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Self-limiting of a wave field following supersonic dispersal of a plasma

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It is shown that in a collisionless dispersing plasma moving at supersonic speed, striction monlinearity imposes an upper limit on the field intensity of the electromagnetic wave propagating in the plasma. This limit is due to the fact that in the supersonic stream the plasma is swept into the region of the strong electric field, and this decreases the refractive index and hinders the wave propagation. The field of s- and p-polarized radiation in a plasma with cubic nonlinearity is considered, where the plasma inhomogeneity is determined entirely by the influence of the ponderomotive forces. Self-confinement of the field is demonstrated in the case of s-polarization also for a plane-inhomogeneous linear plasma layer.

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One of the pressing problems of nonlinear theory of plasma is that of the behavior of a strong electromagnetic field in a moving dispersing plasma. A certain group of questions raised in this problem can be explained by numerically solving the equations of nonlinear electrodynamic and hydrodynamics (see, e.g., Ref. 1). At the same time, the numerous restrictions on the use of numerical methods leave many aspects of this problem uninvestigated. The present communication reports an attempt to investigate the electromagnetic field in a stationary inhomogeneous plasma stream. Recognizing that the case of subsonic flow is similar in many respects to the situation considered in a paper by one of us,[2] and striving to obtain qualitative results of interest for modern experiments, we focus our attention in this article on the case of supersonic plasma flow. With an aim also to make the results as conclusive as possible, we consider in detail the picture corresponding to a maximum deviation of the plasma density from the critical value, when the "striction" increment to the dielectric constant of the plasma can be accounted for by a cubic nonlinearity only.

Both a qualitative investigation of the equations of the nonlinear electrodynamics, and the analytic solutions obtained by us for the field, show that the supersonic flow in a classically transparent (in accord with the linear theory) plasma the striction nonlinearity imposes an upper limit on the electric field intensity. The reason is that in supersonic flow, in contrast to subsonic flow, the plasma is not expelled from the strong-field region but, on the contrary, becomes denser there. In the relatively weak field region the plasma becomes more tenuous. The increased density of the plasma in the strong-field region hinders the wave propagation. This effect takes place for both standing and for traveling waves, and for both s- and p-polarized radiation. What changes qualitatively in the supersonic flow is the structure of the electromagnetic soliton, in which the decrease of the plasma density is accompanied by a weakening of the electric field.

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1. The stationary one-dimensional flow of a non-isothermal plasma acted on by a ponderomotive force due to a high frequency field is described by the hydrodynamic equations (cf. Ref. 1)

$$\frac{\partial (nv)}{\partial x}=0,\quad v\frac{\partial v}{\partial x}=-\frac{v_s^2}{n}\frac{\partial n}{\partial x}-\frac{Ze^2}{4M_i m_e \omega_0^2}\frac{\partial |\mathbf{E}_0|^2}{\partial x}. \tag{1.1}$$

Here n is the density of the number of ions, v is their hydrodynamic velocity, v_s is the speed of sound, $Z|e|$ and M_i are the charge and mass of the ion, e and m_e are the charge and mass of the electrons, ω_0 is the frequency, and $E_0(x)$ is the coordinate-dependent electric-field intensity

$$\mathbf{E}(x,z,t)=\mathrm{Re}\{\mathbf{E}_0(x)\exp[-i\omega_0 t+iz(\omega_0/c)\sin\theta]\}, \tag{1.2}$$

where θ is the incidence angle. In (1.1) we assume the temperature, and hence the speed of sound, to be independent of the coordinates, as is approximately the case in experiment.

