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Radiation from a vortex in a long Josephson junction placed in an alternating electromagnetic field

M. B. Mineev and V. V. Shmidt

Institute of Solid State Physics, USSR Academy of Sciences

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We consider phenomena connected with the motion of one vortex in a long Josephson junction placed in an alternating electromagnetic field. We show that under certain conditions the vortex radiates electromagnetic energy to both sides of the junction, and the radiation frequency is in general not equal to the external frequency applied to the junction, i.e., a single vortex plays the role of a frequency converter. The presence of a threshold rate of vortex radiation leads to resonant singularities on the current-voltage characteristic of the junction.

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1. INTRODUCTION

The electromagnetic properties of long tunnel Josephson junctions have been the subject of many studies. In the present paper we are interested in phenomena connected with the motion of one vortex (or of a strongly rarefied chain of vortices) along a long junction. It is assumed that the alternating and direct currents perpendicular to the junctions are given and are uniformly distributed along the junction. We list in this connection some already known facts.

If a strong magnetic field is applied to the junction, a periodic vortex structure is produced in it.[1] When direct current is made to flow through this junction, the vortices are moved by the Lorentz force. If their velocity coincides with the electromagnetic-wave propagation velocity in the junction, a resonant peak appears on the current-voltage characteristic (CVC).[1,2] This picture is valid only in the presence of sufficiently strong damping, when edge effects can be neglected. If, however, the samping in the junction is weak, then the reflection of the electromagnetic waves from the edges of the junction gives rise to standing waves, i.e., the junction turns into a resonator. A singularity (a Fiske step) appears on the CVC of the junction when the Josephson frequency is equal to one of the natural frequencies of the junction.[1,3-5]

Highly interesting singularities in the form of giant steps were observed on the CVC of a junction by Chen *et al.*[6] in a zero magnetic field. This phenomenon was later investigated by Fulton and co-workers.[7,8] We recall briefly the gist of the phenomenon.

In a long junction to which direct current is applied, a single vortex executes finite motion, being periodically reflected from the edges. In each reflection act, the direction of the current in the vortex is reversed, and in each passage of the vortex (or antivortex) from one edge of the junction to the other the phase difference of the order parameter of the superconductors making up the junction increases by 2π. The average rate of change of the phase shift is thus

$$\omega=\frac{\partial\varphi}{\partial t}=\frac{2\pi v}{W},$$

where V is the vortex velocity and W is the junction length. Since the vortex velocity cannot exceed the maximum velocity c_0 of the electromagnetic wave in the junction (the Swihart velocity),[1)] we have

$$\omega<\omega_m=2\pi c_0/W.$$

This means that if only one vortex moves in the junction the voltage on the junction is

$$V<V_m=\Phi_0\omega_m/2\pi c=\Phi_0 c_0/Wc,$$

where Φ_0 is the magnetic-flux quantum, and c is the speed of light in vacuum. On the other hand, as $v\rightarrow c_0$

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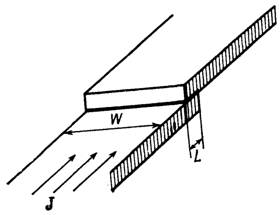

FIG. 1. Configuration of thin-film tunnel Josephson junction, which produces along the side W a uniform distribution of the current J entering the junction. In this case $L \sim \lambda_J$.

(and with it $V \to V_m$) the vortex energy increases without limit, since $E \propto (1 - v^2/c_0^2)^{-1/2}$ (Ref. 9), or, equivalently, as $V \to V_m$ the current that maintains v constant in time also increases without limit, transferring energy from the source to the vortex. It is this which explains the appearance of the current step on the junction CVC at $V = V_m$. On the other hand if the junction contains two vortices, then we get a similar picture, except that when the vortex collides with the antivortex that moves against it, the two can pass through each other under certain conditions.[10] As a result, the limiting value of V_m is doubled and a new branch appears on the CVC, with a new current step at $V = 2V_m$. Similarly, for n vortices moving in the same junction, the CVC will have n current steps at $V_k = kV_m$ $(k = 1, 2, \ldots, n)$. Such CVC were in fact observed in experiment.[6-8]

