[15]O. L. Volkov, V. G. Eselevich, G. N. Kichigin, and V. L. Papernyĭ, Zh. Eksp. Teor. Fiz. 67, 1689 (1974) [Sov. Phys. JETP 40, 841 (1975)].
[16]N. V. Astrakhantsev, O. L. Volkov, V. G. Eselevich, G. N. Kichigin, and V. L. Papernyĭ, *ibid.* 76, 1289 (1979) [48, 649 (1979)].
[17]V. G. Eselevich and V. G. Faĭnshteĭn, SibIZMIR Preprint No. 15, 1980.
[18]K. E. Lonngren and N. Hershkovich, IEEE Trans., Plasma Sciences PS-7, 107 (1979).
[19]S. I. Anisimov and Yu. V. Medvedev, Zh. Eksp. Teor. Fiz. 76, 121 (1979) [Sov. Phys. JETP 49, 62 (1979)].
[20]J. S. Bendat and A. G. Piersol, Random Data: Analysis and Measurement Procedures, Wiley, 1968 (Russ. transl. Mir, 1974, p. 38).
[21]A. Gurevich, D. Anderson, and H. Wilhelmson, Phys. Rev. Lett. 42, 769 (1979).
[22]D. W. Forslund, J. Geophys. Res. Space Phys. 75, 17 (1970).
[23]E. V. Mishin, Dokl. Akad. Nauk SSSR 215, 565 (1974) [Sov. Phys. Doklady 19, 140 (1974)].
[24]V. G. Eselevich and V. G. Faĭnshteĭn, SibIZMIR Preprint No. 35, 1978.

Translated by J. G. Adashko

Coherent Mössbauer scattering in birefringent crystals

E. V. Smirnov and V. A. Belyakov

All-Union Research Institute for Physicotechnical and Radiotechnical Measurements
(Submitted 18 July 1979)
Zh. Eksp. Teor. Fiz. 79, 883–892 (September 1980)

Coherent scattering of Mössbauer γ radiation in magnetically ordered birefringent crystals is theoretically considered. It is shown that Mössbauer scattering by such crystals has a number of features that distinguish it from the case when there is no birefringence of the γ quanta in the crystal. Analytic expressions that describe the intensity and the polarization characteristics of the scattering are obtained for the case of strong birefringence, and their connection with the details of the magnetic structure of the crystal and with the magnitude of the birefringence is analyzed. It is shown that in this case the coefficient of reflection of the γ quanta from the crystal, as a function of the incidence angle, of the orientation of the magnetic fields at the nuclei, and of the energy of the γ quanta assumes in this case a complicated (multihump) form. The possibility of experimentally investigating these effects in α-Fe_2O_3 and $FeBO_3$ crystals are discussed.

PACS numbers: 76.80. + y, 78.20.Fm

INTRODUCTION

The study of the diffraction of resonant γ radiation by crystals is now branching out into an autonomous research trend, and progresses vigorously both theoretically and experimentally.[1] A number of interesting properties of Mössbauer diffraction is observed under conditions of hyperfine splitting of a Mössbauer line in a crystal.[2-9] In particular, theoretical investigations of the diffraction of Mössbauer γ quanta by perfect magnetically ordered crystals have revealed peculiarities in Mössbauer dynamic scattering, viz., complicated polarization properties of the scattered radiation[10] and magnetic Pendollosung beats.[11]

This paper deals with Mössbauer scattering of γ radiation by magnetically ordered crystals having strong birefringence. It is shown that the intensity and polarization of the diffracted radiation has a complicated dependence on the energy of the γ quanta, on the orientation of the magnetic fields at the nuclei, and on the angle of incidence of the radiation on the crystal. This dependence is connected with the birefringence of the γ quanta in the crystal and is due to the possibility of varying the birefringence by changing the beam energy or the orientations of the magnetic fields.

The scattering of Mössbauer radiation is considered in detail in the case of thin (kinematic limit) and thick (dynamic theory) perfect crystals, and a detailed analysis is presented of situations that are of interest from the experimental point of view and lend themselves to an analytic description.

SYSTEM OF BASIC EQUATIONS

We consider the scattering of resonant γ radiation by a crystal in the form of a plane-parallel plate, containing Mössbauer nuclei, with the Bragg condition satisfied or nearly so. The γ-radiation field in the crystal is obtained by solving Maxwell's equations, which in the two-wave approximation can be represented in the form

$$\gamma_1\frac{d\mathbf{E}_1}{dz}=\frac{i\varkappa}{2}\hat{F}_{11}\mathbf{E}_1+\frac{i\varkappa}{2}\hat{F}_{12}\mathbf{E}_2,$$
$$\gamma_2\frac{d\mathbf{E}_2}{dz}=\frac{i\varkappa}{2}\hat{F}_{21}\mathbf{E}_1+\frac{i\varkappa}{2}(\hat{F}_{22}-\alpha\hat{I})\mathbf{E}_2, \qquad (1)$$

