Plasma waves in a randomly inhomogeneous metal[1)]

V. A. Ignatchenko, Yu. I. Mankov, and F. V. Rakhmanov

L. V. Kirenskiĭ Physics Institute, Siberian Division, USSR Academy of Sciences
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The spectrum and damping of volume plasma waves in a randomly inhomogeneous metal are investigated by a method based on the theory of random functions. The inhomogeneity is introduced as a small correction to the periodic lattice potential and is described by a random function with an arbitrary correlation radius $1/k_0$, which depends on the characteristic size of the inhomogeneities and is assumed to exceed considerably the lattice constant. The inhomogeneous potential can interact with the plasma wave either directly or indirectly by modifying the equilibrium distribution function of the conduction electrons. The modification of the plasma-excitation dispersion law is most significant for small wave numbers k. The dispersion curve $\omega'(k)$ has an inflection in the vicinity of $k \approx k_0$. At the same value of k, the plasma wave damping due to the inhomogeneities is a maximum. The theory is compared with experiment.

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INTRODUCTION

The similarities of the excitation spectra in an inhomogeneous medium, in the case when the inhomogeneities constitute localized noninteracting defects in an ideal lattice, have by now been well investigated theoretically (see, e.g., the monographs[1,2] for phonons, Ref. 3 for magnons, and Refs. 4–6 for plasma waves). An increase in the defect density leads to a correlation between them, and this complicates the problem greatly.

In the limiting case of a high defect density, however, the approximation of the random-inhomogeneity medium becomes valid. The parameters of this medium are described by stationary random functions of the spatial coordinates. The presence of a strong interaction between the defects leads of an increase of the correlation radii of such random functions, i.e., to their smoothing. If the correlation radii become much larger than the lattice constant a, the main singularities of the inhomogeneous medium manifest themselves in the region of long waves ($ka \ll 1$, where k is the wave vector of the plasma wave), the excitation spectrum in such a medium can be investigated using the mathematical formalism of the theory of random functions, which has been well developed by now.[7,8] This approach was used in an investigation of magnetic resonance[9] and spin waves in an inhomogeneous ferromagnet[10] and in an antiferromagnet.[11] The methods of the theory of random functions were used also to study surface plasma waves in a semiconductor film of nonuniform thickness,[12] and in metals with rough surfaces.[13,14]

A number of recent experimental papers[15,16] report investigations of plasma waves in polycrystalline and amorphous metals and semiconductors. In the long-wave part of the spectrum of the plasma excitations, deviations were observed from the spectral curves of a single crystal, and have so far found no explanations. There are grounds for assuming that these singularities are connected with the inhomogeneity of the investigated samples.

In the present paper we consider volume plasma waves in an inhomogeneous metal in the case when the inhomogeneity can be simulated by a random electric potential much smaller than the periodic potential of the crystal lattice. The investigation is carried out by the phenomenological method developed in Refs. 9 and 10.

1. HAMILTONIAN AND EQUATIONS OF MOTION

The deviation of the metal crystal lattice from ideal causes the conduction electrons to be in a nonperiodic potential $U(\mathbf{r})$, which can be represented as a sum of a periodic potential $U_0(\mathbf{r}) = U_0(\mathbf{r}+\mathbf{a})$ and a nonperiodic potential $V(\mathbf{r})$. In the approximation in which the electrons do not interact with one another, the single-electron Hamiltonian is of the form

$$\hat{\mathscr{H}} = \hat{p}^2/2m_0 + U_0(\mathbf{r}) + V(\mathbf{r}), \qquad (1.1)$$

where $\hat{p}$ is the momentum operator and m_0 is the electron mass.

