Averaged equations in spin dynamics of superfluid ^{3}He-B

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Nonlinear dissipative spin-dynamics regimes of superfluid ^{3}He-B are investigated in the absence of an external magnetic field, using an auxiliary Hamiltonian system in a space of dimensionality 3/2. An analysis of the interaction of the Leggett-Takagi dissipative mechanism and of the nonlinearity points to the possibility of a nonexponential and nonmonotonic magnetization-relaxation regime.

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1. INTRODUCTION

Nonlinear spin dynamics in superfluid ^{3}He is an important source of information on the dissipation mechanism in a superfluid liquid in the P-pairing state.[1] At the present time, the theoretical explanation of the spin relaxation is based on the so-called internal mechanism proposed by Leggett and Takagi[2] (see also the papers by Fomin[3,4,5] where an approach is developed that takes into account phenomena connected with spatial inhomogeneity).

The Leggett-Takagi (LT) theory was used successfully to study superfluid-liquid configurations that are important from the experimental point of view. One of the most remarkable examples in this respect is the wall-pinned (WP) mode in the B phase of superfluid ^{3}He,[6,7,2] for which a theoretical analysis within the framework of the Leggett-Takagi theory is in good agreement with experiment.[6] A general approach to the study of the LT equations, based on asymptotic methods of nonlinear mechanics, was developed by Fomin for the important case of spin dynamics in a strong magnetic field.[3,4,8,9]

In the present paper, on the basis of the LT equations, we study the spin dynamics of the B-phase of superfluid ^{3}He for a spatially homogeneous configuration when the external magnetic field is turned off. It is assumed that turning off the field has set the initial values of the spin and of the order parameter. The situation considered covers regimes that are important from the points of view of the theory of superfluid ^{3}He, such as the WP mode[6,10] and the driven (D) mode[10] corresponding to the magnetic-ringing regime.

In the nondissipative case, Maki and Ebisawa obtained in quadratures a general solution for the spin dynamics of ^{3}He-B in the absence of an external magnetic field and have shown that the Hamiltonian system specified by the Leggett equations is completely integrable.[11] By virtue of the presence of dissipation, their method cannot be applied to the LT equations. Interest attaches therefore to a qualitative description of the aggregate of the solutions of the LT equations, with which to assess the types of nonlinear regimes.

The study, in this paper, of the aggregate of the solutions of the LT equations is based on the observation that the initial Hamiltonian system with dissipation, obtained from the LT equations, generates the following system for the six dynamic variables (the three coordinates of the spin vectors and the three Euler angles for the order parameter): the rotation angle θ for the order-parameter matrix; the projection $S_\parallel$ of the spin vector on the axis of the rotation specified by the order parameter; $S_\perp=(S^2-S_\parallel^2)^{1/2}$, where $\mathbf{S}$ is the spin vector. The Poisson brackets for θ, $S_\parallel$, and $S_\perp$ are calculated from the initial Leggett brackets. In this sense one can speak of a Hamiltonian system in a phase space of dimensionality $\frac{3}{2}$.

2. THE LEGGETT-TAKAGI EQUATIONS

For our purposes it is convenient to write down the LT equations as a Hamiltonian system with dissipation (see Ref. 12). The basis for this is the Leggett-Takagi energy equation,[2] which is of the form

$$\frac{dE}{dt}=-\mu\left(\frac{dU}{d\theta}\right)^2. \tag{1}$$

Here E is the Leggett energy, $U=U(\theta)$ is the dipole energy, μ is a constant connected with the constant in the right-hand side of Eq. (6.4) of Ref. 2 by the formula

$$\mu=\gamma^2\chi^{-1}\frac{1-\lambda}{\lambda}\tau.$$

Thus, the right-hand side of Eq. (1) specifies the dissipative function in the form $F=(\frac{1}{2})(dU/d\theta)^2$.

