Collision correlation as the cause of the power-law asymptotic behavior of responses and the distinctive features of the spectrum of low-frequency fluctuations

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The collision-correlation-induced power-law tails in the spatially homogeneous response in a gas with binary encounters are investigated. It is shown that the low-frequency response is determined by the interaction between the diffusion-type hydrodynamic modes ("diffusons"), which describe the spreading of the correlation arising between the single-particle quantities upon the disturbance of the state of equilibrium of the system. The coupling between the diffusons may be "strong" or "weak," depending on the type of conservation laws obtaining in the system and the types of hydrodynamic modes. In the latter case the asymptotic behavior of the response is provided by the single-diffuson approximation. But in the "strong"-coupling case the determination of the correct asymptotic behavior requires, generally speaking, the consideration of all the many-diffuson processes, and the problem then acquires the features of the dynamics of critical phenomena. The effect of the collision-induced long-lived correlation on the low-frequency fluctuation spectrum and, in particular, the possible explanation of $1/f$ noise by this effect are considered.

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INTRODUCTION

The investigation of power-law tails in kinetic theory began with the work of Alder and Wainwright,[1] who discovered in numerical experimenta that the asymptotic form of the velocity–velocity correlator, $\overline{v(t)v(0)}$, for a selected particle of a gas of hard spheres is given by a power law, $t^{-d/2}$, where d is the dimensionality of the space, instead of the expected exponential law $e^{-t/\tau}$ ($1/\tau$ is the collision rate). Such a temporal asymptotic behavior (for three-dimensional space, to which we restrict ourselves below, this law is $t^{-3/2}$) is due to the fact that the particle excites in the course of its motion a weakly damped hydrodynamic mode of the diffusional type. The power-law tails continue to be intensively investigated, and there is an extensive literature on this question (see, for example, Refs. 2–10).

The purpose of the present paper is to investigate the temporal asymptotic behavior of the spatially homogeneous response at large times, as well as to ascertain how the phenomena giving rise to the power-law asymptotic behavior of the response affect the low-frequency spectrum of the spatially-homogeneous fluctuations.

As the initial physical system, we shall consider a gas of uncharged particles interacting (i.e., colliding) with each other and with external scatterers forming a thermostat. The initial cause of the appearance of the power-law tails is the violation of the so-called Boltzmann hypothesis on molecular disorder (Stosszahlansatz), which postulates the total noncorrelation of the occupation numbers of the single-particle states. At the same time, these states turn out to be correlated after each collision, since the states of two particles (in the case of a binary encounter) or else the states of a particle and an external scatterer change simultaneously. At thermodynamic equilibrium this correlation on the average vanishes, and the occupation numbers of the single-particle states remain uncorrelated. In the nonequilibrium state the correlation does not vanish even on the average, making, for example, a contribution to the fluctuation phenomena in the steady nonequilibrium state.[11,12]

In the response problem the initial deviation from equilibrium gives rise to a correlation between the single-particle occupation numbers that is proportional to this deviation. This correlation is, however, small (with respect to the parameter of the kinetic equation), and has over periods of time of the order of the relaxation time little effect on the response, which, at these times, can be well described within the framework of the molecular-disorder hypothesis. But the situation changes at large times, since the correlation arising between the single-particle states in momentum space leads to the correlation in coordinate space of such local macroscopic characteristics of the gas as density, temperature, and total momentum, characteristics which are conserved during relaxation in momentum space. The relaxation of the correlation of these quantities is brought about as a result of the slow hydrodynamic processes (diffusion, viscosity, thermal conduction), and therefore the contribution of this "hydrodynamic" correlation to the response turns out to be long-lived. Although this contribution is small, it is just the one that determines the temporal asymptotic behavior of the response. Thus, the low-frequency response problem becomes a problem of the kinetics of the diffusion type of hydrodynamic modes, i.e., of the kinetics of "diffusons."

The papers that have so far been published on the power-law tails are usually devoted only to the single-diffuson approximation, which corresponds to the so-called Kawasaki–Oppenheim ring operator.[13] In this approximation, the relaxation amounts to the diffusional dissipation of a conserved quantity's gradient that has appeared at some point, e.g., an excess momentum surrounding a particle, as in the case considered by Alder and Wainwright.[1] Diffusional dissipation by itself yields an asymptotic $t^{-3/2}$ time dependence. Additional powers of $1/t$ can, however, arise in certain cases. This is precisely what happens in the case of a gas interacting with a thermostat, when there is only one hydrodynamic mode—the diffusional mode—involved. This mode turns out to be weakly coupled to momentum space and the processes of correlation formation and destruction occurring in this space. This weak coupling leads not only to a steeper (than $t^{-3/2}$) power-law tail in the spatially homogeneous response, but also to the smallness of the many-diffuson corrections to the single-diffuson approximation, which, consequently, yields the correct temporal asymptotic form of the spatially homogeneous response. It may be said that in this case the initial perturbation produces weakly interacting diffusons, and that the situation can be described with the required accuracy with the aid of perturbation theory.

But the other situation in which the hydrodynamic modes are more strongly coupled to the relaxation in momentum space is also possible. In the gas considered by us, such a situation arises when the thermostat-induced energy and/or momentum relaxation is "cut off." The new hydrodynamic modes—thermal-conductive and/or viscous modes—that then appear turn out to be more directly coupled to both the processes of correlation formation in momentum space and the single-particle relaxation in this space. As a result, no additional powers of $1/t$ arise in the asymptotic form of the spatially homogeneous response, and we have the $t^{-3/2}$ law in the single-diffuson approximation. It turns out, however, that the $t^{-3/2}$ law is not necessarily the true asymptotic form in this case, since the many-diffuson corrections may increase with increasing t, and a situation somewhat reminiscent of the situation that obtains at the critical point of a second-order phase transition may arise. Thus, we may, in investigating the low-frequency response, get into the field of critical-phenomenon dynamics. The application of the methods of critical dynamics to the response problem is outside the scope of the present paper. Let us only note the following: the true asymptotic form of the response should be obtained by summing the entire many-diffuson series. It can be expected that this will give in the frequency representation a weak frequency dependence of the type ω^{α}, $|\alpha| \ll 1$. If this were not the case, then the kinetic coefficients would be subject to strong dispersion at low frequencies. But low-frequency anomalies are normally not observed in the kinetic coefficients, an argument in favor of the convergence of the series describing the long-range correlations to an almost constant quantity.

At the same time, a low-frequency anomaly in the fluctuation spectrum—the so-called $1/f$ noise—is well known and widespread. It can be shown that the spectrum of the background low-frequency fluctuations is described by a series that is more singular by one power of the frequency than the series for the response. The convergence of the latter series to ω^{α}, $|\alpha| \ll 1$ could thus indicate the existence of a $1/\omega$ type of singularity in the low-frequency fluctuation spectrum.

