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Temperature dependence of the conductivity of point junctions between a superconductor and a normal metal

S. I. Dorozhkin

Institute of Solid State Physics, USSR Academy of Sciences
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The temperature dependences of the current-voltage characteristics of point junctions between a superconductor (aluminum) and a normal metal (silver) were investigated experimentally and estimates were made of the parameters of the junctions and of the mean free path l of the electrons in the region of the junction. The measured temperature dependences are close to those predicted by Zaĭtsev [Sov. Phys. JETP 51, 111 (1978)] for junctions with diameter $d \ll (l, \xi_0)$, although the assumptions of the theory were not realized in the experiments. A simple model is proposed for the conduction of point junctions with $\xi(T) \ll d \ll l_\varepsilon$, where l_ε is the energy relaxation length of the electrons in the normal state of the superconductor. It is shown on the basis of this model that Zaĭtsev's results are valid also for such junctions.

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The current-voltage characteristics of point junctions between normal metals are linear if the junction voltage U is such that $eU \ll k\Theta_D$ (Θ_D is the Debye temperature),[2] and do not depend on the temperature at sufficiently low temperature. The transition of one of the electrodes of the junction into the superconducting state leads to the appearance of noticeable nonlinearities (see Fig. 1). Below the superconducting-transition temperature T_c, a change takes place in the differential resistance $r = dU/dI$ at $U = 0$, and the CVC takes in an appreciable current interval the form

$$I = U/R + I_0 \operatorname{sign} U,$$

where R is the resistance of the junction when both electrodes are in the normal state, I_0 does not depend on U and is called the excess current. These features of the CVC were first noted by Pancove.[3]

The values of r and I_0 depend on temperature below T_c. Chien and Farrell[4] observed a differential-resistance temperature dependence of the form $r \approx R - R_1\Delta(T)/\Delta(0)$, where R_1 is a constant and $\Delta(T)$ and $\Delta(0)$ are the energy gaps in the superconductor at temperatures T and $0K$, respectively. Gubankov and Margolin[3] obtained an experimental dependence that agreed with the corresponding dependence of the energy gap in the superconductor, while the differential resistance varied nonmonotonically with temperature.

Recent theoretical papers[1,6] dealing with the conductivity of S-c-N (superconductor–constriction–normal metal) junctions, which include also the point junctions, have shown that the results depend on the relations between the electron mean free path l, the super-conductor coherence length ξ_0, and the geometric dimensions d and l of the constriction. (The constriction is regarded as a cylinder of diameter d and length L, which joins massive materials.) Both references are devoted to the study of junctions with $(d, L) \ll \xi(T)(1 - T/T_c)^{1/4}$ [$\xi(T)$ is the coherence length in the superconductor at the temperature T], but the restrictions on the electron mean

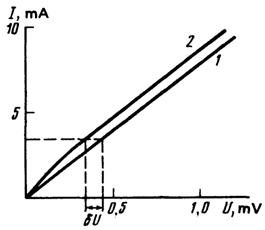

FIG. 1. Current-voltage characteristics of a point junction between aluminum and silver. $R = 0.128\Omega$, 1) $T = 1.4$ K, both electrodes in the normal state; 2) $T = 0.65$ K, the aluminum is superconducting. δU is the change of the junction voltage when the aluminum becomes superconducting (the current through the junction is fixed.

free path of the electrons are different in these papers. In the case $l \ll \max(d, L)$ (Ref. 6) the excess current is proportional to the energy gap in the superconductor, and $r(T)$ is a nonmonotonic function of temperature: $r(T) \leq R$, with $r=R$ at $T=T_c$ and $T=0K$. Temperature dependences of this kind were observed by Gubankov and Margolin.[5] If $L \ll d \ll (l, \xi_0)$ (Ref. 1), then the excess current is proportional as before to the energy gap, and the differential resistance decreases monotonically when the temperature drops below T_c. The last result agrees qualitatively with the $r(T)$ temperature dependence obtained by Chien and Farrell.[4]

The cited experiments were performed on clamped junctions. The question of the relation between d, L, l, and ξ_0 was not investigated at all. This is not a simple problem, since there is every reason for assuming (see, e.g., Ref. 7) that in the course of preparation of point junction there are produced in the junction region contaminations and structural distortions of the crystal lattices of the metals. This leads to an appreciable decrease of the electron mean free path in the region of the junction compared with the value in the interior of the metal.