The Hamiltonian structure of the LT equations is specified by the coordinates of the spin momentum S_i ($i=1, 2, 3$) and of the order parameter A_{ij} ($i,j=1,2,3$) as the dynamic variables, by Poisson brackets of the form

$$\{S_i, S_j\}=\varepsilon_{ijk}S_k,$$
$$\{S_i, A_{jm}\}=\varepsilon_{ijk}A_{km},\quad \{A_{im}, A_{jn}\}=0 \tag{2}$$

and by the Leggett Hamiltonian

$$\mathcal{H}={}^1/{}_2\gamma^2\chi^{-1}\mathbf{S}^2-\gamma\mathbf{HS}+U(A). \tag{3}$$

Here γ is the gyromagnetic ratio, χ is the susceptibility, and $U(A)$ is the dipole energy. For ^{3}He-B we have[13]:

$$U=8g_D[(\cos\theta+{}^1/{}_4)^2-{}^1/{}_4], \tag{4}$$
$$A_{ij}=3^{-1/2}\Delta e^{i\varphi}R_{ij}. \tag{5}$$

Here R_{ij} is the rotation matrix.[13] It is assumed in the present paper that $\varphi=$const.

3. HAMILTONIAN SYSTEM IN DIMENSIONALITY 3/2

The roration matrix R_{ij} can be parametrized by the rotation angle θ and by the unit vector c_i of the rotation axis in accordance with the equations

$$R_{ij}=\cos\theta\delta_{ij}+(1-\cos\theta)c_ic_j-\sin\theta\varepsilon_{ijk}c_k,$$
$$c_i=-(2\sin\theta)^{-1}\varepsilon_{ijk}R_{jk},\quad \cos\theta={}^1/{}_2[\mathrm{Tr}\,\hat{R}-1]. \tag{6}$$

The quantities $S_\parallel$ and $S_\perp$ are then given by

$$S_\parallel=S_ic_i,\quad S_\perp=(S^2-S_\parallel^2)^{1/2}. \tag{7}$$

Summation over repeated indices is implied throughout.

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It can be verified by direct calculation that the following equations hold for the Poisson brackets:

$$\{S_i,\theta\}=-c_i,\quad \{c_i,c_j\}=0,$$
$$\{S_i,c_j\}={}^1/_2(c_ic_j-\delta_{ij})\operatorname{ctg}(\theta/2)+{}^1/_2\varepsilon_{ijk}c_k.$$

It follows from these equations that for $S_\parallel$, $S_\perp$, and θ the following relations hold for the Poisson brackets:

$$\{S_\parallel,\theta\}=-1,\quad \{S^2,\theta\}=-2S_\parallel,\quad \{S^2,S_\parallel\}=-S_\perp^2\operatorname{ctg}(\theta/2). \qquad (8)$$

It should be noted that the Leggett Hamiltonian is a function of only $S_\parallel$, $S_\perp$, and θ. It can thus be stated that $S_\parallel$, $S_\perp$, and θ form a system with a phase space of dimensionality $\frac{3}{2}$ and with a Poisson structure specified by Eqs. (8).

It is important in what follows that the quantity

$$B=S_\perp\sin(\theta/2) \qquad (9)$$

has zero Poisson brackets with all $S_\parallel$, $S_\perp$, θ and is thus an integral of the motion in the nondissipative regime described by the Leggett equations. The integral B was indicated in Ref. 11. In the case when the value $\theta=\pi$ is permitted, as in the D-mode regime, it has the meaning of the minimum of $S_\perp$.