1. THE RESPONSE PROBLEM WITHIN THE FRAMEWORK OF THE BOLTZMANN MOLECULAR-DISORDER HYPOTHESIS

Let us consider a gas of uncharged particles that interact through collisions with each other and with scatterers forming a thermostat, and impart momentum and energy to the scatterers. The state of the gas of N particles is described by a distribution function that satisfies the Boltzmann equation and the normalization condition:

$$\partial_t F_p + I_p F_p + \Pi_p\{F, F\} = 0, \quad \sum_p F_p = N. \tag{1.1}$$

Here the linear operator I_p describes the collisions with the thermostat, while the bilinear operator Π_p describes the binary collisions.

In the state of thermodynamic equilibrium the distribution function is stationary and has a Maxwellian form, with

$$\partial_t F_p = I_p F_p = \Pi_p\{F, F\} = 0. \tag{1.2}$$

Now let the distribution function at the moment of time $t=0$ be different from the equilibrium function, i.e., let

$$F_p(0) = F_p + \delta F_p(0).$$

The regression of the small initial deviation δF_p is given by the formula

$$\delta F_p(t) = \exp(-J_p t)\,\delta F_p(0), \tag{1.3}$$

which is the solution to the equation for the response:

$$(\partial_t + J_p)\,\delta F_p(t) = 0, \tag{1.4}$$

where the symbol J_p denotes the linearized collision operator

$$J_p = I_p + \Pi_p\{\ldots, F\} + \Pi_p\{F, \ldots\}, \tag{1.5}$$

which is made to vanish by the function $\partial_N F_p$, which, at equilibrium, coincides with the Maxwell function[1)]:

$$J_p \partial_N F_p = 0. \tag{1.6}$$

The formula (1.3) describes a fast relaxation (i.e., one occurring over a period of time $\tau_p \sim J_p^{-1}$) in momentum space. We shall, neglecting the possible straggling of the relaxation times τ_p (and assuming that all the τ_p are shorter than some $\tau_p^{\max}$), call this exponential relaxation. Indeed, for the flux (current) generated by the deviation $\delta F_p(t)$, we obtain the expression

$$\mathbf{j}(t) = \sum_p \mathbf{v}\,\delta F_p(t) = \sum_p \mathbf{v} \exp(-J_p t)\,\delta F_p(0) \approx e^{-t/\tau} \sum_p \mathbf{v}\,\delta F_p(0) = e^{-t/\tau}\mathbf{j}_0. \tag{1.7}$$

By choosing the initial deviation in the form

$$\delta F_p{}^{\beta} = v_\beta F_p / N, \tag{1.8}$$

we can find the following velocity–velocity correlation function:

$$\overline{v_\alpha(t) v_\beta(0)} = \frac{1}{N} \sum_p v_\alpha \exp(-J_p t)\, v_\beta F_p. \tag{1.9}$$

The integral over time from zero to infinity of this correlator is the usual diffusion tensor

$$D_{\alpha\beta}=\frac{1}{N}\sum_{p} v_\alpha J_p^{-1} v_\beta F_p=\int_0^\infty v_\alpha(t)v_\beta(0)\,dt. \tag{1.10}$$

The Boltzmann equation thus leads to an exponential decay law for the velocity–velocity correlation, and, consequently, power-law tails of the type found by Alder and Wainwright[1] (i.e., power laws of the type $t^{-3/2}$) should be explained by deviations from this equation, i.e., by violations of the molecular-disorder hypothesis. These violations are caused by the presence of correlation between the single-particle states. But it will be useful to consider the response to a spatially homogeneous perturbation in our system before proceeding to take this correlation into account. For simplicity, we choose this perturbation to be a point perturbation: $\delta F_p(\mathbf{r},0)=\delta F_p(0)\delta(\mathbf{r})$. Its evolution is described by the formula

$$\delta F_p(\mathbf{r},t)=\exp\left\{-t\left(\mathbf{v}\frac{\partial}{\partial\mathbf{r}}+J_p\right)\right\}\delta F_p(\mathbf{r},0), \tag{1.11}$$

which is the solution to the spatially homogeneous Boltzmann equation:

$$\left(\partial_t+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}+J_p\right)\delta F_p(\mathbf{r},t)=0. \tag{1.12}$$

The relaxation then occurs in two stages. First, there occurs (over a period of time of the order of τ_p) a transition to local equilibrium:

$$\delta F_p(\mathbf{r},\ t)\approx F_p\delta n_0(\mathbf{r},\ t), \tag{1.13}$$

where $\delta n_0(\mathbf{r},t)$ is the initial delta function that has spread over a distance of the order of the mean free path $l=v\tau_p$. The subsequent relaxation process will have the character of diffusion, compensating the induced change in the local concentration, $\delta n(\mathbf{r})\equiv\delta n_0(\mathbf{r},t)$:

$$\delta n(\mathbf{r},t)=e^{-tD\Delta}\delta n(\mathbf{r})=\frac{1}{[2(\pi Dt)^{1/2}]^3}\int d\mathbf{r}'\exp\left\{-\frac{|\mathbf{r}-\mathbf{r}'|^2}{4Dt}\right\}\delta n(\mathbf{r}'). \tag{1.14}$$

The function $\delta n(\mathbf{r})$—the initial condition for the diffusion equation—is, to within terms of the order of $l/r\ll 1$, which arise during the fast phase of the relaxation, the initial concentration perturbation

$$\delta n(\mathbf{r})\approx\sum_p \delta F_p(\mathbf{r},0).$$

Let us note that, on account of the law of conservation of particle number,

$$\int d\mathbf{r}\,\delta n(\mathbf{r},t)=\int d\mathbf{r}\,\delta n(\mathbf{r},0)=\delta N. \tag{1.15}$$

Let the particles be introduced at the moment of time $t=0$ into some small region, i.e., at the "point" $\mathbf{r}=\mathbf{r}_0$. Then the concentration at this "point" falls off in time according to the $t^{-3/2}$ law:

$$\delta n(t)\cong\delta N/(Dt)^{3/2}. \tag{1.16}$$

This formula describes the decrease of the concentration in the region of initial localization of the particles as a result of the diffusional expansion of the region of localization. The power-law asymptotic form $t^{-3/2}$, given by (1.16), has a universal character, and does not depend on the form of the initial perturbation; it is only necessary that $\delta N\neq 0$, i.e., that a subsequently-conserved quantity—a number of particles—be introduced at the moment of time $t=0$ into some small region ("point") of the system. We shall see below that a phenomenon described by a formula of the type (1.16)—the decrease of the density of some conserved quantity as a result of its diffusional spreading over the entire system—always underlies $t^{-3/2}$ tails.

