Electron bremsstrahlung in a dipole potential

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Electron bremsstrahlung is considered in a medium of point dipoles of magnitude d less than the critical value $0.639ea_0$ corresponding to absence of fall to the center. The bremsstrahlung cross section $d\sigma/d\omega$ is expressed, just as in the known Sommerfeld theory of radiation in a Coulomb field, in the form of single-parameter functions of the frequency. Analytic expressions are obtained for the radiation in a potential $\pm ar^{-2}$, and numerical solutions are obtained in a potential $-(\mathbf{d}\cdot\mathbf{r})r^{-3}$. The following limiting classical cases are considered: Born, classical, and limits of high and low radiation frequencies. Simple analytic approximations are obtained both for the spectrum and for the total radiation.

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1. INTRODUCTION

The bremsstrahlung of an electron moving in the potential of a point dipole is of interest from the viewpoints of both applications and general physics. The practical significance of this problem is due to its connection with electron radiation in a weakly ionized medium containing molecules that have a constant dipole moment d (such as NH_3). It is easy to estimate that, e.g., for a plasma containing molecules with ~1 a.u., the total radiation of the electron collisions with dipoles predominates over the radiation produced by collision with ions at a temperature $T\sim 1$ eV at a degree of ionization as low as $N_e/N_d \lesssim 10^{-3}$.

The problem is of general physical interest because it admits of an exact analytic solution at sufficiently small dipole values, $d \leqslant 0.639ea_0$ (e.g., for the molecules CO, NH_3, or H_2S). A classical example of the exact solution of the bremsstrahlung problem is the Sommerfeld theory of bremsstrahlung of an electron in a Coulomb field[1] (see also Ref. 2, §90). The general solution,[1] however, is exceedingly complicated in form and its use to obtain the limiting results of the Born and classical approximations is far from a trivial matter (see Ref. 3 and the literature therein).

The distinguishing features of the considered dipole potential are connected, first, with the specific law of its fall-off with distances ($\propto r^{-2}$, just as for a centrifugal potential) and, second, with the possibility that the particle will fall to the center at values $d > d_{cr} = 0.639ea_0$.[4,5] The first circumstance results in a radical simplification of the analytic form of the radial wave functions of the particle compared with the case of the Coulomb potential. The second circumstance leads to a limitation on the value of the scattering dipole ($d \leqslant d_{cr}$). It was precisely this circumstance which was used by Mittleman and von Holdt[4] to calculate the differential cross sections of electron scattering by a point dipole. We note that in the bremsstrahlung case of interest to us it is necessary to know the complete structure of the wave function, and not only its asymptotic form as in the scattering case.

It must be pointed out that the analogy between our problem and the Sommerfeld theory[1] remains in force only in the case of a point dipole, i.e., at $d \leqslant d_{cr}$, when the bremsstrahlung spectrum can be represented in the form of single-parameter functions of the frequency. Any departure from this model (e.g., the case $d > d_{cr}$ or allowance for the quadrupole interaction) calls for the introduction of additional parameter, which are connected with a correct treatment of the fall of the particle to the center.

An aggravating circumstance in the solution of our problem is the complicated angular dependence of the dipole potential. It is clear at the same time that allowance for the angular dependence does not introduce any new parameters in the problem. We consider first therefore the singularities in the limiting case of bremsstrahlung in the field of a spherically symmetrical potential $U=\alpha r^{-2}$ (Sec. 2), and then proceed to calculate the spectrum of the radiation in the real potential $U=-(\mathbf{d}\cdot\mathbf{r})r^{-3}$ (Sec. 3).

The points of physical interest in our problem are the deduction of the classical and Born results from the general quantum solution, as well as the connection between the bremsstrahlung cross section and the diffuse-scattering cross section.

