# Magnetoacoustic phenomena in ferromagnets with itinerant electrons

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A general approach to the description of the magnetoelastic phenomena in ferromagnetic electron liquids is formulated. It is used to analyze the attenuation and the spectrum of the transverse magnetoelastic waves in a two-component electron liquid with $s$–$d(f)$ exchange interaction.

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## 1. INTRODUCTION

The phenomenological theory of ferromagnetism[1,2] requires a generalization necessary for the description of ferromagnets with itinerant electrons, and this has to a certain extent been achieved with the aid of the theory of the electron Fermi liquid.[3-5] In spite of the achievements of this theory,[6-8] no general approach to the description of magnetoelastic phenomena in ferromagnets with itinerant electrons has thus far been formulated. In the present communication we formulate such a general approach. For this purpose, we introduce the concept of magnetodeformation potential, and demonstrate its use in the theory of the magnetoelastic properties of a ferromagnet describable by the new variant, formulated in this paper, of the $s$–$d(f)$-exchange model, in which both the $s$ and $d(f)$ electrons are mobile and the exchange interaction is an indirect one.

The general approach is used to analyze the attenuation and the spectrum of transverse magnetoelastic waves. It is shown that the itinerancy of the $d(f)$ electrons has a considerable influence on the well-known effect of sound absorption by the $s$ electrons,[9] suppressing, in particular, the absorption by the $s$ electrons as the absorption edge ($k_2$) for the $d(f)$ electrons is approached from the side of small wave vectors ($k<k_2$). In the region $k>k_2$ the sound absorption by the $d(f)$ electrons is appreciable, and a magnetoelastic resonance is possible in a narrow region near $k=k_2$. Finally, in the short-wave region $k>k_2$, where there is no cause for resonance because of strong magnon attenuation, the magnetoelastic interactions manifest themselves in the effect of sound (both counterclockwise—and clockwise—polarized sound) absorption by the $d(f)$ electrons.

## 2. THE MAGNETODEFORMATION POTENTIAL

The construction of the theory of magnetoelastic phenomena in a magnetically ordered electron Fermi liquid requires a definite generalization of the known dependence of the electron energy on the nonequilibrium electron-density-matrix correction $\delta\hat{n}$ (see, for example, Refs. 3 and 4):

$$\delta\hat{\varepsilon}(\mathbf{p},\mathbf{r})=-2\beta\hat{\mathbf{s}}\mathbf{B}+\mathrm{Sp}_{s'}\int\frac{d\mathbf{p}'}{(2\pi\hbar)^3}\{\varphi(\mathbf{p},\mathbf{p}')+4\hat{\mathbf{s}}\hat{\mathbf{s}}'\psi(\mathbf{p},\mathbf{p}')\}\delta n(\mathbf{p}'\ \mathbf{r}).\quad(2.1)$$

Here $\beta$, $\hat{\mathbf{s}}$, and $\mathbf{p}$ are respectively the magnetic moment, the spin operator, and the quasimomentum of the electron, $\mathbf{B}$ is the magnetic induction, $\varphi$ and $\psi$ are functions characterizing the spin-dependent and spin-independent Fermi-liquid interactions of the electron, and $\mathrm{Sp}_s$ denotes the trace with respect to the spin variables.

It is well known that a productive role is played in the theory of the elastic properties of metals by the concept[10] of deformation potential $\Lambda_{ik}(\mathbf{p})$, which determines the dependence of the electron energy on the lattice deformation:

$$\delta\varepsilon_1(\mathbf{p},\mathbf{r})=\Lambda_{ik}(\mathbf{p})\,\hat{I}\,\frac{\partial u_i}{\partial x_k},\quad(2.2)$$

where $\hat{I}$ is the unit matrix in spin space and $\mathbf{u}$ is the lattice-displacement vector. In the more complicated case of interest to us here, it is necessary to take into consideration the interaction with the magnetic electrons, whose magnetism is determined by their distribution. The corresponding electron-energy function has the following form:

$$\delta\hat{\varepsilon}_2(\mathbf{p},\mathbf{r})=\frac{\partial u_i}{\partial x_k}\mathrm{Sp}_{s'}\int\frac{d\mathbf{p}'}{(2\pi\hbar)^3}\cdot 4\hat{s}_m\hat{s}_l'\lambda_{ik;ml}(\mathbf{p},\mathbf{p}')\delta\hat{n}(\mathbf{p}',\mathbf{r}),\quad(2.3)$$

where $\lambda_{ik;ml}(\mathbf{p},\mathbf{p}')$ is the magnetodeformation-potential tensor. The formula (2.3) can be used to describe both ferromagnets and antiferromagnets with itinerant electrons.

Because our aim in the present communication is to demonstrate the productiveness of the magnetodeformation-potential concept, we shall limit ourselves below to the consideration of the isotropic ferromagnet. Furthermore, as in the theory of magnets that neglects the momentum dependence of the function $\psi$, we shall also neglect the analogous dependence of $\lambda_{ik;ml}$. Then

$$\lambda_{ik;ml}=2\beta^2\delta\delta_{ik}\delta_{lm}+\beta^2\lambda\{\delta_{il}\delta_{km}+\delta_{im}\delta_{kl}\}.\quad(2.4)$$

For the magnetoelastic phenomena discussed below, we can obtain the basic relationships without making allowance for $\varphi$ and $\Lambda_{ik}$. Then, summing up all the foregoing, according to the formulas (2.1)–(2.4), we can use the following effective electron Hamiltonian:

$$\hat{\varepsilon}(\mathbf{p},\mathbf{r})=\varepsilon(\mathbf{p})\,\hat{I}-2\beta\hat{\mathbf{s}}\mathbf{B}+\frac{2\psi}{\beta}\mathbf{M}\hat{\mathbf{s}}+4\beta\delta\mathbf{M}\hat{\mathbf{s}}\,\mathrm{div}\,\mathbf{u}$$
$$+2\beta\lambda\left\{\left(\mathbf{M}\frac{\partial}{\partial\mathbf{r}}\right)(\hat{\mathbf{s}}\mathbf{u})+M_i\hat{\mathbf{s}}\frac{\partial}{\partial\mathbf{r}}u_i\right\},\quad(2.5)$$

$$\mathbf{M}=2\beta\mathrm{Sp}_s\int\frac{dp}{(2\pi\hbar)^3}\widehat{\widehat{s n}}.\quad(2.6)$$

The parameters $\delta$ and $\lambda$ characterize the magnetostriction energy of the electron.

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It is natural for the magnetodeformation potential to appear also in the equation of motion of the lattice. For our subsequent purposes, it is sufficient to use the equation

$$\rho_m\frac{\partial^2 u_i}{\partial t^2}=\lambda_{iklj}^{(0)}\frac{\partial^2 u_j}{\partial x_k\partial x_l}+\rho^{(i)}E_i$$
$$+\frac{1}{c}[\mathbf{j}^{(i)}\mathbf{B}]_i+\delta\frac{\partial\mathbf{M}^2}{\partial x_i}+\lambda\left\{M_i\operatorname{div}\mathbf{M}+\mathbf{M}\frac{\partial}{\partial\mathbf{r}}M_i\right\}, \tag{2.7}$$

which describes the lattice vibrations, and is, when (2.5) and (2.6) are taken account, a generalization of the well-known equation obtained in Ref. 11 by one of the present authors (see Refs. 12 and 13). Here $\rho_m$ is the mass density, $\rho^{(i)}$ is the lattice-charge density, and $\mathbf{j}^{(i)}=\rho^{(i)}\mathbf{u}$.