In the present study we investigated point junctions obtained by electric breakdown of a narrow layer of an insulator. We observed that in these junctions the temperature dependences of the excess current and of the differential resistance are close to those predicted by the theory for the case $d \ll (l, \xi_0)$. Measurements of the temperature dependence of the junction resistance $R(T)$ in the interval 1.3–200 K have made it possible to estimate the values of d and l. It turned out, however, that in our case the electron mean free path near the junction is small, so that $l \ll \xi(0) \leq d$, where $\xi(0)=(\xi_0 l)^{1/2}$, and ξ_0 is the coherence length in the pure superconductor. Consequently, the conditions for the validity of Zaĭtsev's theory[1] are not satisfied in the region of the junction. By way of explanation of the results we propose a simple model of the conductivity of a point junction with dimensions $L \ll \xi(T) \ll d \ll l_\varepsilon$, where l_ε is the electron energy relaxation length in the normal state of the superconductor. It is shown on the basis of this model that when the foregoing conditions are satisfied we can expect a monotonic dependence temperature dependence of the differential resistance and the presence of an excess current, with $I_0 \sim \Delta$.

EXPERIMENT

The procedure used to prepare the point junctions is analogous to that used in an earlier paper.[8] The superconducting electrode at $T<1.2$ was an aluminum single crystal with resistivity ratio $\rho(300\ \text{K})/\rho(4,2\text{K}) = 2 \cdot 10^4$. The second electrode was a silver film sputtered on the crystal surface. The film thickness varied from sample to sample and amounted to ~1 μm. For the junctions for which the temperature dependence of $R(T)$ was measured, the film thickness was 1.6 μm, and the film resistance ratio was $R_f(300\ \text{K})/R_f(4.2\text{K}) = 10$. The microjunctions were produced by electric breakdown of the oxide layer on the aluminum surface. The breakdown was produced at helium temperatures by a voltage 3–10 V from a 5×10 μF capacitor through a 10 $k\Omega$ resistor. To prevent the contact parameters from changing as a result of diffusion, the sample was constantly kept at a temperature lower than 78 K after the production of the junction. After completing the entire measurement cycle, the temperature dependence of the resistance was measured at higher temperatures.

In the experiment we measured the temperature dependence of the voltage on the junction at a fixed current through the junction. It is clear from Fig. 1 that the change δU of the junction voltage is proportional to the change of the differential resistance $R-r$ at small currents, and to the excess current I_0, at large currents, with $\delta U = R I_0$. The measurements were made by a four-contact method, and the voltage was measured accurate to 1 μV. In a number of measurements of r this accuracy was insufficient. Higher accuracy of the measurements of the differential resistance was ensured by a modulation procedure: an alternating current of approximate frequency 4 kHz was passed through the junction and the amplitude of the current was fixed; the alternating voltage on the junction was less than 20 μV and was amplified with a tuned amplifier and detected. The rectified current is proportional to r in this case.

The transition of the sample into the superconducting state was revealed by the change of the surface impedance. To measure the impedance, a flat coil (20 turns of copper wire of 0.05 mm diameter) was placed on the surface and was connected to the tank circuit of an RF oscillator operating approximately at 1.5 MHz. The change of the sample impedance led to a change of the oscillator voltage measured with an electron-counting frequency meter.