The equations of motion for the Hamiltonian system specified by (3), (8), and by the dissipative function $F=(\frac{1}{2})(dU/d\theta)^2$ are of the form

$$\frac{d}{dt}S_\parallel=-\frac{1}{2}\gamma^2\chi^{-1}S_\perp^2\operatorname{ctg}\frac{\theta}{2}+\frac{dU}{d\theta},$$
$$\frac{d}{dt}S_\perp=\frac{1}{2}\gamma^2\chi^{-1}S_\parallel S_\perp\operatorname{ctg}\frac{\theta}{2}, \qquad (10)$$
$$\frac{d}{dt}\theta=-\gamma^2\chi^{-1}S_\parallel-\mu\frac{dU}{d\theta}.$$

The Leggett configuration corresponds to $S=0$ is described by the equations

$$\frac{d}{dt}S_\parallel=\frac{dU}{d\theta},$$
$$\frac{d}{dt}\theta=-\gamma^2\chi^{-1}S_\parallel-\mu\frac{dU}{d\theta},$$

which can be written as a single second-order equation.

The WP-mode regime corresponds in the nondissipative approximation to the condition $S_\parallel=0$, so that[7]

$$\frac{1}{2}\gamma^2\chi^{-1}S_\perp^2\operatorname{ctg}\frac{\theta}{2}=\frac{dU}{d\theta}. \qquad (11)$$

It should be noted that the perturbations due to the dissipations deform the curve (11) from the plane $S_\parallel=0$.

The line $\theta=\pi$, $S_\parallel=0$ ($S_\perp$ is arbitrary) consists of unstable equilibrium points that are saddle points at $S_\perp < S_{\perp B}=6\gamma^{-2}\chi g_D$, i.e., prior to the intersection of the curve specified by Eqs. (11) for the WP mode with the line $\theta=\pi$, $S_\parallel=0$, and are unstable foci at $S_\perp > S_{\perp B}$.

In the case when the dissipation is small, the configuration of the integral curves—of the solutions of the system (10)—can be obtained by using the two integrals of the conservative system, E (energy) and B, the behavior of the particular solutions corresponding to the WP mode and to the axis $\theta=\pi$, $S_\parallel=0$, and the circumstance that, with the exception of the immediate vicinity of the points $\theta=0$, 2π and $S_\parallel=S_\perp=0$ the considered configuration tends

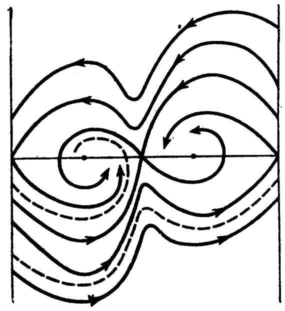

FIG. 1. Phase portrait of Leggett configuration. The thick lines show the separatrices. Depending on the relation between the quantities g_D and μ, they can be wound jointly an integer number of times around a pair of foci corresponding to the angles $\theta=\arccos(-1/4)$ and $2\pi-\arccos(-1/4)$.

to the Leggett configuration (the phase portrait is shown in Fig. 1). The final picture of the behavior of the solutions is shown in Fig. 2.

4. THE D MODE

The phase picture constructed in the preceding section can be considerably refined in the case of the D-mode regime corresponding to rotation of the magnetization. In this case the magnetization of the system is large, so that $\gamma^2\chi^{-1}S \gg \Omega_L$. Here Ω_L is the Leggett frequency of the linear NMR in ^{3}He-B. It is convenient to introduce the characteristic frequency

$$\omega_0=\gamma^2\chi^{-1}S_{\perp\,max},$$

where $S_{\perp max}$ is the largest value of $S_\perp$ during one period of the oscillations. It should be noted that $S_{\perp max}$, and consequently also ω_0, varies slowly with time by virtue of the dissipation. In the D-mode regime we have $\omega_0 \gg \Omega_L$.