In the present study we undertook a theoretical investigation of the phenomena occurring in a long tunnel Josephson function in which one vortex moves, and the effects of these phenomena on the CVC. We assume that the direct and alternating currents are uniformly distributed along the junction. Such a uniform distribution of the current fed to the junction was reported, for example, by Johnson and Barone[11] for the Josephson junction shown in Fig. 1. The experiment[11] and a theoretical calculation[12,13] of the current distribution in such a junction have shown the distribution to be uniform in the direction of the long side of the junction.[2)] In the experiment, the current distribution remained uniform at least up to a dimension $W = 18\lambda_J$. Josephson junctions into which a uniformly distributed direct current enters from the outside were investigated by many workers (see, e.g., Refs. 14–20). We note finally that the junction configuration shown in Fig. 1 is by far not the only one that results in a uniform distribution of the entering current.

We solve the equation for the phase difference φ between the sides of the junction with allowance for the external currents and dissipation, we find that under definite conditions, far from the vortex, plane waves propagate along the junction and carry energy; in other words, the vortex radiates. This radiation leads to certain singularities on the CVC of the junction.

2. CVC OF JUNCTION WITH UNIFORM AND DIRECT BIAS CURRENT

Assume first that no alternating current flows through the junction. The equation for the phase difference between the order parameters of the superconductors that make up the junction is of the form[14-21]

$$\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial t^2} - \sin\varphi = \eta\frac{\partial\varphi}{\partial t} + J. \tag{1}$$

Here η is the coefficient of viscosity in the junction, J is the direct current through the junction per unit length. Equation (1) is written in the relative units $c_0 = \lambda_J = 1$.

The solution of (1) without the right-hand side, corresponding to a single moving vortex (soliton) in an infinitely long junction, is of the form

$$\varphi_0 = 4\,\mathrm{arctg}\, e^{\xi}. \tag{2}$$

Here ξ and τ are the proper coordinate and time of a vortex moving with velocity v:

$$\xi = \frac{x - \beta t}{\gamma}, \quad \tau = \frac{t - \beta x}{\gamma},$$

$$\beta = v/c_0, \quad \gamma = (1 - \beta^2)^{1/2}.$$

Treating the right-hand side of (1) as a perturbation, we seek the solution of (1) in the form $\varphi = \varphi_0 + \varphi_1$, where $|\varphi_1| \ll 1$. To this end we assume $|J| \ll 1$ and $\eta \ll 1$. Linearizing (1) with respect to φ_1, we obtain (in the reference frame of the vortex) the equation

$$L_{\xi\tau}\varphi_1(\xi,\tau) \equiv \left[\frac{\partial^2}{\partial\xi^2} - \frac{\partial^2}{\partial\tau^2} - \cos\varphi_0(\xi)\right]\varphi_1(\xi,\tau) = J - \frac{\beta\eta}{\gamma}\frac{\partial\varphi_0(\xi)}{\partial\xi}, \tag{3}$$

which takes, after taking the Fourier transform with respect to τ, the form

$$L_{\xi\omega}\varphi_1(\xi,\omega) \equiv \left[\frac{\partial^2}{\partial\xi^2} + \omega^2 - \cos\varphi_0(\xi)\right]\varphi_1(\xi,\omega) = 2\pi\delta(\omega)\left[J - \frac{\beta\eta}{\gamma}\frac{\partial\varphi_0(\xi)}{\partial\xi}\right].$$

The Green's function of the operator $L_{\xi\omega}$ is

$$G_\omega(\xi,\xi') = -\frac{\exp[-q_0|\xi-\xi'|]}{2q_0(1-q_0^2)}(\mathrm{th}\,\xi \pm q_0)(\mathrm{th}\,\xi' \mp q_0), \tag{4}$$

$$\begin{aligned} q_0 &= (1-\omega^2)^{1/2} \quad \text{if} \quad |\omega| \leq 1, \\ q_0 &= -i(\omega^2-1)^{1/2} \quad \text{if} \quad \omega > 1, \\ q_0 &= i(\omega^2-1)^{1/2} \quad \text{if} \quad \omega < -1. \end{aligned} \tag{4a}$$