Temperatures 1.3–0.6 K were obtained by pumping on He^3 vapor and measured with a carbon resistance thermometer calibrated against the saturated vapor pressure of He^3. The accuracy with which these temperatures were measured was ~5 mK. The sample was placed in liquid He^3 at a distance 1 cm from the thermometer. Temperatures 1.3–200 K were measured with the cryostat slowly heated. At $T>4.2$ K the temperature was measured with a copper-constantan thermocouple. The earth's magnetic field was reduced to approximately 0.05 Oe with two pairs of Helmholtz compensation coils mounted on the outside of the cryostat.

We prepared 17 junctions with resistances in the range $(3\times 10^{-2} - 1)$ Ω at 4.2 K. The CVC of all the junction revealed an excess current below the superconducting-transition temperature T_c of aluminum, and the differential resistance r at $T \approx 0.5T_c$ was noticeably lower than the normal resistance R. For the measurements of the excess current I_0 and of the differential resistance r, we selected eight junctions whose CVC were linear and independent of temperature from 4.2 K to T_c. This selection was necessary, since the CVC of the remaining junctions starged to change at temperatures noticeably higher than T_c, and revealed nonlinearities on top of those described above. These additional nonlinearities are not discussed in the present

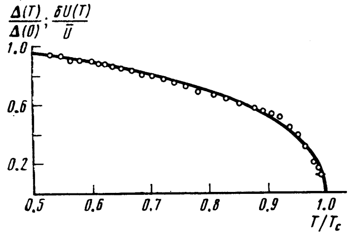

FIG. 2. Change δU of junction voltage vs. temperature (points). $R=0.18\Omega$, $\overline{U}=79\,\mu$V. Solid curve—temperature dependence of the relative gap $\Delta(T)/\Delta(0)$ according to the BCS theory.

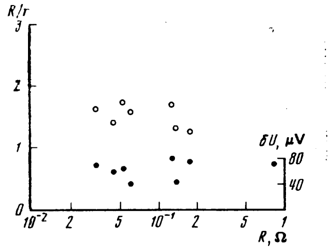

FIG. 4. Values of R/r (light circles) and $\delta U/RI_0$ (dark circles) measured at $T\approx 0.5T_c$, for eight investigated junctions. The theoretical values of these quantities at $T=0.5T_c$ and at an energy gap $\Delta(0)=180\,\mu$keV in aluminum are $R/r\approx 1.1$; $\delta U\approx 126\,\mu$V (Ref. 6); $R/r\approx 2.91$; $\delta U\approx 459\,\mu$V (Ref. 1).

paper.

The temperature dependence of $\delta U(T)=RI_0(T)$ was measured on four junctions. The results $\delta U(T)/\overline{U}$ can be fitted with accuracy not worse than 10% to the temperature dependence of the reduced gap $\Delta(T)/\Delta(0)$ (see Figs. 2 and 3) by selecting the proportionality coefficient $\overline{U}$ for each of the junctions. A run of measurements of $\delta U(T)$, $r(T)$ at $T<T_c$ and $R(T)$ was performed on two of the junctions. The temperature dependences of $\delta U(T)/\overline{U}$ and $[R-r(T)]/[R-r(T=0.57T_c)]$ are shown in Fig. 3. The same figure shows the values of $[R-r(T)]/[R-r(0)]$, calculated by us numerically from formula (45) of Zaĭtsev's paper.[1] We used in the calculation the temperature dependence of the energy gap $\Delta(T)$ in the superconductor and its absolute value $\Delta(0)=1.76\,kT_c$, which follow from the BCS theory. Figure 4 shows the absolute values of $\delta U=RI_0$ and R/r, measured at $T\approx 0.5T_c$ for all the selected junctions. No noticeable dependence of δU on R was observed within the limits of the deviations. This corresponds to an inverse proportionality of the excess current I_0 to the junction resistance R.