Equations (1.1) have the obvious integrals

$$\begin{gathered} n(x)v(x)=NV=\mathrm{const},\\ {}^1/_2 v^2(x)+v_s^2\ln n(x)+[Ze^2|\mathbf{E}_0(x)|^2/4M_i m_e\omega_0^2]={}^1/_2V^2+v_s^2\ln N=\mathrm{const}. \end{gathered} \tag{1.3}$$

Eliminating $v(x)$, we can write down the following equation that connects $n(x)$ with the electric field intensity:

$$ {}^1/_2N^2V^2/n^2(x)+v_s^2\ln n(x)+[Ze^2|\mathbf{E}_0(x)|^2/4M_i m_e\omega_0^2]={}^1/_2V^2+v_s^2\ln N. \tag{1.4}$$

When the hydrodynamic equations (1.1) are valid, the electroneutrality condition holds, meaning that the characteristic distance over which the field changes is large compared with the Debye radius. Under these conditions we can use the following expression for the high-frequency dielectric constant:

$$\varepsilon=1-(n/n_c), \tag{1.5}$$

where $n_c=m_e\omega_0^2/4\pi e^2 Z$ is the critical density of the ions. In our analysis we disregard small dissipative effects, all the more since the general scheme for taking these processes into account in nonlinear electrodynamics was developed earlier by one of us.[2]

Equations (1.4) and (1.5) constitute the material equations of the nonlinear electrodynamics of a stationary plasma stream. The qualitative deviation from the nonlinear electrodynamics of supersonic flow can be seen from the following relation that follows from (1.11):

$$(v^2-v_s^2)\frac{1}{n}\frac{\partial n}{\partial x}=\frac{1}{16\pi n_c M_i}\frac{\partial|\mathbf{E}_0(x)|^2}{\partial x}. \tag{1.6}$$

We see therefore that whereas in subsonic flow the maximum of $|\mathbf{E}_0|^2$ corresponds to a minimum of the density, in supersonic plasma flow this corresponds to maximum density. In other words, in the subsonic flow the strong field expels the plasma and by the same token decreases the nonlinear reaction of the plasma on the field, thus precluding limitations on the field strength in a tenuous plasma. On the contrary, in a supersonic flow the pondermotive force sweeps the plasma into the region of the strong field, thus increasing the nonlinear reaction of the plasma on the electromagnetic field. Inasmuch as in a dense plasma the electromagnetic field is subject to the skin effect, it should be clear that an upper bound on the electromagnetic field intensity should exist in supersonic flow.

We note that nonlinear electrodynamics of a dispersing plasma deals only with a unique regime in which the stream velocity oscillates in space between the sonic and supersonic values.[3] In this regime, the plasma converged on the critical-density region at supersonic velocity. In our analysis that follows we shall deal with supersonic flow in the vicinity of the critical density and with a relatively weak electromagnetic field, when the regime dealt with in Ref. 3 is not realized.

In the case of interest to us, that of fields whose pressure is low compared with the thermal pressure, it is possible to demonstrate distinctly the concrete peculiarity of the nonlinear electrodynamic properties of a supersonic plasma stream. We can assume in this case

$$n(x)=N+\delta n(x),\quad v(x)=V+\delta v(x) \tag{1.7}$$

and regard δn and δv as small compared with N and V respectively. We then have from (1.3) and (1.4)

$$\delta n=\frac{N}{V^2-v_s^2}\frac{|\mathbf{E}_0(x)|^2}{16\pi n_c M_i}, \tag{1.8}$$

$$\delta v=-\frac{V}{V^2-v_s^2}\frac{|\mathbf{E}_0(x)|^2}{16\pi n_c M_i}. \tag{1.9}$$

Equations (1.5) and (1.8) allow us to write down the following material equation

$$\varepsilon=1-\frac{N}{n_c}-\frac{N}{n_c}\frac{|\mathbf{E}_0(x)|^2}{16\pi n_c M_i(V^2-v_s^2)}, \tag{1.10}$$

which we shall use below to investigate the behavior of the magnetic field in a dispersing plasma. Bearing in mind a discussion of the consequences of (1.6), we restrict ourselves to the region of classical (linear) transparency of the plasma, when $N\leqslant n_c$.

