Photon emission by an electron in the field of an intense plane electromagnetic wave, with effects of a constant magnetic field included

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We consider photon emission by an electron moving in a constant magnetic field and in the field of an intense plane electromagnetic wave with circular polarization. With the approach developed here, based on the use of exact solutions of the quantum equations of motion of charged particles, we obtained for the probability of emission a compact representation that is convenient for analyzing the fundamental characteristics of the radiation over a wide range of the parameters. The resonance region is examined, and the limiting expression for the emission probability is obtained; this is new representation for the total probability of the analogous process in a constant crossed field + wave configuration. A calculation of the probability of synchrotron radiation is made which includes the effects of an intense wave ($\xi = eE/mc\omega \gg 1$) and of quantum corrections. An analysis of the resonance region shows that the possibility of realizing resonant modes of particle motion depends on the parameters of the electromagnetic field and the average electron velocity component along the magnetic field.

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1. INTRODUCTION

The study of quantum effects in the interaction of elementary particles with intense electromagnetic fields takes on particular significance owing to the observation near pulsars[1] of magnetic fields with intensities comparable with the quantum-electrodyanmic characteristic field value[1)] $H_c = m^2/e$.

The systematic study of nonlinear quantum processes in constant fields, and also in fields of plane electromagnetic waves, was begun comparatively recently (see Refs. 2 and 3 and papers cited there). There has recently been increasing interest in this sort of research when the action of fields of rather complicated configuration is considered. In our opinion an especially interesting situation is one in which the field taken as external is that of a plane electromagnetic wave propagating along a magnetic field. A distincitive feature of this configuration of fields is the possibility of a resonant action of the field on charged particles. Besides this, the exact solution of the quantum equations of motion of charged particles in such a field is known.[4]

There have been several papers on fundamental quantum-electrodynamical processes in a field consisting of a plane electromagnetic wave and a constant magnetic field; these have dealt with particular aspects of the exact treatment of the action of this complicated external field (see, e.g., Refs. 5–11). It must be pointed out, however, that the consistent use of the exact solution is made difficult by the complexity of the analysis, which is characteristic of many-parameter problems.

In the present paper we examine the emission of a photon by a charged particle[2)] of spin $\frac{1}{2}$, using quantum-mechanical methods of describing the states of the particle in an electromagnetic field and paying close attention to effects associated with the intensity of the electromagnetic fields. As our initial field configuration we consider one which is a superposition of a constant uniform magnetic field and the field of a plane circularly polarized ($g=\pm1$) electromagnetic wave propagated along the magnetic field:

$$A_\mu = A_\mu^{(1)}(H) + A_\mu^{(2)}(\varphi); \quad \varphi = (kx) = \omega(t-z),$$
$$A_\mu^{(1)}(H) = \{0, 0, xH, 0\}; \quad A_\mu^{(2)} = \xi(m/e)\{0, -\sin\varphi, g\cos\varphi, 0\}, \tag{1}$$

taking the z axis as the direction of the magnetic field H; the invariant classical parameter for the intensity of the wave is connected with its amplitude E and the frequency ω:

$$\xi = eE/m\omega. \tag{2}$$

We note that quantum effects of the intensity on the scattering of electromagnetic waves by electrons moving in an external magnetic field have been studied in an earlier paper,[5] where some properties of the radiation of relativistic particles were analyzed. Another paper[6] studied the effects of a strong electromagnetic wave (EMW) on the emission from weakly excited electrons moving in a magnetic field; in particular it was shown that in the neighborhood of the point of cyclotron resonance[3)] the effective parameter of the coupling of an electron of momentum p_μ moving in a magnetic field with the field of an EMW is the quantity

$$\xi/\delta = \xi[1 - m\omega_H/(kp)]^{-1}; \quad \omega_H = eH/m. \tag{3}$$

This conclusion was later confirmed in the fundamental results of Ref. 8. We also note that an essentially analogous parameter determines the behavior of the cross section (as found in the classical approximation) for scattering of a strong EMW by a plasma electron moving in a magnetic field.[12]

Here we have derived a compact representation for the characteristic functions that determine the total probability of the emission process. This representation has not been given previously. It corresponds to an explicit breaking up of the total probability of the process into a sum of partial probabilities corresponding to a fixed numbers of quanta of the wave in the

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reaction, which is extremely convenient for analysis of the characteristics of the emission over a wide range of the parameters. This representation is used to study the resonance region and to derive a limiting expression for the probability of emission in the field (1), which is also a new representation for the total probability of the analogous process in the constant crossed field + wave configuration.[13]

2. THE TRANSITION AMPLITUDE

The effect of a wave propagated along the magnetic field [see Eq. (1)] preserves the symmetry of the electron motion in the magnetic field, so that the probability of emission does not depend on the azimuthal angle φ_0 at which the photon is emitted in a coordinate system in which the angle θ is measured from the direction of the magnetic field.

Let the frequency of the emitted photon be $\varkappa$; the wave vector is

$$\boldsymbol{\varkappa}=\varkappa(\sin\theta\cos\varphi_0,\ \sin\theta\sin\varphi_0,\ \cos\theta).$$

Without loss of generality we can fix the angle φ_0, say, at $\varphi_0=\pi/2$. For a transition of the electron from the state with the quantum numbers $n,\ p_y,\ p_-=(kp)/\omega,\ \zeta$, the probability of the electron emitting a photon of definite polarization can be written in the form[5]

$$W_\rho=\frac{1}{(2\pi)^5}\sum_{n'}\int\frac{d^3\varkappa}{2\varkappa}dq_y\,dq_z|M_\rho|^2, \tag{4}$$

where the matrix element M_p is calculated from the transition current $\mathbf{j}$,

$$M=-ie(4\pi)^{1/2}\mathbf{j}\boldsymbol{\varepsilon}^*,\quad \mathbf{j}=\int d^4x\Psi_{n'}^*\boldsymbol{\alpha}\Psi_n e^{i(\varkappa x)}, \tag{5}$$

where $\boldsymbol{\varepsilon}$ is the polarization vector of the photon and $\boldsymbol{\alpha}$ are Dirac matrices; primes indicate the final state.