To estimate the size of the junction and the electron mean free path in the region of the junction we measured the temperature dependences of $R(T)$ in the range from 1.3 to 200 K (see Fig. 5). Notice should be taken of the linear $R(T)$ dependences at $T>50$ K and of the constancy of the resistance at $T<20$ K. Above 230 K we observed abrupt changes of R, due apparently to migration of the atoms in the region of the junction. In the reduction of these results it was assumed that the bridge length $L\ll d$ and that the electron mean free path l is constant in the region that determines the junction resistance, i.e., over distances on the order of d from the contact. It will be made clear below that the first assumption holds true with large margin, since L is determined by the thickness of the oxide layer on the aluminum, which is less than 100 A. If the electron mean free path varies noticeably over distances on the order of d from the junction, then the estimated value of l is the upper bound of the true electron mean free path in the region where the electric field is concentrated. The data were reduced using an interpolation formula[9] for the resistance of short ($L=0$) junctions of like metals with different electron mean free paths (l_1 and l_2)

$$R=(2d)^{-1}\{\gamma(K_1)\rho_1+\gamma(K_2)\rho_2\}+16\rho l/3\pi d^2. \tag{1}$$

Here ρ is the resistivity of the metal, $K=2l/d$, and $\gamma(K)$ is a slowly varying function: $\gamma(0)=1$, $\gamma(\infty)\approx 0.7$; we assume hereafter $\gamma=1$. The last term in (1) does not depend on temperature, since the quantity ρl does not depend on the electron mean free path but is a characteristic of the electrode material. In the case of a junction of unlike metals, this quantity should contain the averaged parameters of the metals. The concrete type of the averaging is immaterial to us, for it has turned out in our case (see below) that $16\rho l/3\pi d^2\ll R$ for both aluminum and silver.

It is easy to express with the aid of (1) the value of d in terms of the change $R(T)-R(0)$ of the junction resistance with change of temperature:

$$d=\frac{\rho_{i1}(T)+\rho_{i2}(T)}{2[R(T)-R(0)]}. \tag{2}$$

On going from (1) to (2) we used the Matthiessen rule $\rho(T)=\rho_i(T)+\rho_0$, where $\rho_i(T)$ and ρ_0 are respectively the ideal and residual resistances. For a junction with $R(1.3\text{ K})=0.128\ \Omega$ we have (see Fig. 5)

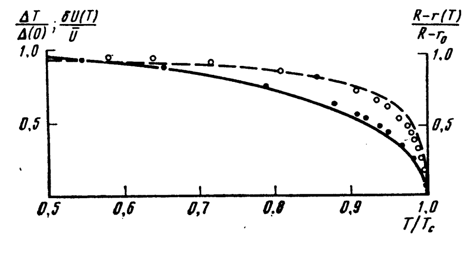

FIG. 3. Temperature dependences of $\delta U(T)/\overline{U}$ (dark points $U=84\,\mu$V) and $[R-r(T)]/[R-r_0]$ [light circles, $r_0=r(T=0.57T_c)$]. $R=0.128\Omega$. Solid curve—temperature dependence of the relative gap $\Delta(T)/\Delta(0)$ according to the BCS theory. Dashed curve—temperature dependence of $[R-r(T)]/[R-r_0]$, $[r_0=r(0)]$ according to the Zaĭtsev theory.[1]

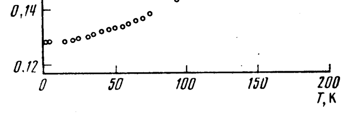

FIG. 5. Temperature dependence of the resistance of a junction whose resistance at low temperature is $R=0.128\ \Omega$.

$$d=\frac{\rho_{i1}(200\text{ K})+\rho_{i2}(200\text{ K})}{2[R(200\text{ K})-R(1.3\text{ K})]}=3.7\cdot10^{-5}\text{ cm},$$

where the ideal resistivity is $\rho_{i1}(200\text{ K})=1.04\cdot10^{-6}$ and $\rho_{i2}(200\text{ K})=1.78\cdot10^{-6}$ Ω-cm for silver and aluminum, respectively. At $d=3.7\times10^{-5}$ cm we have $16\rho l/3\pi d^2 < 1.1\cdot10^{-2}$ Ω, i.e., much less than the junction resistance. We used the values $\rho_1 l_1=0.86\cdot10^{-11}$ and $\rho_2 l_2=0.24\cdot10^{-11}$ Ω-cm for silver and aluminum, respectively. Neglecting the last term of (1) and assuming that the electron mean free paths of the two metals are equal, we obtain from (1)