2. We consider first the case of s-polarization, when $\mathbf{E}_0$ is oriented along the y axis. Assuming

$$E_{0y}(x)=iE(x)\exp[-i\varphi(x)],$$

we can write expression (1.2) in the form

$$E_y(x,z,t)=E(x)\sin[\omega_0 t-(\omega_0/c)z\sin\theta+\varphi(x)]. \tag{2.1}$$

Accordingly, the material equation (1.10) takes on the form

$$\varepsilon=\varepsilon_0-[E(x)/E_V]^2, \tag{2.2}$$

where

$$\varepsilon_0=1-\frac{N}{n_c},\quad E_V^2=\frac{n_c}{N}16\pi n_c M_i(V^2-v_s^2). \tag{2.3}$$

Formula (2.1) makes it possible, in analogy with the procedure in Ref. 2, to write down the following two integrals of the field equations:

$$-E^2(x)\varphi'(x)=M_s=\mathrm{const}, \tag{2.4}$$

$$(E')^2+(M_s^2/E^2)+U(E^2)=\mathscr{E}_s=\mathrm{const}, \tag{2.5}$$

$$U(E^2)=\omega_0^2c^{-2}\{(\varepsilon_0-\sin^2\theta)E^2-(E^4/2E_V^2)\}. \tag{2.6}$$

The left-hand side of (2.5) can be regarded as the sum of the effective kinetic, centrifugal, and potential energies, and the corresponding problem of finding the intensity of the electric field becomes analogous to the problem of particle motion in a central field. Figure 1 shows the dependence of the sum of the centrifugal and potential energy at $\varepsilon_0 > \sin^2\theta$. It is seen from this figure that finite motion in this effective potential field is possible at certain values of $\mathscr{E}_s$. The electric field intensity has then an upper bound. In the particular case when there is no energy flux (standing wave), and $M_s = 0$, the maximum field value

$$E_{s,\,max} = E_V(\varepsilon_0 - \sin^2\theta)^{1/2} \tag{2.7}$$

occurs at

$$\mathscr{E}_s = {}^1/_2\omega_0{}^2c^{-2}(\varepsilon_0 - \sin^2\theta)^2E_V{}^2. \tag{2.8}$$

In accordance with Fig. 1, at $M_s = 0$ we have from (2.5) (cf. Ref. 2)

$$E(x) = E_V(\varepsilon_0 - \sin^2\theta - \varepsilon_1) \times \mathrm{sn}\left(\frac{x\omega_0}{\sqrt{2}c}[\varepsilon_0 - \sin^2\theta + \varepsilon_1]^{1/2}, k\right), \tag{2.9}$$

$$\varepsilon_1 = \left[(\varepsilon_0 - \sin^2\theta)^2 - \frac{2c^2\mathscr{E}_s}{\omega_0{}^2E_V{}^2}\right]^{1/2}, \quad k = \frac{\varepsilon_0 - \sin^2\theta - \varepsilon_1}{\varepsilon_0 - \sin^2\theta + \varepsilon_1}. \tag{2.10}$$

We note that when (2.8) is satisfied the oscillatory dependence (2.9) turns into

$$E(x) = E_V(\varepsilon_0 - \sin^2\theta)^{1/2}\,\mathrm{th}(k_mx), \quad k_m = \frac{\omega_0}{\sqrt{2}c}(\varepsilon_0 - \sin^2\theta_0)^{1/2}. \tag{2.11}$$

This solution corresponds to

$$\varepsilon = \varepsilon_0 - (\varepsilon_0 - \sin^2\theta)\,\mathrm{th}^2(k_mx),$$
$$\delta n = n_c(\varepsilon_0 - \sin^2\theta)\,\mathrm{th}^2(k_mx),$$
$$\delta v = -V\frac{n_c}{N}(\varepsilon_0 - \sin^2\theta)\,\mathrm{th}^2(k_mx).$$

The foregoing formulas allow us to call the solution (2.11) a supersonic soliton. In contrast to the caviton,[1] in which the density well is filled with an intense high-frequency electric field, in our case of (2.11) the bottom of the density well corresponds to a zero electric-field intensity. On the contrary, for the magnetic field we have

$$B_x(x, z, t) = -\sin\theta E_y(x, z, t),$$
$$B_z(x, z, t) = \frac{E_V(\varepsilon_0 - \sin^2\theta)\cos[\omega_0t - (\omega_0/c)z\sin\theta]}{\sqrt{2}\,\mathrm{ch}^2(k_mx)}. \tag{2.12}$$

In particular, at $\theta = 0$ the magnetic field inside the supersonic soliton is a maximum and decreases with increasing distance from the density well. We must point out in this connection the possibility of propagation in the plasma of peculiar magnetoacoustic nonlinear waves constituting traveling density wells filled with high-frequency magnetic field, similar in some respect to electroacoustic nonlinear waves.[4]

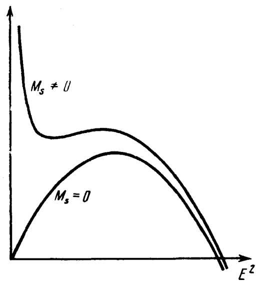

FIG. 1.