Using the explicit forms of the Dirac matrices and of the functions Ψ_n (see Ref. 5), we can put the integrals (5) in the form

$$\left.\begin{matrix}-ij_1\\ j_2\end{matrix}\right\}=(2\pi)^3NN'\delta(p_--p_-'-\varkappa_-)\delta(p_y-p_y'-\varkappa_y)\sum_l\delta(q_z+l\omega-q_z'-\varkappa_z)$$
$$\times[G_1(a_1\Phi_3{}^l\mp a_2\Phi_2{}^l)+H_1(b_1\Phi_1{}^\mp+b_2\Phi_4{}^\mp)],$$
$$j_3=(2\pi)^3NN'\delta(p_--p_-'-\varkappa_-)\delta(p_y-p_y'-\varkappa_y)\sum_l\delta(q_z+l\omega-q_2'-\varkappa_z) \tag{6}$$
$$\times[G_2(b_1\Phi_1{}^l+b_2\Phi_4{}^l)+(H_2/\Delta+H_3/\Delta')(a_1\Phi_3^{l-1}+a_2\Phi_2^{l+1})],$$

where N and N' are the normalization coefficients of the functions Ψ_n and $\Psi_{n'}$, and

$$G_1=c_1'c_1-c_3'c_3,\quad a_1=a'b,\quad a_2=iab^{*\prime},\quad b_1=bb^{*\prime},\quad b_2=aa',$$
$$H_1=0.5g\omega\xi(c_1-c_3)(c_1'-c_3'),\quad G_2=c_1'c_3+c_3'c_1+4(c_1-c_3)(c_1'-c_3')R'R^*,$$
$$H_2=0.5g\omega\xi(c_1-c_3)(c_1'+c_3'),\quad H_3=0.5g\omega\xi(c_1+c_3)(c_1'-c_3'),$$
$$\varkappa_-=\varkappa^0-\varkappa_z,\quad p_-=p^0-p_z=q_-=q^0-q_z,\quad q_y=p_y.$$

The spin coefficients $c,\ a,\ b$ and the functions $R,\ \Psi_n$ are given by the relations

$$c_1=2^{-1/2}(1+\zeta m_\perp/p^0)^{1/2},\quad c_3=2^{-1/2}(1-\zeta m_\perp/p^0)^{1/2},$$
$$a=2^{-1/2}(1+\zeta m/m_\perp)^{1/2},\ b=2^{-1/2}i\zeta(1-\zeta m/m_\perp)^{1/2},$$
$$R=2^{-3/2}ig\omega\xi\Delta^{-1}e^{ig\varphi},\quad \Delta=\alpha\omega-g\omega_H,\quad \gamma=m\omega_H,$$
$$\alpha=m^{-1}q_-=m^{-1}p_-,\quad m_\perp{}^2=p_0{}^2-p_z{}^2,\quad p_0{}^2=m^2+2\gamma n+p_z{}^2,\quad \Psi_n=e^{-iS(\varphi)}u_n(\rho),$$
$$S(\varphi)=m\alpha t+q_z\varphi/\omega+g\xi\Delta^{-1}(m\omega_Hx+p_y)\sin\varphi-0.25m\Delta^{-2}\xi^2\omega_H\sin 2\varphi-p_yy.$$

The general form of the functions $\Phi_i^l(i=1,2,3,4)$ and $\Phi_{1,4}^\pm$

$$\Phi_1{}^l=B_{nn'}^l,\quad \Phi_2{}^l=B_{n-1,n'}^l,\quad \Phi_3{}^l=B_{n,n'-1}^l,\quad \Phi_4{}^l=B_{n-1,n'-1}^l,$$
$$\Phi_1{}^\pm=\Phi_1^{l+1}/\Delta'\pm\Phi_1^{l-1}/\Delta,\quad \Phi_4{}^\pm=\Phi_4^{l+1}/\Delta\pm\Phi_4^{l-1}/\Delta' \tag{7}$$

is determined by integrals of the type

$$M_{nn'}=\int d^4x\,u_{n'}(\rho')u_n(\rho)\exp[iS'(\varphi)-iS(\varphi)+i(\varkappa x)], \tag{8}$$
$$u_n(\rho)=\gamma^{1/4}(2^nn!)^{-1/2}(\pi)^{-1/4}e^{-\rho^2/2}H_n(\rho),\ \rho=\gamma^{1/2}(x-\xi\Delta^{-1}\cos\varphi)+p_y\gamma^{-1/2},$$

(where H_n are Hermite polynomials) after one has separated out from Eqs. (8) the δ functions which arise in the integration over space. In this case the general structural part of the transition amplitude reduces to the form

$$M_{nn'}=(2\pi)^3\delta(p_--p_-'-\varkappa_-)\delta(p_y-p_y'-\varkappa_y)\sum_{l=-\infty}^{\infty}\delta(q_z+l\omega-q_z'-\varkappa_z)B_{nn'}^l.$$

Under the summation sign here there is a δ function which expresses the law of conservation of the systems quasimomentum component along the direction of the magnetic field, $H\|z$: $q_z=p_z+\xi^2m\omega/2\Delta$. The number l labels a partial process involving a fixed number of "wave quanta" that take part in the reaction. This situation is very typical in processes that take place in the field of an EMW. The essential difference from the case of a "pure" EMW (with no effect of a constant field) is that in the present case the number l can be either positive or negative.