2. RADIATION IN THE CASE OF MOTION IN A SPHERICALLY SYMMETRICAL POTENTIAL $\pm\alpha r^{-2}$

An analysis of the spectrum of the radiation of a particle in a potential $U=\alpha r^{-2}$ makes it possible, as already noted, to investigate all the limiting cases of the problem. The wave functions of the particle reduce in this case, as is well known,[5,6] to Bessel functions:

$$R_\nu(r)=(\pi/2qr)^{1/2}J_\nu(qr)\equiv|\nu\rangle;\quad \nu=[(l+1/2)^2+2M\alpha]^{1/2}, \tag{2.1}$$

where l is the orbital angular momentum, q is the momentum, and M is the particle mass. The cross section for bremsstrahlung in a spherically symmetrical potential is expressed in the form of the overlap integral $A_{ll'}$ of the wave functions (2.1) with angular momenta l and $l\pm1$, expanded in a series in terms of the angular momenta of the incident electron[9]

$$\hbar\omega\frac{d\sigma}{d\omega}=\frac{2\pi^2\alpha^2}{3Mc^3a_0}\sum_{l=0}^{\infty}(l+1)[A_{l+1,l}^2+A_{l,l+1}^2]. \tag{2.2}$$

In our case the integrals $A_{ll'}$ are expressed in explicit analytic form in terms of the complete hypergeometric function $F(a,b,c,x)$:

$$A_{vv'}=\left(\frac{q'}{q}\right)^{v'}\frac{\Gamma(a)}{\Gamma(1-b)\Gamma(c)}F(a,b,c,z^2). \tag{2.3}$$

Here q and q' are the initial and final momenta of the electron and are connected by the relations $(q^2-q'^2)/2M=\hbar\omega$; $z=q'/q$, $a=(v'+v-1)/2$, $b=(v'-v-1)/2$, $c=v'+1=a+b+2$, $\Gamma(x)$ is the gamma function, and $a_0\equiv\hbar^2/me^2$.

The problem has three limiting cases: 1) the Born approximation, 2) the diffusion approximation that connects the bremsstrahlung connection with the diffuse-scattering cross section, and 3) the classical approximation. The Born approximation follows from (2.2) and (2.3) in the limit as $M\alpha\to 0$:

$$\left(\hbar\omega\frac{d\sigma}{d\omega}\right)_B=\frac{8\pi^2\alpha^2}{3Mc^3a_0}\frac{q'}{q}. \tag{2.4}$$

The threshold[1)] behavior of the cross section in the high-frequency region ($\hbar\omega\approx\varepsilon=q^2/2M$), which follows from (2.4), is determined by the factor $(1-\hbar\omega/\varepsilon)^{1/2}$.

At low frequencies $\hbar\omega\ll\varepsilon(M\alpha)^{-1/2}$ the general formula (2.2) leads to the unknown connection between the cross section $d\sigma/d\omega$ and the diffuse-scattering cross section σ^*:

$$\hbar\omega\frac{d\sigma}{d\omega}=\frac{8}{3\pi}\frac{e^2}{Mc^3}\varepsilon\sigma^*, \tag{2.5}$$

$$\sigma^*=\frac{4\pi}{q^2}\sum_l(l+1)\sin^2(\delta_l-\delta_{l+1}), \tag{2.6}$$

where the phase shifts δ_l are easily discerned from the structure of the wave function (2.1) as $r\to\infty$:

$$\delta_l={}^1/_2\pi[l+{}^1/_2-((l+{}^1/_2)^2+2M\alpha)^{1/2}]. \tag{2.7}$$

At $M\alpha\gg 1$ and $\hbar\omega\ll\varepsilon$ we obtain from (2.2) and (2.3) the results of the classical analysis. These results can also be obtained from a Fourier analysis of the classical electron trajectory:

$$\hbar\omega\frac{d\sigma}{d\omega}=\frac{2\pi^2\alpha^2}{3Mc^3a_0}\frac{1}{q^2}\int_0^\infty\frac{\rho\,d\rho}{r_0^4}[A(\omega)+A(-\omega)], \tag{2.8}$$

$$A(\omega)=\left[k_{\rho/r_0+1}\left(\frac{\omega r_0}{v}\right)+k_{\rho/r_0-1}\left(\frac{\omega r_0}{v}\right)\right]^2, \tag{2.9}$$

where $k_v(x)=W_{v,1/2}(2x)/\Gamma(1+v/2)$ is the Whittaker function, $r_0^2=\rho^2+\alpha/\varepsilon$, and v is the electron velocity.