At $\mathscr{E}_s$ larger than in (2.8) or less than zero, there are no oscillating solutions, i.e., there are no solutions corresponding to a wave dependence of the field. It can therefore be stated that in a dispersing plasma the electric field intensity of s-polarized standing waves is bounded by the condition

$$E_s{}^2/4\pi \leqslant 4n_cZ\varkappa T_e\{V^2/v_s{}^2 - 1\}\{\cos^2\theta - N/n_c\}(n_c/N). \tag{2.13}$$

It follows from this, in particular, that the maximum possible field amplitude decreases both when V approaches the speed of sound and when N approaches the value $n_c\cos^2\theta$.

We note that for a traveling wave, when $M_s \neq 0$, the maximum electric field intensity turns out to be less than given by formula (2.13) and decreases somewhat with increasing energy flux density in the traveling wave. When the energy flux density reaches a value close to that determined by the right-hand side of (2.13), wave propagation becomes impossible. This can be directly seen from the following solution obtained for Eq. (2.5) at $M_s \neq 0$ (cf. Ref. 2):

$$E^2(x) = E_1{}^2 + (E_2{}^2 - E_1{}^2)\,\mathrm{sn}^2\{[\omega_0(x - x_0)/\sqrt{2}cE_V](E_3{}^2 - E_1{}^2)^{1/2}, [(E_2{}^2 - E_1{}^2)/(E_3{}^2 - E_1{}^2)]^{1/2}\}, \tag{2.14}$$

where $E_1^2 \leqslant E_2^2 \equiv E_{\max}^2 \leqslant E_3^2$ are the three real roots of the equation

$$\frac{M_s{}^2}{E^2} + \frac{\omega_0{}^2}{c^2}\left\{(\varepsilon_0 - \sin^2\theta)E^2 - \frac{E^4}{2E_V{}^2}\right\} = \mathscr{E}_s. \tag{2.15}$$

We note that at $E_1 = E_2$ formula (2.14) describes a nonlinear plane wave with constant $E(x)$. A solution in the form of a plane wave $E(x) = E_{\max}$ exists also at $E_2 = E_3 = E_{\max}$, when formula (2.14) describes a soliton

$$E^2(x) = E_1{}^2 + (E_{max}^2 - E_1{}^2)\,\mathrm{th}^2[x\omega_0c^{-1}(E_{max}^2 - E_1{}^2)^{1/2}/\sqrt{2}E_V],$$

that goes over as $M_s \to 0$ $(E_1 \to 0)$ into the soliton (2.11).

According to (2.15), propagation of the traveling waves becomes impossible at

$$\frac{cM_s}{\omega_0} > \frac{32\pi}{\sqrt{27}}Z\varkappa T_e\frac{n_c{}^2}{N}\left[\cos^2\theta - \frac{N}{n_c}\right]^{3/2}\left(\frac{V^2}{v_s{}^2} - 1\right). \tag{2.16}$$

At the corresponding maximum value of the energy flux density, the maximum electric field intensity is given by

$$E_{max}(M_s) = ({}^2/_3)^{1/2}E_V(\varepsilon_0 - \sin^2\theta)^{1/2}. \tag{2.17}$$

To conclude this section, we emphasize that the analytic formulas obtained in this section for the field are valid under conditions of weak nonlinearity, which call for smallness of $\varepsilon_0 - \sin^2\theta$.