where $\mathbf{E}_1, \mathbf{E}_2, \gamma_1, \gamma_2$ are the amplitudes and direction cosines of the incident and diffracted waves relative to the normal to the surface, $\varkappa$ is the wave vector of the radiation in vacuum, z is the coordinate along the normal to the sample surface, $\hat{I}$ is a unit matrix of second order, F_{ip} is an operator that describes the scattering, by the unit cell of the crystal, of a wave with wave vector $\mathbf{k}_p$ into a wave with wave vector $\mathbf{k}_i$. The elements of this operator are the scattering amplitudes $F_{ip}^{s's}$, where the superscripts $s, s'=1,2$ designate the polarizations of the waves in the primary and secondary directions; the explicit forms of the operators are given in Ref. 10. The wave vectors $\mathbf{k}_1^{(s)}$ and

$\mathbf{k}_2^{(s')}$ of the primary and secondary waves are connected by the Bragg condition

$$\mathbf{k}_2^{(s')}-\mathbf{k}_1^{(s)}=\boldsymbol{\tau}, \qquad (2)$$

where τ is the reciprocal-lattice vector, and the parameter $\alpha=[(\varkappa+\tau)^2-\varkappa^2]/\varkappa^2$ determines the deviation of the angle of incidence of the radiation on the crystal from the Bragg condition.

In the general case the diffraction problem must be solved by numerical methods, especially when it comes to analyzing the condition for the solvability of the system (1), a condition that determines the admissible values of the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ under diffraction conditions (the so-called dispersion surface). Nonetheless, some features of Mössbauer diffraction by birefringent crystals can be discerned without using an explicit form of the solution of the system (1), by qualitatively analyzing the dispersion surface. The form of the corresponding dispersion surface for the case of birefringent crystals is shown in Fig. 1.

The difference between the surface shown in Fig. 1 and the corresponding surface for x-rays[13] lies in the allowance for the birefringence of the γ quanta (for the dependence of the values of the wave vectors $\mathbf{k}_i^{(s)}$ on the wave polarization). It follows from the form of the dispersion surface that four regions of diffractive reflection of γ quanta, with different incidence angles, can coexist. In the kinematic approximation the four values of the diffraction angle (for different intrinsic polarizations of the incident and scattered waves[14,15]) are determined by the points $L_{11}, L_{12}, L_{21}, L_{22}$. In the dynamic theory there are encountered diffractive-reflection regions having different γ-quantum incidence angles and having dimensions determined by the distances between the branches of the dispersion surface (near the points $L_{s's}$). The angular dimensions $\Delta\theta$ and the position of one of these regions are shown with the aid of the construction illustrated in Fig. 1.

The noted peculiarities of scattering in birefringent crystals manifest themselves most clearly under conditions of strong birefringence of the Mössbaer γ quanta, i.e., at

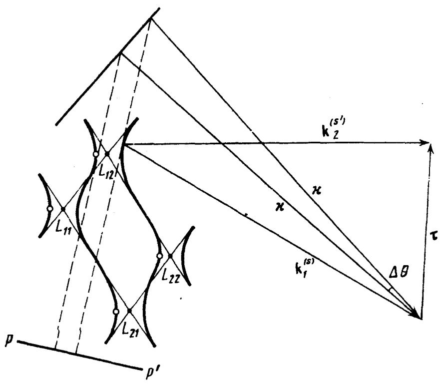

FIG. 1. Qualitative form of dispersion surface for birefringent crystals. The points $L_{s's}$ correspond to scattering in the kinematic approximation, PP'-crystal surface, $\Delta\theta$-width of one of the diffraction maxima, the circles mark the points at which the inelastic channels are suppressed.[4]

$$|\mathrm{Re}(F_{ii}^{ss}-F_{ii}^{s's'})|>|F_{21}^{s's}|. \qquad (3)$$

The condition (3) is a requirement that the structural amplitude of the scattering through the Bragg angle be small compared with the birefringence (the difference between the amplitudes of forward scattering for different intrinsic polarizations), and can be satisfied for both crystal and magnetic diffraction reflections.[2,3] Satisfaction of the condition (3) can be aided by a special choice of the energy of the γ quanta, and of the orientations of the beams relative to the magnetic axes in the crystal.

When inequality (3) is satisfied, the system (1) reduces to four independent second-order systems, each of which describes the scattering of γ quanta with fixed polarizations $\mathbf{n}_1^{(s)}$ and $\mathbf{n}_2^{(s')}$ for a definite interval of incidence angles. The corresponding systems are obtained from (1) replacing the matrices $\hat{F}_{ip}$ with scattering amplitudes $F_{ip}^{s's}$, and the vectors $\mathbf{E}_i$ with the scalar amplitudes $E_i^{(s)}$. The physical cause of the possibility of separating the polarizations in the system (1) is that in birefringent crystals the diffraction scattering of different eigenwaves takes place at different incidence angles on the crystal. This means that for a fixed incidence angle only γ quanta with a definite polarization $\mathbf{n}_1^{(s)}$ (one of the intrinsic polarizations) undergo diffraction scattering, and the scattered radiation is polarized and has a polarization vector $\mathbf{n}_2^{(s')}$. We note that the intrinsic polarizations $\mathbf{n}_i^{(s)}$ depend on the magnetic structure of the crystal, on the energy of the γ quanta, and are elliptic in the general case.[15]