At low frequencies ω, Eqs. (2.8) and (2.9) lead to the results of the diffusion approximation (2.5) and (2.6), except that σ^* is replaced by the classical diffusion cross section $\sigma^*_{\rm cl}$. At high frequencies $\hbar\omega\gg\varepsilon(M\alpha)^{-1/2}$, the cross section (2.8), (2.9) falls off exponentially.

Expressions (2.8) and (2.9) yield a finite result only if $\alpha>0$. At $\alpha<0$ the total radiated energy diverges in the region $\rho\to(|\alpha|/\varepsilon)^{1/2}$, thus directly indicating the limitations of the classical analysis at low impact distances for an attraction potential that causes the particle to fall to the center.

It follows from the foregoing that the classical limit for the electron ($M=1$) calls for satisfaction of the condition $|\alpha|\gg 1$ and is therefore not realized in the considered point-dipole approximation. Since there is no classical limit in the problem, it is convenient to

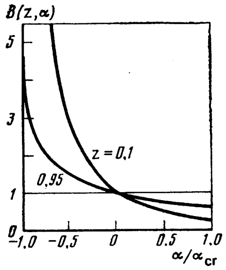

FIG. 1. Dependence of the factor B on the force constant α in the potential $\pm\alpha r^{-2}$ at small ($z=0.95$) and large ($z=0.1$) emission frequencies, $z\equiv q'/q=(1-\hbar\omega/\varepsilon)^{1/2}$.

characterize the behavior of the bremsstrahlung cross section not as usual by the Gaunt factor [equal to the ratio $(d\sigma/d\omega)/(d\sigma/d\omega)_{\rm cl}$], but by a factor B that determines the ratio of $d\sigma/d\omega$ to its Born limit:

$$B(\alpha,\omega)=(d\sigma/d\omega)/(d\sigma/d\omega)_B. \tag{2.10}$$

It is clear from the foregoing that the parameter α (the magnitude of the dipole moment) is the analog of the parameter $Ze^2/\hbar v$ in the Coulomb problem. The results of the calculation of the factor B are shown in Figs. 1 and 2. It is important that, depending on the sign of α, we can have $B(\alpha,\omega)$ larger ($\alpha<0$) or smaller ($\alpha>0$) than unity (Fig. 1).

This circumstance is particularly clear near the threshold $\hbar\omega\approx\varepsilon$. In fact, in the high-frequency limit $z=q'/q\to 0$ we can put $F(a,b,c,z^2)\approx 1$ in (2.3) and retain in the sum (2.2) only on term with the minimum value $v'=v_0=v(l=0)=(\frac{1}{4}+2M\alpha)^{1/2}$, thus obtaining

$$\hbar\omega\frac{d\sigma}{d\omega}=\frac{2\pi^2\alpha^2}{3Mc^3a_0}\left[\frac{\Gamma(a)}{\Gamma(1-b)\Gamma(c)}\right]^2\Bigg|_{l=1,l'=0}(1-\hbar\omega/\varepsilon)^{v_0}. \tag{2.11}$$

In the limit of small $M\alpha$, the law governing the decrease of the cross section at the threshold approaches the Born law corresponding to the value $v_0=\frac{1}{2}$. It is seen that, depending on the sign of α, the decrease of the cross section at the threshold, compared with the Born value, is either more ($\alpha>0$) or less ($\alpha<0$) abrupt.