If $V(\mathbf{r})$ is a slowly varying function compared with the lattice period a and $|V| \ll |U_0|$, then for long waves ($ka \ll 1$) we can go over from the quantum Hamiltonian (1.1) to the classical[17,18]

$$\mathscr{H}(\mathbf{r}, \mathbf{p}) = \varepsilon(\mathbf{p}) + V(\mathbf{r}), \qquad (1.2)$$

where $\varepsilon(\mathbf{p})$ is the dispersion law of the conduction electrons in an ideal crystal lattice. In this quasiclassical approximation, the equation of motion is the classical Liouville equation, which reduces in the case of noninteracting electrons to an equation for the single-particle distribution function $F(\mathbf{r}, \mathbf{p}, t)$:

$$\frac{\partial F}{\partial t} - \frac{\partial \mathscr{H}}{\partial \mathbf{r}}\frac{\partial F}{\partial \mathbf{p}} + \frac{\partial \mathscr{H}}{\partial \mathbf{p}}\frac{\partial F}{\partial \mathbf{r}} = 0. \qquad (1.3)$$

The stationary solution of this equation, satisfying the electron statistics, is the Fermi-Dirac distribution function, which depends in our case both on the momenta and on the spatial coordinates:

$$F_0(\mathbf{r}, \mathbf{p}) = A\left[\exp\left(\frac{\varepsilon - \varepsilon_F + V(\mathbf{r})}{k_B T}\right) + 1\right]^{-1}, \qquad (1.4)$$

where A is the normalization constant and ε_F is the Fermi energy.

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The function $F_0(\mathbf{r},\mathbf{p})$ describes the equilibrium distribution of the electrons in an inhomogeneous metal. The quantity $\varepsilon_F - V(\mathbf{r})$ can be interpreted as the chemical potential of the electrons, which varies continuously from point to point in an inhomogeneous crystal. The normalization constant A is determined in the usual fashion from the condition of the electron-number conservation. We assume the electron gas to be single-band with an isotropic quadratic dispersion law, and approximate the derivative $\partial F_0/\partial\varepsilon$ with the aid of the Dirac δ function:

$$\varepsilon=p^2/2m, \quad \partial F_0/\partial\varepsilon\approx-A\delta(\varepsilon-\varepsilon_F+V(\mathbf{r})), \tag{1.5}$$

where m is the effective mass of the electron. Then

$$A=V_0^{-1}\int[1-V(\mathbf{r})/\varepsilon_F]^{3/2}d\mathbf{r}, \tag{1.6}$$

where V_0 is the volume of the metal.

We shall regard the plasma wave as small deviations from the inhomogeneous equilibrium state (1.4). It is necessary to add to the Hamiltonian (1.2) the energy of the interaction of the electron with the alternating electric fields, both intrinsic and external, and represent the distribution function $F(\mathbf{r}, \mathbf{p}, t)$ in the form

$$F(\mathbf{r}, \mathbf{p}, t)=F_0(\mathbf{r}, \mathbf{p})+\varphi(\mathbf{r}, \mathbf{p}, t). \tag{1.7}$$

Substitution of this expression in (1.3) leads to the following equation for the function φ:

$$\frac{\partial\varphi}{\partial t}+\mathbf{v}\nabla\varphi-\frac{\partial\varphi}{\partial\mathbf{p}}\nabla V=e\mathbf{E}\mathbf{v}\frac{\partial F_0}{\partial\varepsilon}, \tag{1.8}$$

where $\mathbf{v}$ is the electron velocity and $\mathbf{E}$ is the sum of the intrinsic electric fields and the fields from the external sources. This equation was written in an approximation linear in the function φ (and correspondingly in the electric fields $\mathbf{E}$), but the influence of the small inhomogeneous potential $V(\mathbf{r})$ is taken formally into account in it exactly. It is seen that $V(\mathbf{r})$ acts on the plasma wave both directly (the last term in the left hand side of the equation) and indirectly, by changing the ground state, against the background of which the plasma wave propagates. A similar situation was encountered earlier in the analysis of magnetic resonance[9] and of spin waves[10] in a magnetic crystal with inhomogeneous orientation of the anisotropy axis.

Equation (1.8) describes a plasma wave in the presence of the inhomogeneous potential $V(\mathbf{r})$, regardless of whether this inhomogeneity of the electric field was produced by external or internal factors (in a particular case the potential $V(\mathbf{r})$ can also be periodic, but with a period much larger than the lattice period). We confine ourselves hereafter to the rather general case when $V(\mathbf{r})$ is a stationary random function with finite (nonzero) correlation radius. It is then convenient to represent $V(\mathbf{r})$ in the form

$$V(\mathbf{r})=w\rho(\mathbf{r}), \tag{1.9}$$

where w is the mean squared spatial fluctuation of the potential, $\rho(\mathbf{r})$ is a normalized stationary random function with zero expectation value and with unity variance:

$$\langle\rho\rangle=0, \quad \langle\rho^2\rangle=1. \tag{1.10}$$

For a stationary random function, integration over space is equivalent to averaging over the ensemble, so that the normalization constant A is now determined by the expression

$$A=\langle[1-2\gamma\rho(\mathbf{r})]^{3/2}\rangle, \tag{1.11}$$

where $\gamma=w/2\varepsilon_F\ll 1$.