We assume hereafter that the strong-birefringence condition (3) is satisfied and that the described separation of the polarizations takes place in the system (1). In this case we obtain from (1) for the amplitudes of the transmitted and scattered waves the expressions

$$\mathbf{E}_1(\mathbf{e})=E_0\sum_{s'=1,2}\mathbf{n}_1^{(s')}\sum_{s=1,2}(\mathbf{n}_1^{(s)}\mathbf{e})t^{s's},$$
$$\mathbf{E}_2(\mathbf{e})=E_0\sum_{s'=1,2}\mathbf{n}_2^{(s')}\sum_{s=1,2}(\mathbf{n}_1^{(s)}\mathbf{e})r^{s's}, \qquad (4)$$

where E_0 and $\mathbf{e}$ are respectively the amplitude and polarization of the incident wave, while $|t^{s's}|^2$ and $|r^{s's}|^2$ specify the transmission and reflection coefficients for the waves with intrinsic polarizations.

It follows from (4) that $\mathbf{E}_1(\mathbf{e})$ and $\mathbf{E}_2(\mathbf{e})$ contain four terms, each of which describes the scattering of Mössbauer radiation in the already noted different angle regions for the determination of the intrinsic polarizations of the incident and scattered waves. In the absence of hyperfine splitting of the Mössbauer line, formulas (4) reduce to the known expressions[16] for this case and contain only two terms.

The radiation reflection and transmission coefficients are of the form

$$R_B(\mathbf{e})=\frac{|\mathbf{E}_2(\mathbf{e})|^2}{|E_0|^2}, \qquad T_B(\mathbf{e})=\frac{|\mathbf{E}_1(\mathbf{e})|^2}{|E_0|^2}, \qquad (5)$$

where $\mathbf{E}_1(\mathbf{e})$ and $\mathbf{E}_2(\mathbf{e})$ are defined by expressions (4). If the incident beam is unpolarized, then the reflection and transmission coefficients R_B and T_B are obtained by averaging (5) over the polarization. Expression (4) is valid for arbitrary thickness of the scattering crystal, and to obtain the qualitative picture of the scattering we consider first the case of thin crystals.

SCATTERING IN THIN CRYSTALS

For thin crystals, using the smallness of the reflected wave compared with the transmitted one, we obtain for $t^{s's}$ and $r^{s's}$ in Bragg geometry ($\gamma_2/\gamma_1 < 0$) from (1) the following expressions:

$$
\begin{aligned}
t^{s's} &= \exp\left\{\frac{i\varkappa F_{11}^{ss}L}{2\gamma_1}\right\}\left\{1-\frac{A}{2\gamma_1 B^2}\right\}, \quad r^{s's}=\frac{C}{B},\\
A &= F_{12}^{ss'}F_{21}^{s's}\left[1+\frac{i\varkappa L}{2|\gamma_2|}B-\exp\frac{i\varkappa\alpha L}{2|\gamma_2|}\right],\\
B &= \left|\frac{\gamma_2}{\gamma_1}\right|F_{11}^{ss}+F_{22}^{s's'}-\alpha,\\
C &= F_{21}^{s's}\left\{1-\exp\left[\frac{i\varkappa L}{2|\gamma_2|}B\right]\right\},
\end{aligned}
\tag{6}
$$

where L is the crystal thickness.

We analyze now the angular dependence of the reflection coefficient R_B when an unpolarized beam of γ quanta is incident on the crystal. It follows from (4)–(6) that the intensity of the diffracted radiation as a function of the incidence angle has four maxima at the following values of α:

$$
\begin{aligned}
\alpha_1&=\mathrm{Re}\left(F_{22}^{11}+\left|\frac{\gamma_2}{\gamma_1}\right|F_{11}^{11}\right), \quad \alpha_2=\mathrm{Re}\left(F_{22}^{22}+\left|\frac{\gamma_2}{\gamma_1}\right|F_{11}^{11}\right),\\
\alpha_3&=\mathrm{Re}\left(F_{22}^{11}+\left|\frac{\gamma_2}{\gamma_1}\right|F_{11}^{22}\right), \quad \alpha_4=\mathrm{Re}\left(F_{22}^{22}+\left|\frac{\gamma_2}{\gamma_1}\right|F_{11}^{22}\right).
\end{aligned}
\tag{7}
$$

The angle width of each of the maxima depends on the values $\mathrm{Im}F_{ii}^{ss}$ of the imaginary parts of the zero-angle scattering amplitudes, on the thickness L of the scattering crystal, and on the geometry of the scattering (on the parameter γ_2/γ_1). Thus, if

$$
\frac{\varkappa L}{2|\gamma_2|}\mathrm{Im}\left(\left|\frac{\gamma_2}{\gamma_1}\right|F_{11}^{ss}+F_{22}^{s's'}\right)\gg 1,
$$

then the width of the maxima is equal to $2\,\mathrm{Im}(|\gamma_2/\gamma_1|F_{11}^{ss}+F_{22}^{s's'})$.

