Propagation of nonlinear second sound waves in He II

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Experiments are described on the study of the propagation of second sound pulses which evolve in He II with the formation of a temperature discontinuity. The heat flux density from an emitter (resistive film deposited onto a glass backing) reached 135 W/cm², and the pulse duration was varied between 2 and 50 μ sec. The time of pulse propagation calculated by use of Burgers equation agrees well with the experiments. Good agreement between the experimental temperature $T_3 = 1.885 \pm 0.005$ K for which $\alpha_2(T_3) = 0$ (α_2 is the nonlinearity coefficient) and the temperature obtained by numerical calculation of α_2 is also observed.

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INTRODUCTION

Interest has developed recently on nonlinear phenomena in He II, in particular, in the behavior of pulses of second sound of large amplitude.[2-6] Khalatnikov[7] was the first to obtain an expression for α_2—the nonlinearity coefficient, which describes the dependence of the velocity of second sound on its amplitude; this made it possible to explain qualitatively the Osborne experiment.[8] It has been shown[2,9] that the propagation of a temperature perturbation of finite amplitude is subject to Burgers equation, which has been studied in great detail (see, for example, Ref. 10).

In experiments with second-sound pulses, frequent use is made of the expression $\Delta c_2 = c_2 - c_{20}$ (c_2 and c_{20} are the velocities of pulses with final and zero amplitude, respectively). For short pulses, such a quantity is unsuitable for the description of the process. As will be shown below, the velocity of a propagating pulse is not constant and the dependence of Δc_2 on Q, where Q[W/cm²] is the heat flux density, is linear only at certain relations between the parameters; in the general case, it has a complicated character. With the help of the Burgers equation, we have carried out a calculation of the travel time of a short pulse of second sound of known amplitude. We have also performed an experiment on the measurement of this time. We have obtained excellent agreement of the experimental points with the calculated curve.

Using the method of measurement of the travel time of pulses of second sound of finite amplitude, a measurement has been made of T_3—the temperature at which $\alpha_2(T_3) = 0$.

THE CALCULATION

As has already been mentioned, the evolution of a heat pulse in an approximation that is quadratic in the velocity satisfies the Burgers equations:[2,9]

$$\frac{\partial v_n}{\partial t} + \left(c_{20} + \frac{\alpha_2(T)}{2} v_n\right)\frac{\partial v_n}{\partial x} = \gamma \frac{\partial^2 v_n}{\partial x^2},$$

here γ is the absorption coefficient of second sound, x and t are the coordinate and the time, v_n is the velocity of the normal component. This equation has an analytic solution;[9] however, in the case of a pulse of special profile (in the given case, a rectangular shape) we need not write out the cumbersome formulas connected with this solution, and one can describe the evolution of the pulse by using the following three circumstances: 1) in the co-moving system of coordinates v_n, x' ($x' = x - c_{20}t$) the area of the figure bounded by the profile v_n of the pulse remains unchanged (this follows from the Burgers equation)[9]; 2) the velocity of each point of the profile v_n of the pulse is determined by the amplitude of this point: $c_2 = c_{20} + \alpha_2 v_n/2$; 3) the spatial extent ξ of the front of the pulse, on which the discontinuity is formed, is negligible in comparison with the length of the entire pulse. By estimate, at $T = 1.5$ K, the width of the front $\xi \sim \gamma/v_n \approx 10^{-6}$ cm, which is much less than the characteristic length of the pulse.

It follows from these properties that the evolution of the propagating initially rectangular pulse has two stages: in the first stage, at constant amplitude and velocity, its length increases twofold, the profile acquires the shape of a right triangle—on Fig. 1, these are curves 1, 2, and 3. In the second stage, the amplitude and the velocity of the pulse decrease, the length increases—these are curves 3 and 4.

The finite value of the heat capacity of a unit surface area of the emitter C_s and the presence of a Kapitza resistance R_K between the emitter and the He II lead to a relaxation of the increase in the temperature of the heater, with a characteristic time $\tau = C_s R_K$,[11] which naturally leads to a deviation of the real profile of the pulse from rectangular. However, by estimate, $\tau \lesssim 10^{-8}$ sec, and we can assume the profile to be right-angled with this accuracy.

The total time from the initial emission to the arrival of the pulse at the sensor is $t_c = t_1 + t_2$; t_1 is the time of the first stage (on Fig. 1, curves 1, 2, 3), $t_1 = \Delta t + 2\lambda/\alpha_2 v_0$, $\lambda = c_2 \Delta t$ is the initial pulse length, $c_2 = c_{20} + \alpha_2 v_0/$

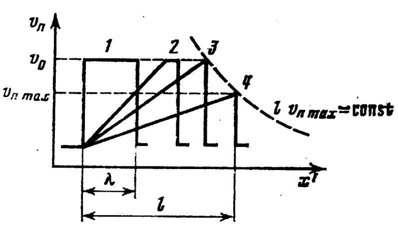

FIG. 1. Evolution of the profile of a pulse of second sound in a co-moving set of coordinates.

