Field configurations with topological number and the mechanism of charge screening in two-dimensional massless electrodynamics

G. S. Danilov, I. T. Dyatlov, and V. Yu. Petrov

V. P. Konstantinov Institute of Nuclear Physics, Leningrad, USSR Academy of Sciences
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The confinement mechanism in the two-dimensional quantum electrodynamics of massless fermions is investigated. The confinement occurs because the quarks make a transition to nonlocalized states and therefore become unobservable. These states consist of a set of chiral and charged vacuums. The delocalization of the quarks has the consequence that the electromagnetic field appears to acquire the ability to produce nonvanishing charges. This effect arises for fields that change the two-dimensional topological number. The participation of vacuum states in such a "topological effect" means that the charge screening processes violate relativistic causality. If the theory is to retain a consistent interpretation, it is necessary that the confinement be such that the charge and chirality of the quarks become unobservable. The physical picture of hadron production and its connection with the topological effect are also discussed.

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1. INTRODUCTION

In this paper, we shall investigate the confinement mechanism in two-dimensional quantum electrodynamics (QED_2) of massless fermions (which we shall call quarks).[1] It will be shown that electromagnetic fields which change the two-dimensional topological number (topological fields) give rise here to physical processes which lead to screening of the charge and chirality of the massless quarks. A feature of processes involving topological fields is the nonconservation of the number of quarks and their right, R, and left, L, charges (i.e., chiralities) in the causal space-time regions determined by the action of the electromagnetic field. Under such conditions, the quarks cease to be physically interpretable objects. If the theory is to have a physical interpretation, it is necessary to have confinement, which is ensured in the model by the existence of a system of degenerate vacuum states: The charge and chirality which disappear from the causal regions go over to these states. The present paper is devoted to the study and physical interpretation of these phenomena, and also the associated space-time picture of the screening and production of hadrons in QED_2.

The existence of field configurations that change the topological numbers of gauge fields has been noted in a number of papers,[2-6] in which phenomena associated with these field configurations were studied. Some interesting properties of systems admitting fields with topological number were found, but an understanding of the physics of the phenomenon was not achieved. In the simplest confinement model—massless QED_2—the processes that occur in the case of topological fields can be studied in detail.

The charge screening in QED_2 has been known since the work of Schwinger.[1] It was shown later[7] that the excitation spectrum of the model consists of neutral massive bosons. Charged and chiral systems have already been studied[8]: Stationary states with quantum numbers exist in QED_2 only in the form of vacuum states (i.e., states with vanishing momentum). This means that with the passage of time a local charge must be neutralized, becoming compensated by the screening charge. The compensating charge can arise only as a result of the action of the electromagnetic field of the original particle. But one would not expect the electromagnetic field to change the right and left charges and chirality, since it should produce only pairs $q_R\bar{q}_R$ or $q_L\bar{q}_L$ of right- or left-handed quarks and antiquarks.

This paradox can be explained in the formulation of the problem in which the electromagnetic field is regarded as an external field and the back reaction of the quarks on the field is ignored. It is then found that when quark–antiquark pairs are produced by topological fields in the mathematical vacuum of the quarks the number of particles with very small momenta $p\sim V^{-1}$ (V is the volume of the system) in the produced state is necessarily nonzero. Since quarks with such momenta make a small contribution $\sim V^{-1}$ to the matrix elements of all local quantities (energy–momentum, charge, and chirality densities), there is in this problem in the limit $V\to\infty$ apparent nonconservation of the right and left charges and the chiralities, and this can explain the appearance of the screening charges.

This phenomenon is the physical explanation of the nonconservation of the right and left charges in QED_2, which follows already from the existence of the Adler anomaly.[9] In the Schwinger model, the axial current $j_\mu^5(x)$ of the massless quarks satisfies the equation[10]

$$\frac{\partial j_\mu^5(x)}{\partial x_\mu}=\frac{g^2}{2\pi}\varepsilon_{\mu\nu}F_{\mu\nu}=-m^2E(x), \tag{1}$$

where $E(x)=F_{10}(x)$ is the intensity of the electromagnetic field, g is the dimensional charge of the model, and $m^2=g^2/\pi$. If, and only if, a topological field is present, Eq. (1) leads to nonconservation of the chirality[3]

$$K=\frac{1}{g}\int j_0^5(x)\,dx,$$

since it follows from (1) that

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$$\frac{g}{2\pi m^2}[K(t=+\infty)-K(t=-\infty)]=-\frac{1}{2\pi}\int_{-\infty}^{+\infty}E(x,t)\,dx\,dt=\Delta Q_T, \qquad (2)$$

where ΔQ_T is the change in the two-dimensional topological charge.

In two-dimensional space, the chiral current j_μ^5 can, because of the properties of the two-dimensional matrices γ_μ $(\gamma_5\gamma_\mu=\varepsilon_{\mu\nu}\gamma_\nu)$, be expressed in terms of the vector current j_μ $(\rho=g^{-1}j_0, j=g^{-1}j_1)$:

$$j_\mu^5(x)=\varepsilon_{\mu\nu}j^\nu(x). \qquad (3)$$

The nonconservation of the ciralty (2) is nonconservation of the right and left charges and is associated with nonconservation of the particle number.

Of course, it is well known[11] that the gauge-invariant definition of the electromagnetic current and the axial current requires further discussion, since products of quark operators taken at one point are not defined (for more detail, see Sec. 2). As a result, the electromagnetic current is not directly related to the particle number in an arbitrary gauge. However, it was shown in Ref. 8 that in the Coulomb gauge the chirality

$$K=\frac{1}{g}\int j_0^5\,dx=\int j(x)\,dx$$

is equal to the difference between the number of right- and left-handed quarks. Then nonconservation of chirality is indeed nonconservation of the number of quarks (minus the number of antiquarks). The absence of nonphysical degrees of freedom in the Coulomb gauge means that this gauge is also convenient for the physical interpretation of the phenomena that occur in the model and for the investigation of the physical states and their variation in time. Therefore, in this paper we shall use the Coulomb gauge.

These results obtain in the problem with an external topological field offer hope that topological fields will arise in the self-consistent problem of interacting massless quarks when there is screening of the quark charges. But the nonconservation of the quantum numbers in processes of screening of local charges is no longer explained by the production in topological fields of particles with small momentum $p\sim V^{-1}$. The role of the topological fields is now to produce quarks directly in one of the vacuum states with quantum numbers. Again there is an apparent nonconservation of the quantum numbers, since the right and left charges are not localized in the vacuum states. Moreover, the interaction of the local and nonlocal states of the system due to the topological fields leads to the existence of noncausal effects like the transfer of charge from one region to another dynamically independent region.

Summarizing what we have said above, we can assert that the phenomenon of effective nonconservation ($V\to\infty$) of the number of quarks in topological fields by itself means that a theory will be physically consistent only if the charge and other quark characteristics are in principle unobservable, since their causal conservation cannot be ensured. Only the complete compensation of the topological fields that arise with the charge prevent the processes in which this charge is not conserved. This means that the phenomenon admits a consistent interpretation only in a theory with confinement.

The development of the system with time and the physical interpretation of the resulting phenomena are most readily studied in the Hamiltonian formulation. This is set forth in Sec. 2, in which we present a convenient computational formalism in which one can readily find the wave functions of the states at all times and the expectation values of the quantities in the model in these states. In the same section, we investigate the spectrum of physical states of the Schwinger model. Important here is the huge degeneracy, which already exists in the system of free massless fermions.[12] In two-dimensional space, this degeneracy has the consequence that the states of the free system can be constructed in two representations: quark and boson. Of course, only the interaction, which lifts the degeneracy, dictates the choice of the boson representation, which signifies charge confinement. But the existence of the two representations of the free system indicates where the charged and chiral states disappear to in a theory with confinement. For massless quarks, all such states have vanishing momentum (or rather, $p\sim V^{-1}$ for finite volume V of the system), i.e., in the limit $V\to\infty$ they form new vacuum states. The total number of states is not changed when the interaction is switched on, and the neutral hadrons are augmented to the complete system by vacuum states with quantum numbers. It is these states that are important for understanding the processes of charge screening and the vanishing of the chirality and other quark characteristics.

Section 3 is devoted to studying the topological effect which arises in an external topological field. We have already discussed the results of this section. In Sec. 4, we consider a QED_2 system in a finite spatial volume V. When the problem is formulated thus, one can verify the fulfillment of the conservation laws for the particle number, charge, and chirality. If a topological effect is present, the conservation laws are satisfied only in the volume as a whole. The conservation laws are noncausal in nature, since the charge and chirality extend beyond the domains of influence of the field of test charges.

The development in time of the particle screening phenomena following $q_R\bar{q}_L$ pair production[10] is investigated in Sec. 5. The topological effect of the field produced by the separating particles effectively transforms the system already at times $t\sim 1/m$ into a neutral system with vanishing right and left charges. The field produces screening particles and limits the growth of the topological charge (2). The time at which the topological charge ceases to grow may be called the confinement time. We investigate the effects that arise from the interaction of the electromagnetic field of test charges with particles of the physical vacuum.

At larger times, $t>1/m$, there is a rapid growth (proportional to t^2) in the number of quarks with momenta $p\gg m$. At times $t\sim p/m^2$, their distribution goes over into the hadron distribution of the parton model ($1/p$). The leading pair of quarks loses energy, and after a time $t\sim p_{\text{in}}/m^2$ participates on an equal footing in the formation of the hadrons. The produced hadrons become spatially separated from the quarks of the $q_R\bar{q}_L$ pair.

The time $t\sim p/m^2$ of establishment of the hadron distribution can be called the hadron production time. The picture of the transition in time of the momentum distribution of the quarks into the momentum distribution of the hadrons is the same as in Refs. 10 and 13, namely, the fastest hadrons are the ones which are produced last.

2. QED$_2$ IN THE HAMILTONIAN FORMULATION

In this section, we consider and solve QED$_2$ in the Hamiltonian formulation, which will give us a simple technique for calculating all characteristics of the model. It will be convenient to consider a QED$_2$ system in a finite volume $V=2L(-L\leqslant x\leqslant L)$. In the Coulomb gauge, the Hamiltonian of such a system is[8]

$$H=H_0+H_C;$$

$$H_0=\frac{1}{2i}\int_{-L}^{L}dx\left\{\psi_R^+\frac{\partial\psi_R}{\partial x}-\frac{\partial\psi_R^+}{\partial x}\psi_R-\psi_L^+\frac{\partial\psi_L}{\partial x}-\frac{\partial\psi_L^+}{\partial x}\psi_L\right\},$$

$$H_C=-\frac{g^2}{4}\int_{-L}^{L}\rho(x)|x-y|\rho(y)\,dx\,dy+\frac{g^2}{4}LQ^2-\frac{g^2}{4}\left(\int_{-L}^{L}x\rho(x)\,dx\right)^2. \tag{4}$$

Here, $\psi_R(x)$ and $\psi_L(x)$ are the operators of the fields of the right- and left-handed particles:

$$\psi_i(x)=\psi_R(x)u_i^{(R)}+\psi_L(x)u_i^{(L)},\quad (1\pm\gamma^5)u^{(R,L)}=0. \tag{5}$$

As is explained in Ref. 8, the Hamiltonian (4) describes a system with total charge

$$Q=\int_{-L}^{L}\rho(x)\,dx.$$

The actual form of the volume-dependent terms in (4) is determined by the choice of the external compensating charge (see Ref. 8 and Sec. 4 of the present paper) and the choice of the periodic boundary conditions for the Schrödinger operators $\psi_{R,L}^{\pm}(x)$.