The functions $B_{nn'}^l$ specified by the integral representation (8) can be written in the form

$$B_{nn'}^l=\frac{1}{2\pi}\int_0^{2\pi}e^{if}I_{nn'}(\rho)d\varphi;$$
$$f=l\varphi-0{,}5(\varepsilon+\varepsilon')g\varkappa_\perp\sin\varphi-g(n-n')\,\mathrm{arctg}\,\tau,$$
$$\tau=(\varepsilon-\varepsilon')\sin\varphi[\varkappa_\perp(m\omega_H)^{-1}+(\varepsilon-\varepsilon')\cos\varphi]^{-1}, \tag{9}$$
$$2\gamma\rho=\varkappa_\perp{}^2+\gamma^2(\varepsilon-\varepsilon')^2-2\gamma\varkappa_\perp(\varepsilon-\varepsilon')\cos\varphi,\quad \varkappa_\perp=\varkappa\sin\theta.$$

These functions have a rather complicated structure; only in the special case when the EMW field is completely absent in Eq. (1) ($\xi=0$) do they reduce to the well known Laguerre functions $I_{nn'}$ of the argument $p=\varkappa_\perp^2/2\gamma$ (here the index l takes the value zero). In the general case, however, we can get for the coefficients $B_{nn'}^l$ a compact operator representation[4)]

$$B_{nn'}^l=J_l(|R|)(R/R_+)^{l/2}H_{nn'}(\eta)|_{\eta=\eta_0},$$
$$\eta=x^2+y^2-2xy\cos\varphi,\quad x^2=\varkappa_\perp{}^2/2\gamma,\quad y^2=(\varepsilon-\varepsilon')^2/2\gamma, \tag{10}$$

where $J_l(|R|)$ is a Bessel function of the operator argument $|R|=(RR_+)^{1/2}$, with

$$\left.\begin{matrix}R\\ R_+\end{matrix}\right\}=\frac{g(\varepsilon+\varepsilon')\varkappa_\perp}{2\gamma}+\frac{g(n-n')(\varepsilon-\varepsilon')}{2\gamma}\pm\frac{\varkappa_\perp(\varepsilon-\varepsilon')}{2\gamma}\frac{\partial}{\partial\eta}.$$

The functions $H_{nn'}$ are given by

$$H_{nn'}=\exp[-ig(n-n')\lambda]I_{nn'}(\eta);$$
$$\lambda=-(y/x)[1-(2xy)^{-2}(\eta_0-\eta)^2]^{1/2}+\mathrm{arctg}\,\tau.$$

Here $I_{nn'}$ is the Laguerre function, related with the Laguerre polynomials by the formula

$$I_{ns}(\rho)=\frac{1}{(n!s!)^{1/2}}e^{-\rho/2}\rho^{(n-s)/2}Q_s^{n-s}(\rho),\quad Q_s{}^l=e^\rho\rho^{-l}\frac{d^s}{d\rho^s}(\rho^{l+s}e^{-\rho}),$$

and it is understood that after the operator $J_1(|R|)(R/R_+)^{1/2}$ has acted on the function $H_{nn'}$ the value is taken at the point $\eta_0 = x^2 + y^2$.

For the squared modulus of the amplitude for a transition of the polarized electron ($\zeta, \zeta' = \pm 1$ give the signs of the spin components in the direction of the magnetic field for the electron in its initial and final states) with emission of a photon characterized by a polarization vector with rectangular components $\mathbf{e}_1$ and $\mathbf{e}_2$ ($\mathbf{e}_1 \cdot \mathbf{e}_2 = 0$, $\mathbf{e}_1 \cdot \varkappa = 0$, $\mathbf{e}_2 \cdot \varkappa = 0$) we get the expression

$$2S_j = (1+\zeta\zeta')S_j^+ + (1-\zeta\zeta')S_j^-, \quad j=1,2;$$
$$S_1^\pm = F_1^\pm = (8q_0^* q_0'^*)^{-1}\{\sigma_1^\pm[\mu_4'^\pm \mu_4^- |\Phi_3^l|^2 + \mu_4'^\mp \mu_4^+ |\Phi_2^l|^2 \mp 2\mu_3\mu_3'\Phi_2^l\Phi_3^l]$$
$$+0.5\xi^2\omega^2 p_- p_-'[\mu_4'^\mp \mu_4^- |\Phi_1^-|^2 + \mu_4'^\pm \mu_4^+ |\Phi_4^-|^2 \pm 2\mu_3\mu_3'\Phi_1^-\Phi_4^-]$$
$$+g\omega\xi(p_-' m_\perp \pm p_- m_\perp')[\mu_3(\mu_4'^\pm \Phi_4^- \Phi_3^l - \mu_4'^\mp \Phi_1^- \Phi_2^l)$$
$$\pm \mu_3'(\mu_4^- \Phi_1^- \Phi_3^l - \mu_4^+ \Phi_4^- \Phi_2^l)]\};$$

$$S_2^\pm = F_2^\pm \cos^2\theta + F_3^\pm \sin^2\theta - F_{2,3}^\pm \sin 2\theta;$$
$$F_2^\pm = F_1^\pm(\Phi_1^l \to \Phi_1^l,\ \Phi_2^l \to -\Phi_2^l,\ \Phi_3^l \to \Phi_3^l,\ \Phi_4^l \to \Phi_4^l,\ \Phi_1^- \to \Phi_1^+,\ \Phi_4^- \to \Phi_4^+);$$
$$F_3^\pm = (8q_0^* q_0'^*)^{-1}\{\sigma_2^\mp[\mu_4'^\mp \mu_4^- |\Phi_1^l|^2 + \mu_4'^\pm \mu_4^+ |\Phi_4^l|^2 \pm 2\mu_3\mu_3'\Phi_1^l\Phi_4^l]$$
$$+{}^1/_2\xi^2\omega^2\sigma_3^\pm[\mu_4'^\pm \mu_4^- |\Phi_3^{l-1}|^2 + \mu_4'^\mp \mu_4^+ |\Phi_2^{l+1}|^2 \pm 2\mu_3\mu_3'\Phi_3^{l-1}\Phi_2^{l+1}]$$
$$+g\omega\xi\sigma_4^\pm[\mu_3(\mu_4'^\pm \Phi_4^l \Phi_3^{l-1} + \mu_4'^\mp \Phi_1^l \Phi_2^{l+1}) \pm \mu_3'(\mu_4^- \Phi_1^l \Phi_3^{l-1} + \mu_4^+ \Phi_4^l \Phi_2^{l+1})]\};$$
$$F_{2,3}^\pm = (8q_0^* q_0'^*)^{-1}\{\sigma_5^\pm[\pm\mu_3'(\mu_4^- \Phi_1^l \Phi_3^l + \mu_4^+ \Phi_2^l \Phi_4^l)$$
$$+\mu_3(\mu_4'^\mp \Phi_1^l \Phi_2^l + \mu_4'^\pm \Phi_3^l \Phi_4^l)] + {}^1/_2\xi^2\omega^2(p_-' m_\perp/\Delta' \pm p_- m_\perp'/\Delta)$$
$$\times[\mu_3(\mu_4'^\mp \Phi_1^+ \Phi_2^{l+1} + \mu_4'^\pm \Phi_4^{l-1}\Phi_4^+) \pm \mu_3'(\mu_4^- \Phi_1^+ \Phi_3^{l-1} + \mu_4^+ \Phi_2^{l+1}\Phi_4^+)]$$
$$+{}^1/_2 g\omega\xi[\sigma_6^\pm(\mu_4'^\mp \mu_4^- \Phi_1^l \Phi_1^+ + \mu_4'^\pm \mu_4^+ \Phi_4^l \Phi_4^+ \pm \mu_3\mu_3'(\Phi_4^l \Phi_1^+ + \Phi_1^l \Phi_4^+))$$
$$+\sigma_7^\pm(\mu_4'^\pm \mu_4^- \Phi_3^l \Phi_3^{l-1} + \mu_4'^\mp \mu_4^+ \Phi_2^l \Phi_2^{l+1} \pm \mu_3\mu_3'(\Phi_3^l \Phi_2^{l+1} + \Phi_2^l \Phi_3^{l-1}))]\}.$$