It follows from (7) that the angle distance Δ between the centers of the first and second maxima is equal to that between the third and fourth

$$
\Delta=\alpha_1-\alpha_2=\alpha_3-\alpha_4=\mathrm{Re}\,(F_{22}{}^{11}-F_{22}{}^{22})
\tag{8}
$$

and is determined by the magnitude of the birefringence in the $\mathbf{k}_2$ direction. The angle distance between the centers of the outermost maxima

$$
\Delta_{14}=\alpha_1-\alpha_4=\mathrm{Re}\left[F_{22}^{11}-F_{22}^{22}-\left|\frac{\gamma_2}{\gamma_1}\right|(F_{11}^{11}-F_{11}^{22})\right]
\tag{9}
$$

depends on the birefringence in the directions of the incident and diffracted waves and on the scattering geometry. If the relation between the birefringences in the directions $\mathbf{k}_1$ and $\mathbf{k}_2$ is such that

$$
|\gamma_2/\gamma_1|\,|\mathrm{Re}\,(F_{11}{}^{11}-F_{11}{}^{22})|=|\mathrm{Re}\,(F_{22}{}^{11}-F_{22}{}^{22})|,
\tag{10}
$$

then the centers of the second and third maxima coincide and $\Delta_{14}=2\Delta$. In particular, for the symmetrical Bragg case ($|\gamma_2/\gamma_1|=1$) the centers of the maxima coincide at equal values of the birefringence in the $\mathbf{k}_1$ and $\mathbf{k}_2$ directions.[10] For the asymmetrical Bragg case at $|\gamma_2/\gamma_1|\gg 1$ the value of Δ_{14} can exceed 2Δ appreciably.

If all the peaks are separated in angle, the radiation scattered into each of them is fully polarized, and in the outermost and in one of the central maxima there are polarized γ quanta with polarization vector $\mathbf{n}_2^{(1)}$, while radiation with polarization vector $\mathbf{n}_2^{(2)}$ is scattered in the two other maxima. For the sake of argument, we assume hereafter that all the scattering peaks are resolved.

MAGNETIC AND ENERGY DEPENDENCES OF THE REFLECTION COEFFICIENT

We consider now the dependence of the reflection coefficient R_B on the orientation of the magnetic field at the nuclei at a fixed value of the incidence angle of the γ quanta on the crystal. Variation of the orientation of the magnetic fields at the nuclei in the crystal (e.g., when an external field is applied or its direction is changed) changes the amplitude $F^{s's(N)}$ of the nuclear resonant scattering, and hence also the refraction coefficient of the eigenwaves. For a fixed incidence angle (fixed α), the maxima of the scattering, as functions of the magnetic-field orientation, are reached, as follows from (6), when the following condition is satisfied

$$
\mathrm{Re}\left(F_{22}^{s's'}+\left|\frac{\gamma_2}{\gamma_1}\right|F_{11}^{ss}\right)=\alpha.
\tag{11}
$$

Relation (11) can be satisfied at several orientations of the magnetic field $\mathbf{H}_n$ at the scattering nuclei. This means that at the corresponding directions of $\mathbf{H}_n$ the conditions of the Bragg reflection are satisfied for definite intrinsic polarizations of the incident and diffracted waves, while the reflected intensity, as a function of the orientation of the magnetic field in the crystal, has several maxima. Their number depends on the value of the parameter α, on the magnetic structure of the crystal, and on the multipolarity of the Mössbauer transition through which the scattering takes place. Thus, in the case of antiferromagnetic ordering in the crystal and scattering through a magnetic dipole transition, the reflection coefficient R_B can have up to four maxima when $\mathbf{H}_n$ is rotated through 90°. The polarization characteristics of the scattering at these maxima are similar to those noted above in the investigation of the angular dependence, and are determined by the vectors $\mathbf{n}_2^{(s)}$.

Similar maxima of the reflected-radiation intensity can appear also when the energy E of the γ quanta is varied. Thus, if the splitting of the Mössbauer line in the crystal is large, and the energy width of the source line is small then, as follows from (4)–(6), up to eight maxima can exist in the general case in the energy dependence of R_B near each resonance, and their positions are determined by the values of E that satisfy the condition (11). All the maxima lie on one side of the resonant value of the energy. Averaging (5) over the source line contour, without assuming that the line is narrow, can cause some of the maxima of the dependence of R_B on E to be smoothed out, but the asymmetry of the reflection coefficient relative to the resonance remains.