The Hamiltonian (4) contains the charge density operator $\rho(x)$. Like the current density operator $j(x)$, it is not properly defined, since the product of two local operators, $\psi_{R,L}^{+}(x)$ and $\psi_{R,L}(x)$, taken at one point is singular.[11] As was noted in Ref. 8, this indeterminacy can be correctly eliminated if one makes the calculations, not with $\psi_{R,L}^{+}(x)$ and $\psi_{R,L}(x)$, but with the operators $a_{R,L}^{\pm}(x)$ and $b_{R,L}^{\pm}(x)$ of creation of particles and antiparticles[1)]:

$$\psi_{R,L}(x)=a_{R,L}(x)+b_{R,L}^{+}(x)=\frac{1}{\sqrt{V}}\sum_{p_n>0}a_{R,L}(p_n)\exp(\pm ip_nx)+b_{RL}^{+}(p_n)\exp(\mp ip_nx). \tag{6}$$

In reality, the operators $a_{R,L}^{\pm}$ and $b_{R,L}^{\pm}$ are nonlocal, which is reflected in the nonlocal nature of the commutation relations between them:

$$\{a_R^+(x),a_R(x')\}=\{b_R^+(x),b_R(x')\}=\frac{1}{2\pi i}\frac{1}{x-x'-i0},$$

$$\{a_L^+(x),a_L(x')\}=\{b_L^+(x),b_L(x')\}=\frac{1}{2\pi i}\frac{1}{x'-x-i0}. \tag{7}$$

It is the nonlocal nature of the commutation relations (7) that makes it possible to eliminate the ambiguity in the operators $\rho(x)$ and $j(x)$. One can show[8] that by the operators $\rho(x)$ and $j(x)$ it is necessary to understand

$$\rho(x)=\rho_R(x)+\rho_L(x),\quad j(x)=\rho_R(x)-\rho_L(x),$$

$$\rho_{R,L}(x)=a_{R,L}^{+}(x)a_{R,L}(x)-b_{R,L}^{+}(x)b_{R,L}(x)+a_{R,L}^{+}(x)b_{R,L}^{+}(x)+b_{R,L}(x)a_{R,L}(x)\pm\frac{g}{2\pi}A_1(x). \tag{8}$$

[Here, $\rho_R(x)$ and $\rho_L(x)$ are the operators of the density of the right and left charges.] These operators are gauge invariant and correctly reproduce the well-known Schwinger anomaly[11]:

$$[\rho(x),j(x')]=\frac{1}{\pi i}\delta'(x-x'). \tag{9}$$

To derive Eq. (9), it is sufficient to use Eqs. (7) and (8). In such an approach, the usually employed separation procedure[7,10] is superfluous.

The operator $\rho(x)$ satisfies the continuity equation

$$\partial\rho/\partial t+\partial j/\partial x=0. \tag{10}$$

To obtain (10), it is sufficient to calculate the commutator of $\rho(x)$ with the Hamiltonian. But if we find the commutator of the current density $j(x)$ with the Hamiltonian and use Eq. (9), we arrive at Eq. (1), i.e., the Adler anomaly in QED$_2$.

In the Coulomb gauge $A_1=0$, which we use throughout in what follows, Eqs. (8) reduce to the usual $j_\mu=\bar{\psi}\gamma_\mu\psi$, where $\psi^{\pm}$ is expressed in terms of the creation and annihilation operators. Therefore, in this gauge the total right and left charges are directly related to the number of right- and left-handed quarks, N_R and N_L, minus the number of antiquarks, $\bar{N}_R$ and $\bar{N}_L$:

$$Q_R=\int_{-L}^{L}\rho_R(x)\,dx=N_R-\bar{N}_R,$$

$$Q_L=\int_{-L}^{L}\rho_L(x)\,dx=N_L-\bar{N}_L. \tag{11}$$

We have already mentioned this circumstance in the Introduction.

A complete solution of the QED$_2$ model is achieved by its bosonization (our approach is close to that of Ref. 14). We define the operators ($p>0$)

$$c_R^{\pm}(p)=\left(\frac{2\pi}{p}\right)^{1/2}\int_{-L}^{L}e^{\pm ipx}\rho_R(x)\,dx,$$

$$c_L^{\pm}(p)=\left(\frac{2\pi}{p}\right)^{1/2}\int_{-L}^{L}e^{\mp ipx}\rho_L(x)\,dx. \tag{12}$$

Then it follows from the expression (9) for the Schwinger anomaly that these operators commute as ordinary boson creation and annihilation operators:

$$[c_R^+(p_n),\,c_R(p_n')]=[c_L^+(p_n),\,c_L(p_n')]=V\delta_{p_np_n'}. \tag{13}$$

It is readily verified that to accuracy $1/V$ the Hamiltonian H_0 in Eq. (4) can be expressed in terms of the operators $c_R^{\pm}$ and $c_L^{\pm}$:

$$H_0=\frac{1}{V}\sum_{p_n>0}p_n[c_R^+(p_n)c_R(p_n)+c_L^+(p_n)c_L(p_n)]. \tag{14}$$

This is proved in Appendix 1. The complete equivalence of the expressions (4) and (14) for H_0 was not noted in Ref. 14, and therefore the model was not completely solved there. The Coulomb energy H_C can also be expressed in terms of the operators $c^{\pm}_{R,L}$. Restricting ourselves to the case $Q=0$, we have

$$H_c=+\frac{g^2}{4\pi}\frac{1}{V}\sum_{p>0}\frac{(c_R^+(p)+c_L(p))(c_R(p)+c_L^+(p))}{p^2}. \quad (15)$$

Equations (12)–(15) show that the QED$_2$ model is equivalent in the limit $V\to\infty$ to the model of neutral massless bosons interacting in accordance with (15). The Hamiltonian $H=H_0+H_C$ can be diagonalized by going over to the operators $C(p)$ and $C^+(p)$:

$$H=\frac{1}{V}\sum_p\omega_pC^+(p)C(p),\quad \omega_p=(p^2+m^2)^{1/2}, \quad (16)$$

where

$$C_R^{\pm}(p)=\frac{1}{2}\left(\frac{p}{\omega_p}\right)^{1/2}\left[C^{\pm}(p)\left(\frac{\omega_p+p}{p}\right)-C^{\mp}(-p)\left(\frac{\omega_p-p}{p}\right)\right],$$

$$c_L^{\pm}(p)=\frac{1}{2}\left(\frac{p}{\omega_p}\right)^{1/2}\left[C^{\pm}(-p)\left(\frac{\omega_p+p}{p}\right)-C^{\mp}(p)\frac{\omega_p-p}{p}\right], \quad (17)$$

$$[C^+(p),C(p')]=V\delta_{pp'}.$$

Thus, we obtain the well-known result of Ref. 7: The physical states of the Schwinger model are formed by free massive bosons.

It is obvious from the form of the Hamiltonians (14) and (15) that in the vacuum state of the model there is a condensate of correlated $c_R^+(p)c_L^+(p)$ pairs of massless excitations. The wave function of the vacuum state is readily found to be

$$\Omega_0=A\exp\left\{-\frac{1}{V}\sum_{p>0}\varphi(p)c_R^+(p)c_L^+(p)\right\}|0\rangle,$$

$$\varphi(p)=(\omega_p-p)/(\omega_p+p),\quad C(p)\Omega_0=0 \quad (18)$$

(A is a normalization constant). In such a vacuum condensate, the boson naturally acquires mass.

The Hamiltonian (16) of the free massive bosons describes the development of the system in time. It is extremely simple in terms of the operators $C(p)$ and $C^+(p)$. The quark structure of the wave functions of the physical states can be determined by Eqs. (8), (12), and (17) if the quark structure of the vacuum is known. In particular, it was calculated in Ref. 8 in the infinite-momentum frame.

The bosonization of the Schwinger model (14)–(16) means that it does not contain charged and chiral excitations, i.e., states with energy and momentum different from the vacuum values. It is convenient to have a representation of the fermion operators $\psi_{R,L}(x)$ and $\psi^+_{R,L}(x)$ in which the boson nature of all the excitations with momentum $p\neq 0$ (as $V\to\infty$) is explicitly taken into account. For this, we must separate from the operators of the fermion fields the part relating to the states with very small momentum ($p_n=n\pi/L, n=0,\pm1,\ldots,\pm N_0$). In the limit $V\to\infty$, this part, σ_R and σ_L, does not depend on the spatial coordinates, since the momentum operator does not contain states with $p\sim V^{-1}$. For $Q=0$, these operators also commute with the Hamiltonians (14) and (15), which are constants of the motion.

We show that the required representation for $\psi_{R,L}(x)$ must be written in the form[2)]

$$\psi_{R,L}(x)=\exp\left\{-\frac{1}{V}\sum_{p_n>0}\exp(-ip_nx)\left(\frac{2\pi}{p_n}\right)^{1/2}c^+_{R,L}(p_n)\right\}$$
$$\times\frac{\sigma_{R,L}}{V^{1/2}}\exp\left\{\frac{1}{V}\sum_{p_n>0}\exp(ip_nx)\left(\frac{2\pi}{p_n}\right)^{1/2}c_{R,L}(p_n)\right\}. \quad (19)$$

Equation (19) is written down in such a way that the operator $\sigma_{R,L}$ which it defines commutes with the operators $c^+_{R,L}(p)$ and $c_{R,L}(p)$:

$$[\sigma_R,c^{\pm}_{R,L}(p)]=[\sigma_L,c^{\pm}_{R,L}(p)]=0. \quad (20)$$

This can be verified directly by using (12) to establish the commutator of $c_{R,L}(p)$ and $\psi_{R,L}(x)$. By virtue of (20), the operators $\sigma_{R,L}$ and $\sigma^+_{R,L}$ also commute with the Hamiltonian and with the momentum operator, since in accordance with Eqs. (14)–(16) these last can (for $Q=0$) be expressed solely in terms of $c_{R,L}(p)$ and $c^+_{R,L}(p)$.

The operators $\sigma_{R,L}$ and $\sigma^+_{R,L}$ must be normalized in accordance with

$$\sigma_R^+\sigma_R=\sigma_L^+\sigma_L=1,\quad \sigma_R^{\pm}\sigma_L^{\pm}+\sigma_L^{\pm}\sigma_R^{\pm}=0, \quad (21)$$

if the sums in (19) are taken over all values of the momentum $p_n=n\pi/L$ except $p_n=0$. If the sum begins with some number N_0, then on the right-hand side of (21) we must write N_0. However, these changes are unimportant in the calculation of all matrix elements with $\psi_{R,L}(x)$.

