Effect of the magnetic moment of a charged particle on its energy loss in optically active materials

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Asymmetry in the energy loss of longitudinally polarized fast charged particles in optically active antipodes (enantiomers) is demonstrated in the framework of classical electrodynamics. For ultrarelativistic electrons and positrons the degree of asymmetry turns out to be of order α^4, and for relativistic electrons and positrons, of order $(v^3/c^3)\alpha^4$, where $\alpha \approx 1/137$ is the fine-structure constant.

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1. Recently Zel'dovich and Saakyan[1] evaluated the degree of asymmetry of the cross section for decay of the L and D forms of optically active molecules on interaction with longitudinally polarized fast electrons. This problem is of immediate interest in connection with attempts to explain the asymmetry which exists in nature between right-handed and left-handed forms of molecules. References on this question are contained in the paper cited,[1] in which the dependence of the decay cross section on the helicity of the electrons was determined in the framework of quantum electrodynamics in the first order of perturbation theory. A degree of asymmetry of the order $\alpha^3 v/c$ was predicted, where v is the electron velocity and α is the fine-structure constant.

In the present work we also consider the asymmetry in the interaction of longitudinally polarized fast charged particles (it is assumed that the particle has a magnetic moment) with optically active antipodes, but this time in the framework of classical electrodynamics. We have calculated the magnetic-moment-dependent part of the retarding force exerted on the particle by the medium and have found the degree of asymmetry η of the energy loss of the particle in the L and D forms of the material: For ultrarelativistic electrons and positrons the value of η turns out to be $\sim\alpha^4$.

The nature of the origin of the effect is as follows. In the Coulomb excitation of the molecules of the medium by a passing charged particle, the electrons in the molecules begin to move along helices with opposite sense for L and D forms. As a result a circular current appears around the particle and a magnetic field begins to act on it.[2] For an isotropic gyrotropic medium only the field component along the direction of motion of the particle turns out to be nonzero, and its sign changes in the transition from the L to D forms of the material. If the particle has a magnetic moment, this field leads to appearance of a force collinear with the velocity vector, $F_\mu = \nabla(\boldsymbol{\mu}\cdot\mathbf{B})$, as a result of which there also appears an asymmetry in the energy loss.

The optical activity introduces into the values of the effects associated with it a factor of the order $a/\lambda \approx (a/a_0)\alpha$, where a is the dimension of the asymmetric chiral center in the molecule and λ is the wavelength of the radiation characteristic of traversal of this center by an optical electron; a_0 is the Bohr radius. Since the intra-atomic and intramolecular magnetic fields are less than the electric fields by α times, the magnetic field acting on a particle moving in an optically active medium is approximately $(a/a_0)\alpha^2$ times smaller than the corresponding electric field. Therefore

$$\eta = \left|\frac{F_\mu}{F_e}\right| \approx \left|\frac{\nabla(\mu\mathbf{B})}{eE}\right| \approx \frac{\mu B}{aeE} \approx \frac{\mu\alpha^2}{ea_0} .$$

However, in a gyrotropic medium the electric field E acting on the particle from the medium turns out also to be dependent on μ. From general considerations it follows that in an isotropic medium the expression for E should contain terms of the form

$$[c_1\mathbf{v}(\boldsymbol{\mu}\mathbf{v})/c^2 + c_2\boldsymbol{\mu}]ga_0^{-3},$$

where c_1 and c_2 are scalars and g is a pseudoscalar associated with the optical activity ($g \sim a\alpha^2/a_0$). These terms also should contribute to the degree of asymmetry of the energy loss.

To find the dependence of E on μ it is necessary to introduce the magnetic moment into the Maxwell equations. In Sec. 2 we show how the presence of a magnetic moment in the particle affects the Maxwell equations. In Sec. 3 we calculate the retarding force and give an expression for the degree of asymmetry of the energy loss in optically active antipodes.