$$l=\frac{\rho_1 l_1+\rho_2 l_2}{2Rd}.$$

The mean free path calculated from this formula at $T=1.3$ K for a junction with $R(1.3\text{ K})=0.128$ Ω is $l=1.2\times10^{-6}$ cm. This is much lower than the electron mean free path in the interior of the metals at this temperature. The reason is the high concentration of the scattering centers in the region of the junction, which occurs in the case of electric breakdown, and justifies the assumption that the mean free paths of the two metals do not differ greatly in the region of the junction.

DISCUSSION OF RESULTS

The temperature dependence in all the investigated junctions between the superconductor and the normal metal agrees with the temperature dependence of the energy gap in the superconductor (see Figs. 2 and 3). This result indicates that the two quantities are proportional, $I_0\sim\Delta$. The results of measurements on a large number of junctions (see Fig. 4) allow us to state that within the limits of the deviations the excess current is inversely proportional to the junction resistance, $I_0\sim1/R$.

The above results for the excess current agree well with the predictions of both theories[1,6]:

$$I_{01}=\Delta\left(\frac{1}{4}\pi^2-1\right)\Big/2eR\ [6],\qquad I_{02}=8\Delta/3eR\ [1].$$

The absolute values of the measured quantities

$$\delta U(T/T_c\approx0.5)=RI_0(T/T_c\approx0.5)$$

are much less than those calculated from the presented formulas (see Fig. 4). The results of the theories of Refs. 1 and 6 for the temperature dependence of the differential resistance are in qualitative agreement. The $r(T)$ dependences measured by us agree well with Zaĭtsev's results[1] (see Fig. 3), although the absolute values of the differential resistance differ significantly from those predicted by the theory (see Fig. 4). It is, however, unclear whether the Zaĭtsev theory,[1] which was developed for the pure case $d\ll(l,\xi_0)$ is valid under our conditions, since $l\ll d$ in the region of the junction. The low value of the electron mean free path leads to another important consequence, namely to a decrease of the coherence length in the region of the contact, which now equals at $T\ll T_c$.

$$\xi(0)=(\xi_0 l)^{1/2}=1.3\cdot10^{-5}\text{ cm}$$

[$\xi_0=1.36\times10^{-4}$ cm (Ref. 10) is the superconducting coherence length in pure aluminum]. We consequently have in the region of the junction

$$l\ll\xi(0)<d.$$

The first possible explanation of these results is that the point defects are concentrated at a distance L_0 from the junction, such that $L_0\ll d$, and Zaĭtsev's results[1] are valid in such a very inhomogeneous situation. Indeed, if $L_0\ll\xi(0)$, then the main change in the order parameter from zero in the normal state to the equilibrium value in the superconductor takes place in the region in which $l\gg d$ and the coherence length is $\xi_0>d$.

Another possibility is that the qualitative results of Ref. 1, namely the monotonic $r(T)$ dependence and the effect of the excess current, remain valid in a larger range of the parameters d, l, and ξ_0. We shall attempt below to show with the aid of a simple model that the same results can be expected also for junctions with dimensions $L\ll\xi(T)\ll d\ll l_\varepsilon$. At $L\ll\xi(T)\ll d$ a point junction can be represented as an opening of diameter d in an infinitely thin impermeable diaphragm that separates the superconductor from the normal metal, and it can be assumed that an abrupt N-S boundary is present in the opening. Artemenko and Volkov,[11,12] as well as Ovchinnikov,[13,14] have shown that when a current flows through an N-S boundary an electric field penetrates into the superconductor, so that the superconductor contributes to the resistance. The change of the electric field from its value in the normal metal to zero takes place in two stages: on the N-S boundary [over a distance $\xi(T)$] the electric field changes by an amount δE, and the further decrease of the field takes place in the superconductor exponentially over a distance $l_E[l_E\sim l_E(kT/\Delta)^{1/2}$ (see the review of Artemenko and Volkov[12]) at $kT\gg\Delta$ and $l_E\sim l_\varepsilon$ at $kT\ll\Delta]$. The reason is that the transformation of the quasiparticle current into a Cooper-pair current proceeds in two stages, on the N-S boundary [this corresponds at $l\gg\xi(T)$ to Andreev reflection[15]], and in the interior of the superconductor. The quasiparticles in the superconductor are scattered by various kinds of defects just as in the normal state, and therefore the electric field in the superconductor E_S is connected with the quasiparticle current density by the usual relation (see Ref. 123):