Indeed, it is readily verified that under the condition (21) the anticommutator of $\psi^+_{R,L}(x)$ and $\psi_{R,L}(x)$, defined in accordance with (19), is

$$\{\psi_R^+(x),\psi_R(x')\}=\{\psi_L^+(x),\psi_L(x')\}=\delta(x-x'), \quad (22)$$

as it must be. To calculate the commutator of the exponential factors, we use here the formula

$$\frac{2\pi}{V}\sum_{p_n>0}\frac{\exp(ip_nx)}{p_n}=-\ln(1-\exp i\pi x/L)=\ln\left[\frac{V}{2\pi i(x-i0)}\right]. \quad (23)$$

The anticommutators $\{\psi_R(x),\psi_R(x')\}$, $\{\psi_L(x),\psi_L(x')\}$, etc., vanish as $V\to\infty(\sim 1/V^3)$.

Further, the currents $\rho_{R,L}(x)$ can, when (19) is substituted in (8) and $a_{R,L}$ and $b_{R,L}$ are calculated in terms of $\psi_{R,L}$ by means of (6), be reduced to the form

$$\rho_R(x)=\frac{1}{V}\sum_{p_n>0}\left(\frac{p_n}{2\pi}\right)^{1/2}[c_R(p_n)\exp(ip_nx)+c_R^+(p_n)\exp(-ip_nx)],$$

$$\rho_L(x)=\frac{1}{V}\sum_{p_n>0}\left(\frac{p_n}{2\pi}\right)^{1/2}[c_L(p_n)\exp(-ip_nx)+c_L^+(p_n)\exp(ip_nx)], \quad (24)$$

which is obviously identical to (12). Finally, substituting (19) in (4), we obtain the expression (14) for H_0.

The representation (19) is very convenient for calculating Green's functions and the expectation values of local quantities and for investigating the development of processes in time. Replacing in (19) the operators $c(p)$ and $c^+(p)$ of the massless bosons in accordance with Eqs. (17), we can solve the same problems in the interacting theory.

The possibility of formulating the model by means of the representation (19) indicates the existence (as $V\to\infty$) of infinite degeneracy of the vacuum with respect to the charge and chirality. Besides the vacuum state Ω_0 (18), one can have states formed by multiplying Ω_0 by an arbitrary number of operators $\sigma_{R,L}$ or $\sigma^+_{R,L}$ (Ref. 7):

$$\Omega_{\pm n,\,\pm m}=(\sigma_R^{\pm})^n(\sigma_L^{\pm})^m\Omega_0. \tag{25}$$

Since the operator σ does not depend on the coordinates, and the Hamiltonian (14)–(16) does not depend on σ, the states Ω_{nm} are indeed vacuum states with vanishing energy and momentum.[3)] In the absence of interaction and for finite V, the states Ω_{nm} would describe the possibility of having quarks with very small momenta $p\sim V^{-1}$. Because of the interaction H_C, their quark structure and physical properties are nontrivial. It is to these states that the charge and chirality go in a model with confinement of the massless charge. Their properties can be investigated by the method set forth in Ref. 8.

The computational formalism based on Eqs. (19) and (17) enables us to find how the charge and chirality are distributed in the vacuums (25). To establish this, it is sufficient to calculate the matrix elements which describe the overlapping of the charge or chirality in the vacuums Ω_{nm} and in locally organized packets $\psi_R(x)\Omega_0$ or $\psi_R(x)\psi_L^+(y)\Omega_0$. Then we obtain

$$\langle\Omega_0|\sigma_R{}^+\psi_R(x)|\Omega_0\rangle=C_1\exp\{-mV/8\pi\}, \tag{26}$$

$$\langle\Omega_0|\sigma_R{}^+\sigma_L\psi_R(x)\psi_L{}^+(y)|\Omega_0\rangle=C_2\exp\left\{-m^2\int\frac{\sin^2 p(x-y)}{p^2\omega_p}dp\right\} \tag{27}$$

($V\to\infty$). The constant coefficients in these expressions do not depend on the volume V. In accordance with (26), the charge in the charged vacuums cannot be distributed over the whole of space of ($\rho\sim V^{-1}$) but goes to the edge of the volume and is screened (see Sec. 4). The chirality corresponding to total charge $Q=0$ is delocalized. Equation (27) also indicates the existence of a chiral condensate, since the factor $1/V$ is absent.

Thus, the huge degeneracy of the states of the free field of the massless quarks in the limit $V\to\infty$ makes possible the existence of the representations as different as (4) and (14). But it is only the interaction that determines the final structure of the theory, i.e., the existence or absence of charged particles, fermion excitations, and confinement or nonconfinement of the charge. The interaction lifts the degeneracy and selects the combination of states that is described by the physical wave function. The existence of the degenerate vacuums (25) in the Schwinger model is a direct consequence of the confinement and augments the hadronic states to a complete system.

3. TOPOLOGICAL EFFECT

The physical explanation of the nonconservation of the particle number in topological fields, which is a direct consequence of Eq. (1), must be sought in the properties of the process of $q_R\bar{q}_R$ and $q_L\bar{q}_L$ pair production by the electromagnetic field, since it follows from this equation that this effect also exists in the case of an external field $E(x,t)$. Therefore, we consider a system of free fermions to which an external electromagnetic field is applied at the time $t=0$ and we calculate the current $j(x)$ and the charge density $\rho(x)$ for $t>0$. They can be determined on the basis of Eq. (1) and the continuity equation (10):

$$\partial j/\partial t+\partial\rho/\partial x=-m^2g^{-1}E,\quad \partial\rho/\partial t+\partial j/\partial x=0. \tag{28}$$

If the initial state of the system was the vacuum, Eq. (28) must be solved with null initial conditions. In this case, we obtain from (28) for the densities $\rho_R(x,t)$ and $\rho_L(x,t)$ of the right and left charges

$$\rho_{R,L}(x,t)=\mp\frac{g}{2\pi}\int_0^t dt'\int dx'\,\delta[(t-t')\mp(x-x')]E(x',t'). \tag{29}$$

The total charges

$$Q_R(t)=-Q_L(t)=-\frac{g}{2\pi}\int_0^t E(x',t')\,dt'\,dx'=Q_T(t) \tag{30}$$

are nonzero if $E(x,t)$ is a topological field [$Q_T(t)$ is the topological charge of the field at the time t], which signifies a change in the chirality, which would appear to be impossible when an electromagnetic field acts.

Let us investigate what state develops under the influence of the electromagnetic field from the mathematical vacuum; to do this, we find its wave function $\Psi(t)$. This is most readily found in the boson representation, in which the Hamiltonian of the free fermions in the external field has the form (Coulomb gauge)

$$\begin{aligned}H&=H_0-g\int\rho(x)A_0(x,t)\,dx=H_0-\frac{g}{V}\sum_{p>0}A_0(p,t)\\&\times[c_R{}^+(p)+c_L(p)]-\frac{g}{V}\sum_{p>0}A_0(-p,t)[c_R(p)+c_L{}^+(p)].\end{aligned} \tag{31}$$

We seek $\Psi(t)$ in the form

$$\Psi(t)=F(t)\exp\left\{-\frac{1}{V}\sum_{p>0}\left(\frac{2\pi}{p}\right)^{1/2}e^{-ipt}\sum_{\lambda=R,L}\Phi_\lambda(p,t)c_\lambda{}^+(p)\right\}|0\rangle. \tag{32}$$

Substituting (32) in the Schrödinger equation with the Hamiltonian (31), we find for the functions F and Φ the expressions

$$\begin{aligned}\Phi_{R,L}(p,t)&=\pm\frac{g}{2\pi}\int_0^t\exp[ip(t'\mp x')]E(t',x')\,dt'\,dx',\\F(t)&=\exp\left\{-\frac{2\pi}{V}\sum_{p>0}\sum_{\lambda=R,L}\frac{\Phi_\lambda(p,t)}{2p}\right\}.\end{aligned} \tag{33}$$

The initial conditions for the functions F and Φ are chosen to make the function $\Psi(t=0)$ describe the mathematical vacuum $|0\rangle$.

The function F in (32) ensures that the normalization of $\Psi(t)$ is conserved in time. The summation in (32) is over all p except $p=0$ (see Sec. 2). Separating from (33) the volume-dependent part, we obtain

$$F(t)=V^{-Q_T{}^2(t)}\tilde{F}(t) \tag{34}$$

($\tilde{F}$ does not depend on the volume), where $Q_T(t)$ is the topological charge defined in (30). The calculation of the expectation values of $\rho_{R,L}(x,t)$ in the state $\Psi(t)$ leads to Eq. (29).

From the expression (32) for $\Psi(t)$, we can separate the part responsible for the nonconservation of the number of right- and left-handed particles:

$$\Psi(t)=\Psi_0(t)\varkappa_K(t)|0\rangle; \tag{35}$$

$\Psi_0(t)$ does not change $Q_{R,L}$, while $\varkappa_K(t)$ changes the charges by $Q_T(t)$. This procedure is not unique, but $\varkappa_K(t)$ always includes the singular parts of the sums with small momenta p in (32). If the separated part has the form

$$\varkappa_K(t)=V^{-|Q_T(t)|}\exp\left\{-\frac{1}{V}\sum_{p>0}\left(\frac{2\pi}{p}\right)^{1/2}\exp(-ipt)Q_T(t)\left(c_R^+(p)-c_L^+(p)\right)\right\}, \tag{36}$$

then, using the representations (19) and the relation $c_{R,L}(p)|0\rangle=0$, we can rewrite this expression in the form

$$\varkappa_K(t)=[\psi_R^+(t)\psi_L(-t)\sigma_R\sigma_L^+]^{Q_T} \quad \text{or} \quad [\psi_R(t)\psi_L^+(-t)\sigma_L\sigma_R^+]^{Q_T} \tag{37}$$

respectively, for $Q_T>0$ and $Q_T<0$.

This method of separating $\varkappa_K(t)$ is fairly arbitrary; for example, the field can lead to the appearance of the factor

$$\psi_R(x_0-t_0+t)\psi_L^+(x_0+t_0-t)$$

at an arbitrary point x_0, t_0 where $E\neq 0$ and not only at the point $x_0=t_0=0$, as in (37). However in any such representation the part which changes Q_R and Q_L will contain the operators $\sigma_{R,L}$ ($\sigma_{R,L}^+$), i.e., quarks of very small momenta.

We consider a simple example that explains the situation. Suppose $E\neq 0$ only at one point x_0, t_0 and that this field corresponds to the production of one right-handed and one left-handed particle ($Q_R=-Q_L=1$):

$$E(x,t)=-2\pi g^{-1}\delta(t-t_0)\delta(x-x_0). \tag{38}$$

Then apart from an unimportant constant we obtain

$$\Psi(t)=\psi_R^+(t-t_0+x_0)\psi_L(x_0+t_0-t)\sigma_R\sigma_L^+|0\rangle. \tag{39}$$

Reversal of the sign of E in (38) would lead to the replacement of particles by antiparticles, as in (37). Thus, the field (38) produces causally (at the point where $E\neq 0$) a separating $q_R\bar{q}_L$ pair of local charges and a state containing the corresponding anticharges with very small momenta (for finite volume, $p\sim V^{-1}$).