Substituting (1.9) and (1.11) in (1.8), we obtain an equation in which the functions $\varphi(\mathbf{r},\mathbf{p},t)$ and $\mathbf{E}(\mathbf{r},t)$, which describe the plasma wave, are now random functions. Adding Maxwell's equations, the continuity equation, and the expression for the current density we obtain a closed system of equations for the plasma wave in a randomly inhomogeneous metal. This system describes both the natural and the forced oscillations. In the present paper we confine ourselves to investigation of the natural oscillations.

2. DISPERSION RELATIONS FOR PLASMA WAVES

To investigate the obtained system of equations by the methods of correlation theory of random functions, it is convenient to represent the expressions with complicated dependence on the random function $\rho(\mathbf{r})$ in the form of a Taylor series in powers of $\rho(\mathbf{r})$. In our case this pertains to the expression for the distribution function $F_0(\varepsilon,\rho(\mathbf{r}))$ in the right-hand side of Eq. (1.8). To find the solution in the first nonvanishing approximation of perturbation theory, it suffices to terminate the expansion with the terms quadratic in w.[10] Taking next in (1.8) the Fourier transforms with respect to $\mathbf{r}$ and t, we obtain

$$(\omega-\mathbf{k}\mathbf{v})\varphi+w\{\nabla_p\varphi, \tilde{\mathbf{k}}\rho\}=ie\mathbf{v}[\mathbf{E}f_0'(1-{}^3/_2\gamma^2\langle\rho^2\rangle) +wf_0''\{\mathbf{E}, \rho\}+{}^1/_2w^2f_0'''\{\{\mathbf{E}, \rho\}, \rho\}]. \tag{2.1}$$

Here φ, $\mathbf{E}$, and ρ now denote the Fourier transforms of the corresponding functions; $f_0=f_0(\varepsilon)$ is the Fermi–Dirac distribution function in a uniform metal ($f_0=F_0$ at $w=0$); the primes denote the corresponding derivatives of this function with respect to the energy ε; $\tilde{\mathbf{k}}=\mathbf{k}-\mathbf{k}'$, and the curly brackets denote convolutions of the type

$$\{a, b\}\equiv\{a(\mathbf{k}'), b(\tilde{\mathbf{k}})\}=\int a(\mathbf{k}')b(\mathbf{k}-\mathbf{k}')d\mathbf{k}',$$
$$\{\{a, b\}, c\}\equiv\iint a(\mathbf{k}'')b(\mathbf{k}'-\mathbf{k}'')c(\mathbf{k}-\mathbf{k}')d\mathbf{k}'\,d\mathbf{k}''. \tag{2.2}$$

The remaining equations of the system, taking into account the longitudinal character of the plasma waves, can be written after taking the Fourier transform in the form

$$\mathbf{E}=-\frac{4\pi i}{\omega}\frac{\mathbf{k}(\mathbf{k}\mathbf{j})}{k^2}, \quad \mathbf{j}=-\frac{2e}{(2\pi\hbar)^3}\int\mathbf{v}\varphi\,d\mathbf{p}. \tag{2.3}$$

The oscillations of a homogeneous ($w=0$) plasma have been theoretically well investigated. We describe now briefly a procedure for obtaining the dispersion relation for the system (2.1), (2.3) in this case. This will enable us subsequently to point out more distinctly those singularities which arise in the derivation of the dispersion equation for an inhomogeneous plasma ($w\neq 0$).