$$E_S=j_N/\sigma,$$

where σ is the conductivity in the normal state. In many cases one can neglect the change of the electric field in the normal metal, due to the existence of the N-S boundary (see Ref. 13).

The distinguishing feature of a point junction of size $d\ll l_E$ is that the decrease of the electric field in the interior of the superconductor follows the same law as in the normal metal, on account of the spatial spreading of the quasiparticle current, i.e., at a distance d from the junction. Since in the normal state the voltage on the corresponding half of the junction is proportional to the electric field in the opening (for details see Refs. 9 and 16), we can write for the voltages U_S^N and U_S^S on the superconductor in the normal and superconducting states

$$U_s^N=\lambda E_s^N(0),\quad U_s^S=\lambda E_s^S(0). \tag{3}$$

Here $E_S^N(0)$ and $E_S^S(0)$ are the values of the electric field on the boundary of the metals in the normal and superconducting states, respectively, and λ is a proportionality coefficient determined by the law that governs the spreading of the current. In the normal state the electric field on the metal interface is continuous, $E_N(0)=E_S{}^N(0)$ [$E_N(0)$ is the electric field in the normal metal at the interface]. The transition into the superconducting state leads to the appearance on the boundary of a discontinuity of the electric field [we recall that $d\gg\xi(T)$]:

$$\delta E=E_N(0)-E_S{}^S(0)=E_S{}^N(0)-E_S{}^S(0). \tag{4}$$

Introducing the resistances R_N and R_S of the normal and superconducting halves of the junction in accord with the formulas

$$R_N=U_N/I,\quad R_S=U_S{}^N/I \tag{5}$$

and recognizing that $R=R_S+R_N$, we easily obtain from (3)–(5) an expression for the voltage on the contact:

$$U=U_N+U_S{}^S=I\left\{R-R_S\frac{\delta E}{E_S{}^S(0)}\left[1+\frac{\delta E}{E_S{}^S(0)}\right]^{-1}\right\}. \tag{6}$$

Artemenko, Volkov, and Ovchinnikov[11-14] have calculated the case when the energy of the particles produced by the electric field in the normal metal is small compared with kT and Δ, so that their results can be used to calculate the differential resistance r at $U=0$. The quantity $\delta E/E_S^S(0)=\beta$ was calculated for temperatures close to T_c (see Ref. 12):

$$\beta=\begin{cases}\dfrac{(3\pi)^{1/2}}{4}\left(\dfrac{\Delta}{kT}\right)^{1/2}\left(\dfrac{l_\varepsilon}{l}\right)^{1/4}\sim\left(1-\dfrac{T}{T_c}\right)^{1/4}\\ \text{at } l\gg\xi_0,\ 1\ll\left(\dfrac{l_\varepsilon}{l}\right)^{1/4}\ll\dfrac{kT}{\Delta},\\ \left[\dfrac{\pi}{3\sqrt{2}}\dfrac{\Delta l_\varepsilon}{kT\xi(T)}\right]^{1/2}\sim\left(1-\dfrac{T}{T_c}\right)^{1/2}\ \text{at}\\ l\ll\xi_0,\ 1\ll\dfrac{l_\varepsilon}{\xi(T)}\ll\left(\dfrac{kT}{\Delta}\right)^2.\end{cases} \tag{7}$$

As a result we obtain from (6) an expression for the differential resistance

$$r=R-\frac{\beta R_S}{1+\beta}\quad\text{at}\quad kT\gg\Delta. \tag{8}$$

At $kT\ll\Delta$ the penetration of the weak electric field into the superconductor can be neglected.[13] As a result, the differential resistance of the junction is determined by the resistance of its normal half: $r\approx R_N$. This result is obtained formally from (8) as $\beta\to\infty$, corresponding to the fact that $E_S^S(0)\to 0$ as $T\to 0$, whereas δE remains finite.