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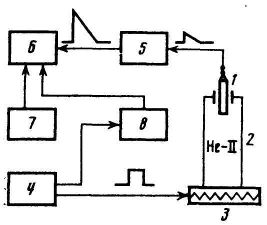

FIG. 2. Experimental setup. 1) Au—Sn—Au film sensor. 2) second sound waveguide, 3) NiCr film emitter, 4) master generator pulse, 5) amplifier, 6) double-beam oscilloscope, 7) time marker generator, 8) adjustable delay unit.

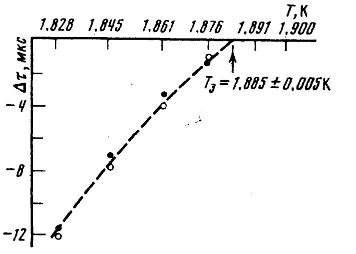

FIG. 4. Dependence of $\Delta\tau = t_B - t_S$ on the temperature, t_S and t_B are respectively the times of flight of pulses with low and high amplitude.

2, α_2 is the coefficient of nonlinearity, $v_0 = Q/ST$ is the initial pulse amplitude (S and T are the entropy per unit volume and the temperature of the helium).

In the second stage, in the produced triangular profile, the amplitude $v_{n\,max}$ decreases and the length l increases. From the condition $v_{n\,max}l$ = const, by a calculation similar to that given in Ref. 12, we obtain

$$v_{n\,max}(t) = v_0[1+\alpha_2 v_0 (t-t_3)/2\lambda]^{-1/2},$$

where t_3 is the moment of conclusion of the first stage. Knowing $v_{n\,max}(t)$ and $B_2 = B - B_1$ (B is the distance from the emitter to the sensor, $B_1 = t_1 c_2$ is the distance traveled by the pulse in the time t_1), t_2 can be calculated:

$$t_2 = \frac{B_2+2\lambda}{c_{20}} + \frac{\alpha_2 v_0 \lambda}{c_{20}^2} - \frac{2\lambda}{c_{20}}\left[\left(1+\frac{\alpha_2 v_0}{2c_{20}}\right)^2 + \frac{\alpha_2 v_0 B_2}{2c_{20}\lambda}\right]^{1/2}.$$

The calculation of T_3 by the formula given in Ref. 1 was carried out on a high-speed computer with the use of thermodynamic data from Ref. 13.

EXPERIMENT

The experimental setup is shown in Fig. 2. A pulse of second sound was excited by the passage of current through the emitter 3—a film of NiCr of thickness ≈500 Å, deposited on glass backing. For detection of the second sound, we used a thermoresistive film sensor 2 of dimensions 0.5 mm×30 μ, the sensitive element of which was a stretched-out superconducting junction of Sn in a sputtered Au–Sn–Au sandwich of total thickness ≈1000 Å on a glass backing. Schematically and structurally, the sensor was prepared by following Ref. 14; with unimportant modifications. The experiment was carried out on the liquid-vapor equilibrium line at a temperature of 1.677 K.

From the master oscillator 4, an electric pulse of rectangular shape is transmitted to the emitter with simultaneous application of a synchronizing pulse to the adjustable delay unit 8. The heating of the emitter excited a pulse of second sound in the He II. As the pulse moved in the waveguide 2 (a cylinder of diameter 2.5 cm) the profile of the pulse, which was initially rectangular, changed in correspondence with what has been said above. When the pulse reached the sensor 1, a signal was generated in it, which went, past the amplifier 5, onto the input of the oscilloscope 6, at the second input of which was applied from the frequency generator 7 a periodic signal (2 in Fig. 3), serving as a time marker. Before the arrival of the pulse of second sound at the sensor, a signal from unit 8, delayed relative to the beginning of the emission by an accurately measured time, triggered the oscilloscope sweep. A typical oscillogram is shown in Fig. 3. The time of traversal by the leading edge of the pulse of the distance from the emitter to the sensor was determined from the magnitude of the delay and the mutual position of the signals of the sensor and the frequency generator 7 on the oscillogram. The accuracy of measurement of t amounted to 1–2 μsec, the accuracy of measurement of the heat flux density Q amounted to ±8%.

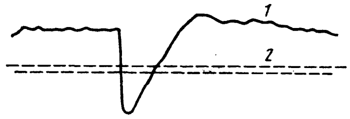

FIG. 3. Typical oscillogram of the experiment. The downward beam deflection indicates an increase in temperature. 1) Detector signal, 2) time marker, frequency 10^5Hz. Pulse amplitude 50W/cm^2, Δt = 10μ sec.

In measurements in the range of temperatures where $\alpha_2 < 0$ (a break at the trailing edge), it must be taken into account that the rate of heating of the emitter (i.e., the slope of the leading edge) was determined by the electrical parameters of the process, and the rate of its cooling, by the properties of heat exchange of the emitter with He II.

An experiment was also carried out on the determination of T_3—the temperature at which the shape of the pulse does not change (i.e., $\alpha_2(T_3) = 0$). For this we measured the times t_S and t_B—of travel of pulses of equal duration but of two different fixed amplitudes as

TABLE I. Time of travel of pulse from emitter to sensor: calculated t_c[μ sec] and experimental t_e [μ sec] as functions of the heat flux density Q[W/cm^2] and pulse duration Δt [μ sec].