Substituting the second equation of (2.3) in the first, and the first in (2.1), we obtain a closed equation for the function $\varphi(\omega, \mathbf{k}, \mathbf{p})$, but it turns out to be integral even for a uniform plasma. An algebraic equation can be obtained only for the characteristics that are integral in $\mathbf{p}$ and depend only on ω and $\mathbf{k}$, namely, either for the current density $\mathbf{j}$ (more accurately, its projection on the vector $\mathbf{k}$), or to the field intensity $\mathbf{E}$ which is proportional to this density. To this end we write down (2.1) with $w=0$ in the form

$$\varphi = ie\mathbf{v}\mathbf{E}f_0'/(\omega-\mathbf{k}\mathbf{v}). \tag{2.4}$$

Substituting here $\mathbf{E}$ from (2.3), we multiply both sides of the equation by $\mathbf{v}$ and by a corresponding coefficient, and integrate with respect to the momentum $\mathbf{p}$. The result is an equation for the current density:

$$\mathbf{j} = -\frac{8\pi e^2(\mathbf{k}\mathbf{j})}{(2\pi\hbar)^3\omega k^2}\int\frac{\mathbf{v}(\mathbf{k}\mathbf{v})f_0'}{\omega-\mathbf{k}\mathbf{v}}d\mathbf{p}. \tag{2.5}$$

Multiplying this equation by $\mathbf{k}$, we obtain an algebraic equation for the scalar function $g=\mathbf{k}\circ\mathbf{j}$, and multiplying the latter by a suitable coefficient, we obtain a perfectly similar equation for $\mathbf{E}$; from the condition $\mathbf{E}\neq 0$ (or $g\neq 0$) we obtain the general form of the dispersion equation in a homogeneous plasma:

$$\omega = -\frac{8\pi e^2}{(2\pi\hbar)^3 k^2}\int\frac{(\mathbf{k}\mathbf{v})^2 f_0'}{\omega-\mathbf{k}\mathbf{v}}d\mathbf{p}. \tag{2.6}$$

The presence in the integrand of a pole on the real axis causes the dispersion relation to be complex—the damping first investigated by Landau[19] appears. This means that the oscillations cannot be exactly represented in the form of sums of independent plane wave with definite dispersion law, since such a representation is valid only approximately in cases when the damping is small enough.

Landau[19] considered a hot plasma, when f_0 can be approximated by a Maxwellian distribution function. In our cold-plasma case, however, when

$$f_0' \approx -\delta(\varepsilon-\varepsilon_F),$$

there is no such damping in a homogeneous plasma. Indeed, upon integration the values v are determined by a δ function. For the values of the wave vectors satisfying the equation $q=kv_F/\omega\ll 1$ (at the parameters typical of metals, this inequality follows from the restriction $ka\ll 1$), the integral in (2.6) has no poles on the real axis in the integration region. Consequently, in our case the waves in a homogeneous plasma can be represented (with the accuracy at which the approximation of f_0' by a δ function is valid) as a sum of plane waves with a real dispersion law that follows from (2.6) (see, e.g., Ref. 20):

$$\omega^2=\omega_p^2\eta(q),\quad \eta(q)=\frac{3}{2q}\left(\frac{1}{q}\ln\frac{1+q}{1-q}-2\right),$$
$$\omega_p^2=\frac{4\pi e^2 n}{m},\quad n=\frac{8\pi}{3}\frac{p_F^3}{(2\pi\hbar)^3},\quad q=\frac{kv_F}{\omega}, \tag{2.7}$$

where ω_p is the plasma frequency; for $q\ll 1$ we have

$$\omega^2\approx\omega_p^2(1+{}^3/_5q^2),\quad q\approx kv_F/\omega_p. \tag{2.8}$$

We turn now to the case $w\neq 0$. The inhomogeneities of the potential lead to the appearance in Eq. (2.1) of integral terms in the form of convolutions in k' and k'' of the form (2.2). These integral terms correspond to wave scattering by inhomogeneities. Now the plasma oscillations cannot be exactly represented, even at $T=0$, by a sum of noninteracting plane waves of the form $\mathbf{E}(\mathbf{k},\omega)$. Approximately, however, it is possible to consider, in the form of a plane wave, the mathematical expectation value $\langle\mathbf{E}(\mathbf{k},\omega)\rangle$ of the random function $\mathbf{E}(\mathbf{k},\omega)$, to the same degree of accuracy with which the correlators that are formed upon averaging of (2.1) are split up. The scattering of the wave by the inhomogeneities causes $\mathbf{E}$ to decrease with time at the expense of the increase of the incoherent components with different values of $\mathbf{k}$. At small w (the inequalities will be written out at the end of the paper), however, such plane waves exist for sufficiently long times and determine the thermodynamic and kinetic properties of the plasma. Therefore the principal characteristics of the inhomogeneous plasma is the dispersion relation for the mathematical expectation value $\langle\mathbf{E}\rangle$. We derive it now.