Here

$$\mu_3 = (2\gamma n)^{1/2}/m_\perp, \quad \mu_3' = (2\gamma n')^{1/2}/m_\perp', \quad m_\perp = (m^2+2\gamma n)^{1/2},$$
$$m_\perp' = (m^2+2\gamma n')^{1/2}, \quad \mu_4^\pm = 1 \pm \zeta m/m_\perp, \quad \mu_4'^\pm = 1 \pm \zeta m/m_\perp',$$
$$\sigma_1^\pm = p_0 p_0' - p_z p_z' \pm m_\perp m_\perp', \quad \sigma_2^\pm = p_0 p_0' + p_z p_z' + {}^1/_2 p_- p_-' \sigma_0(\sigma_0 - 2) \mp m_\perp m_\perp'(1-\sigma_0),$$
$$\sigma_3^\pm = p_- p_+'/\Delta^2 + p_-' p_+/\Delta'^2 \pm 2m_\perp m_\perp'/\Delta\Delta',$$
$$\sigma_4^\pm = [p_+ m_\perp' \mp p_- m_\perp'(1-\sigma_0)]/\Delta - [m_\perp p_-'(1-\sigma_0) \mp p_+ m_\perp']/\Delta',$$
$$\sigma_5^\pm = m_\perp(p_z' + {}^1/_2 p_-' \sigma_0) \pm m_\perp'(p_z + {}^1/_2 p_- \sigma_0),$$
$$\sigma_6^\pm = \pm m_\perp m_\perp' - p_- p_-'(1-\sigma_0), \quad \sigma_7^\pm = (p_- p_+' \pm m_\perp m_\perp')/\Delta + (p_-' p_+ \pm m_\perp m_\perp')/\Delta',$$
$$\sigma_0 = \xi^2\omega^2/\Delta\Delta', \quad p_\pm = p^0 \pm p_z, \quad p_\pm' = p^{0\prime} \pm p_z'.$$

Accordingly, the expression for the probability calculated per unit volume and unit time and for specified polarization characteristics of the particles in the reaction can be written

$$W_j = m^2 e^2 (1-\beta_\parallel) \sum_{l,n'} \int \frac{u\,du}{(1+u)^2} \frac{d\cos\theta}{(1-\cos\theta)^2} S_j \delta(q_0 + l\omega - q_0' - \varkappa_0), \quad (11)$$
$$j=1,2; \quad \beta_\parallel = q_z^*/q_0^*; \quad q_{0,z} = p_{0,z} + \xi^2\omega^2 m\alpha/2\Delta^2,$$

where we have introduced the new variable $u = (\alpha - \alpha')/\alpha'$, where $\alpha = m^{-1}(q_0^* - q_z^*)$.

3. EMISSION AT $H \ll H_c$ PROBABILITY

For $H \ll H_c$ the discrete nature of the electron states in the constant magnetic field manifests itself only weakly. The electron motion in such fields becomes quasclassical and for relativistic particles $[p_0 \gg m,\ (2\gamma n)^{1/2} \gg m]$ the discrete transverse momentum of the electron becomes quasicontinuous ($2\gamma n \approx p_\perp^2 \gg m^2$). Considering this situation and assuming $p_\perp \gg p_z$, which can always be obtained by changing (without loss of generality) to a reference frame that moves in the direction of the magnetic field, we keep in the integrand of Eq. (11) only the principal terms in the expansion in powers of the energy. We also make use of the fact that in this approximation

$$\Phi_1^l = \mathcal{J}_l A_1, \quad \Phi_2^l = \mathcal{J}_l(A_1 - B_1), \quad \Phi_3^l = \mathcal{J}_l[A_1 + (1+u)B_1],$$
$$\Phi_4^l = \mathcal{J}_l(A_1 + uB_1), \quad \Phi_1^\pm = (\alpha\mathcal{J}_{l+1} \pm \alpha'\mathcal{J}_{l-1})A_1/\omega\alpha\alpha',$$
$$\Phi_4^\pm = (\alpha'\mathcal{J}_{l+1} \pm \alpha\mathcal{J}_{l-1})(A_1 + uB_1)/\omega\alpha\alpha'; \quad (12)$$
$$A_1 = A\Phi(t), \quad B_1 = B\Phi'(t), \quad B = \alpha^{-1}(2\chi/u)^{1/3}A,$$
$$A = (\alpha\pi^{1/2})^{-1}(2\chi/u)^{1/3}(1+u)^{1/2},$$

where $\Phi(t)$ and $\Phi'(t)$ are an Airy function and its derivative, with the argument

$$t = (u/2\chi)^{2/3}(1 + \xi^2 + \tau^2 - \lambda_l/u); \quad \mathcal{J}_l = J_l(|R|)(R/R_+)^{1/2},$$

and $J_l(|R|)$ is a Bessel function of order l of the operator argument. In the limit considered ($H \ll H_c$) we can also get a simpler form for the Bessel-function argument, which contains the differential operator $R = h + g_l\hat{a}$:

$$h = 2g\xi\frac{u}{\lambda}\left(\tau - \frac{2g\chi}{\lambda}\right), \quad g_l\hat{a} = 2\xi\frac{u}{\lambda}\left(\frac{2\chi}{u}\right)^{1/3}\frac{\partial}{\partial t},$$
$$\lambda = \frac{2\omega\alpha}{m}, \quad \tau = \alpha\cos\theta, \quad \lambda_l = \lambda l, \quad \chi = \frac{H}{H_c}\alpha.$$