Q	$\Delta t=2$		Q	$\Delta t=5$		$\Delta t=10$		$\Delta t=50$	
	t_c	t_e		t_c	t_e	t_c	t_e	t_c	t_e
4.1	2429.3	2429.5	1.8	2433.4	2431.3	2433.4	2433.5	2433.4	2432.8
7.2	2425.5	2425.0	3.7	2426.3	2426.1	2425.0	2426.0	2424.7	2427.4
12.0	2419.1	2417.5	6.5	2419.0	2420.7	2413.5	2418.6	2411.8	–
18.6	2413.1	2412.8	10.2	2411.7	2415.0	2402.5	2408.9	2395.2	2414.7
24.6	2408.5	2408.2	17.6	2401.1	2405.3	2387.8	2393.3	2362.3	2385.1
46.5	2394.9	2394.0	27.5	2389.6	2393.0	2371.1	2381.9	2320.2	2367.3
53.1	2391.4	2389.0	46.2	2372.2	2379.2	2346.8	2364.0	2261.7	2348.5
80.8	2378.1	2379.8							
98.6	2370.5	2374.9							
135	2355.9	2369.0							

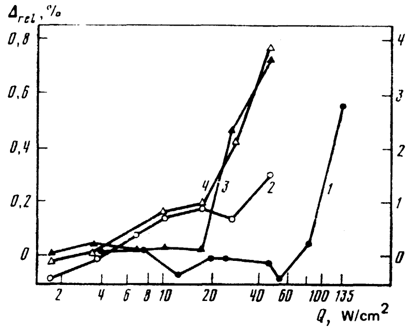

FIG. 5. Dependence of the relative deviation (Δ_{rel} (%) on the flux density Q radiated in the pulse. Curves 1, 2, 3, 4, correspond to pulse durations Δt = 2, 5, 10, 50 sec. Scale at left—for curves 1, 2, 3; at right, for curve 4.

the temperature T_3 was approached from below. If we plot $\Delta\tau(T)$, $\Delta\tau = t_B - t_S$, then $\Delta\tau = 0$ at $T = T_3$.

RESULTS AND DISCUSSION

Values of t_e and t_c—the times measured experimentally and calculated—are given in Table I. A graph of $\Delta_{rel}(Q)$, $\Delta_{rel} = 100(t_e - t_c)/t_c$ is given in Fig. 5. The distance from the emitter to the sensor was determined by multiplication of the tabulated value of the velocity of second sound[13,15] by the time of flight of the pulse, obtained by extrapolation of Q to zero on the $t_e(Q)$ plot. A control experiment in which the directly measured distance was compared with the value calculated in this fashion showed that the quantities agreed within 0.1%. The agreement of the experimental points t_e with the calculated values (which is especially good at $\Delta t = 2$ μsec) is evidence of the validity of the initial premises relative to the evolution and to the applicability of the Burgers equation for the description of the motion of short heat pulses through He II.

The systematic shift of the data at $\Delta t = 5, 10, 50$ μsec (see Fig. 5) is evidently the consequence of the transfer of part of the heat to the backing, with a corresponding decrease in the effective Q; this imposes a limitation on the ranges of Q and Δt that can be used in such experiments.

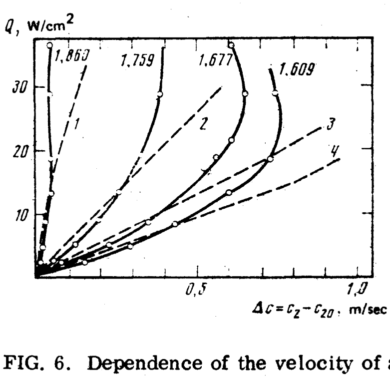

FIG. 6. Dependence of the velocity of a shock wave on the power of the radiated pulse. The solid lines with circles are from Ref. 6 (the numbers on the curves indicate the temperature). The dashed lines are our calculation data.

The agreement of T_3 (1.884 K) calculated by the formula given in Ref. 1 with the measured 1.885 ± 0.005 K attests to the validity of the calculation of α_2 carried out by Khalatnikov.[7] If we construct the graph $\alpha_2(T)$, then there appear three temperatures T_1, T_2, T_3 at which $\alpha_2 = 0$. After suitable verification, the points T_1 and T_2 can be used as reference points for tying-in with the temperature scale. In Ref. 6, a similar experiment is described. A calculation according to the scheme set forth above has also been carried out by us using the parameters of this experiment. Figure 6, the data in which were taken from Ref. 6, shows the experimental and theoretical curves. Taking into account the increase in Δ_{rel} with increase in Δt (in Ref. 6, $\Delta t = 50$ μsec), we have good agreement of the curves. The ranges of Q in which there is no agreement are evidently characterized by overheating of the emitter, by the invalidity of the relation $v_0 = Q/ST$, and by a decreased effective Q radiated in the pulse.

We note in closing that for the calculation of processes connected with the emission of pulses of second sound of finite amplitude, a knowledge is necessary of the detailed picture of the nonstationary heat exchange of the solid with He II, which is lacking at the present time.

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