The upper and lower signs in (4) pertain respectively to the cases $\xi > \xi'$ and $\xi < \xi'$. It follows from (4) and (4a) that the Green's function $G(\xi, \xi'; \tau - \tau')$ is real:

$$G_\omega(\xi,\xi') = G_{-\omega}^*(\xi,\xi').$$

As $\omega \to 0$, the function $G_\omega(\xi, \xi')$, as seen from (4), diverges like ω^{-2}. We represent $G_\omega(\xi, \xi')$ in the form

$$G_\omega(\xi,\xi') = \omega^{-2}F_\omega(\xi,\xi'),$$

where $F_\omega(\xi, \xi')$ is a function regular as $\omega \to 0$, and expand $F_\omega(\xi, \xi')$ in powers of ω^2. Then,

$$\varphi_1(\xi,\omega) = \frac{2\pi\delta(\omega)}{\omega^2}\int_{-\infty}^{\infty} F_0(\xi,\xi')\left[J - \frac{\beta\eta}{\gamma}\frac{\partial\varphi_0(\xi')}{\partial\xi'}\right]d\xi'$$

$$+ 2\pi\delta(\omega)\int_{-\infty}^{\infty}\left[\frac{\partial F_\omega(\xi,\xi')}{\partial\omega^2}\right]_{\omega=0}\left(J - \frac{\beta\eta}{\gamma}\frac{\partial\varphi_0(\xi')}{\partial\xi'}\right)d\xi'. \tag{5}$$

The possible divergence of $\varphi_1(\xi, \omega)$ corresponds to non-uniform motion of the vortex in the junction. For $\varphi_1(\xi, \omega)$ to be finite, and for the vortex motion thus to be uniform, it is necessary that the first integral in the right-hand side of (5) be equal to zero:

$$\int_{-\infty}^{\infty} F_0(\xi,\xi')\left[J - \frac{\beta\eta}{\gamma}\frac{\partial\varphi_0(\xi')}{\partial\xi'}\right]d\xi' = 0. \tag{6}$$

Substituting $\varphi_0(\xi)$ from (2) in (6) and

$$F_0(\xi,\xi')=[\omega^2 G_\omega(\xi,\xi')]_{\omega=0}$$

we obtain from (4) the stationarity condition for the vortex:

$$J=4\beta\eta/\pi\gamma. \tag{7}$$

We now discuss the result. The direct current through the junction interacts with the single Josephson vortex in the junction; the result is a Lorentz force acting on the vortex. This force causes the vortex to move at constant velocity and to overcome the friction force. Equation (7) obtained by us established an umambiguous relation between J and β.

Assuming that the dc voltage on the junction is due to the change of the phase difference φ in the periodic reflection of the vortex from the edges of the junction, as described in the Introduction, we obtain from (7), in dimensional units,

$$J=\frac{2W}{\pi^2\lambda_J R}\frac{V}{(1-(V/V_0)^2)^{1/2}}, \tag{8}$$

where $V_0=\Phi_0 c_0/cW$, and $V=\Phi_0 v/cW$, as noted in the Introduction. Here J is the current density and R is the junction resistance per unit area. The result (8) agrees exactly with a formula given by Fulton and Dynes,[7] where the current was introduced into the junction through the edges. Thus, the CVC should coincide exactly with the one observed in Ref. 7 also in the case of uniform distribution of the current over the long junction.

3. RADIATION OF VORTEX FROM JUNCTION

We consider now the case when in addition to the dc bias voltage the junction carries also an alternating current $f_0\sin\omega_0 t$ uniformly distributed over the junction. The equation for the phase difference takes in this case the form

$$\frac{\partial^2\tilde\varphi}{\partial x^2}-\frac{\partial^2\tilde\varphi}{\partial t^2}-\eta\frac{\partial\tilde\varphi}{\partial t}-\sin\tilde\varphi-J=f_0\sin\omega_0 t. \tag{9}$$

Putting $f_0\ll 1$, we seek the solution of (9) in the form $\tilde\varphi=\varphi+\tilde\varphi_1$, where φ is the solution of the problem considered in the preceding section. Linearizing (9) with respect to $\tilde\varphi_1$ and using the fact that $\varphi=\varphi_0+\varphi_1$, where $\varphi_1\sim\eta\ll 1$, we get

$$\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial t^2}-\eta\frac{\partial}{\partial t}-\cos\varphi_0+\varphi_1\sin\varphi_0\right)\tilde\varphi_1=f_0\sin\omega_0 t. \tag{10}$$