Introducing the dimensionless variables

$$\tau=\omega_0 t,\quad S_\parallel=\omega_0^{-1}\gamma^2\chi^{-1}S_\parallel,$$
$$S_\perp=\omega_0^{-1}\gamma^2\chi^{-1}S_\perp,$$

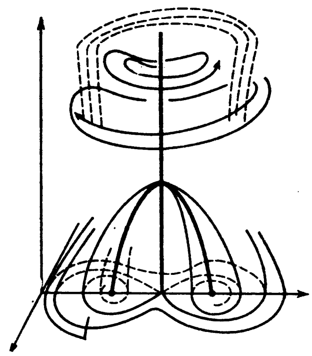

FIG. 2. Phase portrait of the system of Eqs. (10). The thick lines show the WP mode and the unstable equilibrium positions corresponding to $\theta=\pi$. The separatrix surface forms an infinitely-sheeted winding around a trident made up of the lines of the WP mode and part of the axis $\theta=\pi$, $0<\theta<\theta_B=6\gamma^2\chi^{-1}g_D$. In the upper part of the figure, the dashed lines mark the attractor. The two solid lines (one inside and one outside the attractor) are typical representitives of the system trajectories in the region of large magnetization.

we can reduce Eqs. (10) to the form

$$\frac{d}{d\tau}S_\parallel=-\frac{1}{2}S_\perp^2\,\mathrm{ctg}\,\frac{\theta}{2}+(\Omega_L\omega_0^{-1})^2\frac{dU}{d\theta},\quad \frac{d}{d\tau}S_\perp=\frac{1}{2}S_\parallel S_\perp\,\mathrm{ctg}\,\frac{\theta}{2},$$
$$\frac{d}{d\tau}\theta=-S_\parallel-(\mu g_D\omega_0^{-1})\frac{dU}{d\theta},\quad U=(\cos\theta+{}^1/_4)^2. \qquad (12)$$

The quantity $(\mu g_D)^{-1}$ has the meaning of the characteristic relaxation time. For ^{3}He-B it is of the order of 1 msec.[10] Recognizing that $\Omega_L\sim 10^4-10^5$ and that ω_0 in the D-mode regime is 10^7-10^8 and higher, we can assume that

$$\mu g_D\omega_0^{-1}\ll\Omega_L\omega_0^{-1}\ll 1.$$

Thus, the dissipative and nonlinear terms in Eqs. (12) turn out to be small and can be treated by perturbation theory.

As the basic solution of the unperturbed system, which is obtained by discarding the dissipative and linear terms generated by the dipole energy, we choose

$$S_\perp=S_{\perp 0}\sin^{-1}\frac{\theta}{2},\quad S_\parallel=S_{\perp 0}\left(\sin^{-2}\frac{\theta_0}{2}-\sin^{-2}\frac{\theta}{2}\right),$$
$$\cos\frac{\theta}{2}=A\sin\psi,\quad \psi=\psi_0+\frac{\gamma^2}{2\chi}\frac{B}{(1-A^2)^{1/2}}t, \qquad (13)$$
$$\theta_0=\mathrm{const},\quad S_{\perp 0}=\mathrm{const},\quad \psi_0=\mathrm{const}.$$

Here θ_0 is the minimum value of the angle θ during one period. The choice of the functions (13) is dictated by the following considerations. In the absence of dissipation we have the conservation law derived in Ref. 11:

$$\frac{1}{2}\gamma^{-2}\chi\left[\frac{d\theta}{dt}\right]^2+\frac{1}{2}\gamma^2\chi^{-1}S_{\perp 0}^2\sin^{-2}\frac{\theta}{2}+U(\theta)=E. \qquad (14)$$

If the magnetization is large, then we neglect the term $U(\theta)$ in (14) and assume it equal to zero. The resultant conservation law can be integrated exactly. The answer takes the form (13). As an important consequence we have that ψ is a fast variable. We employ an averaging method. To this end we consider the energy integral and the integral (9). It follows from Eqs. (10) that

$$\frac{dE}{dt}=-\mu\left(\frac{dU}{d\theta}\right)^2,\quad \frac{dB}{dt}=-\frac{1}{2}\mu B\,\mathrm{ctg}\,\frac{\theta}{2}\frac{dU}{d\theta}.$$