## 3. GROUND STATE OF THE FERROMAGNETIC ELECTRON LIQUID WITH $s$–$d(f)$-EXCHANGE INTERACTION

Many properties of magnetically ordered conductors can be understood on the basis of Vonsovskiĭ's $s$–$d(f)$-exchange model.[1] But the possibility of the itinerancy of the $d(f)$ electrons in magnetic conductors is usually neglected in the $s$–$d(f)$ model. In the case of the rare-earth metals[1] and semiconductors,[14] as well as in the actinide magnetic substances,[15,16] magnetic ordering occurs on account of the effective indirect exchange that occurs between the $4f\,(5f)$ electrons as a result of the exchange interaction between the $f$ and $s$ conduction electrons. The direct exchange between the $f$ electrons is often neglected in comparison with the indirect exchange because of its weakness. In accordance with the latter, we shall consider in our theory a two-component Fermi liquid with exchange interaction between particles of different types and no interaction between quasiparticles of the same type (in contrast to Ref. 6). Our relatively simple model of a two-component Fermi liquid with a single Fermi-liquid interaction constant $\psi$ corresponding to the $s$–$d(f)$ exchange in the first place corresponds to Vonsovskiĭ's $s$–$d(f)$-exchange model with superexchange interaction between the electrons, and in the second place takes account of the possibility of the itinerancy of the magnetic $d(f)$ electrons.

In this section we formulate the basic propositions of the theory of the ground state of a ferromagnet in the model being discussed by us, to which corresponds, when the magnetoelastic interactions are neglected, the following effective electron Hamiltonian:

$$\hat{\varepsilon}_{1,2}(\mathbf{p})=\varepsilon_{1,2}(\mathbf{p})I-2b_{1,2}\hat{s}_z, \tag{3.1}$$

$$b_1=\beta_1B-\psi(M_2/\beta_2),\qquad b_2=\beta_2B-\psi(M_1/\beta_1). \tag{3.2}$$

Here the subscripts 1 and 2 pertain respectively to the $s$ and $d(f)$ electrons; the $\beta_{1,2}$ are the magnetic moments of the electrons; the $M_{1,2}$ are the partial spontaneous magnetizations,

$$M_i=\beta_i\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}[n_F(\varepsilon_i(\mathbf{p})-b_i)-n_F(\varepsilon_i(\mathbf{p})+b_i)], \tag{3.3}$$

determined by the Fermi distributions $n_F(\varepsilon)$; and $\mathbf{B}$ is the magnetic induction, which is oriented in the equilibrium state in the isotropic model under consideration along the direction of the spontaneous magnetizations. The system of equations (3.2), (3.3) determine the ground state of the two-component electron liquid.

The theory has an especially simple form when the momentum dependence of the electron energy is isotropic and quadratic in the case of weak ferromagnets, in which case the magnetic $b_{1,2}$-level-splitting energies are small compared to the energies $\varepsilon_{1,2}$ [where $\varepsilon_{1,2}=\varepsilon_F-\varepsilon_{1,2}(0)$, $\varepsilon_F$ being the Fermi energy]. The latter obtains when the following inequality is fulfilled:

$$|B_{12}B_{21}-1|\ll B_{12}B_{21};$$
$$B_{12}=\psi m_2p_2/\pi^2\hbar^3,\ B_{21}=\psi m_1p_1/\pi^2\hbar^3, \tag{3.4}$$

where $m_{1,2}$ and $p_{1,2}$ are the masses and the Fermi momenta of the electrons. Then for $\varkappa T\ll\varepsilon_{1,2}$ the formula (3.3) assumes the form

$$M_{1,2}(T)=\beta_{1,2}\frac{m_{1,2}p_{1,2}}{\pi^2\hbar^3}b_{1,2}\left\{1-\frac{b_{1,2}^2}{24\varepsilon_{1,2}^2}-\frac{\pi^2}{24}\frac{(\varkappa T)^2}{\varepsilon_{1,2}^2}\right\}. \tag{3.5}$$

The Eqs. (3.2), which determine the exchange-level-splitting energy, then have the form

$$B_{12}=-\left[1+\frac{b_2^2+(\pi\varkappa T)^2}{24\varepsilon_2^2}-\frac{\Omega_{B1}}{\Omega_1^0}\right]\frac{b_1}{b_2},$$
$$B_{21}=-\left[1+\frac{b_1^2+(\pi\varkappa T)^2}{24\varepsilon_1^2}-\frac{\Omega_{B2}}{\Omega_2^0}\right]\frac{b_2}{b_1}, \tag{3.6}$$
$$\Omega_{1,2}^0=2b_{1,2}/\hbar,\quad \Omega_{B1,2}=2\beta_{1,2}B/\hbar.$$

It follows from the Eqs. (3.6) that, for magnetic order to exist in our model, the following condition must be fulfilled:

$$B_{12}B_{21}>1. \tag{3.7}$$

This condition admits of both parallel and antiparallel ordering of the magnetic moments $\mathbf{M}_1$ and $\mathbf{M}_2$, the former occurring in the two-component electron liquid when the condition (3.7) is fulfilled and the exchange-interaction constant $\psi$ has negative values and the latter being possible at sufficiently low temperatures when $\psi>0$. Neglecting the magnetic induction, we find that at $T=0$

$$\frac{M_1(0)}{M_2(0)}=-\frac{\beta_1b_2(0)}{\beta_2b_1(0)}=\pm\frac{\beta_1}{\beta_2}\left(\frac{m_1p_1}{m_2p_2}\right)^{1/2}. \tag{3.8}$$

Remembering that for $d(f)$ electrons possessing a high density of states at the Fermi level $m_2p_2\gg m_1p_1$, we can see that the spontaneous magnetization of a ferromagnet is determined largely by the $d(f)$ electrons, just as in Vonsovskiĭ's $s$–$d(f)$-exchange model[1] (see also Ref. 6).