This assertion becomes even more convincing when $\Psi(t)$ is expressed in terms of the quark operators $a_{R,L}^+$ and $b_{R,L}^+$. The expression (32) can be disentangled in this manner by means of our procedure in Ref. 8 (see Appendix 2). For the field (38), this leads to the very clear expression

$$\begin{aligned}\Psi(t)&=Aa_R^+(t-t_0+x_0)b_L^+(x_0+t_0-t)\int a_L^+(x)dx\int b_R^+(x)dx|0\rangle\\ &=Aa_R^+(t-t_0+x_0)b_L^+(x_0+t_0-t)a_L^+(p=0)b_R^+(p=0)|0\rangle.\end{aligned} \tag{40}$$

An analogous situation, but with more complicated distributions of the local charged packets and states of the particles with small momenta, arises for other configurations of the topological fields.

To prove that the states with small momenta of the quarks produced by an arbitrary topological field are distinguished, we calculate, for example, the number of right-handed quarks with momenta in the interval from p to $p+dp$ in the state with the wave function (32):

$$n_R(p,t)=\langle\Psi(t)|a_R^+(p)a_R(p)|\Psi(t)\rangle. \tag{41}$$

We can calculate $n_R(p,t)$ by means of the technique of Sec. 2 if we make in accordance with (6) the substitution

$$a_R^\pm(p)=\int e^{\pm ipx}\psi_R^\pm(x)dx. \tag{42}$$

Using Eqs. (19) for $\psi_R^+(x)$ and $\psi_R(x)$ and substituting (32), we reduce (41) to the expression

$$n_R(p,t)=\int\frac{dx\,dy}{2\pi i}\frac{e^{ip(x-y)}}{y-x-i0}\exp\left\{\int\frac{dk}{k}\Phi_R(x,t)\left(e^{-ik(t-x)}-e^{-ik(t-y)}\right)\right\}. \tag{43}$$

The substitution

$$\frac{1}{2\pi i(y-x-i0)}=\int_0^\infty\frac{dk}{2\pi}e^{-ik(y-x)} \tag{44}$$

enables us to reduce (43) to a form in which one can see explicitly that $n_R(p,t)$ is real and positive:

$$n_R(p,t)=\int_p^\infty\frac{dk}{2\pi}\left|\int dx\exp[-ikx-2\pi i\alpha(x,t)]\right|^2. \tag{45}$$

Here

$$\alpha(x,t)=\int\frac{dk}{k}\Phi_R(k,t)e^{-ik(t-x)}=-\frac{g}{2\pi}\int_0^t d^2x_1\theta(t-x-t_1+x_1)E(x_1,t_1). \tag{46}$$

For small p, the main contribution to (45) is made by small k, i.e., large $|x|$. In the limit of large $|x|$, the function $\alpha(x,t)$ is transformed into

$$\alpha(x,t)=\theta(t-x)Q_T(t) \tag{47}$$

[the field $E(x,t)$ is nonvanishing in a finite region of space]. Therefore, as $p\to 0$

$$n_R(p,t)=\int_p^\infty\frac{dk}{2\pi}\left|\int dx\,e^{-ikx}\exp[2\pi iQ_T(t)\theta(t-x)]\right|^2=\frac{2}{\pi}\frac{\sin^2\pi Q_T(t)}{p}, \tag{48}$$

and we obtain a $1/p$ singularity. For integral Q_T, the expression (48) is not defined. The topological effect is discussed in this case in Appendix 3.

Thus, the topological field ensures local nonconservation of the charge and chirality, since the particles produced in this field include a finite fraction of fermions with very small momenta. The fermions with momenta $p\sim V^{-1}$ make a contribution to any local characteristic $f(x)$ of the system which is vanishingly small in the limit $V\to\infty$. Indeed, suppose that in the finite volume V

$$\begin{aligned}f(x)&=\frac{1}{V}\sum_{n=0}^{\infty}f(p_n)\exp(ip_nx)\\ &=\frac{1}{V}\sum_{n=0}^{N_0}f(p_n)\exp(ip_nx)+\frac{1}{V}\sum_{n=N_0}^{\infty}f(p_n)\exp(ip_nx).\end{aligned} \tag{49}$$

In the limit $V\to\infty$, the contribution of the first term tends to zero as $1/V$, and the contribution of the second goes over into the corresponding quantity for infinite volume:

$$\int_0^\infty f(p)e^{ipx}\frac{dp}{2\pi}=\tilde{f}(x). \tag{50}$$

Therefore, an arbitrary finite (and even increasing $N_0<V^\alpha$, where $\alpha<1$) number of fermion states with small momenta is not taken into account in the quantity determined in infinite space. Therefore, if $f(0)\neq 0$, we can encounter nonconservation of quantities such as the charge, chirality, or fermion number, since the sum in (49) may give a finite contribution as a result of integration over the whole of space.

The local charge produced causally by the topological field will be the compensating charge that leads to complete screening in the Schwinger model. A necessary condition for this is the existence of degenerate charged and chiral vacuum states to ensure that the appearance in the system of additional particles carrying charge and chirality does not change the states of the system. For the Schwinger model, such vacuum states play the same part as the states with momenta $p \sim V^{-1}$ for the problem in an external field. We emphasize that because of the interaction the momenta of the quarks in these states have the "normal" order of magnitude $p \sim m$ (Sec. 5).

The states localized in the complete volume of the system can be revealed in the theory only by solving the problem in finite volume and calculating all quantities to accuracy $1/V$. This leads to fulfillment of the conservation laws but, obviously, is accompanied by noncausal effects. We shall see this in the following section.

4. CHARGE SCREENING IN QED$_2$ IN A FINITE VOLUME

Charge screening in the Schwinger model has been known since Schwinger's papers in Ref. 1. In a system of infinite volume, Eqs. (28) for the two components of the current and the Maxwell equation that relates the field intensity **E** to the charge density for the physical states,

$$\operatorname{div} \mathbf{E}(x, t)+g\rho(x, t)=0, \tag{51}$$

lead to the equations

$$(\Box+m^2)\rho(x, t)=0, \quad (\Box+m^2) \operatorname{div} \mathbf{j}(x, t)=0. \tag{52}$$

Equations (52) describe explicitly screening of the charge, whereas for the chirality a uniform distribution in space is also possible.

The Hamiltonian of a two-dimensional charged system ($Q \neq 0$) is not defined in infinite volume. This can already be seen on the basis of Eq. (4) for the Hamiltonian that we have chosen in the Coulomb gauge. We remove the arbitrariness which exists here[8] as follows. A Hamiltonian can be consistently chosen only for a two-dimensional system with total charge equal to zero. We therefore introduce an external charge ($-Q$) at a large distance from the physical charges (Q). The choice

$$\rho_{ex}(x)=-\frac{Q}{2}[\delta(x-L)+\delta(x+L)] \tag{53}$$

does not change the properties of the system within the interval ($-L, L$) and cancels the field outside it.

We shall call the constructed system a two-dimensional charged system with charge Q. Equation (51) is replaced by

$$\operatorname{div} \mathbf{E}(x, t)+g\rho(x, t)=-g\rho_{ex}(x). \tag{54}$$

(The left-hand side commutes with the Hamiltonian, and therefore ρ_{ex} does not depend on the time.) Imposing on the functions periodic boundary conditions, we finally determine the QED$_2$ system in finite volume. Its Hamiltonian takes the form (4), and the equations for $\rho(x)$ and $j(x)$ and Eq. (54) in the momentum representation have the form ($n \neq 0$)

$$\begin{gathered}\dot\rho(p_n, t)+ip_n j(p_n, t)=0,\\ \dot j(p_n, t)+ip_n\rho(p_n, t)=-m^2g^{-1}E(p_n, t),\\ ip_nE(p_n, t)+g\rho(p_n, t)=-g\rho_{ex}(p_n, t),\\ \rho_{ex}(p_n)=-(-1)^nQ=-{}^1/{}_2Q[\exp(ip_nL)+\exp(-ip_nL)].\end{gathered} \tag{55}$$

For $n=0$,

$$\dot Q=0, \quad \dot K=0, \tag{56}$$

since the charge and the total chirality commute with the Hamiltonian (4).

We show that our assertion about screening of the charge in this model follows from Eqs. (55). To do this, we solve these equations for stationary states ($\dot\rho=0, \dot j=0$). In this case, $\rho(x)$ has the form

$$\rho(x)=\frac{1}{V}\sum_{p_n}\frac{\exp(ip_nx)}{\omega_{p_n}^2}m^2\rho_{ex}(p_n)\approx\frac{m}{2}\int_{-L}^{L}\exp(-m|x-x'|)\rho_{ex}(x')dx'. \tag{57}$$

Thus, to exponential accuracy $\rho(x)$ repeats the distribution of the external charge.

For $\rho_{ex}(x)$ (53), the entire charge is concentrated at the edges of the region, which explains the exponentially small overlapping of the charged vacuum and the charged local packet (26). In contrast to this, the chiral stationary states permit only a uniform distribution of the chirality ($j(x)=K/V=\text{const}$). This also occurs in the chiral vacuums (25) in accordance with Eq. (27).

For $p_n \neq 0$, the general solution of the system (55)–(56) has the form

$$\begin{gathered}\rho(p_n, t)=\frac{\sin\omega_{p_n}t}{\omega_{p_n}}\dot\rho(p_n, t=0)+\cos\omega_{p_n}t\left[\rho(p_n, t=0)-(-1)^n\frac{m^2Q}{\omega_{p_n}^2}\right]\\ +(-1)^n\frac{m^2}{\omega_{p_n}^2}Q,\end{gathered} \tag{58'}$$

$$j(p_n, t)=\frac{\sin\omega_{p_n}t}{\omega_{p_n}}\dot j(p_n, t=0)+\cos\omega_{p_n}t\, j(p_n, t=0),$$

while for $p_n=0$

$$\rho(0, t)=Q, \quad j(0, t)=K. \tag{58''}$$

The expressions (58) enable one to follow in time the development of different initial configurations of the local charge. For example, a point charge at the point x_0,

$$\rho(x, 0)=Q\delta(x-x_0), \quad j(x)=Q\delta(x-x_0), \tag{59}$$

leads to the following spatial distribution of the charge at the time t:

$$\begin{gathered}\rho(x, t)=\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial x}\right)\frac{Q}{V}\sum_{p_n}\frac{\sin\omega_{p_n}t}{\omega_{p_n}}\exp(ip_n(x-x_0))\\ +\frac{m^2Q}{V}\sum_{p_n}(-1)^n\exp(ip_nx)\frac{(1-\cos\omega_{p_n}t)}{\omega_{p_n}^2}.\end{gathered} \tag{60}$$

As we shall now show, the first term in the expression (60) is nonzero only in the causal region $t \geq |x-x_0|$ (region I), while the second sum is not zero in the causal region of the external charge $t \geq |L \pm x|$ (region II). The first sum makes a contribution $Q \cos mt$ to the charge, the second $Q(1-\cos mt)$. To show that the charge density $\rho(x, t)$ vanishes outside the causal regions, we represent each of the sums in the expression (60) in the form of a contour integral. Thus, we write the first sum in (60) in the form

$$\int_c \frac{dp}{2\pi i} \operatorname{ctg} Lp \frac{\sin \omega_p t}{\omega_p} e^{ip(x-x_0)} = \int_c \frac{dp}{\pi} \frac{e^{-ipL}}{\sin pL} \frac{\sin \omega_p t}{\omega_p} e^{ip(x-x_0)}. \quad (61)$$

But outside the causal region $|x-x_0|<t$ the contour in (61) can be closed in the lower half-plane, where the integrand has no singularities and the integral vanishes.