2. Consider a charged particle having a magnetic moment μ and moving in a medium with velocity $\mathbf{v}$. The field produced by this particle is described by the Maxwell equations:

$$\operatorname{div}\mathbf{D} = 4\pi\rho; \quad \operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{j}. \tag{1}$$

To find the charge density ρ and current density $\mathbf{j}$ we shall make use of the fact that the quantity $(c\rho, \mathbf{j})$ is a 4-vector. In the system of the moving particle the field is described by the potentials

$$\varphi' = \frac{e}{R'}; \quad A' = \left[\nabla'\frac{1}{R'}, \times\boldsymbol{\mu}\right],$$

which correspond to the densities

$$\rho' = -\frac{1}{4\pi}\Delta'\varphi' = e\delta(\mathbf{R}'), \quad \mathbf{j}' = -\frac{c}{4\pi}\Delta'\mathbf{A}' = c\operatorname{rot}'(\boldsymbol{\mu}\delta(R')). \tag{2}$$

Transforming to the coordinate system attached to the medium, we obtain ($z \parallel \mathbf{v}$)

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$$\rho=e\delta(\mathbf{R})+\mathbf{v}\,\mathrm{rot}\,(\mathbf{M}\delta(\mathbf{R}))/c\gamma,$$
$$\mathbf{j}=e\mathbf{v}\delta(\mathbf{R})+c\gamma\,\mathrm{rot}\,(\mathbf{M}\delta(\mathbf{R}))+\mathbf{v}(\mathbf{v}\,\mathrm{rot}\,(\mathbf{M}\delta(\mathbf{R})))/c\gamma,$$
$$\mathbf{R}=\mathbf{r}-\mathbf{v}t,\quad \gamma=(1-v^2/c^2)^{1/2},\quad \mathbf{M}=\{\gamma\mu_x,\ \gamma\mu_y,\ \mu_z\}. \tag{3}$$

3. The retarding force exerted by an isotropic optically active medium on a charged particle moving in it is $(z\parallel\mathbf{v})$

$$\mathbf{F}=e\mathbf{E}+\nabla(\mu\mathbf{B}). \tag{4}$$

Carrying out a three-dimensional Fourier transformation in this expression and using the equation

$$\mathrm{rot}\,\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},$$

we obtain[1)]

$$F_3(\mathbf{k},t)=eE_3(\mathbf{k},t)+\frac{ik_3}{\omega}c\mu[k_1E_2(\mathbf{k},t)-k_2E_1(\mathbf{k},t)]. \tag{5}$$

The quantities E_i entering here can be found by transforming to the three-dimensional Fourier components in Eq. (1) and solving the system of equations

$$T_{ij}E_j=-i\frac{\omega}{2\pi^2c}\left(\frac{ev_i}{c}+\frac{iv_i(\mathbf{v}[\mathbf{k}\times\mathbf{M}])}{\gamma c^2}+i\gamma[\mathbf{k}\times\mathbf{M}]_i\right)e^{-i\omega t}, \tag{6}$$

where for the isotropic gyrotropic medium considered we have

$$T_{ij}=k_ik_j+\left(\frac{\omega^2}{c^2}\varepsilon(\omega)-k^2\right)\delta_{ij}-ig(\omega)\frac{\omega^2}{c}e_{ijl}k_l,$$
$$g(\omega)=\frac{8\pi N_{\mathrm{mol}}}{9\hbar}(\varepsilon(\omega)+2)\sum_n\frac{R_{n0}}{\omega_{0n}^2-\omega^2}, \tag{7}$$
$$R_{n0}=\mathrm{Im}\,(\langle 0|\mathbf{d}|n\rangle\langle n|\mathbf{m}|0\rangle),$$

where R_{n0} is the optical rotational strength for the transition $0\to n$. The quantity $g(\omega)$ changes sign in the transition from the L form to the D form of the material.