## 4. DISPERSION EQUATION FOR TRANSVERSE MAGNETOELASTIC WAVES

To obtain the dispersion equation for the magnetoelastic waves in a two-component electron liquid, we generalize the relations (2.5)–(2.7) to the case of two kinds of electrons in accordance with the formulas (3.1) and (3.2) of the preceding section and the assumptions made there about the $s$–$d(f)$ exchange. Then the linear approximation to Eq. (2.7), which is suitable for the description of the laws governing the propagation of transverse acoustic waves with frequency $\omega$ and wave vector $\mathbf{k}$ directed along the $z$ axis (which is itself oriented parallel to the spontaneous magnetization vectors $\mathbf{M}_1$ and $\mathbf{M}_2$), can be written in the form

$$\rho_m(\omega^2-s^2k^2\mp\Omega_i\omega)u^\pm(\omega,k)=-\rho^{(i)}e^\pm(\omega,k)$$
$$-ik\lambda_1M_1m_1^\pm(\omega,k)-ik\lambda_2M_2m_2^\pm(\omega,k). \quad (4.1)$$

Here $m_{1,2}^\pm(\omega,k)$, $e^\pm(\omega,k)$, $u^\pm(\omega,k)=u_x\pm iu_y$ are the nonequilibrium circularly-polarized partial-magnetization, electric-field, and lattice-displacement components, $\Omega_i$ is the "ion" cyclotron frequency, $\rho^{(i)}$ and $\rho_m$ are the charge and lattice-mass densities, $s$ is the velocity of of the transverse sound, and $\lambda_1$ and $\lambda_2$ are the parameters characterizing in accordance with (2.4) the magnetodeformation potentials of the two kinds of electrons.

The peculiarity of the model of a ferromagnet with itinerant electrons stems from the laws governing their motion, which, in particular, lead to the following kinetic equations for the nonequilibrium spin density matrices $\delta\sigma_{1,2}^\pm=\mathrm{Sp}_s2(\hat{s}_x\pm i\hat{s}_y)\delta\hat{n}_{1,2}$ of the two kinds of quasi-particles:

$$[\hbar(\omega\mp\Omega_1^0)+\varepsilon_1(\mathbf{p}-\hbar\mathbf{k})-\varepsilon_1(\mathbf{p})]\delta\sigma_1^\pm(\omega,\mathbf{k},\mathbf{p})$$
$$+2\left\{n_F\left[\varepsilon_1(\mathbf{p}+\hbar\mathbf{k})\mp\frac{1}{2}\hbar\Omega_1^0\right]-n_F\left[\varepsilon_1(\mathbf{p})\pm\frac{1}{2}\hbar\Omega_1^0\right]\right\}$$
$$\times\left\{\beta_1b^\pm(\omega,\mathbf{k})-\frac{\psi}{\beta_2}m_2^\pm(\omega,\mathbf{k})-ik\beta_1\lambda_1M_1u^\pm(\omega\mathbf{k})\right\}=0,$$
$$[\hbar(\omega\mp\Omega_2^0)+\varepsilon_2(\mathbf{p}-\hbar\mathbf{k})-\varepsilon_2(\mathbf{p})]\delta\sigma_2^\pm(\omega,\mathbf{k},\mathbf{p}) \quad (4.2)$$
$$+2\left\{n_F\left[\varepsilon_2(\mathbf{p}-\hbar\mathbf{k})\mp\frac{1}{2}\hbar\Omega_2^0\right]-n_F\left[\varepsilon_2(\mathbf{p})\pm\frac{1}{2}\hbar\Omega_2^0\right]\right\}$$
$$\times\left\{\beta_2b^\pm(\omega,\mathbf{k})-\frac{\psi}{\beta_1}m_1^\pm(\omega,\mathbf{k})-ik\beta_2\lambda_2M_2u^\pm(\omega,\mathbf{k})\right\}=0,$$

where $b^\pm(\omega,k)$ is the circularly-polarized nonequilibrium magnetic induction.

From Eqs. (4.2) we have

$$m_{1,2}^\pm(\omega,k)=\beta_{1,2}\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}\delta\sigma_{1,2}^\pm(\omega,\mathbf{k},\mathbf{p})$$
$$=\frac{1}{d^\pm(\omega,k)}\{-a_{1,2}^\pm(\omega,k)b^\pm(\omega,k)+C_{1,2}^\pm(\omega,k)iku^\pm(\omega,k)\}. \quad (4.3)$$

Here

$$d^\pm(\omega,k)=1-\psi^2\Pi_1^\pm(\omega,k)\Pi_2^\pm(\omega,k),$$
$$a_{1,2}^\pm(\omega,k)=\beta_{1,2}^2\Pi_{1,2}^\pm(\omega,k)+\beta_1\beta_2\psi\Pi_1^\pm(\omega,k)\Pi_2^\pm(\omega,k), \quad (4.4)$$
$$C_{1,2}^\pm(\omega,k)=\beta_{1,2}^2\lambda_{1,2}M_{1,2}\Pi_{1,2}^\pm(\omega,k)+\beta_1\beta_2\lambda_{2,1}M_{2,1}\psi\Pi_1^\pm(\omega,k)\Pi_2^\pm(\omega,k),$$

where

$$\Pi_l^\pm(\omega,k)=2\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}\frac{n_F\left[\varepsilon_l(\mathbf{p}-\hbar\mathbf{k})\mp\frac{1}{2}\hbar\Omega_l^0\right]-n_F\left[\varepsilon_l(\mathbf{p})\pm\frac{1}{2}\hbar\Omega_l^0\right]}{\hbar(\omega\mp\Omega_l^0)+\varepsilon_l(\mathbf{p}-\hbar\mathbf{k})-\varepsilon_l(\mathbf{p})+i0}. \quad (4.5)$$

The formulas (4.3)–(4.5) form a constitutive relation relating the nonequilibrium magnetization to the magnetic induction and the lattice displacement.

The occurrence on the right-hand side of Eq. (4.1) of a nonequilibrium electric field and the related induction effects of the magnetoelastic interaction requires the consideration of the Maxwell equations, which, in the model being considered by us, have the form

$$\mp ikh^\pm(\omega,k)=\frac{\omega}{c}e^\pm(\omega,k)+\frac{4\pi\omega}{c}\left[\rho^{(i)}u^\pm(\omega,k)+\frac{i}{\omega}j^\pm(\omega,k)\right], \quad (4.6)$$
$$ike^\pm(\omega,k)=\pm\frac{\omega}{c}b^\pm(\omega,k).$$

Here $h^\pm(\omega,k)$ is the nonequilibrium circularly-polarized magnetic field intensity, $j^\pm(\omega,k)=j_1^\pm(\omega,k)+j_2^\pm(\omega,k)$ is the electronic-current density, to which the two kinds of electrons make, in accordance with the effective Hamiltonian (2.5), contributions of the following form:

$$j_{1,2}^\pm(\omega,k)=\sigma_{1,2}^\pm(\omega,k)e^\pm(\omega,k), \quad (4.7)$$

where the partial conductivity is given by the formula (see Ref. 17)

$$\sigma_l^\pm(\omega,k)=\frac{i\omega_{pl}}{4\pi\omega}-\frac{ie^2}{2\omega}\sum_{\sigma=\pm1}\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}v_{\perp,l}^2$$
$$\times\frac{n_F[\varepsilon_l(\mathbf{p}-\hbar\mathbf{k})-{}^1/{}_2\sigma\hbar\Omega_l^0]-n_F[\varepsilon_l(\mathbf{p})-{}^1/{}_2\sigma\hbar\Omega_l^0]}{\hbar(\omega\mp\Omega_l)+\varepsilon_l(\mathbf{p}-\hbar\mathbf{k})-\varepsilon_l(\mathbf{p})+i0}. \quad (4.8)$$

Here $\Omega_l$ and $\omega_{pl}$ are the cyclotron and plasma frequencies; $v_{\perp,l}^2=v_{x,l}^2+v_{y,l}^2$; $\mathbf{v}_l=\partial\varepsilon_l/\partial\mathbf{p}$; $e$ is the electron charge.