The nonconservation of the charge in the regions I and II is due to the existence of a noncausal current which transmits charge from region I to region II and vice versa instantaneously:

$$j(x,t) = -\frac{Q}{V} \sum_{p\neq 0} \frac{\sin \omega_p t}{ip\omega_p} \left\{ \omega_p^2 e^{ip(x-x_0)} - \frac{1}{2} m^2 [e^{ip(L+x)} - e^{-ip(L-x)}] \right\}$$
$$+ \frac{Q}{V} \sum_{p\neq 0} \cos \omega_p t e^{ip(x-x_0)} + \frac{Q}{V}(1-\cos mt). \quad (62)$$

The first term in (62) is nonzero in the noncausal regions $t<|x-x_0|$ and $t<|L\pm x|$. In the zeroth order in V, the current in these regions is equal to the contribution from the pole in the integral corresponding to the first sum:

$$j_{\text{noncausal}} = \frac{Qm}{2} \sin mt [\theta(x-x_0-t) - \theta(x_0-x-t)]. \quad (63)$$

Therefore, in the causal regions the chirality is not conserved. Conservation of chirality occurs in (62) only because of the $1/V$ terms. These results indicate that nonlocalized objects (i.e., objects with momentum $1/V$) do indeed play a part in screening processes in QED_2.

A similar treatment can be given for other initial charge distributions. For example, for a $q_R \bar{q}_L$ quark pair at different points of space,

$$\rho_R(x, t=0) = \delta(x-x_0), \quad \rho_L(x, t=0) = -\delta(x-y_0), \quad (64)$$

we again find that the charges are concentrated and change in the causal regions around the points x_0 and y_0. The total charges of each region are not conserved and are equal to $\pm\cos mt$. The change in the charges begins instantaneously, since a current of the type (63) arises instantaneously in the noncausal region between x_0 and y_0 (for $t<|x_0-y_0|$).

The charge oscillations observed in both processes mean that the charge of each causal region is on the average zero, i.e., it is screened. The local charged packets go over completely into hadrons; as we shall see, their chirality goes over to nonlocalized vacuum states of the type (25). It is the capacity of such states to acquire chirality that ensures the confinement phenomenon in the Schwinger model.

At the same time, the different behavior of the charge and the chirality should be noted. Charge conservation is limited to the causal regions, and there is transfer of charge only from one region to another. Therefore, the change in the charge in dynamically independent regions occurs synchronously (for example, $\pm\cos mt$). This is due to the circumstance that QED_2 in reality does not have a genuine charged vacuum state, since otherwise the topological effect would make the changes of the charge in the different regions independent. But a charge cannot exist in QED_2 that is not compensated by some other nearby charge [see Eq. (57)], and therefore it is not distributed in the whole of space but is transferred synchronously from one region to the other. The situation with regard to the chirality is different, and the model contains genuine chiral vacuums, in which the chirality is delocalized.

This significant difference between the vacuum states is an important circumstance which enables us to understand how the conservation of the various characteristics of the system can be ensured in more complicated cases despite the loss through the topological effect of particles carrying fermion number and other quantum numbers. Like the charge in our model, they can be localized always only in causal regions because of the absence of corresponding vacuum states.

5. PHYSICAL PROPERTIES OF THE PROCESS OF PRODUCTION OF R AND L QUARKS

The production of $q_R \bar{q}_L$ ($\bar{q}_R q_L$) pairs by an external source[10] can serve as an analog of the process of e^+e^- annihilation into hadrons in the Schwinger model. We wish to establish a consistent space-time picture of the process, and for this purpose we place a source at a definite point x_0, t_0 ($t_0=0$). We are interested in phenomena such as the occurrence of an electromagnetic topological field when the quarks separate and the appearance of screening charge as a result of the topological effect, the complete screening of the initial "test" quarks, the accumulation of hadronic quarks, and the transition of the process to the hadron stage. Bearing in mind now that we shall be dealing with noncausal nonconservation of chirality, we consider the problem in infinite volume.

The simplest way of solving the problem is to use the representation (19) and the Hamiltonian (16) to find the exact wave function of the process at any time:

$$\Psi^{(K)}(t) = e^{-iHt} \psi_R^+(x_0) \psi_L(x_0) \Omega_0. \quad (65)$$

In Eqs. (19), we replace $c^+_{R,L}$ and $c_{R,L}$ in accordance with Eqs. (17). When the operator $\exp(-iHt)$ is applied, the hadron operators in $\psi^+_R(x)$ and $\psi_L(x)$ go over into $C(p)\exp(i\omega_p t)$ and $C^+(p)\exp(-i\omega_p t)$. Bearing in mind that $C(p)\Omega_0=0$, and that σ_R and σ_L commute with the Hamiltonian for $Q=0$, we find

$$\Psi^{(K)}(t) = A(t) \exp\left\{ \frac{1}{V} \sum_{p\neq 0} \left(\frac{2\pi}{\omega_p}\right)^{1/2} \exp(-ipx_0) \right.$$
$$\left. \times \exp(-i\omega_p t)\, \varepsilon(p) C^+(p) \right\} \sigma_R \sigma_L^+ \Omega_0,$$
$$A(t) = \frac{1}{V} \exp\left\{ -\frac{1}{V} \sum_{p>0} \frac{\omega_p - p}{p\omega_p} \right\}, \quad (66)$$
$$\varepsilon(p) = \begin{cases} 1, & p\geq 0 \\ -1, & p<0 \end{cases}.$$

In the limit $V\to\infty$, the value of $A(t)$ does not depend on the volume.

The expression (66) gives a hadronic representation of the wave function, and the factor $\sigma_R^+\sigma_L\Omega_0$ indicates that the transition to hadrons is accompanied by distribution of the local chirality (65) (for $t=0$) over the complete volume of the system with the participation of the chiral vacuums (25). The expression (66) enables us to calculate the R and L currents in this state at any time t and at an arbitrary point $x(x-x_0\to x)$. We use Eqs. (24), replacing $c_{R,L}(p)$ in them by $C(p)$. The expectation value of $\rho_{R,L}(x)$ in the state (66) is

$$\rho_{R,L}^{(K)}(x,t)=\pm\int\frac{dp}{2\pi}\frac{\omega_p\pm p}{\omega_p}\cos(\omega_p t-px). \tag{67}$$

In (67), we have divided by the normalization $|\Psi^{(K)}(t)|^2$. We calculate the integral (67) and represent $\rho_{R,L}^{(K)}(x,t)$ in the explicit form

$$\rho_{R,L}^{(K)}=\pm\left[\delta(t\mp x)-\frac{m}{2}\frac{t\pm x}{(t^2-x^2)^{1/2}}J_1(m(t^2-x^2)^{1/2})\theta(t^2-x^2)\right] \tag{68}$$

or

$$\rho_{R,L}^{(K)}(x,t)=\pm\frac{1}{2}\left(\frac{\partial}{\partial t}\mp\frac{\partial}{\partial x}\right)[J_0(m(t^2-x^2)^{1/2})\theta(t^2-x^2)]. \tag{69}$$

Exactly the same answer is obtained from the solution of the problem (64) of the previous section if $x_0=y_0$ and $V\to\infty$.

The two terms in Eq. (68) are the leading charge (the δ function) and the screening charge, which steadily increases near the light cone. In the limit $|x|\to t$, the second term is equal to $-\frac{1}{2}m^2t$, and for $t\gg 1/m$ there is accumulated in a small region $\sim 1/m^2t$ a charge sufficient for complete local screening, i.e., compensation of the field of the particle.

However, the calculation of the total charges in the causal region at the time t shows that the description of the situation given here is somewhat simplified. In accordance with (68) and (69), the integral (67) over the causal region gives a nonvanishing contribution to $Q_{R,L}$ as $t\to\infty$, and an oscillating

$$Q_{R,L}^{(K)}(t)=\int_{-t}^{t}dx\,\rho_{R,L}^{(K)}(x,t)=\pm\cos mt. \tag{70}$$

We know from Secs. 3 and 4 that the nonconservation of the charges in the causal regions is due to the specific production of quark pairs in topological fields. In the self-consistent problem of interacting particles, a topological field forms a certain number of particles in one of the chiral vacuum states (25), and they do not contribute to the quantities calculated in the limit $V\to\infty$. The oscillating charges in Eq. (70) and Sec. 4 are due to this phenomenon. We calculate the field $E(x,t)$ that arises in the system when the $q_R\bar{q}_L$ charges separate. From the expression (67), we find

$$\rho^{(K)}(x,t)=-\partial E^{(K)}(x,t)/\partial x,$$
$$E^{(K)}(x,t)=g\int_{-\infty}^{+\infty}\frac{dk}{\pi}\frac{\sin\omega_k t}{\omega}e^{ikx}=gJ_0(m(t^2-x^2)^{1/2})\theta(t^2-x^2). \tag{71}$$

The sign of the field (71) is such that its action forces the leading particles to give up energy, which goes over to the hadrons being produced. At short times $t\sim 1/m$, the energy is constant ($E\approx g$) and sufficient for producing by means of the topological effect the charge which compensates in magnitude the charge of the leading quark and which moves in the same direction as it. But local compensation does not occur, and the topological effect continues to change the charges (70). We show that this phenomenon affects only quarks of small momenta $p\sim m$, which cannot in fact be separated from the quarks of the physical vacuum (18). For $t>1/m$, the fraction of quarks of large momenta $p\gg m$, which participate in the phenomena of local screening of the leading particle and in the production of hadrons, becomes appreciable. The charge oscillation effect has here the nature of vacuum fluctuations. It has no relation to the physical processes, which occur already in neutral systems. The time $t\sim 1/m$ should be regarded as the screening time. The value of $\cos mt$ can be averaged over $\Delta t>1/m$, indicating that there is no charge in the system for $t\gg 1/m$.

Since the charge fluctuation effect is explained by the participation of quarks of momenta $p\sim m$ in transitions between the vacuums (25), energy differences $\Delta E\sim m$ of the states whose superposition forms the initial packet are already sufficient for the effect (70) to be manifested. Of course, the amplitude of the oscillations decreases when the accuracy with which the energy is specified increases. Accurate specification of the energy (packet formation time $\Delta t\to\infty$) leads to vanishing of the oscillations. But then the picture of the development of the process in time is lost.