3. We proceed now to consider p-polarized radiation. Bearing in mind the application of the results to a plasma with infrequent collisions and with a density higher than critical in the interior of the plasma, we confine ourselves here to the case of standing waves. The principles of nonlinear electrodynamics of p-polarized waves are treated in a paper by Eleonskii and one of us.[5] Assuming in accordance with that reference $E_{0x} = E_x(x)$ and $E_{0z} = iE_z(x)$, we have

$$E_x(x,z,t)=E_x(x)\cos[\omega_0 t-z(\omega_0/c)\sin\theta],$$
$$E_z(x,z,t)=E_z(x)\sin[\omega_0 t-z(\omega_0/c)\sin\theta]. \quad (3.1)$$

Equation (1.10) then takes the form

$$\varepsilon=\varepsilon_0-[E_x^2(x)+E_z^2(x)]E_V^{-2}. \quad (3.2)$$

The system of field equations reduces to

$$-E_z''+E_x'(\omega_0/c)\sin\theta=\omega_0^2\varepsilon E_z/c^2,$$
$$-\sin\theta E_z'+E_x(\omega_0/c)\sin^2\theta=\omega_0\varepsilon E_x/c,$$
$$(\varepsilon E_x)'=E_z\varepsilon(\omega_0/c)\sin\theta. \quad (3.3)$$

The last of these three equations is the consequence of the first two. It will be useful, however, for the analysis that follows. In accordance with Ref. 5, we have from the field equations (3.3)

$$(E_z')^2+\omega_0^2c^{-2}\{-\sin^2\theta E_x^2+\varepsilon_0(E_x^2+E_z^2)-[(E_x^2+E_z^2)^2/2E_V^2]\}=\mathscr{E}_p=\text{const}. \quad (3.4)$$

With the aid of (3.4) and the first-order equations of the system (3.3) we can both investigate the solutions qualitatively and solve the field equations. To make the exposition more compact, we introduce the notation

$$\xi=\frac{\sin\theta}{(\varepsilon_0)^{1/2}},\quad \mathscr{E}^2=1-\frac{2c^2\mathscr{E}_p}{\omega_0^2\varepsilon_0E_V^2},\quad \zeta=x\frac{\omega_0}{c}(\varepsilon_0)^{1/2}. \quad (3.5)$$

Then, changing to the dimensionless variables

$$E_x(x)=E_V(\varepsilon_0)^{1/2}g(\zeta),\ E_z(x)=E_V(\varepsilon_0)^{1/2}h(\zeta), \quad (3.6)$$

we write down, in accord with (3.3) and (3.4), the following system of equations:

$$(h')^2-\xi^2g^2-{}^1/_2[1-g^2-h^2]^2=-{}^1/_2\mathscr{E}^2, \quad (3.7)$$

$$-\xi h'=g[1-\xi^2-g^2-h^2], \quad (3.8)$$

$$g'(1-3g^2-h^2)-2ghh'=\xi h(1-g^2-h^2). \quad (3.9)$$

Equations (3.7) and (3.8) enable us to express h^2 in terms of g^2:

$$h^2=f_\pm(g^2)=(2g^2-\xi^2)^{-1}\{-2g^4+2g^2-\xi^2-g^2\xi^2\pm\xi[4g^4\xi^2-\mathscr{E}^2(2g^2-\xi^2)]^{1/2}\}. \quad (3.10)$$

Next, with the aid of this formula and (3.7) we ultimately find

$$x\omega_0c^{-1}(\varepsilon_0)^{1/2}=\int_0^g dg(2g^2-\xi^2)f_\pm'(g^2)[f_\pm(g^2)]^{-1/2}\times\{\xi^3\mp[4g^4\xi^2-\mathscr{E}^2(2g^2-\xi^2)]^{1/2}\}^{-1}. \quad (3.11)$$

Formula (3.11) determines the dependence of $E_x(x)$ on the coordinates, and in accord with formula (3.10) also the dependence of $E_z(x)$. In the particular case $\mathscr{E}^2=0$ we have

$$E_x(x)=E_V(\varepsilon_0)^{1/2}\cos(x\omega_0c^{-1}\sin\theta),\quad E_z(x)=E_V(\varepsilon_0)^{1/2}\sin(x\omega_0c^{-1}\sin\theta). \quad (3.12)$$