We have considered above the case of weak fields, and then $\delta E=\beta E_S{}^S(0)$. If δE reaches the saturation value $(\delta E)_0$ when the electric field is increased, then an excess current appears. In fact in this case it is easy to obtain from (3)–(6) the following expression for the current:

$$I=U/R+\lambda(\delta E)_0/R.$$

We shall show below that there is no excess current at $l\gg d$ if the energy of the quasiparticles produced by the electric field in the normal metal exceeds the values kT and Δ.

We represent the current through the junction in the form $I=I_S+I_N$, where I_S is the Cooper-pair current and I_N is the current of the quasiparticles in the superconductor. By virtue of the relation $E_S=j_N/\sigma$ the voltage on the superconducting part of the junction is

$$U_S{}^S=R_SI_N,$$

and the voltage on the junction

$$U=U_N+U_S{}^S=IR_N+I_NR_S=IR-I_SR_S.$$

The current is then given by

$$I=U/R+I_SR_S/R. \tag{9}$$

Saturation of the quantity δE is equivalent to I_S becoming independent of the electric field, i.e., of the voltage on the junction, when the electric field in the normal metal is increased.

We shall assume that the quasiparticles of the normal metal, with energy $\zeta<\Delta$, are reflected from the N-S boundary in accord with Andreev's theory,[15] i.e., that the current transported by them is transformed on the boundary into the Cooper-pair current I_S, and that all the quasiparticles with $\zeta>\Delta$ penetrate into the superconductor. Consequently the current I_S is transported in the normal metal by electrons of energy ε such that $|\varepsilon-\varepsilon_F|<\Delta$, where ε_F is the Fermi energy. The distribution function $f(\varepsilon)$ of the electrons near a point junction of size $d\ll l$ (Ref. 16) is

$$f(\varepsilon)=f_0\left(\varepsilon+\frac{eU}{2}\operatorname{sign}V_z\right),$$

where $f_0(\varepsilon)$ is the Fermi distribution function and V_z is the component of the electron velocity in a direction perpendicular to the plane of the diaphragm. This function satisfies the boundary condition on the interface between the normal and superconducting phases[17] and can be used to calculate the current I_S with $U/2$ replaced by U_N. In the free-electron approximation we have

$$I_S=\frac{\pi d^2}{4}\int\limits_{|\varepsilon-\varepsilon_F|<\Delta} eV_zf(\mathbf{p})\frac{2d^3p}{(2\pi\hbar)^3}=\frac{\pi d^2}{4}\frac{4\pi em}{(2\pi\hbar)^3}\int\limits_{\varepsilon_F-\Delta}^{\varepsilon_F+\Delta}\varepsilon\{f_0(\varepsilon-eU_N)-f_0(\varepsilon+eU_N)\}d\varepsilon, \tag{10}$$

where $\mathbf{p}$ is the electron momentum. We present expressions for I_S in several limiting cases:

$$I_S=\begin{cases}\dfrac{\pi d^2}{4}\dfrac{4\pi e^2m\varepsilon_F}{(2\pi\hbar)^3}\dfrac{\Delta}{kT}U_N=I\dfrac{\Delta}{2kT} & \text{at}\quad \Delta\ll kT,\ eU_N\ll kT; \quad (11)\\ \dfrac{\pi d^2}{4}\dfrac{4\pi e^2m\varepsilon_F}{(2\pi\hbar)^3}2U_N=I & \text{at}\quad \Delta\gg kT,\ eU_N<\Delta; \quad (12)\\ \dfrac{\pi d^2}{4}\dfrac{4\pi em\varepsilon_F}{(2\pi\hbar)^3}2\Delta=\dfrac{\Delta}{eR_N} & \text{at}\quad eU_N\gg kT,\ eU_N>\Delta. \quad (13)\end{cases}$$