The foregoing analysis was made for the case of ideal collimation of the incident beam. If the beam incident on the crystal is not collimated, then to describe the intensity of the reflected radiation we must average (5) over the parameter α. The dependences of the reflection coefficient on the incidence angle, on the magnetic-field orientation, and on the γ-quantum energy are then smoothed out compared with the case of a collimated beam. In particular, if the collimation δ of the incident beam exceeds noticeably the dimensions of the region of the diffraction reflection ($\delta \gg \Delta_{14}$) then the angle-integrated reflection coefficient takes the form

$$R_B^{int} = \frac{\pi\varkappa}{2\gamma} \sum_{s,s'=1,2} |F_{21}^{s's}|^2 \left(\frac{\mu_1^{(s)}}{\gamma_1} + \frac{\mu_2^{(s')}}{\gamma_2}\right)^{-1} \times \left\{1 - \exp\left[-\left(\frac{\mu_1^{(s)}}{\gamma_1} + \frac{\mu_2^{(s')}}{\gamma_2}\right) L\right]\right\}, \tag{12}$$

where $\mu_i^{(s)} = \varkappa \operatorname{Im} F_{ii}^{ss}$ is the linear coefficient of absorption of γ quanta with polarization vector $\mathbf{n}_i^{(s)}$ in the $\mathbf{k}_i$ direction. As seen from (12), in this case the sharp maxima in the energy dependence of the intensity of the reflected radiation average out and do not appear in the integrated scattering.

DYNAMIC SCATTERING

In scattering by thick perfect crystals, the essential role is played by multiple scattering of the radiation in the crystal. In this case the amplitude of the reflected wave E_2 is generally speaking not small compared with the amplitude E_1 of the primary wave, and we must obtain for the system (1) a solution in which it is not assumed that E_2 is small. In the case of strong birefringence of the γ quanta [see condition (3)], the corresponding solution is also given by expression (4), in which now $t^{s's}$ and $r^{s's}$ take the form

$$t^{s's} = (\lambda_1^{s's} - \lambda_2^{s's}) \exp(-i\varkappa L \lambda_1^{s's}/4|\gamma_2|)/D, \tag{13}$$

$$r^{s's} = 2F_{21}^{s's} \{1 - \exp[-i\varkappa L(\lambda_1^{s's} - \lambda_2^{s's})/4|\gamma_2|]\}/D,$$

$$D = \lambda_1^{s's} - 2(F_{22}^{s's'} - \alpha) - [\lambda_2^{s's} - 2(F_{22}^{s's'} - \alpha)] \exp[-i\varkappa L(\lambda_1^{s's} - \lambda_2^{s's})/4|\gamma_2|],$$

$$\lambda_{1,2}^{s's} = \frac{\gamma_2}{\gamma_1} F_{11}^{ss} + F_{22}^{s's'} - \alpha \pm \left\{\left(\frac{\gamma_2}{\gamma_1} F_{11}^{ss} - F_{22}^{s's'} + \alpha\right)^2 + 4\frac{\gamma_2}{\gamma_1} F_{12}^{ss'} F_{21}^{s's}\right\}^{1/2}.$$

Formulas (5), (4) and (13) determine the reflection and transmission coefficients of the Mössbauer radiation for crystals of arbitrary thickness. For thick crystals

$$\frac{\varkappa L}{4|\gamma_2|} \operatorname{Im}(\lambda_1^{s's} - \lambda_2^{s's}) \gg 1$$

the coefficient of reflection of a wave with proper polarization $\mathbf{n}_1^{(s)}$ into a wave with proper polarization $\mathbf{n}_2^{(s')}$ is

$$R_B^{s's} = 4|\gamma_2/\gamma_1| |F_{21}^{s's}|^2 / |\alpha - |\gamma_2/\gamma_1| F_{11}^{ss} - F_{22}^{s's'} + \{(\alpha - |\gamma_2/\gamma_1| F_{11} - F_{22}^{s's'})^2 - 4|\gamma_2/\gamma_1| F_{12}^{ss'} F_{21}^{s's}\}^{1/2}|^2. \tag{14}$$

An analysis of expressions (13) and (14) shows that, just as in the case of scattering by thin crystals, the angular, magnetic, and energy dependences of the intensity of the radiation reflected from a thick crystal have a complicated (multiple-hump) form. The details of these dependences, however, differ for thin and thick crystals.

Consider Mössbauer scattering by a thick crystal. In this case, neglecting absorption, the angular dependence of each of the four maxima in $R_B(\alpha)$ coincides with Darwin reflection curve known from x-ray diffraction.[12] The angular position of the centers of the maxima concides with the value α_i previously obtained for thin crystals, and the width $\Delta\alpha$ of the maxima is

$$\Delta\alpha = 4(|\gamma_2/\gamma_1| F_{12}^{ss'} F_{21}^{s's})^{1/2}.$$

The maximum value of the reflection coefficient for an unpolarized incident beam is equal to 1/2, since one polarization is totally reflected while the other is not reflected at all.