Solving the system (6), substituting the values of E_i into Eq. (5), and carrying out the inverse Fourier transformation for $\mathbf{r}=\mathbf{v}t$, we obtain

$$F=-\frac{ie^2v}{2\pi^2}\int\frac{\omega(\varepsilon/c^2-1/v^2)\,d\mathbf{k}}{\varepsilon(\varepsilon\omega^2/c^2-k^2)}+i\frac{e\mu}{2\pi^2c^2}(1-\gamma)\int\frac{g\omega^5(\varepsilon/c^2-1/v^2)}{(\omega^2\varepsilon/c^2-k^2)^2}\,d\mathbf{k}, \tag{8}$$

where the term proportional to γ is due to the dependence of the field E on μ. In this expression the first integral describes the ordinary ionization loss and a method of calculating it is given in the book by Landau and Lifshitz[3]; the second term is the effect of the magnetic moment of the particle on the energy loss. Calculation of the second integral can be carried out like that of the first, with the only difference that the integrand of the second integral as a function of ω has a second-order pole in the upper half plane, while the first integral has a simple pole.

The final expression for the μ-dependent part of the retarding force has the form

$$F_\mu=\frac{e\mu}{vc^2}(1-\gamma)\left\{\omega^5g(\omega)\left(\frac{\varepsilon}{c^2}-\frac{1}{v^2}\right)\Big/\frac{d}{d\omega}\left[\omega^2\left(\frac{\varepsilon}{c^2}-\frac{1}{v^2}\right)\right]\Bigg|_{\omega=i\xi(0)}^{\omega=i\xi(q_0)}-\int_{i\xi(0)}^{i\xi(q_0)}\omega^3g(\omega)\,d\omega\right\}, \tag{9}$$

where $\xi(q)$ is the imaginary part of the square root of the equation

$$\omega^2(\varepsilon(\omega)/c^2-1/v^2)=q^2,$$

and $q_0\sim 1/a$, where a is the distance between the atoms (ions) of the medium.

For fast particles we have

$$\varepsilon(\xi)\approx 1+4\pi\frac{Ne^2}{m\xi^2},\quad g(\xi)=\frac{8\pi N_{\mathrm{mol}}}{3\hbar}\sum_n\frac{R_{n0}}{\omega_{n0}^2+\xi^2},\quad \xi(q_0)=v\frac{q_0}{\gamma};$$

$$F_\mu=(1-\gamma)e\mu\frac{4\pi N_{\mathrm{mol}}}{3\hbar vc^2}\left(\sum_n R_{0n}\omega_{0n}^2\ln\frac{\omega_{0n}^2+\xi^2(q_0)}{\omega_{0n}^2+\xi^2(0)}-\sum_n R_{0n}\omega_{0n}^2\right)$$
$$\approx(1-\gamma)e\mu\frac{4\pi N_{\mathrm{mol}}}{3\hbar vc^2}\sum R_{0n}\omega_{0n}^2\ln\frac{\omega_{0n}^2+\xi^2(q_0)}{\omega_{0n}^2+\xi^2(0)}. \tag{10}$$

Both for $v^2<c^2/\varepsilon(0)$ and for $v^2>c^2/\varepsilon(0)$—the ultrarelativistic case—the logarithm in Eq. (10) turns out to be identical with the logarithm arising in calculation of the ordinary ionization loss, and drops out in the estimate of the degree of asymmetry. As a result for electrons and positrons we obtain at $v^2>c^2/\varepsilon(0)$

$$\eta\approx(1-\gamma)\frac{N_{\mathrm{mol}}}{3Nc^2e^2}\sum_n R_{0n}\omega_{0n}^2\approx 10^{-9}$$
$$(\gamma\ll 1,\quad N_{\mathrm{mol}}/N\sim 10^{-1},\quad R_{0n}\sim 10^{-37},\quad \omega_{0n}^2\sim 10^{32}),$$

and for $v^2<c^2/\varepsilon(0)$ and $v^2/c^2\ll 1$

$$\eta\approx\frac{v^3N_{\mathrm{mol}}}{c^3N}\sum_n\frac{R_{0n}\omega_{0n}^2}{6c^2e^2}\approx\frac{v^3}{c^3}\cdot 10^{-9}.$$

Thus, the asymmetry in the energy loss of ultrarelativistic electrons in optically active antipodes as a function of the polarization of the particles should be of the order α^4.

[1)] In what follows we shall omit the arguments in the Fourier components.

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