \approx 4\cdot 10^4$ G ($\Omega \approx 6\cdot 10^{11}$ sec^{-1}), $\varphi \approx 10^{-2}$, and $k\Omega \approx 30$, there are two waves in the ferromagnet. For one of them $\omega_0 - \Omega \approx 0.02\Omega$, and for the other $\Omega - \omega_0 \approx 0.8\Omega$, so that the last inequality is satisfied at a collision frequency $\nu \lesssim 10^{10}$ sec^{-1}, which is typical of sufficiently pure materials at low temperature. It can be noted that the system of inequalities (10) is likewise easily satisfied at the chosen values of the physical parameters. The most convenient for observation is the right-polarized wave in cases when it is the only mode near the cyclotron resonance. Far from antiresonance, this is the wave b_1, and in particular at $\omega_0 \approx \omega_s$ its dispersion law is determined by (18), while at $\omega_a \approx N\Omega$ it is the wave b_3', the dispersion law for which at $\rho \gg 1$ is of the form (23).

The existence of a weakly damped wave in a metal in an oblique magnetic field leads to inversion of the cyclotron-resonance line. By determining the position of the inverted peak as a function of b and φ, we can reconstruct the spectrum of the wave. The value of the wave vector is specified by the condition (11). At a fixed frequency of the external electromagnetic field, the resonant value of the induction B_r, which corresponds to the inverted peak, is given by the expression

$$B_r^{-1} = B_N^{-1}(1 + \rho\varphi\alpha^{1/2}/N),$$

where $B_N^{-1} = eN/\omega mc$, and determines the cyclotron-resonance line in a magnetic field parallel to the sample surface. The expression for B_r makes it also possible, given ω, to estimate the maximum interval ΔB of the change of the induction field, i.e., the interval near B_N in which one should expect the appearance of the inverted peak. Since $\rho \ll \alpha^{1/2}$, it follows that $\Delta B \ll 2B_N\varphi\alpha/N$. We note also that the intersection of the spectral curves of the weakly damped wave and the magnetostatic wave can apparently be used to observe weakly damped waves at $\omega_0 \approx \omega_s$, by exciting the magnetic subsystem of the ferromagnet.

[1]A. Ya. Blank, M. I. Kaganov, and Yui Lu, Zh. Eksp. Teor. Fiz. **47**, 2168 (1964) [Sov. Phys. JETP **20**, 1456 (1965)].
[2]A. Ya. Blank and M. I. Kaganov, Usp. Fiz. Nauk **92**, 583 (1967) [Sov. Phys. Usp. **10**, 536 (1968)].
[3]É. A. Kaner and V. G. Skobov, Fiz. Tverd. Tela (Leningrad) **6**, 1104 (1964) [Sov. Phys. Solid State **6**, 851 (1964)].
[4]V. G. Bar'yakhtar, M. A. Savchenko, and K. I. Stepanov, Zh. Eksp. Teor. Fiz. **50**, 576 (1966) [Sov. Phys. JETP **23**, 383 (1966)].
[5]V. P. Silin and A. Z. Solontsov, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 375 (1974) [JETP Lett. **20**, 168 (1974).
[6]A. Ya. Bland and É. A. Kaner, Phys. Stat. Sol. **22**, 47 (1967).
[7]I. E. Aronov and É. A. Kaner, Fiz. Nizk. Temp. **2**, 1277 (1976) [Sov. J. Low Temp. Phys. **2**, 622 (1976)].
[8]I. E. Aronov and É. A. Kaner, *ibid.* **4**, 76 (1978) [4, 38 (1978)].
[9]Yu. I. Man'kov, L. M. Gorenko, and R. G. Khelbopros, Zh. Eksp. Teor. Fiz. **77**, 2142 (1979) [Sov. Phys. JETP **50**, 1028 (1979)].
[10]Yu. I. Man'kov, Fiz. Tverd. Tela (Leningrad) **22**, 408 (1980) [Sov. Phys. Solid State **22**, 239 (1980)].
[11]G. Dewar, B. Heinrich, and J. F. Cochran, J. Appl. Phys. **50**, 2437 (1979).
[12]R. V. Coleman, W. H. Lowrey, and J. A. Polo, Jr., Phys. Rev. **B23**, 2491 (1981).
[13]V. M. Zverev, V. P. Silin, and A. Z. Solontsov, Fiz. Met. Metallov. **42**, 704 (1976).
[14]E. M. Chudnovskii, Ukr. Fiz. Zh. **18**, 485 (1973).
[15]L. M. Gorenko, Yu. I. Man'kov, and R. G. Khlebopros, Zh. Eksp. Teor. Fiz. **71**, 627 (1976) [Sov. Phys. JETP **44**, 329 (1976); Fiz. Tverd. Tela (Leningrad) **19**, 56 (1977) [Sov. Phys. Solid State **19**, 31. (1977)].
[16]É. A. Kaner and V. G. Skobov, Usp. Fiz. Nauk **89**, 367 (1966) [Sov. Phys. Usp. **9**, 480 (1967)].
[17]A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskiĭ, Spinovye volny (Spin Waves), Nauka, 1967, p. 62. [Wiley, 1968].
[18]É. A. Kaner and V. G. Skobov, Zh. Eksp. Teor. Fiz. **45**, 610 (1963) [Sov. Phys. JETP **18**, 419 (1964)].
[19]A. B. Pippard, Phil. Mag. **2**, 1147 (1957).

Translated by J. G. Adashko