The considered character of the spectrum of $d\sigma/d\omega$ and all its limiting cases, except for the classical case, apply also to the case of a real dipole, with the stipulation, however, that the magnitude and sign of the effective interaction constant α are far from obvious beforehand. It is only clear that the values of the true factor

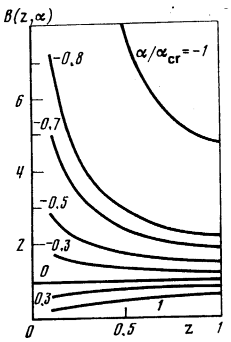

FIG. 2. Dependence of the factor B on the frequency $z=(1-\hbar\omega/\varepsilon)^{1/2}$ at different α in a field $U(r)=\pm\alpha r^{-2}$.

B must lie between the considered cases of "pure" attraction and "pure" repulsion, but the difference between the latter can be quite appreciable (Fig. 2).

3. ELECTRON RADIATION IN THE POTENTIAL $U=-(\mathbf{d}\cdot\mathbf{r})r^{-3}$ OF A POINT DIPOLE

We obtain, first, the bremsstrahlung cross section in the Born approximation:

$$\left(\hbar\omega\frac{d\sigma}{d\omega}\right)_B=\frac{32}{9}\frac{(de)^2}{Mc^3a_0}\frac{q'}{q}. \tag{3.1}$$

A comparison with (2.4) shows that the correct value of (3.1) is obtained from (1.4) by substituting $\alpha=\alpha_{eff}=2d/\pi\sqrt{3}$.

To find $d\sigma/d\omega$ in the general case it is necessary to set up the system of wave functions of the electron in the potential $-(\mathbf{d}\cdot\mathbf{r})r^{-3}$. The Hamiltonian of the system is of the form

$$\hat{H}=\frac{\hat{p}_r^2}{2M}+\frac{\hat{l}^2}{2Mr^2}-\frac{\mathbf{dr}}{r^3}\equiv\frac{\hat{p}_r^2}{2M}+\frac{\hat{\Lambda}}{2Mr^2}, \tag{3.2}$$

where $\hat{p}_r$ is the radial momentum, $\hat{l}$ is the orbital angular momentum of the electron, $\hat{\Lambda}\equiv\hat{l}^2-2M\mathbf{d}\cdot\mathbf{n}$, and $\mathbf{n}\equiv\mathbf{r}/r$.

It is seen from (3.2) that the entire angular dependence is contained in the operator $\hat{\Lambda}$, which does not depend on r. It is therefore possible to separate directly the radial and angular motions of the electron, using wave function with a definite value of $\hat{\Lambda}$. This approach was used effectively in a number of studies with a dipole potential.[4,10-13] The eigenvalue λ of the operator $\hat{\Lambda}$, which serves as the variable-separation constant in (3.2), is obviously a conserved quantity.[4,10,11] Also conserved is the projection m of the angular momentum on the direction of the dipole $\mathbf{d}$. Since $[\hat{l}_z,\hat{\Lambda}]=0$, we can introduce functions with definite λ and m:

$$\psi^{\lambda m}=R^{\lambda m}(r)\,|\lambda m\rangle,$$
$$\hat{\Lambda}|\lambda m\rangle=\lambda|\lambda m\rangle,\ \hat{l}_z|\lambda m\rangle=m|\lambda m\rangle. \tag{3.3}$$