Using the properties of the functions $\tilde{J}_l = J_l(R/R_+)^{1/2}$, which are analogous in their general properties to the Bessel functions of c-number argument, we can bring the expression for the probability of photon emission by the electron, in this approximation, to the form

$$W = \frac{2e^2m^2}{\pi q_0}\sum_l \int_0^\infty \frac{du}{(1+u)^2}\left(\frac{u}{2\chi}\right)^{1/3}\int_{-\infty}^\infty d\tau\,\bar{S}_l,$$
$$\bar{S}_l = -(\mathcal{J}_l\Phi)^2 + (1 + u^2/2(1+u))\{(2\chi/u)^{2/3}[(\mathcal{J}_l\Phi')^2 + \mathcal{J}_l\Phi\mathcal{J}_l\Phi'']$$
$$+\xi^2[(\mathcal{J}_l'\Phi)^2 + h^{-2}(g_l\mathcal{J}_l'\Phi' - l\mathcal{J}_l\Phi)^2 - (\mathcal{J}_l\Phi)^2]$$
$$+2g\xi(2\chi/u)^{1/3}(\mathcal{J}_l\Phi'\mathcal{J}_l'\Phi + \mathcal{J}_l\Phi\mathcal{J}_l'\Phi')\}, \quad (13)$$

where $\tilde{J}_l'\Phi$ and $\tilde{J}_l'\Phi'$ are to be taken as meaning that the derivative is taken with respect to the argument: $\tilde{J}_l'\Phi = \partial J_l\Phi/\partial h$ and $J_l'\Phi' = \partial J_l\Phi'/\partial h$.

The resulting expression (13) is a generalization of known results,[2,3] which hold either when the field of a plane electromagnetic wave acts on the electron or when there is only a constant field. Consideration of the limit of ultrarelativistic energies of the emitting electron ($\alpha \gg 1$, $\alpha \gg \omega_H/\omega$) enabled us at $H \ll H_c$ to get from the general expression (11) for the probability the exact value of the probability of photon emission by an electron moving in the superposition of a constant crossed field $\mathbf{E} \perp \mathbf{H}$, $E = H$ and the field of a plane electromagnetic wave propagated along the Poynting vector of the constant field.

We note that an expression for the probability in such a configuration of electromagnetic fields in the form of a multiple integral was first obtained in Ref. 13. The representation (13) of the total probability of the emission process as a sum of partial probabilities, corresponding to definite numbers of wave quanta taking part in the reaction, is more convenient in many cases. In particular, this is true for the treatment of cases involving a small (or, conversely, a very large) number of wave quanta. For example, at $l = 0$ the expression (13) determines the probability of synchrotron radiation as affected by the action of an EMW of arbitrary intensity:

$$W=\frac{2e^2m^2}{\pi q_0}\int_0^\infty\frac{du}{(1+u)^2}\left(\frac{u}{2\chi}\right)^{1/3}\int_{-\infty}^{\infty}d\tau\, w(u,\tau),$$

$$w(u,\tau)=-(J_0\Phi)^2+(1+u^2/2(1+u))\{(2\chi/u)^{2/3}[(J_0\Phi')^2+J_0\Phi J_0\Phi'']$$
$$+\xi^2[(J_0'\Phi)^2+(g_1/h)^2(J_0'\Phi')^2-(J_0\Phi)^2]$$
$$+2g\xi(2\chi/u)^{1/3}(J_0\Phi'J_0'\Phi+J_0\Phi J_0'\Phi')\},\quad J_0=J_0(|R|). \qquad (14)$$

If $\xi\ll 1$, we can confine ourselves to the first terms of the expansion of the zeroth-order Bessel function

$$J_0(|R|)\Phi={}^1/_4(4-h^2+g_1{}^2t)\Phi,$$

where the second derivative is given by the equation $\Phi''=t\Phi$ that defines the Airy function. In this way, after integrating over τ we get an expression for the probability in the form of single integral

$$W=-\frac{e^2m^2}{3\pi^{1/2}p_0}\int_0^\infty\frac{\Phi'(y)\,du}{y(1+u)^3}(f_1+\xi^2 f_2);$$

$$f_1=5+7u+5u^2,\quad f_2=-\frac{7-5u+11u^2-4u^3}{2(1+u)}+u^{-1}\ln(1+u)(1+u)^3$$
$$+\frac{2}{\lambda^2}\left(1-\frac{4\chi^2}{\lambda^2}\right)[4+10u+11u^2+5u^3+3u^4-4u^{-1}(1+u)^3\ln(1+u)],$$
$$y=\left(\frac{u}{\chi}\right)^{2/3}. \qquad (15)$$

At $\xi=0$ this expression agrees with the well known integral representation for the probability W^{sr} of synchrotron radiation in the quasiclassical limit.[2] At $\xi\neq 0$ it contains corrections to W^{sr} caused by the influence of the wave (cf. also Refs. 10 and 15).