We transform first the integro-differential convolution in the left-hand side of (2.1). We express formally φ of the first term of the equation in terms of the remaining ones and substitute in this convolution. Retaining then the terms with w raised to a power not higher than the second, we obtain now in place of (2.4) the following expression for φ in terms of $\mathbf{E}$:

$$\varphi=\frac{ie}{L}\mathbf{v}\mathbf{E}f_0'\left(1-\frac{3}{2}\gamma^2\right)+\frac{ie}{L}w\Big[f_0''G_0-\nabla_p f_0'\mathbf{R}_0$$
$$+w\left(\frac{1}{2}f_0'''\{G_0,\rho\}-\nabla_p f_0''\{L^{-1}G_0\mathbf{v}\}+\nabla_p\{L^{-1}\nabla_p f_0'\mathbf{R}_0,\mathbf{v}\}\right)\Big], \tag{2.9}$$
$$L=\omega-\mathbf{k}\mathbf{v},\quad \mathbf{v}=\mathbf{k}\rho,$$

where G_0 and F_0 stand for the convolutions

$$G_0=\{\mathbf{v}\mathbf{E},\rho\},\quad \mathbf{R}_0=\{\mathbf{v}\mathbf{E}/L,\mathbf{v}\}.$$

Performing with these expressions the same operations as in (2.4) above, we obtain an integral equation for the scalar function $g=\mathbf{k}\circ\mathbf{j}$ in the form

$$\left[D_0+\frac{3}{2}\gamma^2\omega_p^2\eta(q)\right]g(\mathbf{k},\omega)=-\frac{8\pi e^2}{(2\pi\hbar)^3\omega}\int\frac{\mathbf{k}\mathbf{v}}{L}P\,d\mathbf{p}, \tag{2.10}$$
$$D_0=\omega^2-\omega_p^2\eta(q),$$

where P is the expression in the square brackets of (2.9), in which G_0 and R_0 are replaced respectively by $G=\{(g\mathbf{k}'\cdot\mathbf{v}/\mathbf{k}'^2),\rho\}$ and $\mathbf{R}=\{(\mathbf{k}'\cdot\mathbf{v}g/k'^2L),\nu\}$.

This equation contains, besides the terms quadratic in w, two terms linear in w, which also make a contribution $\propto w^2$ when (2.10) is averaged. It is convenient to change them directly into a form quadratic in w. We determine formally g from Eq. (2.10) and substitute it in the first two terms of the right-hand side of the same equation. Retaining at the same time terms of degree not higher than the second in w, we obtain the final form of the integral equation for the random function $g(\mathbf{k},\omega)$:

$$(D_0+{}^3/_2\gamma^2\omega_p^2\eta)g(\mathbf{k},\omega)=w^2 4\pi e^2\omega\hat{I}[4\pi e^2\omega(\{\mu D_0^{-1}\hat{I}\Phi,\rho\}-\{\mathbf{l}D_0^{-1}\hat{I}\Phi,\mathbf{v}\})$$
$$-{}^1/_2f_0'''\{G,\rho\}+\nabla_p f_0''\{GL^{-1},\mathbf{v}\}-\nabla_p\{L^{-1}\nabla_p f_0'\mathbf{R},\mathbf{v}\}]; \tag{2.11}$$

$$\Phi=f_0''G-\nabla_p f_0'\mathbf{R},\quad \mu\equiv\mu(k)=\frac{\mathbf{k}\mathbf{v}}{k^2}f_0'',\quad \mathbf{l}\equiv\mathbf{l}(k)=\frac{\nabla_p f_0'\mathbf{k}\mathbf{v}}{k^2L}, \tag{2.12}$$

and $\hat{I}$ is the operator of integration, with respect to the momentum, of the function multiplied by $\mathbf{k}\cdot\mathbf{v}/L$:

$$\hat{I}f\equiv\hat{I}(\mathbf{k})f(\mathbf{p})=\frac{2}{(2\pi\hbar)^3}\int d\mathbf{p}\frac{\mathbf{k}\mathbf{v}}{L}f(\mathbf{p}). \tag{2.13}$$