The radial functions $R^{\lambda m}$ are expressed, as in (2.1) above, in terms of Bessel functions:

$$R^{\lambda m}(r)=(q/r)^{1/2}J_\nu(qr). \tag{3.4}$$

The connection between the wave functions $|l_m\rangle$ and the spherical functions $|lm\rangle$ ($m=\text{const}$) is given by

$$|\lambda m\rangle=\sum_{l=|m|}^{\infty}\langle lm|\lambda m\rangle|lm\rangle, \tag{3.5}$$

where the transformation coefficients of the basis $\langle lm|\lambda m\rangle$ and the eigenvalues λ are determined by diagonalizing the matrix $\hat{\Lambda}$ in the basis $|lm\rangle$:

$$\langle l'm|\hat{\Lambda}|lm\rangle=l(l+1)\delta_{ll'}-2Md\langle l'm|\cos\vartheta|lm\rangle, \tag{3.6}$$

where ϑ is the angle between $\mathbf{r}$ and $\mathbf{d}$. The problem of diagonalizing matrices of this type is considered in Ref. 13.

To calculate $d\sigma/d\omega$ we must establish the connection between the wave functions $|\lambda m\rangle$ and the wave functions $|\pm\mathbf{q}\rangle$ corresponding to a definite value of the momentum. Writing down the connection between the two bases in the form

$$|\pm\mathbf{q}\rangle=\sum_{\lambda m}\langle\lambda m|\pm\mathbf{q}\rangle R^{\lambda m}|\lambda m\rangle \tag{3.7}$$

and equating the coefficients of the factors $e^{\pm i\mathbf{qr}}$ in the asymptotic expansions of the right- and left-hand sides of (3.7), we get[2)]

$$\langle\lambda m|\pm\mathbf{q}\rangle=\frac{(2\pi)^{3/2}}{q}\sum_{l=|m|}^{\infty}(-1)^l\,Y^*_{lm}(\mathbf{n}_q)\,e^{\mp i(\nu+1/2)\pi/2}\langle\lambda m|lm\rangle, \tag{3.8}$$

where $Y_{lm}(n)$ are spherical functions. The functions $|\pm\mathbf{q}\rangle$ are normalized to $(2\pi)^{-3}\delta(\mathbf{q})$.

Separating the diverging part of the function $|+\mathbf{q}\rangle-e^{i\mathbf{qr}}$, we can obtain an explicit expression for the scattering amplitude

$$f=4\pi\sum_{ll'm}Y^*_{lm}(\mathbf{n}_q)\,Y_{l'm}(\mathbf{n}_r)\langle l'm|\hat{f}_l|lm\rangle, \tag{3.9}$$

where f_l is the partial scattering amplitude, which is diagonal in the λ representation:

$$\langle l'm|\hat{f}_l|lm\rangle=\sum_\lambda\langle l'm|\lambda m\rangle\langle\lambda m|lm\rangle\frac{\exp(2i\delta_{\lambda l})-1}{2iq}, \tag{3.9'}$$

where $\delta_{\lambda l}=\frac{1}{2}\pi(l+\frac{1}{2}-\nu)$ are the scattering phase shifts, which turn out to be independent of energy by virtue of the specifics of the dipole potential (see Ref. 14).

The obtained wave functions $|\pm\mathbf{q}\rangle$ can be used to calculate the matrix elements of the coordinate $\mathbf{r}_{qq'}$, which determine the bremsstrahlung cross section. Direct calculation yields

$$\hbar\omega\frac{d\sigma}{d\omega}=\frac{(\pi de)^2}{6Mc^3a_0}\sum_{\lambda\lambda'mm'}|\langle\lambda m|\mathrm{N}|\lambda'm'\rangle|^2|A^{\lambda m}_{\lambda'm'}|^2, \tag{3.10}$$

where $d\equiv|\mathbf{d}|$, $\mathrm{N}d=\mathbf{d}-3\mathbf{n}(\mathbf{dn})$, and the overlap integrals A are determined by the same formulas (2.3) as before, except for the substitution $\nu=(\lambda+\frac{1}{4})^{1/2}$.

An investigation similar to that in Sec. 2 shows that at small d the general formula (3.10) leads to the results of the Born approximation (3.1). At low frequencies we obtain from (3.10) the connection (2.5) between the bremsstrahlung and diffusion-scattering cross sections. The latter is determined by the general formulas (3.9) and (3.10) for the scattering amplitude.