Let us simplify this equation. The operator in the parenthesis contains two terms that are small in η. However, if the term $\eta/\partial t$, as will be seen from the result, ensures cutoff of the resultant divergences and damping of the obtained waves, then the second term $\varphi_1\sin\varphi_0$ leads to insignificant changes of the Green's function (4). We leave out therefore the last term in the parentheses. Going over to the reference frame of the vortex and taking the Fourier transform with respect to time, we obtain the following equation, which is simpler than (10):

$$\tilde L_{\xi\varepsilon}\tilde\varphi_1(\xi,\varepsilon)=\left[\frac{\partial^2}{\partial\xi^2}+\frac{\eta\beta}{\gamma}\frac{\partial}{\partial\xi}+\left(\varepsilon^2+\frac{i\eta\varepsilon}{\gamma}-1\right)+\frac{2}{\operatorname{ch}^2\xi}\right]\tilde\varphi_1(\xi,\varepsilon)=\frac{\pi f_0}{i}\left(e^{ik\xi}\delta(\varepsilon+\omega)-e^{-ik\xi}\delta(\varepsilon-\omega)\right). \tag{11}$$

Here $\omega=\omega_0/\gamma$ and $k=\beta\omega_0/\gamma$.

Thus,

$$\tilde\varphi_1(\xi,\tau)=-f_0\operatorname{Im}\left\{e^{-i\omega\tau}\int_{-\infty}^{\infty}\tilde G_\omega(\xi,\xi')e^{-ik\xi'}d\xi'\right\}, \tag{12}$$

where $\tilde G_\omega(\xi,\xi')$ is the Green's function of the operator $\tilde L_{\xi\omega}$. We reduce the operator $\tilde L_{\xi\omega}$ to normal form (without the first derivative). To this end we represent $\tilde\varphi_1(\xi,\omega)$ in the form

$$\tilde\varphi_1(\xi,\omega)=\exp(-\beta\eta\xi/2\gamma)\Phi(\xi,\omega).$$

Then

$$\tilde L_{\xi\omega}\tilde\varphi_1(\xi,\omega)=\exp(-\beta\eta\xi/2\gamma)L_{\xi\Omega_0}\Phi(\xi,\omega),$$

where $L_{\xi\Omega_0}$ is the Fourier component of the operator $L_{\xi\tau}$ introduced in (3), and

$$\Omega_0^2=\omega^2+i\eta\omega/\gamma-(\eta\beta/2\gamma)^2.$$

Knowing the Green's function of the operator $L_{\xi\Omega_0}$, we easily obtain the Green's function of the operator $\tilde L_{\xi\omega}$:

$$\tilde G_\omega(\xi,\xi')=\exp(-\eta\beta(\xi-\xi')/2\gamma)G_{\Omega_0}(\xi,\xi')=-\frac{\exp(-\eta\beta(\xi-\xi')/2\gamma-Q|\xi-\xi'|)}{2Q(1-Q^2)}(\operatorname{th}\xi\pm Q)(\operatorname{th}\xi'\mp Q); \tag{13}$$

$$Q=(1-\Omega_0^2)^{1/2}=\begin{cases}(q_0^2-i\eta\omega/\gamma)^{1/2}, & \text{where } q_0^2=1-\omega^2 \text{ if } |\omega|\leqslant 1,\\ -i(q^2+i\eta\omega/\gamma)^{1/2}, & \text{where } q^2=\omega^2-1 \text{ if } \omega>1,\\ i(q^2+i\eta\omega/\gamma)^{1/2}, & \text{where } q^2=\omega^2-1 \text{ if } \omega<-1.\end{cases} \tag{13a}$$

The equation (13a) was set up with allowance for the fact that $\tilde G(\xi,\xi';\tau-\tau')$ is real. It is remarkable that the function $\tilde G_\omega(\xi,\xi')$ is finite as $\omega\to 0$, so that the indicated method of finding the Green's function with allowance for dissipation eliminates the divergence of φ_1 as $\omega\to 0$.