We have taken it into account here that

$$\frac{\pi d^2}{4}\frac{4\pi e^2m\varepsilon_F}{(2\pi\hbar)^3}=(2R_N)^{-1}.$$

The last relation can be easily obtained with the aid of (10) by integrating over all the energies, i.e., from 0 to $+\infty$. From (9), (11), and (12) we easily calculate the differential resistance of the junction at $U=0$:

$$r=\begin{cases} R-R_s\Delta/2kT & \text{at} \quad \Delta\ll kT, \\ R-R_s=R_N & \text{at} \quad \Delta\gg kT. \end{cases}$$

For the excess current we obtain from (9) and (13) $I_0=R_S\Delta/R_NeR$. It appears that the results remain qualitatively the same also when the electron mean free path is small, $l\ll d\ll l_\varepsilon$, for in this case the electron drifting towards the contact in the electric field also acquires an additional energy equal to eU_N.

The arguments advanced above show that the class of junction for which the qualitative results of Zaĭtsev's theory[1] are valid can be quite large. Indeed, at low temperature the large electron mean free path l_{ph} with respect to elastic scattering by phonons leads to a large $l_\varepsilon=(l_0 l_{ph})^{1/2}$, where l_0 is the mean free path for scattering by point defects, $l_0\ll l_{ph}$. In aluminum at $T=1$ K, $l_{ph}>10^{-1}$ cm, so that even in very dirty aluminum ($l_0=10^{-7}$ cm) we have $l_\varepsilon>10^{-4}$ cm, and in the pure metal this quantity is many times larger. This explains possibly the fact that the excess-current effect and the monotonic temperature dependence of the differential resistance were observed in junctions made by a great variety of methods: in clamped junctions of various types[3,4,18] and in the junctions produced by electric breakdown of a thin insulator layer in the present study. We note in particular that the resistance of the contacts used in these studies varies greatly, from 10^{-3} to 10 ohms. It is quite probable that the conditions of the applicability of the model used above for the calculations were satisfied in the study of Chien and Farrell,[4] who investigated low-resistance junctions ($R\sim10^{-2}\Omega$), and the superconducting electrodes were, besides aluminum, metals with small coherent length ξ_0 (tin, indium, and an alloy of tin with indium).

In our case, within the framework of the assumptions made in the estimate of d and l, we have $l_\varepsilon>3\cdot10^{-4}$ cm near the junction, and consequently the relations $l, L\ll\xi(0)<d\ll l_\varepsilon$ are fulfilled. When T_c is approached, the coherence length $\xi(T)$ increases and can exceed the junction dimension d, so that our experimental situation pertains to a case intermediate between those considered in the theory of Artemenko, Volkov, and Zaĭtsev[6] and the model proposed above.

CONCLUSION

From among our experimental results we must single out the proportionality of the excess current of a point junction between a superconductor and a normal metal to the value of the energy gap in the superconductor. We note that the same result was obtained by Gubankov and Margolin[5] for high-resistance clamped junctions. This property of point junctions, together with their small size, makes the junctions a convenient tool for the investigation of inhomogeneous states in a superconductor.

A theory of the conduction of point junctions of the S-c-N type of small size $d\ll\xi(T)(1-T/T_c)^{1/4}$ is at present available.[1,6] Our experimental results indicate that the qualitative predictions of Zaĭtsev's work are applicable in a larger number of cases. The simple model of the conduction of a point junction, considered in the present paper, indicates that junctions with dimensions $L\ll\xi(T)\ll d\ll l_\varepsilon$ should have analogous properties. A more accurate calculation of the conductivity of such junctions is of undisputed interest.

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