In accordance with (13), we substituted in the right-hand side the expressions

$$\cos(\theta/2)=A\sin\psi,\quad A=\cos(\theta_0/2),$$

where A has the meaning of the slowly varying amplitude, and ψ is the fast variable over which the averaging is carried out. The values of E and B averaged over the period should satisfy the equations

$$\frac{dE}{dt}=-\frac{\mu}{2\pi}\int_0^{2\pi}\left(\frac{dU}{d\theta}\right)^2d\psi,$$
$$\frac{dB}{dt}=-\frac{\mu B}{2\pi}\int_0^{2\pi}\mathrm{ctg}\,\frac{\theta}{2}\frac{dU}{d\theta}d\psi. \qquad (15)$$

After integration with respect to ψ, Eqs. (15) take the form

$$\frac{dE}{dt}=-{}^1/_2\mu g_D{}^2A^2F(A),\quad \frac{dB}{dt}=-{}^3/_4\mu g_DA^2B(1-2A^2),$$
$$F(A)=-35A^2+70A^4-42.75A^2+9. \qquad (16)$$

From (16) we can obtain also an equation for A. We note for this purpose that E is expressed in terms of A and B in accord with the formula

$$E=\frac{1}{2}\gamma^2\chi^{-1}\frac{B^2}{1-A^2}+U(A).$$

It follows therefore from Eqs. (16) that

$$\frac{1}{2}\gamma^2\chi^{-1}\frac{d}{dt}\frac{B^2}{1-A^2}=-\frac{1}{2}\mu g_D{}^2A^2F(A)-\frac{dA}{dt}\frac{1}{2\pi}\int_0^{2\pi}\frac{dU}{dA}d\psi. \qquad (17)$$

As will be seen from what follows, the second term in (17) can be neglected. Combining Eqs. (16) and (17) (with the second term discarded) we obtain the following equations for A and B:

$$\frac{dA}{dt}={}^3/_4\mu g_DA(1-A^2)(1-2A^2)+{}^1/_2\gamma^2\chi^{-1}\mu g_D{}^2AB^{-2}(1-A^2)^2F(A),$$
$$\frac{dB}{dt}=-{}^3/_4\mu g_DA^2B(1-2A^2). \qquad (18)$$

To determine the orders of magnitude of the quantities, we use the dimensionless variables $\tau=\omega_0t$ and $b=\gamma^2\chi^{-1}\omega_0^{-1}B$. Equations (18) take the form

$$\frac{dA}{d\tau}=\frac{3}{4}\frac{\mu g_D}{\omega_0}A(1-A^2)(1-2A^2)-\frac{4}{15}\left(\frac{\Omega_L}{\omega_0}\right)^2\frac{\mu g_D}{\omega_0}Ab^{-2}(1-A^2)^2F(A)$$
$$\frac{db}{d\tau}=-\frac{3}{4}\frac{\mu g_D}{\omega_0}A^2(1-2A^2)b. \qquad (19)$$

If $1-2A^2$ is large compared with $(\Omega_L\omega_0^{-1})^2$, then the second term in the right-hand side of (19) can be neglected, since $\mu g_D\omega_0^{-1}\ll(\Omega_L\omega_0)^2$. On the contrary, as $A\to 2^{-1/2}$ the principal term of the asymptotic relation is determined by the second term in (19). The discarded second term in (17) would yield terms of order

$$(1-2A^2)(\Omega_L\omega_0^{-1})^2(\mu g_D\omega_0^{-1}),$$

therefore both as $A\to 2^{-1/2}$ and at $1-2A^2\gg(\Omega_L\omega_0^{-1})^2$ its contribution is negligible.