Just as in the case of thin crystals, the condition under which the angle positions of the two central maxima do not coincide is the violation of relation (10), but for an actual separation of the maxima it is necessary that the difference between the angle positions exceed the width of the maxima, i.e.,

$$|\operatorname{Re}[F_{22}^{11} - F_{22}^{22} - |\gamma_2/\gamma_1|(F_{11}^{11} - F_{11}^{22})]| > 4(|\gamma_2/\gamma_1| F_{12}^{ss'} F_{21}^{s's})^{1/2}, \quad s \neq s'. \tag{15}$$

The condition for separation of the first and second maxima, as well as of the third and fourth, is determined by relation (3), in which the right-hand side must be multiplied by $4(|\gamma_2/\gamma_1|)^{1/2}$. The inequality obtained in this case, together with relation (15), determines the condition for the resolution of the maxima also in the dependences of the reflection coefficient on the magnetic field and on the energy.

It follows from (13) and (14) that the maxima in the magnetic and energy dependences of the intensity of the reflected γ quanta are similar in form to the maxima of the angular dependence. By way of example, Fig. 2 shows the calculated dependences of R_B on the orientation of the magnetic field in a hematite ($\alpha - Fe_2O_3$) crystal. At the chosen values of the parameter (see the caption of Fig. 2), the reflection coefficient as a function of the field orientation has two distinctly separated maxima. We note that the positions and widths of the maxima in Fig. 2 depend on the orientations of the magnetic fields at the nuclei, and in particular on the rate of change of the scattering ampltidue $F_{ip}^{s's}$ with changing field direction, while the quantity R_B characterizes the absorption of the proper polarizations under diffraction condition as a function of the magnetic-field orientation. Thus, the results of the experimental measurement of the discussed magnetic dependences, in accord with formulas (4), (5), and (13), can be used to determine the magnetic dependence of the nuclear structural

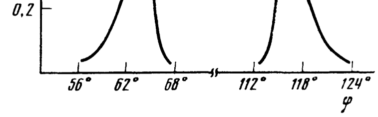

FIG. 2. Reflection coefficient vs. orientation of the antiferromagnetic axis in hematite crystal for the reflection (111) (φ is the angle between the antiferromagnetic axis and the scattering plane, enrichment with ^{57}Fe 85%, γ-quantum energy $E = E_{3/2-1/2} - 5\Gamma$, $\alpha = 7.3 \times 10^{-5}$, Γ-natural line width.

amplitude of the scattering and of the details of the magnetic ordering in the crystal.

THE LAUE CASE

We consider now the scattering of Mössbauer radiation under conditions of birefringence in Laue geometry ($\gamma_2/\gamma_1 > 0$). The field in the crystal is described in this case by Eqs. (4), in which $t^{s's}$ and $r^{s's}$ are of the form

$$t^{s's} = (\lambda_1^{s's} - \lambda_2^{s's})^{-1} \{ [\lambda_1^{s's} - 2(F_{22}^{s's'} - \alpha)] \mathscr{E}_1 - [\lambda_2^{s's} - 2(F_{22}^{s's'} - \alpha)] \mathscr{E}_2 \},$$

$$r^{s's} = 2F_{21}^{s's} (\mathscr{E}_1 - \mathscr{E}_2) / (\lambda_1^{s's} - \lambda_2^{s's})$$

$$\mathscr{E}_{1,2} = \exp\left\{ \frac{i\varkappa L}{4\gamma_2} \lambda_{1,2}^{s's} \right\},$$

where $\lambda_{1,2}^{s's}$ is defined in (13). In the presence of absorption in the crystal, an analysis of the dependence of the imaginary part $\mathrm{Im}\, k_i^{(s)} = \varkappa\, \mathrm{Im}\lambda^{s's}$ of the wave vector on the angle of incidence of the radiation on the crystal reveals certain peculiarities of the effect of suppression in birefringent crystals. Thus, if the following conditions are satisfied:

$$\mathrm{Re}\, F_{12}^{ss'}\, \mathrm{Im}\, F_{21}^{s's} = \mathrm{Re}\, F_{21}^{s's}\, \mathrm{Im}\, F_{12}^{ss'},$$

$$\mathrm{Im}\, F_{11}^{ss}\, \mathrm{Im}\, F_{22}^{s's'} = \mathrm{Im}\, F_{12}^{ss'}\, \mathrm{Im}\, F_{21}^{s's},$$

then $\mathrm{Im}\lambda^{s's}$ can vanish at incidence angles determined by the expression

$$\alpha = \mathrm{Re}\, F_{22}^{s's'} - \frac{\gamma_2}{\gamma_1} \mathrm{Re}\, F_{11}^{ss} + \frac{\mathrm{Im}(\gamma_2 F_{11}^{ss}/\gamma_1 - F_{22}^{s's'})\, \mathrm{Im}(F_{12}^{ss'} F_{21}^{s's})}{2\, \mathrm{Im}\, F_{11}^{ss}\, \mathrm{Im}\, F_{22}^{s's'}}. \quad (16)$$

In accordance with (16), the onset of the suppression effect for different proper polarization is possible in the general case at four unequal values of the angle of incidence of the radiation on the crystal. The points at which the suppression effect takes place are marked by circles on the dispersion surface (Fig. 1).