The solution (3.12) corresponds, according to (3.2), to a zero nonlinear dielectric constant. This means that in this state the plasma density has become equal to critical. The peculiarity of the solution (3.12) can be understood by recalling the singular behavior of the solutions of linear electrodynamics near $\varepsilon=0$. It is important that the limiting state (3.12) is realized at a finite electric field intensity, when

$$E_{p,max}^2/4\pi=4n_cZ\varkappa T_e\{(V^2/v_s^2)-1\}\{1-(N/n_c)\}(n_c/N). \quad (3.13)$$

Just as in the case of (2.12), the left-hand side decreases here when V approaches the speed of sound, and also when N approaches the critical density. It should be noted that the solution (3.12) can take place both in the classical region of the linear transparency, when $N<n_c\cos^2\theta$, and in the region of linear opacity $n_c\cos^2\theta<N<n_c$.

The general discussion of the question of the maximum field of a p-polarized wave in a plasma with a dielectric constant (3.2) is best carried out on the basis of equations (3.7), (3.8), and (3.9). We note first that according to (3.8) the extremum of the field E_z can be realized at points of two types. At the extremal points ζ_t of the first type E_x vanishes:

$$g(\zeta_t)=0,\quad h^2(\zeta_t)=1\pm|\mathscr{E}|. \quad (3.14)$$

At the extremal points of the second type we have

$$g^2(\zeta_t)+h^2(\zeta_t)=1-\xi^2 \quad (3.15)$$

and

$$g^2(\zeta_t)={}^1/_2\xi^{-2}(\mathscr{E}^2-\xi^4),\quad h^2(\zeta_t)={}^1/_2\xi^{-2}\{[1-(1-\xi^2)^2]-\mathscr{E}^2\}. \quad (3.16)$$

The largest extremal point, corresponding to E_z^2 is given by Eq. (3.14) with the plus sign. This point corresponds to the minimum value of h^2 on the segment $(+\infty,1+|\mathscr{E}|)$, which does not correspond to spatially oscillating solutions. The remaining two possible extremal points, as seen from (3.14) and (3.15), are such that in them

$$g^2(\zeta_t)+h^2(\zeta_t)\leqslant 1. \quad (3.17)$$

This means that the intensity of the electric field at these points cannot exceed the value determined by (3.13).

We note next that from the fact that the left-hand side of (3.15) is positive it follows that the extremal points of the second type are possible only under conditions corresponding to the classical region of linear transparency $\xi^2<1$ or, equivalently, $N<n_c\cos^2\theta$. In addition, it follows from (3.16) that the following inequalities should hold:

$$1>1-(1-\xi^2)^2>\mathscr{E}^2>\xi^4. \quad (3.18)$$

Therefore we have according to (3.16) and (3.18)

$$h^2(\zeta_t)<1-\xi^2,\quad g^2(\zeta_t)<{}^1/_2\xi^{-2}(1-\xi^4). \quad (3.19)$$

We see thus that the z component of the electric field intensity cannot exceed the value determined by (3.13). We note that the smaller extremal point (3.14) can be realized, just as that of the points of the second type, only under the condition $\mathscr{E}^2<1$. The largest of all possible values of the field h, corresponding to solutions that oscillate in space, will take place at $\mathscr{E}=0$, when $h^2(\zeta_t)=1$, i.e., in the case of the solution (3.12).

We turn now to a discussion of the limitation imposed on the longitudinal field E_x. At the extremal points ζ_x corresponding to this field, when $g'(\zeta_x)=0$, we have in accordance with (3.9)

$$-2g(\zeta_l)h(\zeta_l)h'(\zeta_l)=\xi h(\zeta_l)[1-g^2(\zeta_l)-h^2(\zeta_l)]. \quad (3.20)$$

If we assume that $h(\zeta_l)\neq 0$, it can be easily verified that a simultaneous solution of Eqs. (3.8) and (3.20) leads only to values $g^2(\zeta_l)<1$. It remains therefore to consider $h(\zeta_l)=0$. It follows then from (3.7) and