Thus, the D mode admits of the presence of two spin relaxation regimes. For values θ_0 far enough from $\pi/2$, namely such that the estimate $|\cos\theta_0|\gtrsim(\Omega_L\omega_0^{-1})^2$ holds, the magnetization remains unchanged, since the dependence of $S_\perp$ and $S_\parallel$ on θ and θ_0 is given by the equations

$$S_\perp=S\sin\frac{\theta_0}{2}\sin^{-1}\frac{\theta}{2},\quad S_\parallel=S\sin^{-1}\frac{\theta}{2}\left(\sin^2\frac{\theta}{2}-\sin^2\frac{\theta_0}{2}\right)^{1/2},$$

for which it follows directly that $S_\parallel^2+S_\perp^2=S^2=\mathrm{const}$ and

$$S_{\perp\,min}=S\sin\frac{\theta_0}{2},\quad S_{\parallel\,max}=S\cos\frac{\theta_0}{2}. \qquad (20)$$

The time dependence of $\theta_0=\theta_0(t)$ then takes the form

$$\theta_0=\mathrm{arctg}\exp({}^3/_4\mu g_Dt),\quad 0<\theta_0<\pi/2,$$
$$\theta_0=\pi-\mathrm{arctg}\exp({}^3/_4\mu g_Dt),\quad \pi/2<\theta_0<\pi. \qquad (21)$$

The system stays in the region described by Eqs. (20) and (21) for several time intervals $(\mu g_D)^{-1}$, i.e., for several measures. It then lands in a region where the second term of Eq. (19) is decisive. At $\theta_0<\pi/2$ the threshold curve is given by the condition that the right-hand side of (19) vanish, and can be described by the function

$$S_{\perp\,min}=0.26\Omega_L\omega_0^{-1}\left(0.25-\cos^2\frac{\theta_0}{2}\right)^{-1/2}. \qquad (22)$$

For $\theta_0>\pi/2$, the threshold curve is specified by the equality $\theta_0=\pi/2$. It separates the region where $S_{\perp\min}$ increases from the region where it decreases (see Fig. 3),

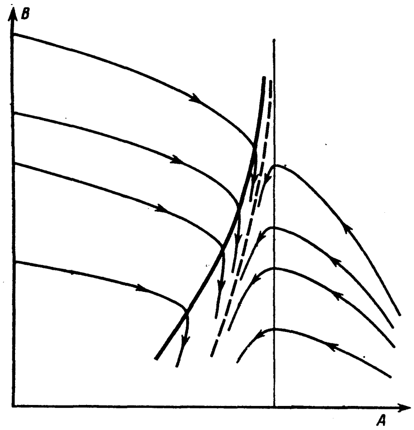

FIG. 3. Phase protrait of the D-mode regime on the plane $B = S_{\perp min}$, $A = \cos(\theta_0/2)$. The thick line represents the threshold curves, and the dashed line the separatrix between the regime of the monotonic increase and the regime of the monotonic increase of B.

which shows a plot of $B=S_{\perp\min}$ against $A=\cos(\theta_0/2)$. After passing through the threshold curve (22), the system passes during each period near the lines $S_{\parallel}=0$, $\theta = \pi/4$, $3\pi/2$ in the space of the variables $S_{\parallel}$, $S_{\perp}$, θ, and the relaxation is determined by the second term of Eq. (19).

On the whole, the trajectory of the system that entered in the indicated regime falls into an attractor located near the surface

$$S_{\parallel}^2+S_{\perp}^2\cos\theta=0,$$

which corresponds to the condition $\theta_0=\pi/2$. From the form of the second term of (19) it follows that the effective dimensionless parameter that determines the relaxation is not $\mu g_D/\omega_0$, but the much smaller number

$$\eta=(\Omega_L/\omega_0)^2\mu g_D/\omega_0.$$

5. CONCLUSION

The relaxation of the spin vector of superfluid ^{3}He-B from the initial state with large magnetization to a state of complete equilibrium goes in succession through several essentially different stages.