In the region of the threshold frequencies $\hbar\omega\approx\varepsilon$ ($z=q'/q\ll 1$) it follows from (3.10) and (2.3) that

$$\hbar\omega\frac{d\sigma}{d\omega}=\frac{(\pi de)^2}{6Mc^3a_0}\frac{(1-\hbar\omega/\varepsilon)^{\nu_0}}{\Gamma^2(\nu_0+1)}\sum_{\lambda,m}\left[\frac{\Gamma(a)}{\Gamma(1-b)}\right]^2|\mathrm{N}^{\lambda m}_{\lambda_0 m}|^2, \tag{3.11}$$

where $\nu_0=(\lambda_0+\frac{1}{4})^{1/2}$ is expressed in terms of the smallest eigenvalue λ_0.

In the limit of small d we can obtain an approximate expansion for λ_0:$\lambda_0\approx -Md/\sqrt{3}$, from which we get

$$\hbar\omega\frac{d\sigma}{d\omega}=\left(\hbar\omega\frac{d\sigma}{d\omega}\right)_B z^{\nu_0-1/2}=\left(\hbar\omega\frac{d\sigma}{d\omega}\right)_B z^{\lambda_0}. \tag{3.12}$$

It is seen from a comparison of (3.12) with (2.11) as $\alpha\to 0$ that near the threshold a small real dipole behaves like an attraction potential with constant $\alpha_{eff}=-d/2\sqrt{3}$. Thus, the effective attraction constant α turns out to be much less than the dipole d.

Numerical calculations were performed for the factor B:

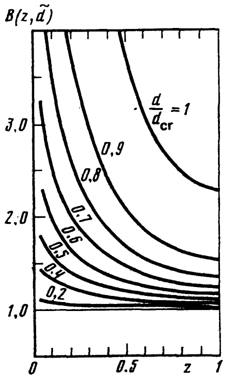

FIG. 3. Dependence of the factor B on the frequency in the field of a real dipole $U(r)=-(\mathbf{d}\cdot\mathbf{r})r^{-3}$ at different values of d/d_{cr}.

$$B(\tilde d, z)=\frac{d\sigma/d\omega}{(d\sigma/d\omega)_B}=\frac{3\pi^2}{64}\frac{1}{z}\sum_{\lambda\lambda' mm'}|\mathbf{N}^{\lambda m}_{\lambda' m'}|^2|A^{\lambda m}_{\lambda' m'}|^2, \tag{3.13}$$

where $z=q'/q=(1-\hbar\omega/\varepsilon)^{1/2}$ and $\tilde d\equiv d/d_{cr}$ ($d_{cr}=0.639ea_0$).

The general behavior of the factor B is clear from the foregoing analysis:

$$B(z, \tilde d\to 0)\to 1,\ B(z\ll 1, \tilde d)\approx f(\tilde d)z^{2\nu_0-1};$$
$$B(z\approx 1, \tilde d)=\sigma^*(\tilde d)/\sigma_B^*. \tag{3.14}$$

The last ratio of the diffusion cross section to its Born limit was calculated earlier[4]; our calculations of the factor B agree well (within 3–5%) with these results in the limiting case $z\to 1$.

The results of the calculation of the factor B are shown in Fig. 3. It is seen that on the whole the character of the bremsstrahlung spectrum corresponds to the case of attraction. The effective constant α_{eff} of the attraction potential $-\alpha r^{-2}$, however, is much smaller than the dipole potential. This is qualitatively understandable: the real dipole potential corresponds to attraction as well as repulsion, so that the variation of B is subject to opposite tendencies that cancel each other to a considerable degree.