(3.8) that

$$g^2(\zeta_l)[1-g^2(\zeta_l)]^2=\xi^2[g^2(\zeta_l)+{}^1/_2(1-\mathscr{E}^2)-{}^3/_2g^4(\zeta_l)]. \quad (3.21)$$

In order for the right-hand side of this equation to be nonnegative, it is necessary to satisfy the inequality

$$g^2(\zeta_l)\leqslant{}^1/_3\{1+[1+3(1-\mathscr{E}^2)]^{1/2}\}\leqslant 1. \quad (3.22)$$

Account was taken here of the fact that $0 \leqslant \mathscr{E}^2 \leqslant 1$. It follows from (3.22) that the intensity of the longitudinal field is also limited to the value (3.13), which is realized only at $\mathscr{E}^2 = 0$.

Thus, when p-polarized radiation interacts with a supersonic plasma stream, both electric-field components are bounded, and the maximum possible field values of both components decrease when the unperturbed density approaches the critical value [see (3.13)].

4. The foregoing analysis allows us to state that when radiation propagates in a supersonic stream of a nonuniform plasma, the amplitude of the possible stationary solutions is also limited, at least in the case of sufficiently gently sloping profiles of the unperturbed density. To make this conclusion even more obvious, we present here results pertaining to the case of a linear layer, when $\varepsilon_0 = \sin^2\theta - x/L$, and neglect the dependence of E_Y on the coordinate. This is permissible near the critical density (reflection points), when the parameter $(L\omega_0/c)^{-1/3}$ is small compared with unity. For the case of s-polarization [see (2.1)], the field equation takes the Painleve form[6]:

$$u''(z)=zu(z)+u^3(z)\,\mathrm{sign}(V^2-v_s^2). \quad (4.1)$$

We have used here the following dimensionless variables

$$z=x(\omega_0/c)(L\omega_0/c)^{-1/3}, \quad u(z)=E(x)(L\omega_0/c)^{1/3}/|E_Y|. \quad (4.2)$$

We consider for this equation solutions that decrease as $z\to+\infty$, corresponding to the same asymptotic form of the solutions of the two possible values of $\mathrm{sign}(V^2 - v_s^2)$:

$$u(z)={}^1/_2A\pi^{-1/2}z^{-1/4}\exp(-{}^2/_3z^{3/2}), \quad z\to+\infty. \quad (4.3)$$

For small values of the amplitude $A \ll 1$, Eq. (4.1) corresponds to the linear Airy equation and its solutions using the asymptotic form (4.3) go over into the Airy function regardless of the sign of $V^2 - v_s^2$, as seen from Fig. 2a, which shows the solution of Eq. (4.1) for $A = 0.1$ with the asymptotic form (4.3).

In the case of subsonic flow $V^2 < v_s^2$, the solution of Eq. (4.1) retains the essential properties of the Airy function with increasing amplitude A, although the deformation of the density profile does cause an obvious shift of the reflection point and a shift of the maximum of the field towards the denser layers of the plasma. This is clearly demonstrated by comparison of Figs. 2a and 2b, the latter showing the solution of Eq. (4.1) for $A = 10$ and $\mathrm{sign}(V^2 - v_s^2)$. In this case, naturally, there is no restriction on the amplitude of the field.

The situation is qualitatively different in the case of supersonic plasma flow, when $V^2 > v_s^2$. Above all, in this case the deformation of the density profile causes the maximum of the field to shift towards the more tenuous plasma layers, as seen from Fig. 2c, which shows the solution of Eq. (4.1) for $(V^2 - v_s^2) = +1$ and $A = 1.4169$ using the asymptotic form (4.3). The most important property of the supersonic plasma flow in our analysis, however, is the absence of solutions of the Painleve equation (4.1) for a field with decreasing asymptotic form (4.3) at an amplitude $A > 1.417$; this obviously means a limitation on the possible value of the electric field intensity of the electromagnetic wave propagating in an inhomogeneous supersonic plasma stream. We emphasize that according to (4.2) the maximum possible electric field intensity decreases like $(L\omega_0/c)^{-1/3}$ with increasing characteristic dimension of the plasma inhomogeneity.