To prove the above assertions about the parts played by the quarks with different momenta, we investigate the time dependence of the number of quarks produced by the field (71) in the considered process. These calculations are given in Appendix 4. The density (in a momentum cell $dp/2\pi$) of the right-handed quarks is

$$n_R(p,t)=\frac{1}{2\pi i}\int\frac{dx\,dy}{y-x-i0}f(x-y)\{\exp 2\pi i[\alpha(x,t)-\alpha(y,t)]-1\}e^{ip(x-y)}. \tag{72}$$

Here, the function

$$f(x-y)=\exp\left\{-\int_0^\infty\frac{dk}{\omega}\left(\frac{\omega-k}{k}\right)^2\sin^2\frac{k(x-y)}{2}\right\} \tag{73}$$

determines the mean density of the quarks in the vacuum Ω_0 (see Appendix 4). We can express $\alpha(x,t)$ in terms of the field intensity $E(x,t)$ (71) by means of Eqs. (46).

The results of the calculations show that oscillations in the particle number do indeed occur only with quarks of small momenta $p\leq m$ [Eq. (A4.18)]. As regards quarks with large momenta, study of (72) shows that there is a rapid increase in their number at times $t<p/m^2$:

$$n_R(p,t)=\frac{\pi}{6}\left(\frac{m^2t}{p}\right)^2\frac{1}{p};\quad p\gg m,\quad t<\frac{p}{m^2}, \tag{74}$$

the quarks of the hadrons produced later being prepared during these times. At times $t>p/m^2$, a stationary distribution arises from (72):

$$n_R(p,t)=\frac{2\pi}{p},\quad n_R(p,t)\frac{dp}{2\pi}=\frac{dp}{p}. \tag{75}$$

The absence of oscillations in Eqs. (74) and (75) indicates that quarks with large momenta no longer participate in the transitions to the chiral vacuums induced by the field (71). The system of physical quarks $p > m$ becomes neutral for $t \sim 1/m$, and this time must be regarded as the screening time ($1/m$ is the confinement radius).

We now show that the stationary distribution (75) is the distribution of the quarks grouped into hadrons. In the parton model, the density of the quarks, which is related to their distribution over the hadrons, must be ($P, p \gg m$)

$$n_R(p,t)=\int N_R(P,t)n_R(P,p,t)\frac{dP}{2\pi}. \qquad (76)$$

Here, $N_R(P,t)dP/2\pi$ is the number of hadrons with momentum P. For the wave function (66), this is obviously equal to

$$N_R(P,t)=\frac{\langle\Psi^{(K)}(t)|C^+(P)C(P)|\Psi^{(K)}(t)\rangle}{\langle\Psi^{(K)}(t)|\Psi^{(K)}(t)\rangle}=\frac{2\pi}{\omega_P}, \qquad (77)$$

where $n_R(P,p,t)$ is the distribution of the R quarks with respect to the momentum p within a hadron with momentum P.

The parton wave function $C^+(P)\Omega_0$ of such a hadron in the quark representation can be readily constructed by means of Eqs. (8), (12), and (16) ($P>0$):

$$\Psi_{\text{had}}(P)=C^+(P)\Omega_0=\left(\frac{\omega_P}{P}\right)^{1/2}c_R{}^+(P)\Omega_0=\frac{(2\pi\omega_P)^{1/2}}{P}\rho_R(P)\Omega_0. \qquad (78)$$

In accordance with (18), the vacuum Ω_0 contains basically quarks with momenta $p \le m$, and therefore for large $P>0$ we can, when Eqs. (8) are substituted in (78), retain only the terms with two creation operators. The parton wave function of the hadron has the form ($P \gg m$)

$$\Psi(P)=\frac{1}{V}\sum_{P>k>0}a_R{}^+(P-k)b_R{}^+(k)\Omega_0. \qquad (79)$$

The distribution of the R quarks in the hadron (79) is

$$n_R(P,p,t)=\langle\Psi_{\text{had}}(P)|a_R{}^+(p)a_R(p)|\Psi_{\text{had}}(P)\rangle=2\pi\theta(P-p)/P. \qquad (80)$$

Substitution of (77) and (80) in Eq. (76) leads to the distribution (75). Therefore, the formation of the hadrons terminates after the hadron formation time

$$t_{\text{had}}\sim P/m^2, \qquad (81)$$

when the distributions (74) and (75) are comparable in magnitude. Our treatment leads to the picture of the development in time in the rapidity space of the quark process of e^+e^- annihilation that has already been frequently discussed in the literature,[10,13] namely, it is the most energetic hadrons which are formed last.

The leading particles carry infinite energy and momentum in the packet (65) that we have formed. The process of their transition into hadrons "for ever," and there is never a time at which the distribution (75) of the produced quarks, which determines the screening charge in (68), can exactly compensate the δ function in Eq. (68). However, it is completely clear that the leading particles must lose energy, giving it up to the hadrons. If their initial momentum (p_{in}) were finite, then after a time $t \sim p_{\text{in}}/m^2$ they would merge with the produced quarks (74), participating on an equal footing in the final formation of the hadron spectrum. The problem of the neutralization of their charges would already have been solved at times $t \sim 1/m$.

The picture of the loss of energy by the leading quark can be clearly deduced from a calculation of the energy density in the state (66). If this density is defined as the corresponding component of the symmetric energy-momentum tensor (this definition agrees with the physical definition of energy density adopted in the general theory of relativity[15]), then a standard calculation by means of the representations (19), the expression for $H_0(x)$ from (4), and the operator of the energy density of the electromagnetic field

$$H_{\text{em}}(x)=\frac{1}{2}E^2(x)=\frac{1}{2}\left[\frac{g}{2}\int\varepsilon(x-x')\rho(x')dx'\right]^2 \qquad (82)$$

leads to the result

$$\langle H_0(x,t)\rangle=\pi|\rho_R^{(K)}(x,t)|^2+\pi|\rho_L^{(K)}(x,t)|^2,$$
$$\langle H_{\text{em}}(x,t)\rangle={}^1/_2|E^{(K)}(x,t)|^2. \qquad (83)$$

Here, $\rho_{R,L}(x,t)$ and $E(x,t)$ are given by Eqs. (67)–(69) and (71).

The integration of (83) over the whole of space is readily performed by means of the Fourier representations (67) and (71) and gives conservation of the energy in the causal region:

$$\int_{-t}^{t}\langle H_0(x,t)\rangle dx=\int_{-\infty}^{+\infty}\omega_k\frac{dk}{\omega_k}-m^2\int_{-\infty}^{+\infty}\frac{\sin^2\omega_k t}{\omega_k{}^2}dk,$$
$$\int_{-t}^{t}\langle H_{\text{em}}(x,t)\rangle dx=m^2\int_{-\infty}^{+\infty}\frac{\sin^2\omega_k t}{\omega_k}dk; \qquad (84)$$

$\langle H\rangle=\langle H_0\rangle+\langle H_{\text{em}}\rangle$ does not depend on l. The total energy of the packet (65) is infinite and equal to the energy of the hadron distribution (77). The electromagnetic energy remains finite ($\frac{1}{2}\pi m$) in the limit $t\to\infty$ and makes possible vacuum fluctuation transitions with particles with $p \le m$.

Equation (83) indicates a decrease in the kinetic energy of the leading particle just as Eq. (68) indicates a decrease in its charge. The energy $H_0(x,t)$ decreases near the light cone with increasing t.

To conclude this section, we discuss the correspondence between the calculations made here of $\rho_{R,L}$ (67)–(69) and the usual calculations of quantities in the S-matrix formalism: the Green's functions, Feynman diagrams, etc. The correspondence is readily established by recalling that our expectation value for the state (65) can be written in the form

$$\langle\Psi(t)|\rho_{R,L}(x)|\Psi(t)\rangle=\lim_{t_1\to t_2}\langle\Omega_0(t_2)J_s(x_2,t_2)$$
$$\times S^+(t,t_2)\rho_{R,L}(x,t)S(t,t_1)J_s(x_1,t_1)|\Omega_0(t_1)\rangle. \qquad (85)$$

Here, $J_s(x)=\psi_R^+(x)\psi_L(x)$ and $\rho_{R,L}(x,t)$ are, respectively, the scalar and vector currents in the interaction representation, and $S(t,t_1)$ is the operator of the S matrix at a finite time. Using $\Omega_0(t_1)=S(t_1,-\infty)\Omega_0$ and the unitarity of the operator S: $S^+(t_1,t_2)=S(t_2,t_1)$, we reduce the calculation of the matrix element (85) to the vacuum Green's function

$$G(x_1;\ x_2;\ x)=\langle 0|T\{J_s(x_2)\rho_{R,L}(x)J_s(x_1)\}|0\rangle \qquad (86)$$

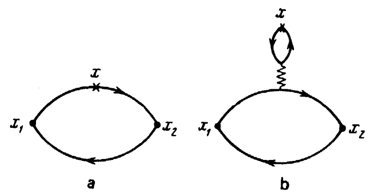

FIG. 1. The two diagrams that determine the current $\rho_{R,L}(x)$.

(x is the two-dimensional coordinate), which must be taken under the conditions $t_2 > t > t_1$, and we then set $x_2 = x_1$. The expectation value of the current is obtained by dividing (85) by the square of the modulus, i.e., $|\Psi(t)|^2$.

The complexity of such a problem is due to the separation from (86) of the square of the modulus of the wave function:

$$|\Psi(t)|^2 = \lim_{t_1\to t_2+\tau} \langle 0|T\{J_s(x_2)J_s(x_1)\}|0\rangle. \tag{87}$$

This can be done in general form if G is calculated as a functional integral. Let us obtain Eq. (68). The calculation of (86) associates Feynman diagrams with each of the terms of Eq. (68). For example, the δ function in (68) is obtained from the diagrams in which the current $\rho_{R,L}(x)$ is included in the line of the charge that directly connects the points x_1 and x_2 [Fig. 1(a)]. The second term in (68), which describes the screening charge, appears here in the form of the two-dimensional invariant integral

$$m^2\int \frac{d^2k}{(2\pi)^2 i}\,\frac{e^{ik(x_2-x)}-e^{ik(x_1-x)}}{(k^2+i0)(k^2-m^2+i0)}\,k_\pm. \tag{88}$$

It arises from the contribution of the diagrams in which the inclusion of $\rho_{R,L}(x)$ occurs necessarily through a quark loop [Fig. 1(b)]. Since this unique divergent diagram of the model leads to an Adler anomaly in the model, this diagram describes the occurrence of the screening charge. The Adler anomaly played the same role in Secs. 3 and 4. However, the physics of the topological effect which arises here is revealed clearly only in the Hamiltonian formalism.

6. CONCLUSIONS

Usually, the hopes for explaining confinement are based on establishing particular physical properties of the vacuum state that prevent the existence of charged particles in it. We draw attention here to a different aspect of the question, namely, the topological fields themselves have the consequence that a theory without confinement is physically inconsistent, this being due to the noncausal nonconservation of the charge associated with the topological fields. The Adler anomaly in the expression for the divergence of the axial quark current can be taken as an indication of the existence of an analogous phenomenon in four-dimensional quantum chromodynamics. The properties of the vacuum must adjust to the topological effect in such a way that the degeneracy of the vacuums with respect to the quantum numbers makes it possible to interpret the theory with "confinement" of these numbers. The physical properties of the vacuums in the given model that ensure this possibility will be described elsewhere.