CONCLUDING REMARKS

The results of the preceding sections reveal the characteristic features of coherent scattering of Mössbauer radiation by birefringent magnetic crystals. It must be noted first that the angular, magnetic, and energy dependences of the intensity of the diffracted beam have a qualitatively different character than in the absence of birefringence. The reflection coefficient as a function of the incidence angle, of the magnetic-field orientation, and of the γ-quantum energy has a fine structure (see, e.g., Fig. 2), which contains information both on the amplitudes $F_{ip}^{s's}, i \neq p$ of the scattering through the Bragg angle and on the amplitudes F_{ii}^{ss} of forward scattering. This makes it possible in principle to determine these amplitudes by a diffraction experiment.

It is also possible to make a birefringent crystal diffracting or nondiffracting without changing its orientation relative to the incident beam, merely by changing the directions of the magnetic fields at the nuclei. It is then possible to study the reflection curves for a fixed scattering geometry by merely varying the orientation of the magnetic fields. It is similarly possible to investigate the reflection curves by varying the energy of the γ quanta.

Under strong birefringence conditions, the radiation scattered in each separated peak is fully polarized. Thus, reflection of Mössbauer radiation from birefringent crystals makes it possible in principle to obtain polarized beams of γ quanta, the type of polarization depending on the magnetic structure of the specimen.

The possibility of experimentally investigating the effects discussed in the present paper is connected with the use of crystals of high degree of perfection and of narrowly collimated beams of γ quanta. Possible objects for investigation can be, for example, perfect antiferromagnetic crystals of hematite (α-Fe_2O_3) and iron borate ($FeBO_3$)[17]. If these compounds are 85% enriched with ^{57}Fe, the birefringence for a Mössbauer line of energy 14.4 keV is $\mathrm{Re}(F_{ii}^{11} - F_{ii}^{22}) \sim 10^{-5}$, and the characteristic angle distance between the fine-structure peaks is $\Delta \approx 2''$. The strong-birefringence condition (3) is satisfied in these crystals for reflections with large values of the Bragg angle (for example, (10 10 10) in hematite, $\theta_B = 72°$), and also for the magnetic diffraction maxima. The structural scattering amplitude, depending on the γ-quantum energy and on the orientation of the magnetic field, can be of the order of $|F_{ip}^{s's}| \sim 10^{-6}$, which ensures in this case separation of the fine-structure peaks in the angular, magnetic, and energy dependences of the reflection coefficient. By choosing an asymmetrical scattering geometry (the case $|\gamma_2/\gamma_1| \gg 1$) it is possible to obtain an appreciable increase of the distance between the individual peaks compared with the value of Δ cited above. Taking into account the existing limitations on γ_2/γ_1 ($|\gamma_2/\gamma_1| \lesssim (F_{ii})^{-1/2}$), we obtain for the maximum angle separation of the peaks the estimate $\Delta_{14}^{max} \leqslant 3'$. The same estimate determines the requirements with respect to collimation of the beams and the mosaic structure of the crystal.

It should be noted that the discussed features of the angular and magnetic dependences of the coefficient of diffraction reflection can manifest themselves not only in the scattering of resonant γ quanta, but also in scattering of other types of radiation, such as neutrons, under birefringence conditions.

In conclusion, the authors thank V. P. Orlov, V. E. Dmitrenko, and R. Ch. Bokun for a helpful discussion of the questions touched upon in this paper.

[1]V. A. Belyakov, Usp. Fiz. Nauk 115, 552 (1975) [Sov. Phys. Usp. 18, 267 (1975)].
[2]V. A. Belyakov and Yu. M. Aĭvazyan, Pis'ma Zh. Eksp. Teor. Fiz. 7, 477 (1968) [JETP Lett. 7, 368 (1968)].
[3]M. A. Andreeva and R. N. Kuz'min, Dokl. Akad. Nauk SSSR 185, 1282 (1969) [Sov. Phys. Dokl. 14, 301 (1969)].
[4]A. M. Afanas'ev and Yu. Kagan, Zh. Eksp. Teor. Fiz. 64, 1958 (1969) [Sov. Phys. JETP 37, 987 (1973)].
[5]G. V. Smirnov, V. V. Sklyarevskiĭ, R. A. Voskanyan, and A. N. Artem'eva, Pis'ma Zh. Eksp. Teor. Fiz. 9, 123 (1969) [JETP Lett. 9, 70 (1969)].
[6]A. N. Artem'ev, I. P. Perstnev, V. V. Sklyarevskiĭ, G. V. Smirnov, and E. P. Stepanov, Zh. Eksp. Teor. Fiz. 64, 261 (1973) [Sov. Phys. JETP 37, 136 (1973)].