The system of equations (4.1), (4.3), (4.6), and (4.7) with allowance for the relation between the induction and the magnetic field $b^\pm=h^\pm+4\pi(m_1^\pm+m_2^\pm)$ allows us to write down the following dispersion equation for the magnetoelastic waves:

$$\left\{\omega^2-s^2k^2\mp\Omega_i\omega-\frac{k^2[\lambda_1M_1C_1^\pm(\omega,k)+\lambda_2M_2C_2^\pm(\omega,k)]}{\rho_md^\pm(\omega,k)}\right\}$$
$$\times\left\{\frac{\omega^2}{k^2c^2}\varepsilon^\pm(\omega,k)-\frac{1}{\mu^\pm(\omega,k)}\right\}=\frac{4\pi k^2}{\rho_m}\left\{\frac{C_1^\pm(\omega,k)+C_2^\pm(\omega,k)}{d^\pm(\omega,k)}\pm\frac{\omega\rho^{(i)}}{ck^2}\right\}^2, \quad (4.9)$$

where

$$\varepsilon^\pm(\omega,k)=1+(4\pi i/\omega)[\sigma_1^\pm(\omega,k)+\sigma_2^\pm(\omega,k)]$$

is the complex permittivity and

$$\mu^\pm(\omega,k)=d^\pm(\omega,k)\{d^\pm(\omega,k)+4\pi[a_1^\pm(\omega,k)+a_2^\pm(\omega,k)]\}^{-1}$$

is the magnetic permeability.

Equation (4.9) describes not only magnetoelastic, but also electromagnetic waves. In this case for relatively short wavelengths, when $\omega/k$ is small compared to the electron velocities $v_{1,2}$ at the Fermi surface, we can use the following simple formula to make estimates:

$$\sigma_{1,2}^\pm\sim\frac{e^2p_{1,2}^2}{4\pi\hbar^3|k|}. \quad (4.10)$$

From this it follows that the effect of the conductivity on the magnetoelastic-wave frequency can be neglected if (see Ref. 18)

$$\omega\ll k^3c^2\{\omega_{p1}^2/v_1+\omega_{p2}^2/v_2\}^{-1}. \quad (4.11)$$

Below the condition (4.11) will be considered fulfilled.

## 5. THE SPECTRUM AND ATTENUATION OF THE MAGNONS

In this section we discuss the properties of the magnons described by the model of an electron liquid with the $s$-$d(f)$-exchange interaction. In this case, in the limit $\rho_m=\infty$, when, as follows from Eq. (4.9), the magnetoacoustic interaction disappears, the dispersion equation for the magnons corresponds to the poles of the magnetic permeability:

$$d^\pm(\omega,k)+4\pi[a_1^\pm(\omega,k)+a_2^\pm(\omega,k)]=0. \quad (5.1)$$

In the limit of sufficiently long waves (i.e., for $kv_{1,2} \ll |\Omega^0_{1,2}|$) and low frequencies ($\omega \ll |\Omega^0_{1,2}|$)

$$\Pi^{\pm}_{1,2}(\omega, k) = -\frac{2M_{1,2}}{\beta_{1,2}\hbar(\Omega^0_{1,2}\mp\omega)} - \frac{k_i k_j \alpha^{(1,2)}_{ij}}{\psi\Omega^0_{1,2}}, \quad (5.2)$$

where the tensors $\alpha^{(1,2)}_{ij}$ are given by the expressions

$$\alpha^{(1,2)}_{ij} = \frac{\psi}{\Omega^0_{1,2}} \int \frac{d\mathbf{p}}{(2\pi\hbar)^3} v_i v_j \left\{ \frac{1}{b_{1,2}} [n_F(\varepsilon_{1,2}(\mathbf{p}) - b_{1,2}) - n_F(\varepsilon_{1,2}(\mathbf{p}) + b_{1,2})] + \frac{\partial}{\partial\varepsilon} [n_F(\varepsilon_{1,2}(\mathbf{p}) - b_{1,2}) + n_F(\varepsilon_{1,2}(\mathbf{p}) + b_{1,2})] \right\}. \quad (5.3)$$

From (5.1) we have, in conformity with the formulas (5.2) and (5.3), a quadratic dispersion law for the magnons with polarization (+):

$$\omega(\mathbf{k}) = \frac{\Omega_{H1}\Omega_2^0 + \Omega_{H2}\Omega_1^0 + (\Omega_1^0\alpha^{(1)}_{ij} + \Omega_2^0\alpha^{(2)}_{ij}) k_i k_j}{\Omega_1^0 + \Omega_2^0} \approx \Omega_{H2} + k_i k_j [\alpha^{(1)}_{ij} + (\Omega_2^0/\Omega_1^0)\alpha^{(2)}_{ij}]. \quad (5.4)$$

Here we have taken account of the smallness of the ratio (3.8), a fact which corresponds to the fulfillment of the condition $|\Omega^0_1| \gg \Omega^0_2$, and have used the notation

$$\Omega_{H\,1,2} = 2\beta_{1,2}H/\hbar, \quad H = B - 4\pi(M_1 + M_2).$$

The magnon dispersion is then determined by both the $s$ and $d(f)$ electrons, while the gap in the spectrum $\Omega_{H2}$ is determined by the $d(f)$ electrons.

To demonstrate the main properties of the magnons, below we limit ourselves to the isotropic and quadratic energy-dispersion law for the $s$- and $d(f)$ electrons. Then at $T = 0$ K we have the following general formula:

$$\Pi^{\sigma}_{1,2}(\omega, k) = \frac{m^3_{1,2}}{2\pi^2\hbar^4 k^3} \sum_{\sigma'=\pm} \sigma' \left\{ \left(\omega - \sigma\Omega^0_{1,2} - \sigma'\frac{\hbar k^2}{2m_{1,2}}\right) k v^{\sigma'}_{1,2} \right. \quad (5.5)$$

$$\left. + \frac{1}{2}\left[(kv^{\sigma'}_{1,2})^2 - \left(\omega - \sigma\Omega^0_{1,2} - \sigma'\frac{\hbar k^2}{2m_{1,2}}\right)^2\right] \ln\frac{\omega - \sigma\Omega^0_{1,2} + kv^{\sigma'}_{1,2} - \sigma'\hbar k^2/2m_{1,2}}{\omega - \sigma\Omega^0_{1,2} - kv^{\sigma'}_{1,2} - \sigma'\hbar k^2/2m_{1,2}} \right\};$$

$$v^{\sigma}_{1,2} = v_{1,2}(1 + \sigma b_{1,2}/\varepsilon_{1,2})^{1/2}, \quad v_{1,2} = (2\varepsilon_{1,2}/m_{1,2})^{1/2}.$$

In the region of frequencies and wave vectors satisfying the condition

$$\omega \geqslant \Omega^0_{1,2} - kv_{1,2} \pm \hbar k^2/2m_{1,2}, \quad (5.6)$$

the imaginary part of (5.5) is nonzero:

$$\mathrm{Im}\,\Pi^{+}_{1,2}(\omega, k) = \frac{m^3_{1,2}}{4\pi\hbar^4 k^3} \sum_{\sigma=\pm} \sigma \left[(kv^{\sigma}_{1,2})^2 - \left(\omega - \Omega^0_{1,2} - \sigma\frac{\hbar k^2}{2m_{1,2}}\right)^2\right] \theta\left(\omega - \Omega^0_{1,2} + kv^{\sigma}_{1,2} - \sigma\frac{\hbar k^2}{2m_{1,2}}\right),$$

where $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x \leqslant 0$.