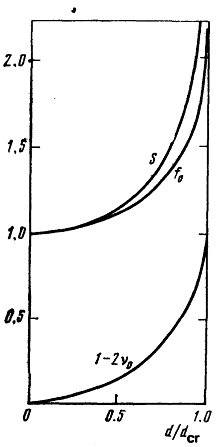

FIG. 4. Dependence of the quantities $2\nu_0-1$, f, and S, which determine respectively the behavior and magnitude of the cross section at the threshold and the total effective radiation, on d/d_{cr}.

We note an interesting and important circumstance that follows from the numerical calculations of the factors B for a spherically symmetrical potential αr^{-2} and a point-dipole potential $-(\mathbf{d}\cdot\mathbf{r})r^{-3}$. In both cases the effective variable of the force interaction is the quantity $\tilde\alpha=\alpha/\alpha_{cr}=8\alpha$ or $d=d/d_{cr}=d/0.639$. It turns out in this case that the following equality holds within ~5% in a wide frequency transition region ($0.1<z\leqslant 1$):

$$B(\tilde d, z)={}^1\!/_2[B(\tilde\alpha=\tilde d, z)+B(\tilde\alpha=-\tilde d, z)]. \tag{3.15}$$

The factor B for radiation in a noncentral potential of a point dipole with a given $\tilde d$ can thus be approximately represented in the form of two terms with weights $\frac{1}{2}$, which correspond to radiation in a spherically symmetrical attraction potential ($\tilde\alpha=-\tilde d$) and repulsion potential ($\tilde\alpha=\tilde d$).

For an analysis of the near-threshold cross section behavior that follows from (3.11) and (3.14), Fig. 4 shows the minimum values of $2v_{min}-1=(4\lambda_{min}+1)^{1/2}-1$ as functions of $\tilde d=d/d_{cr}$, as well as the values of the function $f(\tilde d)=\lim(B/z^{2\nu_0-1})$ as $z\to 0$.

4. TOTAL EFFECTIVE RADIATION AND EMISSIVITY OF ELECTRONS IN A DIPOLE MEDIUM

We consider now the radiation characteristics averaged over the spectrum and over a Maxwellian distribution of the electron velocities. We calculate first the effective radiation $\varkappa$ (Ref. 5):

$$\varkappa=\int_0^{\omega_{max}}\hbar\omega\frac{d\sigma}{d\omega}d\omega;\qquad \hbar\omega_{max}=\varepsilon. \tag{4.1}$$

Expression (4.1) is easily expressed in terms of the factor B:

$$\varkappa=\frac{32}{9}\frac{(de)^2}{Mc^3a_0}\varepsilon S(\tilde d), \tag{4.2}$$

$$S(\tilde d)=3\int_0^1 B(\tilde d, z)z^2\,dz. \tag{4.3}$$

A plot of S against the ratio $d/d_{cr}\equiv\tilde d$ is shown in Fig. 4. As $\tilde d\to 0$ the quantity $S(d)$ tends to unity, as it should.

Further averaging over (4.2) over the Maxwellian distribution of the electron velocities is trivial and reduces to replacement of ε in (4.2) by $\langle\varepsilon\rangle=(\frac{3}{2})kT$, where T is

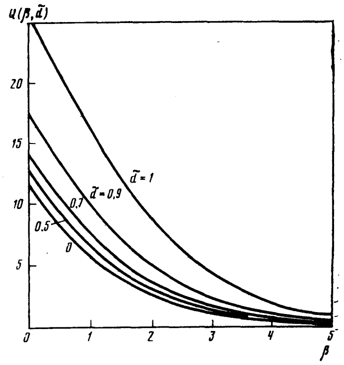

FIG. 5. The function $Q(\beta, \tilde d)$, which determines the emissivity of the electrons in a dipole medium ($\beta=\hbar\omega/kT$, $\tilde d\equiv d/d_{cr}$).