For the calculation of the integral (15) it is convenient to represent its characteristic parts as Mellin-Barnes integrals[5)]:

$$I_{m,n}(x)=-(3\pi)^{1/2}\int_0^\infty\frac{\Phi'(y)u^n}{y(1+u)^m}$$
$$=\frac{2^{m-3}}{\pi\Gamma(m)}\frac{1}{2\pi i}\int_{\nu-i\infty}^{\nu+i\infty}\Gamma\left(s+\frac{m-n-1}{2}\right)\Gamma\left(s+\frac{m-n}{2}\right)\Gamma(s+{}^1/_3)$$
$$\Gamma(s-{}^1/_3)\Gamma\left(-s+\frac{n+1}{2}\right)\Gamma\left(-s+\frac{n+2}{2}\right)x^{-s}\,ds,$$
$${}^1/_3<\nu<m/2;\quad m\geqslant 1,\quad n\geqslant 0;\quad x=1/9\chi^2. \qquad (16)$$

At $x>1$ the integral (16) can be calculated in terms of the sum of the "right side" residues at the points $s=\frac{1}{2}(n+1)+k$, $s=\frac{1}{2}(n+2)+k$, $k=0,1,2,\ldots$:

$$I_{m,n}(x)=\frac{2^{m-3}}{\pi\Gamma(m)}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\left\{\Gamma\left(k+\frac{m}{2}\right)\Gamma\left(k+\frac{m+1}{2}\right)\right.$$
$$\times\Gamma\left(-k+\frac{1}{2}\right)\Gamma\left(k+\frac{n}{2}+\frac{5}{6}\right)\Gamma\left(k+\frac{n}{2}+\frac{1}{6}\right)x^{-\frac{n+1}{2}-k}$$
$$+\Gamma\left(k+\frac{m+1}{2}\right)\Gamma\left(k+\frac{m+2}{2}\right)\Gamma\left(-k-\frac{1}{2}\right)\cdot$$
$$\left.\times\Gamma\left(k+\frac{n}{2}+\frac{4}{3}\right)\Gamma\left(k+\frac{n}{2}+\frac{2}{3}\right)x^{-\frac{n+2}{2}-k}\right\}. \qquad (17)$$

In the other region $x<1$ we get a series of inverse powers of χ by closing the contour of integration on the left. The terms in Eq. (15) that contain $\ln(1+u)$ are calculated analogously. Some complication in this case arises in finding the residue at a double pole in the region $x<1$.

Using these results, we present expressions for the probability (15) in two limiting cases ($\xi^2\ll 1$, $\xi^2\chi^2/\lambda^2\ll 1$, $\xi^2\chi^4/\lambda^4\ll 1$):

$$\chi\ll 1,\quad W=\frac{5e^2m^2}{2\sqrt{3}\,p_0}\chi\left\{1-\frac{8\sqrt{3}}{15}\chi-\frac{\xi^2}{2}+\frac{11\xi^2\chi^2}{6\lambda^2}\left(1-\frac{4\chi^2}{\lambda^2}\right)+\frac{16\sqrt{3}}{15}\xi^2\chi\right\},$$

$$\chi\gg 1,\quad W=\frac{14e^2m^2\Gamma(^2/_3)}{27p_0}\left\{(3\chi)^{2/3}-\frac{45}{28\Gamma(^2/_3)}-\frac{9}{8}\xi^2-\frac{54\xi^2\chi^4}{7\lambda^4}\left(1-\frac{\lambda^2}{4\chi^2}\right)\right\}. \qquad (18)$$

Since a classical limit of the probability in a constant field exists [the terms proportional to χ for $\chi\ll 1$ in Eq. (18)] the corrections due to the effects of the field can be divided into two types: quantum terms proportional to $\xi^2\chi^2$, and classical terms $\sim\chi\xi^2$ and $\sim\xi^2\chi^3/\lambda^2$. The quantum correction leads to an increase of the probability, while the classical terms can give either an increase or a decrease. In the region of large $\chi\gg 1$ the corrections are decidely nonlinear functions of the field strength.

The contributions of the other partial terms can be calculated analogously. It must be pointed out that for $\xi\ll 1$ the main contribution to the l-th term is proportional to $\xi^{2|l|}$, whereas in a weak constant field (with $\chi\ll\xi\ll 1$) it is either of the same order as in the field of an EMW (for $l>0$), or exponentially small (for $l<0$).

4. THE REGION OF CYCLOTRON RESONANCE

The range of parameter values for an electron radiating in the field (1) for which the frequency of the wave, with the Doppler shift taken into account, is close to the cyclotron frequency $\omega_H=eH/m$, is known as the region of the cyclotron resonance. We shall characterize the amount of deviation from the point of exact resonance with a parameter $\delta=(\alpha\omega)^{-1}\Delta=1-g\omega_H/\alpha\omega$, where $g=\pm 1$ is, as usual, a parameter corresponding to the two directions of rotation of the polarization vector in a circularly polarized EMW. In what follows we shall consider the case in which δ can take values much smaller than unity (for definiteness $0<\delta<1$), so that we fix $g=1$. In contrast with the case considered before, we also assume that in its initial state the electron has a small transverse momentum, $2\gamma n\ll m^2$.

Let us introduce a new variable $r=(\Delta-\Delta')/\Delta'=u\alpha'/(\alpha'-\omega_H/\omega)$. It will be useful in the region considered. In this case we have

$$r=\frac{u}{\delta-u+u\delta},\quad \alpha'=\alpha\left(1-\frac{\delta}{1+1/r}\right).$$

Accordingly, in the quasiclassical region, where the role of quantum effects is not important, $\alpha'\approx\alpha$ and when the condition $0<\delta\ll 1$ is satisfied we have $r=u/\delta\geqslant 0$. It must be emphasized that we are neglecting here terms of order u/δ in comparison with unity.

Let us now estimate the effective value of the number of field photons taking part in the reaction. In particular, for the minimum value of the number l we get

$$l_{min}=\frac{1}{2}\xi^2 m\left(\frac{1}{\Delta'}-\frac{1}{\Delta}\right)=\xi^2\frac{m}{2\alpha\omega}\frac{r}{\delta}.$$

Satisfaction of the conditions $0<\delta\leqslant 1$, $r>0$ allows us to conclude that the final state of the electron, characterized by the parameter δ^1, is also a resonance state, i.e., $\delta'=\Delta'/\alpha'\omega\sim\delta\ll 1$. Then the estimate of the quantity $l_{\min}$ leads to the result

$$l_{min}\approx\xi^3\frac{u}{2\chi}\delta^{-2};\quad u=r\delta',\quad \chi=\frac{H}{H_c}\frac{\xi}{\delta}.$$

In accordance with the results of Ref. 6, where an expression is given for the spectrum of the emitted photons, for the motion of the electron in the field (1) we have

$$\varkappa=\frac{(1-\beta_\parallel)\omega[l+(\omega_H/\alpha\omega)(n-n')]}{1-\beta_\parallel\cos\theta},\qquad(19)$$

where $\beta_\parallel$ is the average value of the velocity of the electron's motion along the z axis. It follows from Eq. (19) that the quantity $s=l-n'+n$, which under resonance conditions is the number of the harmonic that is emitted, is given by the expression

$$s=l-n'+n=\frac{u\xi^3}{\chi\delta^3}\left(1+\frac{\delta^2}{\xi^2}+\cos\theta_0+\cos^2\theta_0\right)\frac{2+r}{2(1+r)},\qquad(20)$$

where the angle θ_0 is connected with the angle θ; $\cos\theta_0=(\cos\theta-\beta_\parallel)/(1-\beta_\parallel\cos\theta)$, and $\cos\theta_0\sim\delta/\xi$ with $\xi/\delta>1$.