The regions in which $\Pi^+_1$ and $\Pi^+_2$ have appreciable imaginary parts, which describe the collisionless dissipation caused respectively by the $s$ and $d(f)$ electrons, are bounded on the $\omega = 0$ axis by the points

$$k_{1,2} = (m_{1,2}/\hbar)(v^+_{1,2} - v^-_{1,2}), \quad \bar{k}_{1,2} = (m_{1,2}/\hbar)(v^+_{1,2} + v^-_{1,2}).$$

In the simplest case of a weak ferromagnet (i.e., for $|b_{1,2}| \ll \varepsilon_{1,2}$), to the consideration of which we limit ourselves below,

$$k_{1,2} \approx |\Omega^0_{1,2}|/v_{1,2} \ll \bar{k}_{1,2} \approx p_{1,2}/\hbar.$$

Below we shall all the time be interested in not too short waves, i.e., waves for which $\hbar k \ll p_{1,2}$. Then for $k$ not too close to $k_{1,2}$, when

$$\omega \ll \frac{v_{1,2}}{k}|k^2 - k^2_{1,2}| \ln\left|\frac{k + k_{1,2}}{k - k_{1,2}}\right|, \quad (5.7)$$

we have the following approximate expression:

$$\Pi^{+}_{1,2}(\omega, k) = \frac{m^2_{1,2}v_{1,2}}{\pi^2\hbar^3}\left\{-1 + \frac{\omega}{2kv_{1,2}}\ln\frac{k - k_{1,2}}{k + k_{1,2}} + \frac{\hbar^2k^2}{12p^2_{1,2}} + \frac{(\hbar\Omega^0_{1,2})^2}{24\varepsilon^2_{1,2}}\right\}. \quad (5.8)$$

Hence we find in accordance with the formulas (4.4) that

$$a^+_{1,2} = -\Omega_{\mathrm{mag}\,1,2}/4\pi\Omega^0_{1,2}, \quad (5.9)$$

$$\mathrm{Re}\,d^+(\omega, k) = \frac{\Omega_{B1}}{\Omega_1^0} + \frac{\Omega_{B2}}{\Omega_2^0} + \frac{\hbar^2k^2}{12}\left(\frac{1}{p_1^2} + \frac{1}{p_2^2}\right) + \frac{\omega}{2k}\left(\frac{1}{v_1}\ln\left|\frac{k - k_1}{k + k_1}\right| + \frac{1}{v_2}\ln\left|\frac{k - k_2}{k + k_2}\right|\right), \quad (5.10)$$

$$\mathrm{Im}\,d^+(\omega, k) = -\frac{\pi\omega}{2k}\left(\frac{1}{v_1}\theta(k - k_1) + \frac{1}{v_2}\theta(k - k_2)\right); \quad (5.11)$$

$$\Omega_{\mathrm{mag}\,1,2} = 8\pi\beta_{1,2}(M_1 + M_2)/\hbar.$$

The formulas (5.1), (5.9)–(5.11) allow us to write down the following expressions: for the frequency

$$\omega_{\mathrm{mag}}(k) = \omega_{\mathrm{sp}}(k)\left\{\frac{\Omega_2^0}{\Omega_1^0}\frac{k_1}{2k}\left[\ln\left|\frac{k + k_1}{k - k_1}\right| + \frac{v_1}{v_2}\ln\left|\frac{k + k_2}{k - k_2}\right|\right]\right\}^{-1} \quad (5.12)$$

and for the magnon-damping constant

$$\gamma_{\mathrm{mag}}(k) = \pi\omega_{\mathrm{mag}}(k)\left[\theta(k - k_1) + \frac{v_1}{v_2}\theta(k - k_2)\right] \times\left[\ln\left|\frac{k + k_1}{k - k_1}\right| + \frac{v_1}{v_2}\ln\left|\frac{k + k_2}{k - k_2}\right|\right]^{-1}, \quad (5.13)$$

$$\omega_{\mathrm{sp}}(k) = \Omega_{H2} + \Omega_2^0\frac{\hbar^2k^2}{12}\left(\frac{1}{p_1^2} + \frac{1}{p_2^2}\right).$$

For comparable $s$- and $d(f)$-electron concentrations, a high density of $d(f)$-electron states implies that $m_2 \gg m_1$. Therefore, $k_1 \ll k_2$. Consequently, as the wave vector increases, the magnon-dispersion curve at first approaches the edge of the collisionless absorption by the $s$ electrons. Remembering that $k \ll k_2$, we can assert that the deviation from the quadratic dispersion law (5.4) occurs in a small neighborhood $k \approx k_1$, where $\ln|2k_1/(k - k_1)| \gg (m_2/m_1)^{1/2}$,

$$\omega_{\mathrm{mag}}(k) = \omega_{\mathrm{sp}}(k)\left\{1 + \frac{\Omega_2^0 k_1}{2\Omega_1^0 k}\ln\left|\frac{k + k_1}{k - k_1}\right|\right\}^{-1}. \quad (5.14)$$

The magnon damping that occurs in the region $k_1 < k < k_2$ satisfies the inequality $\omega_{\mathrm{mag}}(k) \gg \gamma_{\mathrm{mag}}(k)$, which allows us to speak of magnons in this region. Let us note that the interaction between the magnons and the $s$ electrons in our model is in many respects similar to the interaction studied in the model that takes account of the direct exchange between the $d(f)$ electrons.[6]

Finally, let us discuss the short-wave region $k_1 \ll k$, where

$$\omega_{\mathrm{mag}}(k) = \omega_{\mathrm{sp}}(k)\frac{2k}{k_2}\left\{\ln\left|\frac{k + k_2}{k - k_2}\right|\right\}^{-1}. \quad (5.15)$$

The spectrum (5.15) is similar to the spectrum studied in the theory of the one-component electron liquid.[7] The formula (5.13) in the short-wave region describes, in particular, the phenomenon whereby the $s$-electron-induced magnon damping decreases as we approach the edge, on the small-$k$ side, of the spin-flip absorption by the $d(f)$ electrons. Let us emphasize that there exists for $k > k_2$ a narrow region in which

$$\ln\left|\frac{2k_2}{k-k_2}\right|\gg\pi,$$

owing to which the absorption of the magnons by the $d(f)$ electrons is relatively weak, and does not forbid the existence of magnons.