Let us now consider what happens when $\delta N=0$. To derive the temporal asymptotic form, let us use the hydrodynamic algorithm for solving the Boltzmann equation (see, for example, Refs. 12 and 14). Instead of $\delta F_p(\mathbf{r},t)$, it is more convenient to consider its Fourier transform $\delta F_p(q,t)$:

$$\delta F_p(\mathbf{q},t)=e^{-tq^2D}(F_p-i\mathbf{q}J_p^{-1}\mathbf{v}F_p)\left(\frac{1}{N}\sum_{p'}\delta F_{p'}-\frac{i\mathbf{q}}{N}\sum_{p'}\mathbf{v}'J_{p'}^{-1}\widetilde{\delta F}_{p'}\right)+\delta(t)J_p^{-1}\widetilde{\delta F}_p. \tag{1.17}$$

We assume the initial perturbation $\delta F_p(\mathbf{r})$ to be a delta-function perturbation. Standing under the sign J_p^{-1} is the "current" part, $\widetilde{\delta F}_p$, of the perturbation:

$$\widetilde{\delta F}_p=\delta F_p-\frac{F_p}{N}\sum_{p'}\delta F_{p'}. \tag{1.18}$$

For $\delta N=\Sigma_p\delta F_p\neq 0$, the algorithm (1.17) immediately yields the $t^{-3/2}$ law:

$$\sum_p\delta F_p(\mathbf{r},t)\big|_{r=0}=\sum_{qp}\delta F_p(\mathbf{q},t)=\sum_q e^{-q^2Dt}\sum_p\delta F_p\propto\frac{\delta N}{(Dt)^{3/2}}.$$

Now let a current

$$\delta\mathbf{j}_0=\sum_p\mathbf{v}\delta F_p\neq 0,\quad \sum_p\delta F_p=0, \tag{1.19}$$

rather than particles, be introduced at the point $\mathbf{r}=0$ at $t=0$. Let us find, using the algorithm (1.17), the time variation of the current at this point:

$$\delta\mathbf{j}(\mathbf{r},t)\big|_{r=0}=\sum_{qp}\mathbf{v}\delta F_p(\mathbf{q},t)=\delta\mathbf{j}(t),$$
$$\delta\mathbf{j}(t)=\sum_q e^{-q^2Dt}q^2D\sum_p\mathbf{v}J_p^{-1}\delta F_p\approx\delta\mathbf{j}_0\frac{\tau_p}{t(Dt)^{3/2}}\propto\frac{\delta\mathbf{j}_0}{t^{5/2}}. \tag{1.20}$$

We have obtained a $t^{-5/2}$ law, which, like the $t^{-3/2}$ law, does not depend on the specific form of the perturbation at $t=0$, but corresponds to a local introduction of a nonconserved quantity. This quantity—in the present case, a local current—relaxes rapidly, creating a concentration gradient:

$$\widetilde{\delta n}(\mathbf{r},t)=-\int_0^t\operatorname{div}\delta\mathbf{j}(\mathbf{r},t)\,dt,$$

$$\widetilde{\delta n}(\mathbf{q},t)=-i\mathbf{q}\int_0^t\sum_p\mathbf{v}\exp(-J_pt)\delta F_p\,dt\cong-i\mathbf{q}\sum_p\mathbf{v}J_p^{-1}\delta F_p.$$

Subsequently, the evolution proceeds according to the diffusion law, the $\widetilde{\delta n}$ serving as the initial condition for the diffusion equation. The quantity $\widetilde{\delta n}$ is proportional to $ql\ll 1$; thus, the initial current $\delta\mathbf{j}_0$ creates a small concentration gradient. One more factor, q, appears in the computation of the local long-lived current $\delta\mathbf{j}$, since the greater part of the diffusional perturbation—the "diffuson"—is proportional to F_p [see (1.17)] and does not generate a current. It is these two q which occur in the numerator of the algorithm (1.17) which change the $t^{-3/2}$ law into a $t^{-5/2}$ law. Thus, the $t^{-3/2}$ law is characteristic of a perturbation that violates a conservation law obtaining in the system (here the particle-number conservation law), whereas the $t^{-5/2}$ law is characteristic of a perturbation that does not violate the conservation laws (i.e., of a perturbation that does not introduce conserved quantities into the system). There is only one conserved quantity (the particle number) in our system, a circumstance which is due to the presence of a momentum- and energy-collecting thermostat. If the thermostat is switched off, then new hydrodynamic—acoustic heat-conductive, and two viscous—modes appear. The last three of these modes are "slow" modes, and can participate in the generation of the long-lived response.

2. CORRELATIONS AND THE RING OPERATOR

The corrections to the Boltzmann equation (1.1) or to the ensuing response equation (1.4) are proportional to powers of the ratio $\Delta t/\tau_{\mathbf{p}} \ll 1$, where Δt is the collision time ("duration"). Of these corrections we should choose those which describe long-range correlations. Onuki[5] first pointed out the connection between long-range correlations in the response problem and fluctuations in the nonequilibrium state, in which binary collisions give rise to an additional correlation.[11,12] By using the apparatus of the theory of fluctuations in the nonequilibrium state, we can routinely construct those corrections to the Boltzmann equation which are responsible for the appearance of power-law tails. We shall first derive the Kawasaki–Oppenheim[13] ring operator, which takes account of the correlation in the single-difuson approximation. In Sec. 3 we shall give the diagrammatic derivation; here the corresponding result will be obtained from simple physical arguments. To derive the Boltzmann equation (1.1), we substitute the product of the single-particle distribution functions into the binary-collision operator, thereby assuming the total absence of correlation of the particles participating in the collision. The correlation can be taken into account by substituting the "instantaneous" distribution functions, and then averaging the entire expression over the ensemble. In view of the short duration of a collision event, the low-frequency correlation of interest to us here should have no effect on it, and should therefore not change the transition probabilities. But the occupation numbers can now be correlated; therefore, the binary-collision operator should be written as $\Pi_{\mathbf{p}}\{\overline{FF}\}$, i.e., we should substitute into this operator the two-particle distribution function

$$\overline{F_{\mathbf{p}}F_{\mathbf{p}_1}}=\bar{F}_{\mathbf{p}}\bar{F}_{\mathbf{p}_1}+\overline{\delta F_{\mathbf{p}}\delta F_{\mathbf{p}_1}}.$$