For the coefficients $B^l_{nn'}$ we get from the integral (9)

$$B^l_{nn'}=\frac{1}{2\pi}\int_0^{2\pi}F(\varphi)e^{il\varphi}d\varphi=\frac{1}{2\pi}\int_0^{2\pi}e^{if}I_{nn'}(\eta)d\varphi,\qquad(21)$$

where in the limit considered, $\xi\gg\delta$ $(\alpha\ll\delta^{-1}\xi)$, the arguments f and η of the two factors reduce to the simple functions

$$f=\varphi(l-n'+n)+\frac{\varepsilon+\varepsilon'}{2\gamma}\varkappa_\perp\sin\varphi,\quad \eta=(2\gamma)^{-1}(\varepsilon-\varepsilon')^2.$$

Accordingly we have for $B^l_{nn'}$ the expression

$$B^l_{nn'}=J_{l-n'+n}(z)I_{nn'}(x),\qquad(22)$$

where $J_s(z)$ is the Bessel function of the argument

$$z=(2\gamma)^{-1}(\varepsilon+\varepsilon')\varkappa_\perp=\frac{u}{\chi}\left(\frac{\xi}{\delta}\right)^3\left(1+\frac{\delta^2}{2\xi^2}+\cos\theta_0\right)\frac{2+r}{2(1+r)},$$

and the argument of the Laguerre function $I_{nn'}$ is

$$x=\frac{1}{2\gamma}(\varepsilon-\varepsilon')^2=\frac{m^2}{2\gamma}\left(\frac{\xi}{\delta}\right)^2r^2.$$

It must be emphasized that kinematically the condition that the system be close to the resonance point, $\delta\ll\xi$, means that the average value of the rate of motion of the electron along the direction of the magnetic field in the laboratory system is relativistic, $1-\beta_\parallel^2\ll1$. The connection between the basic parameters in the neighborhood of resonance is given by

$$\alpha^{-1}\left(\frac{1-\beta_\parallel}{1+\beta_\parallel}\right)^{1/2}=\left(1+\frac{\xi^2}{\delta^2}+\frac{2nH}{H_c}\right)^{-1/2}.\qquad(23)$$

By comparing the expression (19) with the argument z of the Bessel function, one can see that for $\xi\gg\delta$ we have $z\to s$, with $z/s=(1+\delta^2/\xi^2)^{-1/2}\sin\theta_0$, $\theta_0\sim\pi/2$, and $2nH/H_c\ll1$. Accordingly, with increasing parameter ξ/δ the argument and index of the Bessel function increases like $(\xi/\delta)^2$, but their ratio approaches unity: $1-z^2/s^2=\cos^2\theta_0+\delta^2/\xi^2\ll1$. It can be verified that in the limit $\delta\ll\xi$ the expression

$$\frac{\xi^2}{\delta^2}\left(1-\frac{z^2}{s^2}\right)=1+\frac{\xi^2}{\delta^2}\cos^2\theta_0$$

remains finite, and to describe the angular distribution of the probability it is convenient to introduce the quantity

$$v=(\xi/\delta)\cos\theta_0.$$

These results indicate that there is qualitative agreement between this limiting case and the situation found in an intense EMW,[3] but here the meaning of the symbols is different. Using the asymptotic formula for the Bessel function for $z\gg1$ $(s\to z)$, we have where $\Phi(t)$ is

$$J_s(s\varepsilon)\approx\pi^{-1/2}(2/s)^{1/3}\Phi[(s/2)^{2/3}(1-\varepsilon^2)],$$

an Airy function with the argument

$$t=\left(\frac{s}{2}\right)^{2/3}\left(1-\frac{z^2}{s^2}\right)=\left(\frac{u}{2\chi}\right)^{2/3}(1+v^2).\qquad(24)$$

After summing and averaging over the polarizations of the particles of the reaction the expression for the probability reduces in the region now considered to

$$W=\frac{2e^2m^2}{\pi q_0}\int_0^\infty\frac{du}{(1+u)^2}\left(\frac{u}{2\chi}\right)^{1/3}\int_{-\infty}^{\infty}dvG(\chi,u,y),$$

$$G(\chi,u,y)=-\Phi^2(y)+\left(\frac{2\chi}{u}\right)^{2/3}\left(1+\frac{u^2}{2(1+u)}\right)(\Phi'^2(y)+y\Phi^2(y)),$$
$$y=(u/2\chi)^{2/3}(1+v^2),\quad u=(\alpha-\alpha')/\alpha'.\qquad(25)$$

It must be pointed out that in deriving the expression (25) we have carried out a summation over the index n', on which only the Laguerre function $I_{nn'}(x)$ depends. The argument of this function is a purely quantum variable $x\propto(m/2\omega_H)r^2$. It is not hard to verify that the main contribution to the classical part of the probability comes from terms with $n'=n$ (the so-called coherent or unshifted scattering, cf. Ref. 6). The contributions of the other terms are due to quantum effects. In Eq. (25) we have also substituted $\alpha\omega=\omega_H(1-\delta)^{-1}$ and dropped unimportant terms $\sim\delta\ll1$.[6)]

The resulting expression is the exact result for the probability of the emission process in a constant crossed field. Under resonance conditions the effective parameter for the coupling of the electron and the wave is ξ/δ, which for even a weak wave $(\xi\ll1)$ can be large owing to the small detuning from of the quantities $\alpha\omega$ and ω_H from the point of exact resonance:

$$\omega_H/\alpha\omega=1-\delta,\quad\delta\ll1,\quad\delta\ll\xi.$$

We note that the maximum of the spectral distribution of the emission probability for an electron whose initial state is close to the point of cyclotron resonance occurs at the frequency $\varkappa=s\omega\sim(\xi/\delta)^2\omega=(\xi/\delta)^3\omega_H/\alpha$, where ω is the frequency of the wave, or $\varkappa\approx m(H/H_c)(\xi/\delta)^2$ in the reference system in which $\beta_\parallel=0$; this is slightly different from an earlier[8] estimate.