Knowing the Green's function (13), we obtain $\tilde\varphi_1(\xi,\tau)$ from (12) at $|\xi|\gg 1$:

$$\tilde\varphi_1(\xi,\tau)=\pm\frac{\pi f_0}{2}\operatorname{Im}\left\{\exp\left[-i\omega\tau+i\left(q^2+\frac{i\eta\omega}{\gamma}\right)^{1/2}|\xi|-\frac{\eta\beta}{2\gamma}\xi+i\operatorname{arctg}\left(q^2+\frac{i\eta\omega}{\gamma}\right)^{1/2}\right]\left[\left(\omega^2+\frac{i\eta\omega}{\gamma}\right)^{1/2}\left(q^2+\frac{i\eta\omega}{\gamma}\right)^{1/2}\times\operatorname{sh}\frac{\pi}{2}\left(k\pm\left(q^2+\frac{i\eta\omega}{\gamma}\right)^{1/2}-\frac{i\beta\eta}{2\gamma}\right)\right]^{-1}\right\}, \tag{14}$$

where $q=(\omega^2-1)^{1/2}$, and the upper and lower signs pertain respectively to the cases $\xi\gg 1$ and $\xi\ll -1$.

In this formula and hereafter we omit that part of $\tilde\varphi_1$ which correspond to the trivial phase oscillation due to the flow of the alternating current $f_0\sin\omega_0 t$, and which is in no way connected with the presence of the vortex in the junction. It is easy to show that the discarded part of $\tilde\varphi_1$ does not influence the CVC of the junction. We proceed to a discussion of Eq. (14)—the main result of this section. It follows from it that $\tilde\varphi_1$, which is a wave that travels along the junction, attenuates at a distance $\sim(\gamma/\eta)^{1/2}$, and has a phase $\theta=\Omega t-Kx$, where

$$\Omega=\frac{\omega_0}{\gamma^2}\pm\frac{\beta q}{\gamma},\quad K=\frac{\beta\omega_0}{\gamma^2}\pm\frac{q}{\gamma}. \tag{15}$$

Since $\omega^2=q^2+1$, we get from general relativistic considerations

$$\Omega^2=K^2+1, \tag{16}$$

as can also be easily verified by direct substitution.

This is not surprising, since the propagation of the wave $\tilde{\varphi}_1$ in the junction is equivalent to propagation in the junction of an electromagnetic wave with

$$E \propto \frac{\partial \tilde{\varphi}_1}{\partial t}, \quad H \propto \frac{\partial \tilde{\varphi}_1}{\partial x}.$$

The dispersion law of electromagnetic waves in the Josephson junction, however, is that of a plasma, i.e., it takes the form (16).

It is thus clear that radiation in the form (14) exists only at $\omega = \omega_0/(1-\beta^2)^{1/2} > 1$. From this it follows directly that at an external frequency $\omega_0 < 1$ the radiation (14) can start only when the vortex velocity exceeds a certain threshold $\beta_c = (1-\omega_0^2)^{1/2}$. If, however, $\omega_0 > 1$, then the radiation (14) exists at any vortex velocity. The value of $\tilde{\varphi}_1(\xi, \omega)$ at $\beta = \beta_c$ can easily be obtained from (14) by putting in it $q = 0$:

$$\tilde{\varphi}_1(\xi,\tau) = -\frac{\pi f_0 \exp[-(\eta/2\gamma)^{1/2}|\xi|]}{2\,\mathrm{sh}\frac{\pi\beta}{2}}\left[-\frac{\pi}{2}\mathrm{cth}\frac{\pi\beta}{2}\sin\theta \mp \cos\theta \pm \frac{\sin(\theta+\pi/4)}{(\eta/\gamma)^{1/2}}\right], \quad \theta = \tau - (\eta/2\gamma)^{1/2}|\xi|. \tag{17}$$

It is seen that $\tilde{\varphi}_1$ tends to infinity as $\eta \to 0$.

It must be noted that the radiation observed far from the vortex has, generally speaking, a frequency Ω not equal to the external frequency ω_0. Thus, a long Josephson junction in which a vortex moves constitutes a unique frequency converter, whose conversion coefficient can be controlled with an external direct current J in accordance with formulas (7) and (15). Since $\tilde{\varphi}_1$ must be small if perturbation theory is to hold, we must regard (17), assuming the damping to be small, only as an estimate.