The correction $\overline{\delta F_{\mathbf{p}}\delta F_{\mathbf{p}_1}}$ describes the correlation of the single-particle states. The particle collisions occur rapidly ("instantaneously"), and in a small region of space (at a "point"). Therefore, we need a function that describes the correlation of the occupation numbers $\mathbf{p}$ and $\mathbf{p}_1$ at one instant and at one point in space (at a point on the kinetic scales). For a spatially homogeneous system the equal-time correlator $\overline{\delta F_{\mathbf{p}}(\mathbf{r})\delta F_{\mathbf{p}_1}(\mathbf{r}_1)}$ depends only on the difference $\mathbf{r}-\mathbf{r}_1$. It is convenient to represent it by its Fourier transform:

$$\Phi^{\mathbf{q}}_{\mathbf{p}\mathbf{p}_1}=\int e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}_1)}\overline{\delta F_{\mathbf{p}}(\mathbf{r})\delta F_{\mathbf{p}_1}(\mathbf{r}_1)}\,d(\mathbf{r}-\mathbf{r}_1). \tag{2.1}$$

The function $\Phi^{\mathbf{q}}_{\mathbf{p}\mathbf{p}_1}$ satisfies an equation similar to the Boltzmann equation[11,12]:

$$[\partial_t+i\mathbf{q}(\mathbf{v}-\mathbf{v}_1)+J_{\mathbf{p}}+J_{\mathbf{p}_1}]\Phi^{\mathbf{q}}_{\mathbf{p}\mathbf{p}_1}=-\Pi_{\mathbf{p}\mathbf{p}_1}\{F,F\}. \tag{2.2}$$

The left-hand side of this equation describes the evolution of the correlation of the occupation numbers $\mathbf{p}$ and $\mathbf{p}_1$ as a result of the collisions of the particles in these states with other particles, as well as their kinematics. The right-hand side is the source of the correlations—the binary-collision operator with one summation omitted,[11,12] describing the simultaneous departure of particles from, and the arrival of others in, the pair of states during their intercollision. At equilibrium, $\Pi_{\mathbf{p}\mathbf{p}_1}\{F,F\}=0$ because of the equality of the arrivals and departures. But any deviation from equilibrium immediately disturbs the balance, and causes the appearance of a correlation proportional to this deviation. In the response problem, $F_{\mathbf{p}}(t)=F_{\mathbf{p}}+\Delta F_{\mathbf{p}}(t)$, and therefore

$$\Phi^{\mathbf{q}}_{\mathbf{p}\mathbf{p}_1}[\Delta F]=-\frac{1}{\partial_t+i\mathbf{q}(\mathbf{v}-\mathbf{v}_1)+J_{\mathbf{p}}+J_{\mathbf{p}_1}}J_{\mathbf{p}\mathbf{p}_1}[\Delta F], \tag{2.3}$$

where

$$J_{\mathbf{p}\mathbf{p}_1}[\Delta F]\equiv\Pi_{\mathbf{p}\mathbf{p}_1}\{F,\ \Delta F\}+\Pi_{\mathbf{p}\mathbf{p}_1}\{\Delta F,\ F\}$$

is the linearized source of the correlation induced by the perturbation $\Delta F_{\mathbf{p}}$. The equal-time correlation at a given point is described by the function[2)] $\sum_{\mathbf{q}}\Phi^{\mathbf{q}}_{\mathbf{p}\mathbf{p}_1}$. Substituting this function into the binary-collision operator, we obtain a ring operator describing the correlation produced by the perturbation and the inverse effect of the correlation on the evolution of the perturbation:

$$\hat{R}_{\mathbf{p}}[\Delta F]\equiv\Pi_{\mathbf{p}}\left\{\sum_{\mathbf{q}}\frac{1}{\partial_t+i\mathbf{q}(\mathbf{v}-\mathbf{v}_1)+J_{\mathbf{p}}+J_{\mathbf{p}_1}}J_{\mathbf{p}\mathbf{p}_1}[\Delta F]\right\}. \tag{2.4}$$

The ring operator is linear in the perturbation ΔF. The presence of $\mathbf{q}$ in its denominator guarantees the participation of the hydrodynamic modes and the appearance of a diffuson pole. The time derivative in the denominator indicates the presence of memory. Another distinctive feature of the ring operator is its two-particle character: it describes the simultaneous creation and simultaneous annihilation of two spatially inhomogeneous excitations. Notice that, for the ring process to occur, the particles participating in it need not be identical. From the expression (2.4) we can derive the ring operator that describes the long-range correlation that arises upon scattering of gas particles by scatterers forming a thermostat.[6,7] To do this, we should regard the index $\mathbf{p}_1$ as pertaining to these scatterers, and drop the corresponding terms in (2.4). The ring operator then becomes in form (but not in fact) a single-particle operator:

$$\hat{R}_{\mathbf{p}}[\Delta F]\equiv I_{\mathbf{p}}\left\{\sum_{\mathbf{q}}\frac{1}{\partial_t+i\mathbf{q}\mathbf{v}+J_{\mathbf{p}}}\right\}I_{\mathbf{p}}\Delta F_{\mathbf{p}}. \tag{2.5}$$

The correction to the distribution function obtained when the ring process is taken once into account has (in the frequency representation) the form

$$\Delta F_{\mathbf{p}}^{\omega}=\frac{1}{-i\omega+J_{\mathbf{p}}}\hat{R}_{\mathbf{p}}(\omega)\frac{1}{-i\omega+J_{\mathbf{p}}}\Delta F_{\mathbf{p}}(0). \tag{2.6}$$

This correction is small with respect to the parameter

$\Delta t/\tau_p$: the estimate for $q \lesssim 1/l$ yields

$$\frac{\hat{R}}{J} \sim \left(\frac{q}{p}\right)^3 \frac{1}{qv\tau_p} \sim \left(\frac{\Delta t}{\tau_p}\right)^3. \tag{2.7}$$

Despite the smallness, the ring operator determines the evolution of the system at large times because of the fact that it separates out the diffuson pole, which can compensate for this smallness. We separate out the diffuson pole in the ring operator (2.5) with the aid of the algorithm (1.17). Let us again discuss the structure of the latter. The universal hydrodynamic-response function is multiplied by an "amplitude" determined by the perturbation y_p. The universal response function contains the diffuson propagator e^{-tq^2D}, or, in the frequency representation, $(-i\omega + q^2D)^{-1}$. The numerator of the universal response function contains two terms: the equilibrium distribution function F_p and a small correction, which is proportional to q. The correction is the current part of the universal response. The diffuson constant—"amplitude"—is also made up of two parts, determined respectively by the "concentration," $\Sigma_p y_p$, and "current," $q\Sigma_p v J_p^{-1}\tilde{y}_p$, parts of the perturbation y_p. The "current" part is small, and proportional to q. In the ring operator (2.5) the particle number-conserving operator I_p serves as the perturbation y_p:

$$\sum_p I_p \ldots = 0. \tag{2.8}$$

Because of this, the diffuson amplitude is determined by the small current part, which is proportional to q. Furthermore, upon substitution of the hydrodynamic algorithm into the ring operator, the universal response function appears under the sign of the operator I_p, and loses its major, q-independent part:

$$I_p F_p = 0. \tag{2.9}$$

The vanishing of the major part of the response when it is substituted into the ring operator is due to the insensitivity of the relaxation in momentum space to local concentration changes that do not alter the equilibrium character of the momentum distribution of the particles. Only the small correction due to the diffusional current arising during the compensation of the concentration gradients relaxes. As a result, the factor q^2 in the ring operator appears also in the numerator:

$$\hat{R}_p[\Delta F] = f_p^{\alpha} \sum_q \frac{q_\alpha q_\beta}{-i\omega + q^2 D} A_\beta[\Delta F]. \tag{2.10}$$