Now all the terms in the right-hand side of (2.11) are proportional to w^2 and have the general structure of double convolutions. We average (2.11) over the ensemble of realizations of the random functions. The procedure of approximate splitting of the correlations will follow the scheme

$$\begin{aligned}&\langle\{a\{bg, c\rho\}, h\rho\}\rangle\\&=\iint a(\mathbf{k}')b(\mathbf{k}'')c(\mathbf{k}'-\mathbf{k}'')h(\mathbf{k}-\mathbf{k}')\langle g(\mathbf{k}'')\rho(\mathbf{k}'-\mathbf{k}'')\rho(\mathbf{k}-\mathbf{k}')\rangle d\mathbf{k}'\,d\mathbf{k}''\\&\approx b(\mathbf{k})\langle g(\mathbf{k})\rangle\int a(\mathbf{k}')c(\mathbf{k}'-\mathbf{k})h(\mathbf{k}-\mathbf{k}')S(\mathbf{k}-\mathbf{k}')\,d\mathbf{k}'.\end{aligned}\tag{2.14}$$

Here a, b, c, and h are arbitrary functions of $\mathbf{k}$. Here, as earlier,[9,11] it was assumed that

$$\langle g(\mathbf{k}'')\rho(\mathbf{k}'-\mathbf{k}'')\rho(\mathbf{k}-\mathbf{k}')\rangle\approx\langle g(\mathbf{k}'')\rangle\langle\rho(\mathbf{k}'-\mathbf{k}'')\rho(\mathbf{k}-\mathbf{k}')\rangle,\tag{2.15}$$

and the known[7] property of stationary random functions was used, namely

$$\langle\rho(\mathbf{k})\rho(-\mathbf{k}')\rangle=S(\mathbf{k})\delta(\mathbf{k}-\mathbf{k}'),\tag{2.16}$$

where $S(\mathbf{k})$ is the spectral density of the correlation function.

It is seen from (2.14) that we obtain for the mathematical expectation value of the function $g(\mathbf{k}, \omega)$ an algebraic equation of the form

$$D(\mathbf{k}, \omega)\langle g\rangle=0;$$

from the requirement $\langle g\rangle\neq 0$ we obtain the dispersion relation for the plasma waves in a randomly inhomogeneous metal:

$$\omega^2=\omega_p^2\eta(q)\left(1-\gamma^2\sum_{i=1}^{4}J_i\right),\tag{2.17}$$

where

$$\begin{aligned}J_1&=-u^2\int\frac{d\mathbf{k}'S(\mathbf{k}-\mathbf{k}')}{D_0(k')}\hat{I}\mu'\hat{I}'\mu,\\J_2&=\frac{3}{2}-u\frac{\varepsilon_F}{\omega_p k^2}\hat{I}\mathbf{k}\mathbf{v}f_0''',\\J_3&=u^2\int\frac{d\mathbf{k}'S(\mathbf{k}-\mathbf{k}')}{D_0(k')}(\mathbf{k}'-\mathbf{k})\hat{I}[\mu'\hat{I}'\mathbf{l}-\mathbf{l}'\hat{I}'\mu+\mathbf{l}'\hat{I}'(\mathbf{k}'-\mathbf{k})\mathbf{l}],\\J_4&=2u\frac{\varepsilon_F}{\omega_p}\int d\mathbf{k}'S(\mathbf{k}-\mathbf{k}')\hat{I}\nabla_p\frac{\mathbf{k}-\mathbf{k}'}{L(\mathbf{k}')}[\mu-(\mathbf{k}'-\mathbf{k})\mathbf{l}];\\u&=-8\pi e^2\varepsilon_F,\quad \mu'\equiv\mu(k'),\quad \mathbf{l}'\equiv\mathbf{l}(k'),\quad \hat{I}'f\equiv\hat{I}(\mathbf{k}')f.\end{aligned}$$