[7]G. V. Smirnov, N. A. Semioskhina, V. V. Sklyarevskiĭ, S. Kadeckova, and B. Sestak, *ibid.* 71, 2214 (1976) [44, 1167 (1976)].
[8]P. P. Kovalenko, V. G. Labushkin, and V. A. Sarkisyan, Pis'ma Zh. Eksp. Teor. Fiz. 28, 599 (1978) [JETP Lett. 28, 552 (1978)].
[9]V. G. Baryshevskiĭ, Yadernaya optika polyarizovannykh sred (Nuclear Optics of Polarized Media), Izd-vo BGU, Minsk, 1976.
[10]V. A. Belyakov and E. V. Smirnov, Zh. Eksp. Teor. Fiz. 68, 608 (1975) [Sov. Phys. JETP 41, 301 (1975)].
[11]E. V. Smirnov and V. A. Belyakov, Fiz. Tverd. Tela (Leningrad) 20, 2515 (1978) [Sov. Phys. Solid State 20, 1455 (1978)].
[12]V. L. Indenbom and F. N. Chukhovskiĭ, Usp. Fiz. Nauk 107, 229 (1972) [Sov. Phys. Usp. 15, 298 (1972)].
[13]Z. G. Pinsker, Dinamicheskoe rasseyanie rentgenovskikh lucheĭ v ideal'nykh kristallakh (Dynamic X-Ray Scattering in Ideal Crystals), Nauka, 1974.
[14]M. Blume and O. S. Kistner, Phys. Rev. 171, 417 (1968).
[15]Yu. M. Aivazyan and V. A. Belyakov, Fiz. Tverd. Tela (Leningrad) 13, 968 (1971) [Sov. Phys. Solid State 13, 808 (1971)].
[16]Yu. Kagan, A. M. Afanas'ev, and A. P. Perstnev, Zh. Eksp. Teor. Fiz. 54, 1530 (1968) [Sov. Phys. JETP 27, 819 (1968)].
[17]V. G. Labushkin, A. A. Lomov, N. N. Faleev, and V. A. Figin, Abstracts, 12th All-Union Conf. on the Use of X Rays for Single-Crystal Research, Zvenigorod, April, 1979.

Translated by J. G. Adashko

Radiation from a vortex in a long Josephson junction placed in an alternating electromagnetic field

M. B. Mineev and V. V. Shmidt

Institute of Solid State Physics, USSR Academy of Sciences
(Submitted 29 November 1979)
Zh. Eksp. Teor. Fiz. 79, 893–901 (September 1980)

We consider phenomena connected with the motion of one vortex in a long Josephson junction placed in an alternating electromagnetic field. We show that under certain conditions the vortex radiates electromagnetic energy to both sides of the junction, and the radiation frequency is in general not equal to the external frequency applied to the junction, i.e., a single vortex plays the role of a frequency converter. The presence of a threshold rate of vortex radiation leads to resonant singularities on the current-voltage characteristic of the junction.

PACS numbers: 74.50. + r

1. INTRODUCTION

The electromagnetic properties of long tunnel Josephson junctions have been the subject of many studies. In the present paper we are interested in phenomena connected with the motion of one vortex (or of a strongly rarefied chain of vortices) along a long junction. It is assumed that the alternating and direct currents perpendicular to the junctions are given and are uniformly distributed along the junction. We list in this connection some already known facts.

If a strong magnetic field is applied to the junction, a periodic vortex structure is produced in it.[1] When direct current is made to flow through this junction, the vortices are moved by the Lorentz force. If their velocity coincides with the electromagnetic-wave propagation velocity in the junction, a resonant peak appears on the current-voltage characteristic (CVC).[1,2] This picture is valid only in the presence of sufficiently strong damping, when edge effects can be neglected. If, however, the samping in the junction is weak, then the reflection of the electromagnetic waves from the edges of the junction gives rise to standing waves, i.e., the junction turns into a resonator. A singularity (a Fiske step) appears on the CVC of the junction when the Josephson frequency is equal to one of the natural frequencies of the junction.[1,3-5]

Highly interesting singularities in the form of giant steps were observed on the CVC of a junction by Chen *et al.*[6] in a zero magnetic field. This phenomenon was later investigated by Fulton and co-workers.[7,8] We recall briefly the gist of the phenomenon.

In a long junction to which direct current is applied, a single vortex executes finite motion, being periodically reflected from the edges. In each reflection act, the direction of the current in the vortex is reversed, and in each passage of the vortex (or antivortex) from one edge of the junction to the other the phase difference of the order parameter of the superconductors making up the junction increases by 2π. The average rate of change of the phase shift is thus

$$\omega=\frac{\partial\varphi}{\partial t}=\frac{2\pi v}{W},$$

where V is the vortex velocity and W is the junction length. Since the vortex velocity cannot exceed the maximum velocity c_0 of the electromagnetic wave in the junction (the Swihart velocity),[1)] we have

$$\omega<\omega_m=2\pi c_0/W.$$

This means that if only one vortex moves in the junction the voltage on the junction is

$$V<V_m=\Phi_0\omega_m/2\pi c=\Phi_0 c_0/Wc,$$

where Φ_0 is the magnetic-flux quantum, and c is the speed of light in vacuum. On the other hand, as $v\to c_0$

 0038-5646/80/090453-05$02.40