The function f_p^{α} does not depend on the form of the perturbation:

$$f_p^{\alpha} = I_p J_p^{-1} v_\alpha F_p. \tag{2.11}$$

The diffuson amplitude A_β is a linear functional of the perturbation ΔF_p:

$$A_\beta[\Delta F] = \frac{1}{N} \sum_p v_\beta J_p^{-1} I_p \Delta F_p. \tag{2.12}$$

The sum over q in (2.10) determines the asymptotic forms of the ring operator and its contribution to the response. In the frequency representation this will be the law $\omega\omega^{1/2}$; in the temporal representation, the law $t^{-5/2}$. For example, in the simple case in which the binary collisions are unimportant, and $J_p \equiv I_p$, we obtain the contribution to the current in the form

$$\Delta J_\alpha(t) = D_{\alpha\beta} \sum_q q_\beta q_{\beta'} e^{-q^2Dt} \sum_p v_{\beta'} I_p^{-1} \Delta F_p(0) \propto t^{-5/2}. \tag{2.13}$$

This formula coincides with the formula (1.20) for the current, generated by a point perturbation, at the point of perturbation. But the expression (2.13) gives the asymptotic form of the total current generated by a spatially homogeneous perturbation ΔF_p. The mechanism guaranteeing the slowness of the relaxation of $\Delta J_\alpha(t)$ is the diffusional dissipation of the density–density correlation gradients that arise during the relaxation of the particle–particle correlation in momentum space. Thus, the difference between the spatially homogeneous processes (or the description with the aid of quantities pertaining to the gas as a whole) and the spatially inhomogeneous processes is obliterated when we go outside the limits of the single-particle description. Indeed, the two-particle correlation corrections introduce into the spatially homogeneous response features that are characteristic of the spatially inhomogeneous processes—in particular, the slowness of the relaxation. A comparison of the formulas (2.13) and (1.20) clearly demonstrates this.

The additional factor $q^2 \to 1/t$ in comparison with the Alder–Wainwright $t^{-3/2}$ law appeared in the asymptotic form of the spatially homogeneous response in a way similar to the way in which it appeared in the asymptotic form (1.20) in the case of the decay of the current at the point of perturbation. First, the density correlation gradients that arise are small: the collisions producing the correlation do not create the gradients directly; they arise later as a result of the rapid relaxation in momentum space. Secondly, the local changes in the density correlation again do not directly affect the relaxation in momentum space, the influence is exerted only through a small correction due to the diffusional correlation currents. It is precisely this that we had in mind when we spoke in the Introduction of a weak coupling between the diffusional mode and momentum space.

The $t^{-5/2}$ law is characteristic of the correlation produced by collisions with external scatterers. The binary collisions also produce a correlation, but its influence, exerted through the diffusional mode, turns out to be even weaker. We can [using a two-particle hydrodynamic algorithm (see Refs. 11 and 12) similar to the one-particle algorithm (1.17)] show that, in the vicinity of the thermodynamic equilibrium, there appears in the numerator of the ring operator (2.4) not q^2, but q^4. This is due to the additional symmetry characterizing the particle–particle collisions in comparison with the particle–external scatterer collisions. (Let us note that this symmetry does not play a role near a stationary nonequilibrium state, and we again obtain for the response the asymptotic form $t^{-5/2}$.)

Let us now consider whether it is generally impossible to get rid of the powers of q in the numerator of the ring operator, i.e., whether we can do something that will make the hydrodynamic algorithm take on only terms not containing q. For this to happen, it is necessary that the following two conditions be fulfilled. First, the fast relaxation in momentum space should not destroy the collision-induced correlation between the single-particle states and, secondly, the hydrodynamic modes coupled to the preserved correlation should have a direct influence on the single-particle relaxation in momentum space. In a gas with binary collisions, such modes can be the modes connected with viscosity and thermal conduction, which arise when the thermostat is "switched off." Let us, for simplicity, consider the case in which only the relaxation in terms of energy is "switched off," i. e., in which the scattering by the thermostat is purely elastic. A new integral of the motion—the total energy of the gas—then arises, and the single-particle relaxation operator J_p acquires a new zero:

$$\sum_{p} \varepsilon_p J_p \ldots = 0, \quad J_p \frac{\partial F_p}{\partial T} = 0. \tag{2.14}$$

The operator $J_p + J_{p_1}$ now has four zeros, of which only one, namely, $(\partial F_p/\partial T)(\partial F_{p_1}/\partial T)$, is important to us. To it corresponds the conservation law

$$\sum_{pp_1} \varepsilon_p \varepsilon_{p_1} (J_p + J_{p_1}) \ldots = 0. \tag{2.15}$$

At the same time, the source of the correlation—the two-particle operator Π_{pp_1}—violates this law:

$$A = \sum_{pp_1} \varepsilon_p \varepsilon_{p_1} \Pi_{pp_1} \{\ldots\} \neq 0. \tag{2.16}$$

Thus, the binary collisions now directly produce correlating "heat-conductive" diffusons:

$$\frac{\partial F_p}{\partial T} \frac{\partial F_{p_1}}{\partial T} \frac{A}{-i\omega + 2q^2 \varkappa}, \tag{2.17}$$

where $\varkappa$ is the thermal conductivity coefficient:

$$\varkappa_{\alpha\beta} = \frac{1}{N} \sum_{p} \varepsilon_p v_\alpha J_p^{-1} v_\beta \frac{\partial F_p}{\partial T}. \tag{2.18}$$

The property (2.16) allowed us to get rid of one q in the numerator of the ring operator. We can get rid of the second q factor because the function $(\partial F_p/\partial T)(\partial F_{p_1}/\partial T)$ does not make the binary-collision operator, under whose sign the hydrodynamic mode in the ring operator (2.4) stands, vanish:

$$\Pi_p \left\{ \frac{\partial F}{\partial T}, \frac{\partial F}{\partial T} \right\} \neq 0. \tag{2.19}$$