## 6. MAGNETOACOUSTIC RESONANCE AND SOUND ATTENUATION

In considering the magnetoelastic coupling of waves with the aid of Eq. (4.9), we shall, as in the preceding section, assume the relation (4.11) to be fulfilled and neglect the effect of the conduction. Then, introducing the magnetostrictive coupling parameters $\zeta_{1,2}=(\lambda_{1,2}^2M_{1,2}^2/4\pi\rho_m s^2)$ and the inductive coupling parameter $\xi=(4\pi\rho^{(i)2}/\rho_m c^2k^2)$ for the waves, we can write Eq. (4.9) in the following form:

$$\frac{\omega^2(1+\xi)-s^2k^2\mp\Omega_i\omega}{s^2k^2}\{d^\pm(\omega,k)+4\pi[a_1^\pm(\omega,k)+a_2^\pm(\omega,k)]\}$$
$$=4\pi a_1^\pm\left(\zeta_1^{1/2}\mp\frac{\omega}{sk}\xi^{1/2}\right)^2+4\pi a_2^\pm\left(\zeta_2^{1/2}\mp\frac{\omega}{sk}\xi^{1/2}\right)^2$$
$$+(\zeta_1^{1/2}-\zeta_2^{1/2})^2\frac{4\pi\beta_1\beta_2}{\psi}\left(\frac{4\pi\beta_1\beta_2}{\psi}-1\right)[1-d^\pm(\omega,k)]. \qquad (6.1)$$

Below we shall be interested in the conditions under which $|\omega-sk|/sk\ll1$. Furthermore, the sound attenuation discussed below is connected with the magnon attenuation, and this requires the fulfillment of the condition $k>k_1\sim|\Omega_1^0|/v_1$. This implies, in particular, that

$$\xi^{1/2}\leqslant A^{-1/2}\cdot10^{-4}\varepsilon_1/b_1,$$

(where $A$ is the atomic weight of the atoms of the lattice), which allows us to neglect $\xi$ on the left-hand side of Eq. (6.1). Further, we have

$$\Omega_i/sk<(v_1/s)\Omega_i/|\Omega_1^0|\sim10^{-9}(\varepsilon_1/b_1)B,$$

where $B$ is in gausses. This allows us to neglect $\Omega_i$. Further, let us also take into account the smallness of $4\pi\beta_1\beta_2/\psi$ in comparison with unity. In the case, being considered by us, of spontaneous magnetizations, $\Omega_{\mathrm{mag}1,2}\ll|\Omega_{1,2}|$, and

$$\frac{4\pi\beta_1\beta_2}{\psi}=-\frac{1}{2}\left(\frac{\Omega_{\mathrm{mag}1}}{\Omega_1^0}\frac{M_2}{M_1+M_2}+\frac{\Omega_{\mathrm{mag}2}}{\Omega_2^0}\frac{M_1}{M_1+M_2}\right). \qquad (6.2)$$

Let us take into account in the particular case of weak ferromagnetism the smallness of $d^\pm$ in comparison with unity, as well as the formula (5.9). As a result, Eq. (6.1) assumes the following form:

$$(\omega^2-s^2k^2)[d^\pm(\omega,k)+4\pi(a_1^\pm+a_2^\pm)]=s^2k^2\tilde{F}^\pm, \qquad (6.3)$$
$$\tilde{F}^\pm=-\frac{\Omega_{\mathrm{mag}1}}{\Omega_1^0}\left[(\zeta_1^{1/2}\mp\xi^{1/2})^2-\frac{1}{2}(\zeta_1^{1/2}-\zeta_2^{1/2})^2\frac{M_2}{M_1+M_2}\right]$$
$$-\frac{\Omega_{\mathrm{mag}2}}{\Omega_2^0}\left[(\zeta_2^{1/2}\mp\xi^{1/2})^2-\frac{1}{2}(\zeta_1^{1/2}-\zeta_2^{1/2})^2\frac{M_1}{M_1+M_2}\right].$$

As follows from Eq. (6.3), allowance for the motion of the itinerant electrons leads to the occurrence of specific phenomena during the propagation of sound in a ferromagnet. Thus, collisionless spin-flip absorption of counterclockwise-polarized sound by electrons turns out to be possible in the regions of frequencies and wave vectors corresponding to the inequality (5.6). The absorption of clockwise-polarized sound is possible in the regions

$$-\omega\geqslant\Omega_{1,2}^0-kv_{1,2}^\pm\pm\hbar k^2/2m_{1,2}. \qquad (6.4)$$

Furthermore, there occur at the boundaries of the regions (5.6) and (6.4) in the wave-vector dependence of the sound frequency singularities of the form

$$\sum_{\sigma=\pm}\sigma\left(\pm sk-\Omega_{1,2}^0+kv_{1,2}^\sigma-\frac{\sigma\hbar k^2}{2m_{1,2}}\right)\ln\left|\pm sk-\Omega_{1,2}^0+kv_{1,2}^\sigma-\frac{\sigma\hbar k^2}{2m_{1,2}}\right|.$$

Let us note that Kontorovich and Oleĭnik[9] have discussed the singularity of the sound spectrum at the edge of the collisionless absorption of sound by $s$ electrons, using the model of a ferromagnet with mobile $s$ electrons and $d(f)$ electrons localized at the lattice sites.

For the $s-d(f)$-exchange model, when $m_2\gg m_1$ and $n_2\sim n_1$, Eq. (6.3) reduces, according to (5.9)–(5.13), to the following equation

$$(\omega^2-s^2k^2)\{\pm\omega[1\pm i\gamma_{\mathrm{mag}}(k)/\omega_{\mathrm{mag}}(k)]-\omega_{\mathrm{mag}}(k)\}=s^2k^2\omega_{\mathrm{mag}}(k)F^\pm. \qquad (6.5)$$

Here $F^\pm=-[\Omega_2^0/\omega_{\mathrm{sp}}(k)]\tilde{F}^\pm$ is the wave-coupling constant. Under the assumption that the magnetostriction constant $\lambda_2$ is not too small in comparison with $\lambda_1[\lambda_1(m_1/m_2)^{1/2}\ll\lambda_2]$, $F^\pm$ has the form

$$F^\pm=\frac{\Omega_{\mathrm{mag}2}}{\omega_{\mathrm{sp}}}(\zeta_2^{1/2}\mp\xi^{1/2})^2. \qquad (6.6)$$

Equation (6.5) has been written in a form similar to the one used in the phenomenological approach.[2] In view of this, we shall not discuss the spectrum of coupled magnetoelastic waves, noting only that, instead of the quadratic magnon spectrum obtaining in the phenomenological theory,[2] we have in our case the magnon spectrum $\omega_{\mathrm{mag}}(k)$, (5.12). On the other hand, the properties of a magnetic substance with itinerant electrons manifest themselves in Eq. (6.5) not only in the magnon spectrum $\omega_{\mathrm{mag}}(k)$, but also in the wave-vector dependence of the $s$- and $d(f)$-electron-induced spin-flip attenuation $\gamma_{\mathrm{mag}}(k)$ of the magnons, which is naturally neglected in the phenomenological approach.[2]