In a similar way we can also get an expression for the intensity of the radiation. Multiplying the spectral and angular probability distribution by the energy of the emitted photon

$$\varkappa=m\alpha u(1+u)^{-1}(1+\beta_\parallel\cos\theta_0)/(1-\beta_\parallel)(1-\cos\theta_0)$$

and carrying out the integration over the angular variable $v=(\xi/\delta)\cos\theta_0$, we get for the spectral intensity distribution

$$I=-e^2m^2\int_0^\infty\frac{udu}{(1+u)^3}\left\{F(y)+\frac{2}{y}\left(1+\frac{u^2}{2(1+u)}\right)\Phi'(y)\right\},$$

$$y=\left(\frac{u}{\chi}\right)^{2/3},\quad F(y)=\int_y^\infty\Phi(x)dx.$$

This is also the well known expression for the intensity of emission by an electron moving in a constant crossed field. The role of the quantum parameter χ, however, is here played by the quantity $\chi=(H/H_c)(\xi/\delta)$. In the region $\chi\ll 1$ we have

$$I(\chi)=\frac{2}{3}e^2m^2\chi^2\left(1-\frac{55\sqrt{3}}{16}\chi+\ldots\right), \tag{26}$$

where the first term agrees with the result obtained earlier[6] in an exact calculation of the contribution to the probability from coherent transitions of unexcited electrons $n'=n=0$ in the field (1). The second term is a quantum correction to the classical limit of the intensity (see also Ref. 8).

As has been noted previously (see Refs. 2, 3) many characteristic features of the radiation of relativistic particles are not directly due to the type of external electromagnetic field in which the particles are moving. With the relativistic motion of an electron in a constant magnetic field and the field of an intense plane electromagnetic wave as examples, it was shown that in these cases the radiation is produced on a small segment of the particle trajectory, so that the specific nature of the external field is manifested only rather weakly. The situation is analogous for the region of the cyclotron resonance in the case of an electron moving in an external field having the configuration (1). In fact, in the cyclotron-resonance region the transverse momentum of the electron is determined by the parameter ξ/δ ($2\gamma n\ll m^2$) and can be relativistic owing to the resonant action of the field (1) on the particle:

$$p_\perp\sim m\xi/\delta\gg m.$$

It is not hard to verify that in this case the region where the radiation process occurs is a small portion of the particle trajectory

$$\Delta l/R\sim\delta/\xi\ll 1,$$

where R is the instantaneous radius of curvature of the trajectory, in complete analogy with the well known results of Refs. 2 and 3 for the relativistic motion of an electron in a pure magnetic field or the field of an EMW. Because the region in which the process occurs is small compared with the characteristic period of change of the field, in the limit $\xi/\delta\gg 1$ we can, as the results show, completely neglect the change of the field (1) within the region where the radiation is produced. This fact is illustrated in Eqs. (25) and (26), where the effect of the electromagnetic field (1) has been reduced to the effective action of a constant field on the particle.[7)]

Using the expression (23), we can see that as the magnetic field increases from zero, beginning at values

$$\frac{\omega_H}{\omega}>\left(\frac{\omega_H}{\omega}\right)_\kappa=\left(\frac{1-\beta_\parallel}{1+\beta_\parallel}\right)^{1/2}\frac{(1+2\mu n)^{1/3}+\xi^{2/3}}{(1+2\mu n)^{1/3}}[1+2\mu n+\xi^{2/3}(1+2\mu n)^{2/3}]^{1/2},$$
$$\mu=\omega_H/m \tag{27}$$

there are three possible values of the energy parameter α for each value of ω_H/ω. Subject to the condition (27), we can judge as to the possibility of realizing various types of resonant behavior. For example, if $\xi\ll 1$ and $2\mu n\ll 1$, then in a system where $\beta_\parallel=0$ we have $(\omega_H/\omega)\gg 1$. For $\xi^2\gg 1+2\mu n$ in the same system, calculation with Eq. (27) gives $(\omega_H/\omega)\geqslant\xi$, which agress with the analogous condition of Ref. 12 and corresponds to relativistic resonance in the intense wave. If the opposite inequality holds

$$\xi^2\ll 1+2\mu n,$$

the main factor in the determination of the resonance region is the energy of the particle in the magnetic field $(\omega_H/\omega)\gtrsim(1+2\mu n)^{1/2}$. The determination of the upper boundary of the resonance region is based on consideration of the energy lost by the system in radiation.

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[1)] Units in which $\hbar=c=1$ are used here throughout.

[2)] For definiteness, we consider here throughout an electron with charge e and mass m.

[3)] By the point of cyclotron resonance we mean the parameter value at which the frequency of the wave is equal to the frequency of the particle motion in the magnetic field, with the Doppler effect taken into account.

[4)] We point out that operator representations for some special functions are well known. For example, for Bessel functions there is the expression

$$J_n(z)=\frac{z^n}{2^{n-1}\Gamma(n+{}^1/_2)\Gamma({}^1/_2)}\left(1+\frac{d^2}{dz^2}\right)^{n-1/2}\frac{\sin z}{z}.$$

[5)] We note that by a simple change of parameters this integral reduces to the so-called G function of Meijer.

[6)] We also note that in obtaining the argument (24) of the Airy function, which is a classical quantity, quantum terms of order $u\ll\delta$ were dropped. Consequently in this approximation there is no dependence on the sign of δ.

[7)] We note that without the condition $2\gamma n\ll m^2$ the electron motion is more complicated, and this region calls for further study. However, in this case also these qualitative conclusions are apparently still valid.

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1053