The property (2.19) indicates that the presence of the "heat-conductive" diffusons has a direct effect on the relaxation processes in momentum space. The absence of the additional q factors immediately leads to the $t^{-3/2}$ law for the asymptotic form of the spatially homogeneous response:

$$\sum_{q} e^{-2q^2 \varkappa t} \propto t^{-3/2}. \tag{2.20}$$

The presence in the system of viscous modes leads to the same effect, for

$$\sum_{pp_1} p_\alpha p_{1\beta} \Pi_{pp_1} \{\ldots\} \neq 0, \quad \Pi_p \left\{ \frac{\partial F}{\partial \mathbf{V}}, \frac{\partial F}{\partial \mathbf{V}} \right\} \neq 0 \tag{2.21}$$

(here V is the velocity of the gas). In this case the viscous-mode pole in the ring operator is separated out. Also possible is a correlation with the participation of a source of the type

$$\sum_{pp_1} p \varepsilon_{p_1} \Pi_{pp_1} \{\ldots\}.$$

In all these cases the picture of the phenomenon remains the same: there begins to be generated at each point in space upon the appearance there of a deviation from equilibrium a subsequently preserved correlation that then diffusionally spreads out over the entire system. The asymptotic form $t^{-3/2}$ is due to the fact that the correlation decreases in "density" at the point of its production as a result of the expansion of the volume in which it occurs: the radius of the diffusion sphere increases like $t^{1/2}$; consequently, the volume increases like $(t^{1/2})^3$, and the density of the conserved quantity decreases like $t^{-3/2}$. The correlation is produced at all points in the system, and spreads out over the system without being destroyed [cf. (1.15)]. In view of the preservation and continuous generation of the correlation in the system, a correlation "buildup" should occur in the system, and a distinctive correlation instability should develop. Since all the points in space are equivalent, the buildup should occur in the diffusional mode with $\mathbf{q} = 0$, i. e., the spatially homogeneous two-particle distribution function should begin to increase upon the appearance of a deviation from equilibrium. And, indeed, for $\Phi^q_{pp_1}|_{q=0} \equiv \varphi_{pp_1}$ we have [see (2.2)]:

$$(\partial_t + J_p + J_{p_1}) \varphi_{pp_1} = -\Pi_{pp_1} \{F, F\}, \tag{2.22}$$

from which we find, for example, for $\Sigma_p \varepsilon_p J_p \ldots = 0$ that

$$\partial_t \sum_{pp_1} \varepsilon_p \varepsilon_{p_1} \varphi_{pp_1} = -\sum_{pp_1} \varepsilon_p \varepsilon_{p_1} \Pi_{pp_1} \{F, F\}. \tag{2.23}$$

As long as the distribution function differs from Maxwellian, the source on the right-hand side of (2.23) "operates," resulting in the growth of the correlation of the single-particle energies.[3] In the following section we shall see that the correlation instability can make the consideration of the many-diffuson processes of all orders necessary, and that the allowance for these processes may yield for the response a temporal asymptotic form different from $t^{-3/2}$.

3. THE DIAGRAMS FOR THE RING OPERATOR. HIGHER APPROXIMATIONS

It is convenient to use for the study of the long-wave, low-frequency correlations, an example of which is the correlation described by the ring operator (2.4), the diagrammatic technique proposed for classical systems in Ref. 15 (where the interaction of electrons with long-wave, low-frequency phonons is investigated). When we go over to a classical system, the operators figuring in the quantum-statistical expressions commute, and therefore the double ordering—in time and "along the contour" (i. e., in the order in which the operators appear in the original expression)—which is characteristic of quantum diagrammatic techniques,[16,17] reduces to only ordering in time. Two propagator lines representing an observable then merge into one classical propagator. Figure 1 shows two such lines that have merged

FIG. 1.

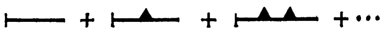

FIG. 2.

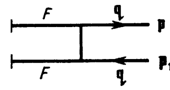

FIG. 6.

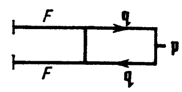

FIG. 7.

FIG. 8.

into one distribution-function line. We consider this line to be saturated by points—collisions with the thermostat (see Fig. 2). We shall represent an external field that introduces a momentum q in the following manner (see Fig. 3): to the point of interaction corresponds the derivative $\mathbf{q}\partial/\partial\mathbf{p}$ (which in the figure acts to the left); to the directed segment with the momentum q corresponds the propagator $B_{\mathbf{p}}^{-1}(\mathbf{q},\omega)$ of the spatially inhomogeneous excitation:

$$B_{\mathbf{p}}(\mathbf{q},\omega)=-i\omega+i\mathbf{q}\mathbf{v}+J_{\mathbf{p}}.$$

The diagram in Fig. 3 evidently solves the spatially-inhomogeneous response problem. The interaction of a particle with a scatterer can, in the case of a sufficiently "mild" potential, also be described classically. In this case the corresponding collision operator should be associated with a set of diagrams describing the motion of the particle in the field of the scatterer (see Fig. 4). Quantum collisions are characterized by a large momentum transfer at the vertex, which makes it impossible to describe a collision, using only the symbols of the classical diagrammatic technique, in which the momentum transfer is always small, and the derivative $\mathbf{q}\partial/\partial\mathbf{p}$ occurs. At the same time, if the Born approximation, i.e., only one diagram, is often sufficient in the quantum description, in the classical treatment the collision is represented in the form of an infinite sum. Let us note that, in view of the condition $\Delta t\ll\tau_{\mathbf{p}}$, the structure of the collision will not be of interest to us, and a collision will, consequently, always be represented as an instantaneous event. Thus, the series in Fig. 2 describes a series of instantaneous collisions with the thermostat, the time interval between collisions being of the order of the relaxation time $\tau_{\mathbf{p}}$.

A binary collision is shown in Fig. 5(a). The vertical line corresponds to an instantaneous exchange of momenta: the particle momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ go over into the momenta p and p′. Joining from the left the distribution functions $F_{\mathbf{p}_1}$ and $F_{\mathbf{p}_2}$, and summing over $\mathbf{p}_1$ and $\mathbf{p}_2$ (as always, the summation is assumed to be over all

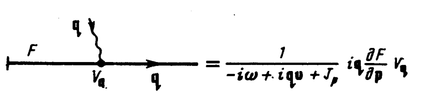

FIG. 3.

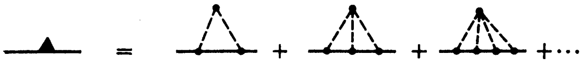

FIG. 4.