Let us discuss the consequences arising from Eq. (6.5), and characterizing sound absorption. Let us begin the discussion with the wave-vector region $k_1<k<k_2$ where the sound attenuation is determined by the spin-flip absorption by the $s$ electrons. In this region of wave vectors there can be strong coupling between counter clockwise-polarized sound and the magnons. When this occurs, the splitting of the sound- and magnon-dispersion curves in the vicinity of the intersection point exceeds the magnon damping constant:

$$(F^+)^{1/2}\gg\gamma_{\mathrm{mag}}(k)/\omega_{\mathrm{mag}}(k). \qquad (6.7)$$

Under this condition, counterclockwise-polarized sound can be in resonance with the magnons. In the vicinity of this resonance

$$|\omega_{\mathrm{mag}}(k)-sk|\leqslant(F^+sk\omega_{\mathrm{mag}}(k))^{1/2},$$

and the counterclockwise-polarized-sound damping constant $\gamma_s^+(k)$ attains the value

$$\gamma_s^+(k)={}^1/_2[\gamma_{\mathrm{mag}}(k)/\omega_{\mathrm{mag}}(k)]sk.$$

Using (5.13) in the wave-vector region $k_1<k<k_2$, excluding the small neighborhood of the wave vector $k_1$, where

$$\ln\left|\frac{k+k_1}{k-k_1}\right| \ll \frac{v_1}{v_2}\ln\left|\frac{k+k_2}{k-k_2}\right|, \tag{6.8}$$

we obtain for the $s$-electron-induced-damping constant for counterclockwise-polarized sound at resonance the expression

$$\gamma_s^+(k)=\frac{\pi}{2}\frac{v_2}{v_1}sk\Big/\ln\left|\frac{k+k_2}{k-k_2}\right|. \tag{6.9}$$

Then, remembering that $v_2/v_1 \sim m_1/m_2$, we find from (6.9) that at points far from the edge of the absorption by the $d(f)$ electrons (i.e., that for $k \ll k_2$) the damping constant

$$\gamma_s^+(k)\approx\frac{\pi}{4}\left(\frac{m_1}{m_2}\right)^{1/2}sk_1. \tag{6.10}$$

For the purpose of estimating $\gamma_s^+(k)$, we can write the condition (6.7) for strong coupling of the waves with $k \sim k_1$ in the form

$$F^+\gg(\gamma_s^+/sk_1)^2\approx m_1/m_2,$$

which is fulfilled in, for example, the rare-earth metals and the actinide magnetic substances with gigantic magnetostriction,[15] for which the wave-coupling parameter attains the value

$$F^+=\frac{\Omega_{\text{mag}\,2}}{\omega_{\text{sp}}}(\zeta_2^{1/2}-\xi^{1/2})^2\sim\zeta_2\sim10^{-3},\quad\left(\frac{m_1}{m_2}\right)^{1/2}\sim\left(\frac{b_1}{\varepsilon_1}\right)\sim10^{-2}.$$

For $k_1 \ll k < k_2$ the condition (6.7) is fulfilled in ferromagnets in which the wave-coupling parameter

$$F^+\gg(v_2/v_1)^2\sim(m_1/m_2)^2\sim(b_1/\varepsilon_1)^4.$$

At points far from resonance, where

$$|\omega_{\text{mag}}(k)-sk|/sk\gg F^+, \tag{6.11}$$

the damping constant for counterclockwise-polarized sound is significantly smaller than the damping constant at resonance, and is, according to (5.13), (6.5), and (6.8) for $k_1 < k < k_2$, given by the expression

$$\gamma_s^+(k)=\pi\Omega_{\text{mag}\,2}(\zeta_2^{1/2}-\xi^{1/2})^2\frac{v_2}{v_1}\frac{k}{k_2}\left(\frac{sk}{\omega_{\text{mag}}(k)-sk}\right)^2\Big/\ln^2\left|\frac{k+k_2}{k-k_2}\right|. \tag{6.12}$$

Let us note that the $s$-electron-induced sound damping effect in the model with the $d(f)$ electrons localized at the lattice sites is discussed in Ref. 9. As follows from (6.9) and (6.12), in the case of strong wave coupling, i.e., when the condition (6.7) is fulfilled, the itinerancy of the $d(f)$ electrons becomes important for sound absorption by the $s$ electrons as the edge of the absorption by the $d(f)$ electrons is approached, and leads to the decrease of the absorption of sound by the $s$ electrons. Thus, the results of Ref. 9, as applied to ferromagnets with a finite $d(f)$-electron band width, are valid at points far from the edge of the absorption by the $d(f)$ electrons in the long-wave region $k \ll k_2$.

Let us consider the case in which the inequality that is the inverse of (6.7) is fulfilled, and the counterclockwise-polarized sound and the magnons are weakly coupled. Then the $s$-electron-induced damping constant for counterclockwise-polarized sound in the case in which the difference between the sound and magnon frequencies is smaller than the magnon damping constant, i.e., in which

$$\frac{|\omega_{\text{mag}}(k)-sk|}{sk}\ll\pi\frac{v_2}{v_1}\Big/\ln\left|\frac{k+k_2}{k-k_2}\right|, \tag{6.13}$$

can be determined from (5.13) and (6.5), and is given by the expression

$$\gamma_s^+(k)=\frac{1}{\pi}\Omega_{\text{mag}\,2}(\zeta_2^{1/2}-\xi^{1/2})^2\frac{v_1}{v_2}\frac{k}{k_2}. \tag{6.14}$$

The $s$-electron-induced damping constant (6.14) for counterclockwise-polarized sound does not exceed the quantities

$$\gamma_s^+\ll\Omega_{\text{mag}\,2}(m_1/m_2)^{1/2}\quad\text{for}\quad k\sim k_1,$$
$$\gamma_s^+\ll\Omega_{\text{mag}\,2}(m_1/m_2)\quad\text{for}\quad k\sim k_2$$

because of the fulfillment of the inequalities

$$(\zeta_2^{1/2}-\xi^{1/2})^2\ll m_1/m_2\quad\text{for}\quad k\sim k_1$$

and, correspondingly,

$$(\zeta_2^{1/2}-\xi^{1/2})^2\ll(m_1/m_2)^2$$

in the region $k \sim k_2$, which constitute the condition for weak coupling of the waves.

To estimate (6.14), let us note that the following limitations on the magnitude of the parameter $(m_1/m_2)^{1/2}$ follow from the condition for the intersection of the sound and magnon dispersion curves:

$$(m_1/m_2)^{1/2}\leqslant(6s/v_1)^{1/2}\sim2\cdot10^{-1}\quad\text{for}\quad k\sim k_1,$$
$$(m_1/m_2)^{1/4}\leqslant(6s/v_1)^{1/4}\sim8\cdot10^{-2}\quad\text{for}\quad k\sim k_2.$$

The corresponding sound frequency attains the value $\Omega_1^0 s/v_1 \sim 10^{11}$ sec$^{-1}$ for $k \sim k_1$ and $(m_2/m_1)^{1/2}$ times this value for $k \sim k_2$.