At $\gamma=0$ this leads to the dispersion law for plasma waves in a homogeneous metal (2.7). The modification of the dispersion law in inhomogeneous metal is expressed in (2.17) in terms of the characteristics of the inhomogeneous potential $V(\mathbf{r})$, namely the relative mean squared fluctuation γ and the normalized spectral density $S(\mathbf{k})$ of the random function $\rho(\mathbf{r})$. The dispersion law turns out to be complex: the integrals J_1 and J_3, which have a pole $D_0=0$ on the real axis, cause, besides modification of the real part of the dispersion law, also the appearance of an imaginary increment to the frequency. We recall that in (2.17) the modification of the dispersion law is due to two physical mechanisms. The integrals J_1 and J_2 are connected with the change of the ground state and are determined by the right-hand side of Eq. (1.8). The integral J_4 is due to direct interaction of the plasma wave with a random potential [third term in the left-hand side of (1.8)]. The integral J_3 is connected with the action of both these mechanisms.

For estimates, we specify the correlation function and the spectral density connected with it by a Fourier transformation in the form[9,10]

$$K(\mathbf{r})=\exp(-k_0 r),\quad S(\mathbf{k})=\frac{1}{\pi^2}\frac{k_0}{(k_0^2+k^2)^2},\tag{2.18}$$

where k_0 is the characteristic wave number [$b=2/k_0$ is the characteristic dimension of the inhomogeneity, $r_0=k_0^{-1}$ is the correlation radius of the random function $\rho(\mathbf{r})$]. This form of $S(\mathbf{k})$ describes a sufficiently large class of inhomogeneities [at $k\ll k_0$ this is white noise, and at $k>k_0$ the function $S(\mathbf{k})$ is cut off quite abruptly].

The integrals with respect to $\mathbf{p}$ in J_i are easily evaluated. We shall integrate with respect to $\mathbf{k}'$ by residue theory, confining ourselves to the case

$$q\approx kv/\omega_p\ll 1,\quad q_0\approx k_0 v/\omega_p\ll 1,\tag{2.19}$$

but retaining, naturally, the arbitrary relation between q and q_0. Functions of q' [with the exception of $S(\mathbf{q}')$] can then be expanded in powers of q', since the integrands are cut off at $q'\gg q_0$ by the function $S(\mathbf{q})$.

As a result, the modified dispersion relation $\omega'(k)$ and the damping $\omega''(k)$ are obtained in the form

$$\begin{aligned}\left(\frac{\omega'}{\omega_p}\right)^2&=1+\frac{3}{5}q^2-15\gamma^2\left\{\frac{1}{q_0^2+4q^2}+\frac{q_0^3}{2q^5}\left[\operatorname{arctg}\frac{q}{q_0}\right.\right.\\&\qquad\left.\left.-\frac{q_0^2+2q^2}{q_0^2}\operatorname{arctg}\frac{qq_0}{q_0^2+2q^2}\right]+\frac{1}{2}\right\},\\\frac{\omega''}{\omega_p}&=15\gamma^2\frac{q_0}{q^3}\left[\frac{(q_0^2+2q^2)^2-2q^4}{q_0^2(q_0^2+4q^2)}-\frac{q_0^2+2q^2}{4q^2}\ln\frac{q_0^2+4q^2}{q_0^2}\right].\end{aligned}\tag{2.20}$$

The contribution made to ω' by the first and second terms in curly brackets is proportional here to γ^2/q_0^2 and is connected with J_1; contributions to the terms $\sim\gamma^2$ are made not only by J_1 but also by J_2 and J_3. The terms of higher order in ω' were neglected. This left in ω' only the terms determined by the integrals J_1, J_2, and J_3, which are due, as noted above, to the change of the ground state. In the expression for the damping ω'' we have neglected the terms $\sim\gamma^2q^3/q_0^5$ compared with γ^2q/q_0^3; the damping is then entirely due to the integral J_1.