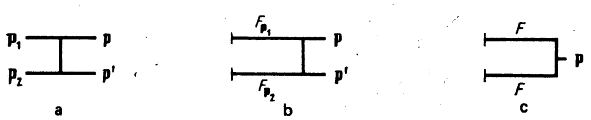

FIG. 5.

internal lines), we obtain a binary-collision operator without one summation, $\Pi_{\mathbf{p}\mathbf{p}_1}\{F,F\}$, depicted in Fig. 5(b). The operator $\Pi_{\mathbf{p}}\{F,F\}$, which has only one outcome, p, is depicted in Fig. 5(c). The function $\Phi_{\mathbf{p}\mathbf{p}_1}^{\mathbf{q}}$ is depicted in Fig. 6. A cross section drawn through both propagator lines to the right of the vertical—the binary collision—corresponds to $[-i\omega+i\mathbf{q}(\mathbf{v}-\mathbf{v}_1)+J_{\mathbf{p}}+J_{\mathbf{p}_1}]^{-1}$. Joining the symbol for $\Phi_{\mathbf{p}\mathbf{p}_1}^{\mathbf{q}}$ to the symbol for $\Pi_{\mathbf{p}}$, we obtain a diagrammatic representation of the ring operator (2.4) (see Fig. 7). We have omitted all the p indices, and retained only the momentum q. It is not difficult to see why this operator is a "ring:" the ring along which the spatial-homogeneity-destroying momentum q moves can be seen. Also evident is the correlative character of the generated diffusons. Notice that by itself the ring operator is spatially homogeneous: q does not get outside (a summation has been performed over it). The diagram in Fig. 7 can be interpreted as follows: at some moment of time a binary collision produces a correlation in the system, i.e., introduces the momenta q and −q. This correlation is taken into account in another collision (and spatial homogeneity is reestablished). If the period between the collisions is sufficiently long, then the response to the first collision assumes the form of a hydrodynamic mode—a diffuson. If this period is short, then the "ring process" is a small correction to the binary collision. Thus, the "ring process" can in a sense be regarded as a prolonged (in time) collision in which there is enough time for the "bare" particle lines to "get dressed" in other collisions and be transformed first into excitations described by the kinetic equations (i.e., into "relaxons") and then into hydrodynamic modes (diffusons). This is especially clearly illustrated by the impurity ring operator computed in the Born approximation (see Fig. 8). We see that the fast dynamical correlation—the collision—and the slow statistical correlation—the "ring process"—are both described by topologically identical diagrams. The only difference between them is that the ring process is "prolonged" in time, and we must insert collision operators in the propagator line under the impurity cap, which changes the corresponding energy denominator: instead of $-i\omega+i\mathbf{q}\cdot\mathbf{v}$, we have first $-i\omega+i\mathbf{q}\cdot\mathbf{v}+J_{\mathbf{p}}$, and then $-i\omega+q^2D$, appearing. [In order to avoid any misunderstanding, let us note that we cannot limit ourselves in the classical description to the Born approximation: the ring operator depicted in Fig. 8 will contain an additional smallness q^2/p^2 in comparison with the exact impurity ring operator (see Fig. 9) because of the small momentum transfer that occurs at the vertices. A ring operator for the two-body interaction can be constructed at the vertices in the Born approximation with allowance for the exchange, which will give rise to a large momentum transfer.[9]]

It is convenient to investigate the higher-order correlations with the aid of the diagrammatic technique.

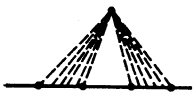

FIG. 9.

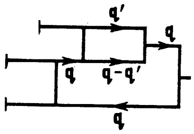

FIG. 10.

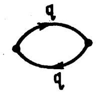

FIG. 11.

An example of these correlations for binary collisions is shown in Fig. 10. More complicated constructions can be imagined. Since only the diffuson poles are important for the asymptotic form, we can simplify the diagrammatic technique by contracting to a point-vertex anything connected with the spatially homogeneous relaxation. The spatially homogeneous state will then play for us the role of a "vacuum" from which spatially inhomogeneous excitations—diffusons—will be created under the action of a perturbation. The simplest ring operator will then assume the shape of a loop (see Fig. 11). The process depicted in Fig. 10 will assume in the abridged notation the form of a loop with an inset [Fig. 12(a)]. A similar process is depicted in Fig. 12(b). Notice that the multiple application of the hydrodynamic algorithm gives rise not only to third-order, but also to fourth-order, etc., vertices. At small q the dominant contribution will be made by third-order vertices. Here we shall not specify the type of vertices: the important thing is to determine whether they are proportional to q or not. Let us estimate the orders of magnitude of diagrams of various degrees of complexity. The simplest loop gives one summation (q^3) and one diffuson propagator. This yields $q^3/q^2=q$ ($q\sim\omega^{1/2}$) in the case of q-independent vertices and $q^2q^3/q^3\sim\omega\omega^{1/2}$ for vertices that are proportional to q, i.e., the already known $t^{-3/2}$ and $t^{-5/2}$ laws. Let us consider how the order of magnitude of a diagram will change upon the addition of a second diffuson (Fig. 12). One summation, i.e., q^3 in the numerator, and two diffuson propagators (corresponding to two new cuts between the vertices) are added, which gives q^4 in the denominator and a contribution from the two vertices to the numerator. The role of the "internal" vertices turns out to be decisive: if each of them contributes q, then the diagram in Fig. 12 is smaller than the simplest diagram; their ratio $q^2q^3/q^4=q\to 0$. Thus, the many-diffuson corrections turn out to be insignificant at large times, and the simplest ring operator yields the correct asymptotic response. On the other hand, if the two internal vertices do not depend on q, then each new complication in the diagram will give rise to factors $q^3/q^4=1/q\sim 1/\omega^{1/2}$, and the behavior of the response at large times will be determined precisely by the many-diffuson corrections describing the many-particle correlation. But in our theory the left and right third-order internal

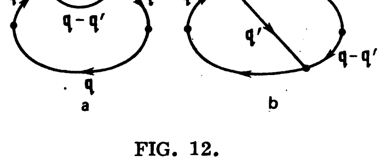

FIG. 12.