If the difference between the sound and magnon frequencies satisfies the inequality that is the inverse of (6.13), then the $s$-electron-induced-damping constant for counterclockwise-polarized sound in the case of weak coupling between the waves is given by the expression (6.12).

As can be seen from (6.5), the interaction of clockwise-polarized sound with magnons is not a resonance interaction. The $s$-electron-induced-damping constant for clockwise-polarized sound in the region $k_1 < k < k_2$ will be given by the expression (6.12) with $s$ and $+\xi^{1/2}$ respectively replaced by $-s$ and $-\xi^{1/2}$.

In the region of short wavelengths $k > k_2$ the damping of the magnetoelastic waves is determined by the spin-flip absorption by the $d(f)$ electrons. Assuming that the condition for weak coupling of the counterclockwise-polarized sound with the magnons,

$$F^+\ll\pi^2\Big/\ln^2\left|\frac{k+k_2}{k-k_2}\right|, \tag{6.15}$$

is fulfilled in this region, we find from (5.13) and (6.5) for the $d(f)$-electron-induced-damping constants of counterclockwise- and clockwise-polarized sound the expressions

$$\gamma_s^\pm(k)=\frac{1}{\pi}\Omega_{\text{mag}\,2}(\zeta_2^{1/2}\mp\xi^{1/2})^2\frac{k}{k_2}\left\{1+\left(\frac{\omega_{\text{mag}}(k)\mp sk}{\pi sk}\ln\left|\frac{k+k_2}{k-k_2}\right|\right)^2\right\}^{-1}. \tag{6.16}$$

As noted in the preceding section, the magnons exist as a collective mode near the edge of the absorption by the $d(f)$ electrons, where

$$\ln\left|\frac{2k_2}{k-k_2}\right| \gg \pi, \qquad (6.17)$$

In this case, when the difference between the sound and magnon frequencies is smaller than the magnon-damping constant, i.e., when

$$\frac{|\omega_{\rm mag}(k)-sk|}{sk} \ll \pi \Big/ \ln\left|\frac{2k_2}{k-k_2}\right|, \qquad (6.18)$$

the counterclockwise-polarized-sound damping constant (6.16) attains its maximum value, equal to

$$\gamma_{\bullet}^{+}(k)=\frac{1}{\pi}\Omega_{\rm mag\,2}(\zeta_2^{1/2}-\xi^{1/2})^2\frac{k}{k_2}. \qquad (6.19)$$

A comparison of (6.19) with the $s$-electron-induced-damping constant (6.14) for counterclockwise-polarized sound shows that, in the case of weak coupling of the waves under the conditions (6.13) and (6.18), the damping constant for counterclockwise-polarized sound decreases by a factor of $v_1/v_2 \sim m_2/m_1$ as we go through the edge of the absorption by the $d(f)$ electrons into the region $k>k_2$. Such a decrease of the damping constant is due to the fact that, when the waves are weakly coupled under the conditions (6.13) and (6.18), the damping constant for counterclockwise-polarized sound is inversely proportional to the magnon-damping constant (5.13), which increases by a factor of $v_1/v_2$ when the wave vector becomes greater than $k_2$. On the other hand, when the difference between the sound and magnon frequencies satisfies the condition that is the inverse of (6.18), the $d(f)$-electron-induced sound damping constant (6.16) is $v_1/v_2$ times greater than the $s$-electron-induced sound damping constant (6.12).

Let us, using (6.19), estimate the maximum relative $d(f)$-electron-induced sound damping constant:

$$\frac{\gamma_{\bullet}^{+}}{sk}\sim\frac{\Omega_{\rm mag\,2}}{\Omega_2^0}\frac{v_2}{s}(\zeta_2^{1/2}-\xi^{1/2})^2\sim\frac{\Omega_{\rm mag\,2}}{\Omega_1^0}\frac{v_1}{s}\left(\frac{m_1}{m_2}\right)^{1/2}\zeta_2\sim 8\cdot 10^{-5}$$

in magnetic substances with gigantic magnetostriction $\zeta_2 \sim 10^{-3}$ and parameters $(m_1/m_2)^{1/2} \sim 8\times 10^{-2}$, $M_2 \sim 10^3$ G, and $\Omega_1^0 \sim 10^{14}$ sec$^{-1}$. The $d(f)$-electron-induced damping constant (6.16) for clockwise-polarized sound near the absorption edge (6.17) is

$$\left(\frac{\omega_{\rm mag}(k)+sk}{\pi sk}\ln\left|\frac{k+k_2}{k-k_2}\right|\right)^2$$

times smaller than (6.19). When we go from the edge of the absorption by the $d(f)$ electrons into the region of greater $k$, where

$$\ln\left|\frac{k+k_2}{k-k_2}\right| \ll \pi,$$

the magnons become strongly damped, and the difference between the damping constants (6.16) for counterclockwise- and clockwise-polarized sound reduces to a dependence of the coupling constant (6.6) on the polarization.

Let us note that, in deriving the Eq. (6.1) for coupled magnetoelastic waves, we neglected the effect of the conduction on the frequency of the magnetoelastic waves, which is justified on account of the fact that the inequality (4.11) is fulfilled. On the other hand, the conductivity (4.10) corresponds to the region of the anomalous skin effect, where magnetoelastic waves can undergo $s$- and $d(f)$-electron-induced nonspin-flip collisionless damping.[19] In this case, according to (19), we can neglect the effect of the conduction on both the frequency and the damping of magnetoelastic waves in the $k>k_1$ region if the following inequality is fulfilled:

$$\frac{\Omega_{\rm mag\,2}}{\Omega_2^0}<\left(\frac{c\Omega_1^0}{v_1\omega_{p1}}\right)^2\left(\frac{k}{k_1}\right)^2. \qquad (6.20)$$

This condition is easily fulfilled.

We also neglected in our investigation the lattice damping of sound, assuming it to be weak compared to the considered $s$- and $d(f)$-electron-induced spin-flip damping at fairly low temperatures in perfect crystals.[20]

Thus, the analysis, performed with the aid of the magnetodeformation potential concept, of the magnetoelastic phenomena occurring in a two-component electron Fermi liquid with $s$–$d(f)$ exchange demonstrates the productiveness of such a model for the study of the magnetoelastic phenomena occurring in the rare-earth metals and the actinide ferromagnets, in which the magnetic ordering of the $4f(5f)$ electrons occurs as a result of the $s$–$f$ exchange.

The experimental investigation of the effect of the conduction $s$ electrons on the spectrum and attenuation of magnetoelastic waves requires the excitation of acoustic waves with frequency of the order of $10^{11}$ sec$^{-1}$. Let us note that hypersound with angular frequency $5.8\times 10^{10}$ sec$^{-1}$ has been excited in dysprosium plates in experiments.[21] The study of the manifestation of effects with finite $d(f)$-electron band width in the spectrum and attenuation of magneto elastic waves requires the use of sound of frequency $(m_2/m_1)^{1/2}$ times higher than this. It may be inferred that magnetoelastic waves with frequency higher than $10^{11}$ sec$^{-1}$ can be studied, in particular, with the aid of inelastic neutron scattering (cf. Ref. 8).


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