1. During the first stage of the relaxation, which lasts for several milliseconds, the modulus of the spin vector remains practically unchanged, and its transverse part $S_{\perp}$ increases if $\theta_0<\pi/2$, or decreases if $\theta_0>\pi/2$, i.e., the dynamics of the magnetization is not monotonic or exponential. In the phase space of the variables $S_{\parallel}$, $S_{\perp}$, θ the trajectory of the system tends to reach an attractor characterized by the fact that the amplitude θ_0 of the angle θ tends to $\pi/2$. Observation of this relaxation regime may be made difficult by its short time intervals (if it is recognized that the relaxation time of quantum interferometers is of the order of 2.5 msec) and by the need for a rapid turning off of strong magnetic fields (of the order of 100 G). It should be noted, however, that nonmonotinicity of spin relaxation was successfully observed for ^{3}He-A.[10]

2. The next stage of the relaxation sets in when the trajectory of the system in phase space $S_{\parallel}$, $S_{\perp}$, θ lands in the attractor W specified by the condition $\theta_0\approx\pi/2$. In this case, in the complete phase space of the variables of the spin and of the order parameter S_i, A_{ij} $(i,j=1,2,3)$, the system is also in a certain attractor W_1. If W_1 is a strange attractor, i.e., the motion of the trajectories in it is random (this can possibly take place if the system buildup is caused by an alternating external magnetic field), then one can expect the spin relaxation to be turbulent, and in particular, to be spatially inhomogeneous. A similar phenomenon (orientational turbulence in the space of the order parameter) is known in the physics of liquid crystals.[14] The study of similar regimes in superfluid ^{3}He is an interesting problem. It is not connected with the presence of vortices of the superfluid velocity and in this respect is not similar to the turbulence in superfluid ^{4}He. It is worth noting here also that within the context of the present paper the spatial inhomogeneity of the spin relaxation, previously indicated by Fomin,[5] and the possible existence of nonmonotonic and non-exponential regimes are mutually related phenomena.

3. An interesting regime sets in the transition region, when the system goes off the attractor in the phase space $S_{\parallel}$, $S_{\perp}$, θ and lands in the region of the WP mode. This region is easiest to investigate experimentally, since it is characterized by relatively low values of the magnetization. One can also expect here an inhomogeneous and nonmonotonic behavior of the relaxation because of the more complicated motion of the system trajectories in the $S_{\parallel}$, $S_{\perp}$, θ space.

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[1]L. P. Pitaevskiĭ, Zh. Eksp.Teor. Fiz. 37, 1794 (1959) [Sov. Phys. JETP 10, 1267 (1960)].

[2]A. J. Leggett and A. J. Takagi, Ann. Phys. (NY) 106, 79 (1977).

[3]I. A. Fomin, Pis'ma Zh. Eksp. Teor. Fiz. 30, 179 (1979) [JETP Lett. 30, 164 (1979)].

[4]I. A. Fomin, *ibid.* 28, 679 (1978) [28, 631 (1978)].

[5]I. A. Fomin, Zh. Eksp. Teor. Fiz. 78, 2393 (1980) [Sov. Phys. JETP 51, 1203 (1960)].

[6]R. A. Webb, R. E. Sager, and J. C. Wheatley, J. Low Temp. Phys. 26, 439 (1977).

[7]W. F. Brinkman, Phys. Lett. A49, 411 (1974).

[8]I. A. Fomin, Zh. Eksp. Teor. Fiz. 71, 791 (1976) [Sov. Phys. JETP 44, 416 (1976)].

[9]I. A. Fomin, *ibid.* 77, 279 (1979) [50, 144 (1979)].

[10]R. A. Sager, R. L. Kleinberg, R. Warkentin, and J. C. Wheatley, J. Low Temp. Phys. 32, 263 (1978).

[11]K. Maki and H. Ebisawa, Phys. Rev. B13, 2924 (1976).

[12]I. E. Dzyaloshinskii and G. E. Volovik, Ann. Phys. (NY) 125, 67 (1980).

[13]S. A. Pikin, Strukturnye prevrashcheniya v zhidkikh kristallakh (Structural Transitions in Liquid Crystals), Nauka, 1980.

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