We are grateful to V. N. Gribov and L. L. Frankfurt for numerous helpful discussions. The discussions with Gribov led to the consideration of the problem in an external field.

APPENDIX 1

To prove the equivalence of the representations (4) and (14) for H_0, we express $c_R^+(p)$ and $c_R(p)$ in terms of the quark creation and annihilation operators $a_R^+(x)$ and $b_R^+(x)$ in accordance with (8) and (12) and substitute the result in Eq. (14):

$$H_0=\int_0^\infty dp\int dx\,dy\,e^{ip(x-y)}[a_R^+(x)a_R(x)-b_R^+(x)b_R(x)+a_R^+(x)b_R^+(x)+b_R(x)a_R(x)][a_R^+(y)a_R(y)-b_R^+(y)b_R(y)+a_R^+(y)b_R^+(y)+b_R(y)a_R(y)]. \tag{A1.1}$$

When the expressions in the square brackets in (A1.1) are multiplied, we obtain 16 terms containing products of four creation and annihilation operators. It will be clear from the following calculations (which are carried out similarly for all the terms) that a contribution to H_0 is made by only two of them:

$$H_0=\int_0^\infty dp\int dx\,dy\,e^{ip(x-y)}[a_R^+(x)a_R(x)a_R^+(y)a_R(y)+b_R^+(x)b_R(x)b_R^+(y)b_R(y)]. \tag{A1.2}$$

We represent H_0 as the half-sum of the integral (A1.2) and the same integral with the substitution $x\to y$:

$$H_0=\frac{1}{2}\int dx\,dy\int_0^\infty dp\,(e^{ip(x-y)}+e^{ip(y-x)})\,[a_R^+(x)a_R^+(y)a_R(y)a_R(x)+b_R^+(x)b_R^+(y)b_R(y)b_R(x)]+\frac{1}{2}\int\frac{dx\,dy}{2\pi i}\int_0^\infty dp\,\frac{e^{ip(x-y)}}{y-x-i0}[a_R^+(x)a_R(y)+b_R^+(x)b_R(y)]+\frac{1}{2}\int\frac{dx\,dy}{2\pi i}\int_0^\infty dp\,\frac{e^{ip(y-x)}}{x-y-i0}[a_R^+(y)a_R(x)+b_R^+(y)b_R(x)]. \tag{A1.3}$$

We have here commuted all the operators to bring the two operators a_R (and b_R) next to each other [the commutator is given in (7)]. In the first term, we now take the integral over p. The integral gives $\delta(x-y)$, and the first term in (A1.3) goes over into

$$\pi\int dx[a_R^+(x)a_R^+(x)a_R(x)a_R(x)+b_R^+(x)b_R^+(x)b_R(x)b_R(x)], \tag{A1.4}$$

which vanishes by virtue of the Pauli principle. We now take the integral over p in the remaining two terms in (A1.3). As a result of the integration, a pole of second order arises at $y=x\pm i0$. Bearing in mind now that $a_R(y)$ [$b_R(y)$] is analytic in the upper half-plane, and $a_R^+(y)$ [$b_R^+(y)$] in the lower, we can also integrate with respect to y, closing the integral around the pole. Finally, we obtain

$$\frac{1}{2i}\int dx\left[a_R^+(x)\frac{\partial a_R(x)}{\partial x}-\frac{\partial a_R^+(x)}{\partial x}a_R(x)+b_R^+(x)\frac{\partial b_R(x)}{\partial x}-\frac{\partial b_R^+(x)}{\partial x}b_R(x)\right], \tag{A1.5}$$

which is the term associated with the R quarks in Eq. (4) for H_0.

APPENDIX 2

To derive the expression (40), we use the expression for the evolution operator $S(t)$ at finite time for a system of two-dimensional fermions in an external field. Equations (36) and (37) of Ref. 8 give us an expression for

$$S(t)=D(A)\exp\{a_i^+(p)G_{++}^{(i)}(pt,p'0)a_i(p')-a_i^+(p)G_{+-}^{(i)}(pt,p't)b_i^+(p')$$
$$-b_i(p)G_{-+}^{(i)}(p0,p'0)a_i(p')+b_i(p)G_{--}^{(i)}(p0,p't)b_i^+(p')\},\quad (A2.1)$$

where $i=R, L$, and summation over p and p' is understood. In (A2.1), we have used the notation of Ref. 8, $D(A)$ is the determinant of the Dirac equation in an external field, the form of which is unimportant for us, and G_{++}, etc., are the parts of the Green's function of the same equation corresponding to different signs of the frequency (+, -) of the time arguments. The values of the Green's functions are taken at the times 0 and t, as is indicated in the arguments. The expression (A2.1) is an "untangled" operator form, i.e., all the annihilation operators commute with the creation operators.

In the wave function $S(t)|0\rangle$, where $|0\rangle$ is the mathematical vacuum of the free quarks, only one term in the exponential of (A2.1) is nonzero:

$$\Psi(t)=D(A)\exp\{-a_i^+(p)G_{+-}^{(i)}(pt,p't)b_i^+(p')\}|0\rangle. \quad (A2.2)$$

The Green's function $G^{(R)}$ is equal to

$$G_{+-}^{(R)}(pt,p't)=-\int\frac{dx\,dx'}{2\pi i}e^{-ipx}e^{-ip'x'}\exp\left\{ig\int_0^t\frac{d^2x_1}{2\pi i}\right.$$
$$\left.\times[A_0(x_1)+A_1(x_1)]\left[\frac{1}{t-x-t_1+x_1-i0}-\frac{1}{t-x'-t_1+x_1}\right]\right\}. \quad (A2.3)$$

The formula for $G^{(L)}$ is obtained from (A2.3) by replacing x and x' by $-x$ and $-x'$. In the Coulomb gauge, for the field intensity (38) we have

$$A_0=-2\pi g^{-1}\delta(t-t_0)\theta(x_0-x),\quad A_1=0, \quad (A2.4)$$

Substituting (A2.4) in (A2.3), we obtain

$$G^{(R)}(pt,p't)=-\int\frac{dx\,dx'}{2\pi i}\frac{e^{-ipx}e^{-ip'x'}}{x-x'-i0}\frac{t-x'-t_0+x_0}{t-x-t_0+x_0-i0}$$
$$=2\pi\delta(p')e^{-ip(t-t_0+x_0)}. \quad (A2.5)$$

Therefore, using (6), we obtain

$$\int\frac{dp\,dp'}{(2\pi)^2}2\pi a_R^+(p)\delta(p')e^{-ip(t-t_0+x)}=a_R^+(t-t_0+x)b_R^+(p=0). \quad (A2.6)$$

Making similar calculations for the L particles, we arrive at the expression (40). A reversal of the sign of the field obviously interchanges particles and antiparticles.

APPENDIX 3

The indeterminate form in Eq. (48) can be correctly evaluated in the case of integral topological charge Q_T only if we calculate $n_R(p,t)$ in a finite volume. For this, it is sufficient to use Eqs. (32), (33), and (19) written down for finite V. Making the calculations, we obtain instead of (43)

$$n_R(p_n,t)=\frac{1}{V}\int_{-L}^{L}dx\,dy\frac{e^{i\pi(x-y)/L}}{1-e^{i\pi(x-y)/L}}\exp[ip_n(x-y)]$$
$$\times\exp\left\{\frac{1}{V}\sum_{k_n\neq0}\frac{\Phi_R(k_n,t)}{k}[\exp(-ik_n(t-x))-\exp(ik_n(t-x))\right.$$
$$\left.+\exp(ik_n(t-y))+\exp(-ik_n(t-y))]\right\}. \quad (A3.1)$$

The expression (A3.1) gives the particle density in the momentum space. To obtain the number of particles $N(p_n,t)$ with momentum $p_n=n\pi/L$, we must divide $n(p_n,t)$ by the volume V. We separate from (A3.1) the contribution associated with the topological effect, for which we replace $\Phi_R(k_n,t)$ by its value at $p_n=0$ (Q_T). Summing further the series in the argument of the exponential by means of Eq. (23), we obtain

$$N_R(p_n,t)=\int_{-L}^{L}\frac{dx\,dy}{V^2}\exp(ip_n(x-y))\frac{e^{i\pi(x-y)/L}}{1-e^{i\pi(x-y)/L}}\exp\left[-iQ_T\frac{\pi}{L}(x-y)\right]. \quad (A3.2)$$

The further calculation is easy. For positive integral Q_T, the number of particles N_R is

$$N_R(p_n,t)=\begin{cases}1, & n=0,1,\ldots,Q_T-1\\ 0, & n\geqslant Q_T\end{cases}, \quad (A3.3)$$

i.e., because of the topological effect there are produced Q_T quarks with momenta 0, π/L... $(Q_T-1)\pi/L$, which tend to zero as $V\to\infty$. In the special case of the field (38), we arrive at the result already discussed in Sec. 3.

For integral $Q_T<0$ (or, which is the same thing, for antiquarks when $Q_T>0$), $N_R(p_n,t)=0$. Thus, for integral Q_T only particles with corresponding sign of the charge carry small momenta.

APPENDIX 4

To calculate

$$n_R(p,t)=\frac{\langle\Psi(t)|a_R^+(p)a_R(p)|\Psi(t)\rangle}{\langle\Psi(t)|\Psi(t)\rangle}-\langle\Omega_0|a_R^+(p)a_R(p)|\Omega_0\rangle \quad (A4.1)$$

in the state with the wave function (66), we express $a_R^+(p)$ and $a_R(p)$ in accordance with Eqs. (42) by analogy with the derivation of Eq. (43). We replace $\psi_R^+(x)$ and $\psi_R(x)$ by their representation (19), replacing $c_R^+(p)$ and $c_R(p)$ in them by the exact hadron operators $C^+(p)$ and $C(p)$, using the relation (17) for this. Disentangling the obtained expression, we arrive at the formula

$$n_R(p,t)=\frac{1}{2\pi i}\int\frac{dx\,dy}{y-x-i0}e^{ip(x-y)}f(x-y)$$
$$\times\left[\exp\left\{\int_{-\infty}^{+\infty}\frac{dk}{k}\frac{\omega+k}{k}(e^{i\omega t}e^{-ik(x-x_0)}-e^{-i\omega t}e^{ik(x-x_0)})-(x\to y)\right\}-1\right]. \quad (A4.2)$$

In calculating (A4.2), we used Eqs. (20) and (21). The function $f(\xi)$ is defined by (73).