We note one more feature of (14). At $\omega_0 = 1$ (corresponding in dimensional units to $\omega_0 = c_0/\lambda_J$) resonance sets in at the natural frequency of the system, as a result of which $\tilde{\varphi}_1(\xi, \tau)$ increases abruptly at the frequency $\omega_0 = 1$. An estimate of $\tilde{\varphi}_1$ yields in this case

$$\tilde{\varphi}_1(\xi,\tau) = \begin{cases} \dfrac{\pi f_0 \gamma^2}{2\beta\,\mathrm{sh}(\pi\beta/\gamma)}\sin\left(\dfrac{\beta\xi-\tau}{\gamma} + \mathrm{arctg}\dfrac{\beta}{\gamma}\right) & \text{if} \quad \xi \gg 1, \\ -\dfrac{2f_0}{\eta}\dfrac{(1-\beta^2)^{3/4}}{1+\beta^2}\cos\left(t - \mathrm{arctg}\dfrac{\beta}{\gamma}\right) & \text{if} \quad \xi \ll -1. \end{cases} \tag{17a}$$

Waves with a plasma dispersion law (16) have the property that their group velocity is the reciprocal of the phase velocity:

$$v_g^{\pm} = \frac{\partial\Omega}{\partial K} = \frac{K}{\Omega} = \frac{\beta\omega_0 \pm q\gamma}{\omega_0 \pm \beta q\gamma} \tag{18}$$

(the upper and lower signs correspond to $\xi \gg 1$ and $\xi \ll -1$, respectively). It is seen from (17) that $v_g^+ > 0$ at all β and ω_0, and that $\mathrm{sign} v_g^- = \mathrm{sign}\,(1-\omega_0)$, i.e., at the frequencies $\omega_0 < 1$ the radiation is dragged behind the vortex. It is easy to verify that at $\omega_0 < 1$ we have $v_g^- < \beta$, i.e., the radiation lags the vortex in this case.

We note finally that at $\omega_0/\gamma < 1$ there is no radiation of electromagnetic waves. $\tilde{\varphi}_1$ is then localized near the vortex in a region of the order of $(1-\omega^2)^{-1/2}$.

4. CURRENT-VOLTAGE CHARACTERISTICS

The CVC of a Josephson junction without an external alternating current were investigated in a number of studies, referred to in the Introduction. In the absence of an external magnetic field the dependence of the current on the voltage is determined in this case by Eq. (8). Let us see how the CVC are influenced by a uniform alternating current $f_0 \sin\omega_0 t$. We consider Eq. (9), for which the corrections $\tilde{\varphi}_2$ to φ in terms of the parameter f_0 are:

$$\tilde{L}_{\xi\tau}\tilde{\varphi}_2(\xi,\tau) = -{}^1/_2 \sin\varphi_0(\xi)\,\tilde{\varphi}_1^2(\xi,\tau); \tag{19}$$

the operator $\tilde{L}_{\xi\tau}$ is defined in (11). It follows from (12) that $\tilde{\varphi}_1(\xi, \tau)$ is of the form

$$\tilde{\varphi}_1(\xi,\tau) = f_0 A(\xi)\sin(\omega\tau + \alpha(\xi)).$$

Thus, (19) takes the form

$$\tilde{L}_{\xi\tau}\tilde{\varphi}_2(\xi,\tau) = -{}^1/_4 f_0^2[A(\xi)]^2[1-\cos(2\omega\tau + 2\alpha(\xi))]\sin\varphi_0(\xi). \tag{20}$$

The second term in the right-hand side of (20) leads to radiation similar to $\tilde{\varphi}_1(\xi, \tau)$, but of second order in f_0. We shall not be interested hereafter in this small correction. On the other hand, the first term in the right-hand side of (20), which does not depend on the time, can lead to divergence of $\tilde{\varphi}_2$. As already mentioned in the derivation of (7), this divergence is due to the decelerated (or accelerated) motion of the vortex. For $\tilde{\varphi}_2$ to be finite (and the vortex motion uniform) we must introduce an additional direct current J_1 that cancels out the deceleration (or acceleration) of the vortex. By the method described in the derivation of (7), we obtain

$$J_1 = -\frac{f_0^2}{2\pi}\int_{-\infty}^{\infty}\frac{[A(\xi)]^2\,\mathrm{sh}\,\xi}{\mathrm{ch}^3\xi}d\xi. \tag{21}$$

(Allowance has already been made here for the fact that $\sin\varphi_0(\xi) = -2\sinh\xi/\cosh^2\xi$.)

It follows from (21) that $J_1 \neq 0$ only if $A(\xi)$ has no definite parity with respect to ξ, i.e., if the amplitudes of the radiating waves ahead and behind the vortex are different.