In the limiting cases we have:

$$\begin{aligned}&\text{a) } q\ll q_0:\\&\left(\frac{\omega'}{\omega_p}\right)^2=1+\frac{3}{5}q^2-5\gamma^2\times\left(1-\frac{18}{5}\frac{q^2}{q_0^2}\right)q_0^{-2}\\&\frac{\omega''}{\omega_p}=10\gamma^2\frac{q}{q_0^3}\\&\text{b) } q\gg q_0:\\&\left(\frac{\omega'}{\omega_p}\right)^2=1+\frac{3}{5}q^2-\frac{15}{4}\gamma^2\frac{1}{q^2},\\&\frac{\omega''}{\omega_p}=\frac{15}{2}\gamma^2\frac{1}{qq_0}.\end{aligned}\tag{2.21}$$

The dependences of ω' and ω'' on k are shown schematically in the figure. It is seen that the dispersion curve has an inflection at $k=\sqrt{3}k_0/2$, and the attenuation has a maximum at the same value of k. The largest influence of the inhomogeneity on ω' turns out to occur at small k: in this region, the plasma frequency turned out to be smaller than its value in a

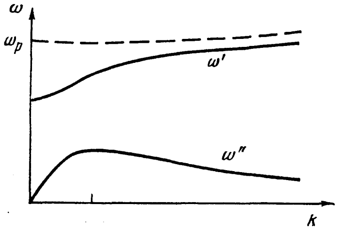

FIG. 1. Dispersion ω' and damping ω'' of plasma waves in a randomly inhomogeneous metal. The dashed curve shows the plasma-wave dispersion in a homogeneous medium. The bar on the abscissa axis marks the point $k=\sqrt{3}k_0/2$.

homogeneous metal. We note that the frequency at the local levels in the plasma spectrum is also lower than ω_p.[4-6]

CONCLUSION

Thus, the action of a randomly inhomogeneous potential leads to a complex modification of the dispersion relation for a plasma wave. A correlation wave number appears in the modified relation and corresponds to the characteristic dimension of the inhomogeneity. We note that the complex modification of the dispersion relation for the plasma waves is close in character to the modification of the dispersion relation for spin waves in the case of anisotropy fluctuation[10] (and differs from the modification corresponding, for example, to the exchange or magnetization fluctuation, see the same reference).

We discuss now the inequalities within the framework of which the problem was solved:

$$ka\ll 1,\quad k_BT\ll w,$$
$$\gamma=w/2\varepsilon_F\ll 1,\quad \gamma/q_0\ll 1,$$
$$q_0=k_0v/\omega_p\ll 1,\quad q=kv/\omega_p\ll 1.$$

The first inequality is the usual condition for introducing the long-wave approximation. The second inequality requires that the principal role be played not by the thermal noise but by the spatial noise connected with $\rho(\mathbf{r})$; the problem is solved actually at $T=0$. The third inequality expresses the smallness of the random potential compared with the lattice potential, and was used for an approximate splitting of the correlations.

The fourth follows from the final result—from the requirement that the corrections (both real and imaginary) to the plasma-wave frequency by much smaller than the frequency itself. This inequality thus entered implicitly together with $\gamma\ll 1$ in the splitting of the correlations. With these four inequalities satisfied, the general dispersion (2.17) was obtained, in which the frequency of the plasma wave was expressed in terms of the spectral density of the random function $\rho(\mathbf{r})$ in integral form.

The next two inequalities were needed to carry out the approximate integration in (2.17) with the model spectral function and to obtain analytic expressions for the modified dispersion equation. The inequality $q_0\ll 1$ led to the result that the stronger of the two preceding inequalities turned out to be $\gamma/q_0\ll 1$; the inequality $q\ll 1$ at parameters typical for metals is equivalent to the inequality $ka\ll 1$.

The experiments known to us[15] reveal a tendency of the plasma frequency in a polycrystalline metal to be lower than in single crystals, and also an increase of the slope of the dispersion curve at small k. This is in qualitative agreement with the results of the theory proposed here. For a correct comparison of the theory with experiment, however, a thorough experimental investigation of the long-wave region of the plasma-wave spectrum is necessary. This investigation would help observe the characteristic influction on the dispersion curve and make it possible to obtain information on the correlation radius (characteristic dimension) of the inhomogeneity and on its value γ.

Particular interest attaches to the performance of such experiments with amorphous metals. It is possible that observation of plasma waves might turn out to be one of the important methods of investigating these unique substances.

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