Each term in the argument of the second exponential in (A4.2) can be transformed by means of the readily verified relation

$$\frac{\omega+k}{2\omega}e^{i(\omega-k)t}=-\frac{g^2}{2\pi}\int_{-\infty}^{t}e^{-ik(t_1-x_1)}dt_1\,dx_1\int\frac{dk'}{2\pi i}\frac{e^{i\omega' t_1}e^{-ik'x_1}}{\omega'}$$
$$=-\frac{g}{2\pi}\int_{-\infty}^{t}e^{-ik(t_1-x_1)}\mathscr{E}(x_1,t_1)dx_1\,dt_1. \quad (A4.3)$$

Here, it must be borne in mind that $g^2/\pi=m^2=(\omega-k)(\omega+k)$. We obtain the field $\mathscr{E}(x_1,t_1)$ from the field (71) by expressing $\sin\omega t$ in the form of a difference of the exponentials $\exp(\pm i\omega t)$: $E(x,t)=\mathscr{E}(x,t)+\mathscr{E}^*(x,t)$.

Substituting (A4.3) in (A4.2), we represent $n_R(p,t)$ in the form

$$n_R(p,t)=\frac{1}{2\pi i}\int\frac{dx\,dy}{y-x-i0}e^{ip(x-y)}f(x-y)\{\exp\{2\pi i[\alpha(x,t)-\alpha(y,t)]\}-1\},$$ (A4.4)

where $\alpha(x,t)$ is defined by (46) with the field $E(x,t)$ given by the expression (71). In deriving (A4.3), we assumed that the field (71) has been applied since $t=-\infty$. It is readily verified that application of the field at a certain time $t=0$ would lead to the appearance in (A4.4) of the additional factor

$$\exp\{2\pi i[\theta(t-x)-\theta(t-y)]\}=1.$$ (A4.5)

Therefore, the integration over t_1 in the integral (46) for $\alpha(x,t)$ is over only the interval in which the field (71) is nonvanishing ($t\geq 0$).

The first exponential in (A4.4), i.e., $f(\xi)$, which is given by the expression (73), determines the density of quarks in the vacuum Ω_0 [-1 in the square brackets in (4.2)]. It leads to a contribution proportional to the volume V of the system. For $p\gg m$, we have $|x-y|\ll 1/m$ and $f(\xi)\approx f(0)=1$, which means that there are no quarks with large momenta in the vacuum Ω_0 (18). The formula for $n_R(p,t)$ can then be represented in the form (45):

$$n_R(p,t)=\frac{1}{2\pi}\int_p^\infty dk\left|\int d\xi\,e^{-ik\xi}e^{-2\pi i\alpha(\xi,t)}\right|^2.$$ (A4.6)

In this case, the integration over t_1 and x_1 in the expression for

$$\alpha(\xi,t)=-\frac{g^2}{2\pi}\int_0^t dt_1\,dx_1\,J_0(m(t_1^2-x_1^2)^{1/2})\theta(t_1^2-x_1^2)\theta(\xi-t_1+x_1)$$ (A4.7)

is restricted to the region near the light cone

$$0<\xi_1=t_1-x_1<\xi\sim 1/p;$$ (A4.8)

therefore $t_1^2-x_1^2\approx 2t_1\xi_1$ and

$$\alpha(\xi,t)=-\frac{m^2}{2}\int_0^t dt_1\int_0^\xi d\xi_1\,J_0(m(2\xi_1t_1)^{1/2})=[J_0(m(2\xi t)^{1/2})-1].$$ (A4.9)

For $t\ll p/m^2$, we obtain from

$$\alpha(\xi,t)\approx-m^2\xi t\theta(\xi)/2,$$ (A4.10)

and substitution in (A4.6) leads to the formula

$$n_R(p,t)=\frac{\pi}{6}\left(\frac{m^2t}{p}\right)^2\frac{1}{p},\quad t<\frac{p}{m^2}.$$ (A4.11)

In the other limiting case, $t\gg p/m^2$ and the argument of the Bessel function satisfies $m^2\xi t\gg 1$ in the main region, and therefore the exponential in (A4.6) can be expanded[16]:

$$\int d\xi\,e^{-ik\xi}\exp\{2\pi i\theta(\xi)[J_0(m(2\xi t)^{1/2})-1]\}$$
$$=2\pi\delta(k)+2\pi i\int d\xi\,e^{-ik\xi}J_0(m(2t\xi)^{1/2})=\frac{2\pi}{k}\exp\left(-\frac{2m^2t}{4ik}\right).$$ (A4.12)

Substituting (A4.12) in (A4.6), we obtain a distribution that does not depend on the time,

$$n_R(p,t)=2\pi/p,\quad t\gg p/m^2,$$ (A4.13)

and a number of particles in the interval dp equal to the hadron distribution (77):

$$n_R(p,t)dp/2\pi=dp/p.$$ (A4.14)

Finally, for $p\ll m$ we must use the exact expression (A4.4) for $n_R(p,t)$. The function $f(\xi_1-\xi_2)$ ($t-x=\xi_1$, $t-y=\xi_2$) restricts the difference $\xi=\xi_1-\xi_2$, but these quantities themselves are now large. Therefore, the integration over t_1 and x_1 in the expression for $\alpha(x,t)$ is in practice over all possible values of t_1 and x_1, i.e.,[16]

$$\alpha(x,t)=-\frac{g^2}{2\pi}\int_0^t dt_1\,dx_1\,J_0(m(t_1^2-x_1^2)^{1/2})\theta(t_1^2-x_1^2)\theta(t-x)$$
$$=(\cos mt-1)\theta(t-x).$$ (A4.15)

We again use (44) in (A4.4) and integrate with respect to the variables $\xi=\xi_1-\xi_2$ and ξ_2. Then we obtain from (A4.2)

$$n_R(p,t)=\frac{1}{2\pi}\int_p^\infty dk\int d\xi\,e^{-ik\xi}f(\xi)F(\xi).$$ (A4.16)

Here, $F(\xi)$ is given by the expression

$$F(\xi)=\int d\xi_2\{\exp[2\pi iQ_T(t)(\theta(\xi+\xi_2)-\theta(\xi_2))]-1\}$$
$$=2i\exp[i\pi Q_T(t)]\sin\pi Q_T(t).$$ (A4.17)

We substitute (A4.17) and (A4.16) and obtain

$$n_R(p,t)=A(p)\sin^2\pi Q_T(t)+B(p)\sin 2\pi Q_T(t),$$ (A4.18)

where

$$A(p)=-\frac{2}{\pi}\int_p^\infty dk\int_0^\infty d\xi\,\xi\cos k\xi f(\xi)=\frac{2}{\pi}\int_0^\infty d\xi\sin p\xi f(\xi),$$
$$B(p)=\frac{1}{\pi}\int_p^\infty dk\int_0^\infty d\xi\,\xi\sin k\xi f(\xi)=\frac{1}{\pi}\int_0^\infty d\xi\cos p\xi f(\xi).$$ (A4.19)

Equation (A4.18) clearly demonstrates the oscillating nature of the density of the quarks produced with small momenta. The value of (A4.1) can also be negative, which corresponds to stripping of quarks from the physical vacuum Ω_0. For $f(\xi)=\exp(-\delta\xi)$, $\delta\to 0$, we obtain the previous result (48).

[1)] In finite volume, the integrals $\int dp/2\pi$ are replaced by the sums $(1/V)\sum_{p_n}$ over the discrete values $p_n=n\pi/L$ of the momentum. We normalize the creation and annihilation operators in accordance with

$$\{a^{\pm}_{R,L}(p_n),\ a_{R,L}(p_n')\}=V\delta_{p_np_n'}.$$

[2)] Our expression for $\psi_{R,L}(x)$ differs from the one given in Ref. 7, where the operators $\sigma_{R,L}$ were introduced formally for the first time. In contrast to the operators $\psi(x)$ defined in the Ref. 7, our operators $\psi(x)$ have the correct commutation properties for different times.

[3)] Strictly speaking, the case $Q\neq 0$ ($[Q,\sigma^\pm]=\pm\sigma^\pm$) requires more careful study. We defer this to another place (see also Sec. 4).

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Kinetic equation for atoms interacting with laser radiation

V. G. Minogin

Spectroscopy Institute, USSR Academy of Sciences
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The motion of atoms in a resonant light wave that excites the atoms in a transition from the ground state to an excited state is considered. A kinetic equation of the Fokker-Planck type is obtained to describe the motion of the atoms due to the recoil of the induced and spontaneous transitions. The equation is used to analyze velocity monochromatization and focusing of an atomic beam in a laser beam.

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1. INTRODUCTION. FORMULATION OF PROBLEM

The recent calculations and the results of the first experiments (see the reviews[1-3]) have demonstrated convincingly the effectiveness of using laser-radiation pressure to act on the spatial motion of neutral particles. Thus, by using the pressure of laser light it becomes possible to deflect[4,5] focus,[6] and slow down[7] atom beams.

On the theoretical level, the investigations of light pressure were based so far on the study of the motion of atoms either in plane light waves or in light waves of constant bounded cross section. These approaches have revealed the role played by the main processes responsible for the existence of light pressure, and the character of the motion of the atoms in the simplest field configurations. At the same time, the use of these models is insufficient for the analysis of experimental situations in which an essential role is played by the laser-beam divergence. This, in one of the most promising applications, that of radiative cooling and dragging of atoms,[3] it is expedient to use light beams both with bounded cross section and with definite angle divergence.[8,9] The need for analyzing such problems calls for knowledge of the laws of motion of atoms in real laser beams.

The present paper presents a derivation of a kinetic equation that describes the evolution of the distribution function of atoms interacting with diverging or converging laser beams. The equation is derived for laser radiation of the fundamental TEM_{00q} mode and for atoms whose interaction with the laser field can be described by a two-level scheme. Particular attention in the analysis is paid to the conditions under which the equation is valid. Velocity monochromatization of an atomic beam in a plane light wave and the focusing of an atomic beam in a light wave with an inhomogeneous transverse distribution of the field are considered by way of examples of the derived equation.

2. INITIAL EQUATIONS

To obtain the equation of motion of an ensemble of atoms in a laser beam, we start from the equation

$$i\frac{\partial\hat{\rho}}{\partial t}=(\hat{H}'-\hat{H}''^{*})\hat{\rho}-i\hat{\Gamma}\hat{\rho} \tag{1}$$

For the density matrix $\hat{\rho}(\mathbf{r}', \mathbf{r}'', t)$ that describes the interaction of the atom with a classical light field E. In this equation, the Hamiltonian of the interaction consists of three terms:

$$\hat{H}=\hat{H}_0-(\hbar^2/2M)\nabla^2+\hat{V}. \tag{2}$$

The first determines the internal states of the atom, the second the translational state of the atom, and the third the dipole interaction of the atom with the field:

$$\hat{V}=-\hbar^{-1}\hat{\mathbf{d}}\mathbf{E}. \tag{3}$$

The relaxation operator $\hat{\Gamma}$ describes the change of the state of the atom on account of spontaneous decays.

We specify the laser radiation in the form of a fundamental TEM_{00q} mode (Fig. 1). The corresponding field $\mathbf{E}$ takes a cylindrical coordinate system with z axis along the beam axis the form[10]

$$\mathbf{E}(\mathbf{r},t)=\mathbf{e}E_0\frac{q_0}{q}\exp\left(-\frac{\rho^2}{2q^2}\right)\cos\left[\omega t-\left(k+\frac{\rho^2}{bq^2}\right)z\right], \tag{4}$$