vertices in which the diffuson line inserted in the diagram terminates have different characters (see Figs. 10–12). The left vertex corresponds to the creation of a pair of diffusons (as a result of the correlation arising during the collisions between the single-particle energies and/or the momenta of the colliding particles). The fact that this vertex does not vanish as $q\to 0$ physically indicates that there arises at some point in the system a collision-conserving quantity that then diffusively spreads out over the entire system. The appearance in the system of a correlation between the single-particle quantities is not prohibited by the single-particle conservation laws obtaining in the system. Thus, the q-independent vertex from which two diffuson lines are drawn out to the right (into the future) is possible; an example of such vertices is the left third-order internal vertex in Fig. 12. As for the right internal vertex it corresponds to conversion of two diffusons into one (only one diffuson line goes out from it to the right). The nonvanishing, as $q\to 0$, of a vertex with one diffuson line going out to the right (into the future) would indicate the introduction into the system (with subsequent spreading) of a conserved single-particle quantity. But the single-particle conservation laws admit only of the redistribution in the system of the single-particle conserved quantities, and not their production. Thus, a vertex with a single diffuson line going out to the right (the right internal vertex in Fig. 12 is such a vertex) must go to zero as $q\to 0$, which can be shown by a direct calculation with the use of the hydrodynamic algorithm (1.17). This circumstance saves the low-frequency response problem from a power-law discontinuity as $\omega\to 0$. Furthermore, the expression for a "two-into-one" vertex (i.e., a vertex at which two lines from the "past" are converted into one line going into the future) may turn out to be proportional not to q, but to q^2, which makes the many-diffuson corrections generally insignificant for $q\to 0$. The correct asymptotic form of the response, $t^{-3/2}$, is then given (as in the case of the $t^{-5/2}$ law) by the simplest ring diagram. But in the presence of a scattering asymmetry in the system the expansion of the vertex begins with the first power in q. We obtain the same result when we take account of the interaction between the various symmetrically different hydrodynamic modes (e.g., the viscous and heat-conductive modes). In this case there may exist a series of diagrams that increase like powers of $\ln\omega$. To obtain the correct asymptotic form of the response in such a situation, we must sum the entire series of diagrams, i.e., take the many-particle correlations into account. It should be noted that at low frequencies the properties of such a series and, consequently, of the response described by it, should be fairly universal, a fact which draws the low-frequency response problem and the problem of phase transitions together. Since the kinetic coefficients normally do not exhibit any universal dispersion at low frequencies, we can assume that this series, when summed, will yield a slowly varying function of the frequency (i.e., a function of the type ω^α, $|\alpha|\ll 1$; not excluding, naturally, the possibility that $\alpha=0$).

At the same time, experiment indicates the existence

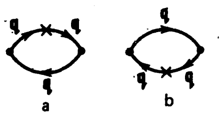

FIG. 13.

of an anomaly in the low-frequency fluctuation phenomena for a very broad class of systems—the so-called $1/f$ noise. Therefore, it would be interesting to analyze how the above-described correlations affect fluctuation phenomena. The pertinent diagrams for the long-lived corrections to the double-time two-particle correlation function, which gives the fluctuation spectrum, differ from the diagrams, considered by us above, for the single-particle function in that they have a second "exit" (the two particles are observed at different moments of time). The "exits" may be the "ends" of the diagrams shown in Figs. 11 and 12; in the case of the two-particle function, such diagrams will give a series describing the equilibrium fluctuations related to the response by the fluctuation-dissipation theorem. But more singular with respect to the frequency are the diagrams in which the left end corresponds to the introduction of some state of nonequilibrium into the system and the second exit is located on a diffuson line (see Fig. 13). The resulting additional cross section adds an extra diffuson pole. The section of the diagram between the exits then describes the evolution of the fluctuation, i.e., its decay; the observable frequency ω appears in the corresponding cross section. The section of the diagram to the left of the exits describes the appearance and evolution of the correlations from the moment of the appearance of the nonequilibrium single-particle distribution (the left end of the diagram) to the commencement of the observation of the fluctuation. The cross section of this part of the diagram contains not $-i\omega$, but the Laplace parameter s (cf. Ref. 11), which can be made to go to zero or tend to the frequency of the external field, depending on the formulation of the problem. In particular, if s is small (the state of nonequilibrium was switched on long ago), the left sections in the diagrams in Fig. 13 will give in the denominator an extraneous—in comparison with the corresponding diagram in the response (Fig. 11)—q^2 factor.

Thus, the first ring correction to the nonequilibrium-fluctuation spectrum contains an extra factor of $1/\omega$ in comparison with the first ring correction to the response, i.e., it gives an anomaly of the $1/\omega^{1/2}$ type in the excess-noise spectrum.

The property whereby an extra section arises in the diagrams for the two-particle distribution function is retained in the higher approximations, in which it may also lead to the appearance of an extra factor of $1/\omega$ in comparison with the corresponding diagram for the response, although the situation here is more complex. The point is that the second-exit point can be inserted in any of the diffuson lines; therefore, each response diagram generates several diagrams for the nonequilibrium two-particle function, some of which will, for $s \to 0$, contain in the denominator such a number of q^2 factors that the corresponding integral will diverge at the lower limit. This fact is related to the possibility, noted in the preceding section, of the correlations' growing in the presence of a state of nonequilibrium.

For all the complexity of the resulting picture, we can clearly see that the singularity of the series describing the nonequilibrium fluctuations is stronger than that of the series for the response, which prompts us to postulate that the experimentally observed $1/f$ excess noise may have a "ring character," i.e., be due to the above-described difficulty in damping out the perturbations in the system, the slowness of the system in returning to equilibrium, the fact that the correlations in the system have a tendency to build up. (Notice that the integral of $1/\omega$ over the spectrum diverges at the lower limit, which agrees with the tendency, described by (2.23), of the equal-time two-particle distribution function's to increase in time in the nonequilibrium state.)

The kinetics of interacting diffusons, which describes the low-frequency response and low-frequency fluctuations, is somewhat reminiscent of the kinetics of matter at the critical point, which, as is well known, is characterized by power dependences. Thus, if we assume that the $1/f$ noise is indeed due to the above-considered long-lived correlations, then a power dependence of the type $1/\omega^{\beta}$ for the excess noise, with β acting as some dynamical critical exponent [its experimental value ranges from 0.8 to 1.2 (Ref. 18)], appears to be natural.

The authors are grateful to V. L. Gurevich for a discussion and valuable comments.

[1)]Instead of the equilibrium state, we could have considered the stationary nonequilibrium state (see Refs. 11 and 12). The term with the external force creating the state of nonequilibrium would then be included in $I_{\mathbf{p}}$. In this case $N\partial_N F_{\mathbf{p}} \neq F_{\mathbf{p}}$, and the distribution function $F_{\mathbf{p}}$ would make only the sum of the operators in (1.1), and not each of them, vanish.

[2)]Naturally, we are talking about the coincidence of the points $\mathbf{r}$ and $\mathbf{r}_1$ only to within the dimension of the region over which the large-scale averaging is performed, a dimension which is large compared to the range, a, of the forces acting between the particles. Equation (2.2) is applicable only in the case of kinetic scales significantly larger than a. Since the long-lived correlation of interest to us is, as will be seen below, determined by small q, the upper limit of the integration over q has no effect on the result at all.

[3)]This fact, which has thus far apparently remained unnoticed (although it follows directly from the theory of fluctuations in the nonequilibrium state[11,12]), may have a direct bearing on the experimentally observed $1/f$ type of excess noise, which is discussed in detail at the end of the following section.

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