The function $A(\xi)$ is known only in quadratures [see (12)]. At $|\xi| \gg 1$ we determine $A(\xi)$ from (14). We obtain now a qualitative estimate of J_1. At $q \neq 0$ we easily obtain $A(\xi)$ from (14) by extrapolating $\tilde{\varphi}_1(\xi, \tau)$ from (14) into the region $|\xi| \lesssim 1$ and by leaving out the terms with η:

$$A(\xi) = \pi/2\omega q\,\mathrm{sh}\,[{}^1/_2\pi(q + k\,\mathrm{sign}\,\xi)].$$

Substituting $A(\xi)$ in (21) we get

$$J_1(\beta) = \frac{\pi f_0^2}{4\omega^2 q^2}\left(\mathrm{sh}^{-2}\left[\frac{\pi}{2}(k-q)\right] - \mathrm{sh}^{-2}\left[\frac{\pi}{2}(k+q)\right]\right). \tag{22}$$

We thus have a small correction (of the order of f_0^2), given by the formula (7), to the CVC; this correction vanishes at $\beta = 0$ and $\beta = 1$.

If $q = 0$ [in this case $\omega = 1$ and $\beta = \beta_c = (1-\omega_0)^{1/2}$] then, by expanding the Green's function (13) in a Laurent series in Q near the point $\omega = 1$ and using (12) and (21), we obtain ultimately

$$J_1(\beta_c) = \frac{\pi f_0^2 (2\omega_0)^{1/2}}{16(\eta)^{1/2}\,\mathrm{sh}^2({}^1/_2\pi\beta_c)}. \tag{23}$$

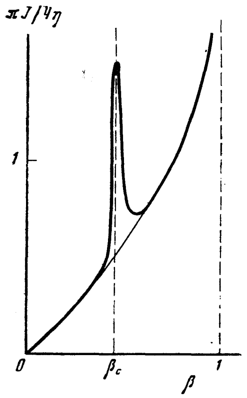

FIG. 2. Current-voltage characteristic of Josephson junction having one vortex and placed in an alternating electromagnetic field. Here $\beta = v/c_0 = V/V_0$.

We now discuss the result. If $\omega_0 > 1$ (in this case we always have $q \neq 0$), then the correction J_1 is small in the entire velocity interval $0 \leqslant \beta < 1$. If $\omega_0 < 1$, however, the situation is more interesting. If the vortex velocity $\beta \neq \beta_c$, then $J_1(\beta)$ is given by (22) and is a small correction to $J_0 = 4\beta\eta/\pi\gamma$. But as the vortex velocity β approaches its threshold value β_c, the correction $J_1(\beta)$ increases rapidly, reaching at $\beta = \beta_c$ a maximum value $J_1(\beta_c) \propto \eta^{-1/2}$ in accord with (23). It is seen thus that $\beta = \beta_c$ the correction $J_1(\beta)$ diverges in a nondissipative junction. To maintain the vortex velocity constant under conditions of such strong radiation a large energy must flow into the junction from the dc source, and it is this which explains the noted divergence of $J_1(\beta_c)$. Since the total direct current through the junction is $J_0 + J_1$, where J_1 is given by (22) and (23), the expected CVC takes the form shown schematically in Fig. 2.

On the other hand if $\omega_0 = 1$, then the perturbation theory constructed for φ is not valid, since $\bar{\varphi}_1 \sim 1/\eta > \varphi_0$ on the entire velocity interval $0 \leqslant \beta < 1$, but the qualitative behavior of $\bar{\varphi}_1(\beta)$ seems to be correctly described by formula (17a).

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1) Vortices with $v > c_0$ are unstable.

2) Such homogeneity can of course be realized only if the current is uniformly distributed in the films that lead to the junction. This can be attained by many methods. These conducting films can be made so thin that the depth of penetration $\lambda_\perp$ of the perpendicular magnetic field turns out to be larger than the Josephson penetration depth λ_f, and $\lambda_\perp \gg W$. It is possible to place the junction shown in Fig. 1 on a superconducting flat screen. It is possible, finally, the produce such a junction by depositing superconducting films on the surface of a dielectric cylinder. Of course, many other methods of realizing a uniform distribution of the current in a long Josephson junction can be proposed.

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