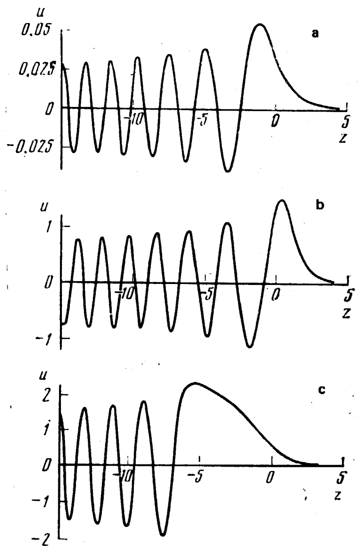

FIG. 2.

Summarizing all the foregoing, we can draw the following conclusion. In a supersonic plasma stream, the plasma striction nonlinearity that leads to an increase of the particle density in the region with stronger fields, a limitation is imposed on the possible field intensity, inasmuch as the plasma transparency decreases with increasing field. This conclusion allows us to state that when radiation propagates in an inhomogeneous supersonic stream under condition when the plasma is subject to cubic nonlinearity (i.e., when the pressure of the field is insufficient to slow down the supersonic flow), the stationary value of the electric field in the vicinity of the reflection point turns out to be limited and decreases with increasing characteristic dimension of the inhomogeneity, in contrast to the situation in linear electrodynamics. This conclusion distinguishes qualitatively the foregoing results from those obtained earlier in the theory of a quiescent plasma[2] and in the theory devoted to the case of subsonic influx of plasma into the critical-density region.[3] At the same time, our analysis is in qualitative agreement with an experiment[7] that shows the field in the plasma to decrease on going from subsonic to supersonic flow.

APPENDIX I

We consider here the non-wave solution of Eq. (2.5), corresponding to $M_s = \mathscr{E}_s = 0$. We then have from (2.5)

$$E(x) = E_V[2(\sin^2\theta - \varepsilon_0)]^{1/2}/\mathrm{sh}[x(\omega_0/c)(\sin^2\theta - \varepsilon_0)^{1/2}]. \quad (A.1)$$

This solution is possible only in the opacity region, when $\sin^2\theta > \varepsilon_0$. For a plasma with an abrupt boundary, Eq. (A.1) can describe the nonlinear screening of a high-frequency electromagnetic field. The field that decreases monotonically inside the plasma can in this case exceed the value (2.7), inasmuch as at $x(\omega_0/c)(\sin^2\theta - \varepsilon_0)^{1/2} \ll 1$ formula (A.1) yields

$$E(x) = \sqrt{2}\, cE_V/\omega_0 x. \quad (A.2)$$

Accordingly, the maximum field can be exceeded only in the very unusual penetration region, when the intensity of the electric field turns out to be inversely proportional to the coordinate. In this region we have for the dielectric constant

$$\varepsilon = (-2c^2/\omega_0^2 x^2) + {}^1/_3\varepsilon_0 + {}^2/_3 \sin^2\theta. \quad (A.3)$$

This expression, while not containing E_V, corresponds to an essentially nonlinear region. In our approximation (1.8) we must assume $\sqrt{2}c/(\omega_0 x) \ll 1$, for which purpose $\sin^2\theta - \varepsilon_0$ must be small.

APPENDIX II

The material equation (1.10) corresponds to a stationary picture of the field. On the other hand, if we are interested in the nonstationary problem, we can write in the same approximation the following equation

$$\left\{\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)^2 - v_s^2\frac{\partial^2}{\partial x^2}\right\}\frac{\delta n}{N} = \frac{1}{16\pi n_e M_i}\frac{\partial^2}{\partial x^2}|\mathbf{E}_0(x,t)|^2.$$

It is easily seen that the intensity of the nonstationary field need not necessarily become self-limited. This can be easily verified using as the example the functions $\delta n(x - vt), \mathbf{E}_0(x - vt)$, for which

$$\frac{\delta n}{N} = \frac{1}{(V-v)^2 - v_s^2}\frac{|\mathbf{E}_0|^2}{16\pi n_e M_i}.$$

It is clear therefore that in such a case of a nonstationary dependence of the field on the time and on the coordinate the self-limitation effect will take place under the condition $(V - v)^2 > v_s^2$.

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