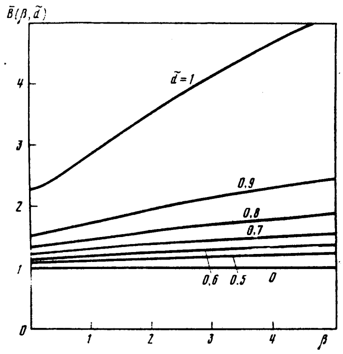

FIG. 6. The function $\bar{B}(\beta,\tilde{d})$, which determines the ratio of the emissivity to its Born limit.

the electron temperature. This circumstance is closely connected with the specifics of the dipole potential, in which S turns out to be independent of the electron energy.

We obtain next the electron emissivity j_ω in a dipole medium[15]:

$$j_\omega = N_d \hbar\omega \langle v(d\sigma/d\omega)\rangle, \tag{4.4}$$

where N_d is the dipole concentration and $\langle\ldots\rangle$ denotes averaging over the Maxwellian velocity distribution. In the Born approximation we have according to (3.1)

$$j_\omega^B = N_d v_T c_B Q_B(\hbar\omega/kT), \tag{4.5}$$

$$Q_B(\beta) = \frac{4\beta^2}{\pi^{1/2}}\int_0^1 dz\,\frac{z^2}{(1-z^2)^3}\exp\left(-\frac{\beta}{1-z^2}\right),$$

$$c_B = {}^{32}/_9\,(de)^2/Mc^3a_0. \tag{4.6}$$

In the general case the function $Q_B(\beta)$ in (4.5) is replaced by the function $Q(\beta,\tilde{d})$, which contains the factor B:

$$Q(\beta,\tilde{d}) = \frac{4\beta^2}{\pi^{1/2}}\int_0^1 dz\,\frac{z^2 B(z,\tilde{d})}{(1-z^2)^3}\exp\left(-\frac{\beta}{1-z^2}\right). \tag{4.7}$$

The limiting expressions for $Q(\beta,\tilde{d})$ are

$$Q(\beta,\tilde{d}) \approx \begin{cases} 2B(1,\tilde{d})\pi^{-1/2}, & \beta \ll 1 \\ 2\pi^{-1/2} f(\tilde{d})\Gamma(\nu_0+1)\beta^{1-\nu_0}e^{-\beta}, & \beta \gg 1 \end{cases}. \tag{4.8}$$

It is seen from (4.8) that at $\beta \ll 1$ the emissivity j_ω is expressed in terms of the diffusion cross section (3.14), and at $\beta \gg 1$ the main contribution is made by the near-threshold region.

The form of the function $Q(\beta,\tilde{d})$ is shown in Fig. 5. Figure 6 shows the ratio $\bar{B} = Q(\beta,\tilde{d})/Q_B(\beta)$ and different values of $\tilde{d}$; from this ratio we can assess the extent to which the emissivity differs from the Born approximation. The limiting values of this ratio are of the form

$$\bar{B} = \begin{cases} B(1,\tilde{d}), & \beta \ll 1 \\ \Gamma(\nu_0+1)\Gamma^{-1}(3/2) f(\tilde{d})\beta^{-(\nu_0-1/2)}, & \beta \gg 1 \end{cases}. \tag{4.9}$$

5. ANALYTIC APPROXIMATION. CONCLUSION

For practical estimates it is convenient to have simple analytic approximations of various parameters that characterize the bremsstrahlung. We begin with the description of the behavior of the cross section $d\sigma/d\omega$ at the threshold. According to (3.11) and (3.14), the cross section near the threshold $z = q'/q = (1-\hbar\omega/\varepsilon)^{1/2} \ll 1$ is of the form

$$\left(\hbar\omega\frac{d\sigma}{d\omega}\right)_{\text{thr}} \approx f(\tilde{d})z^{2\nu_0-1}, \tag{5.1}$$

where the functions $f(\tilde{d})$ and $v_0(\tilde{d})$ are approximated, with ~5% accuracy, by the formulas

$$f(\tilde{d}) \approx 2-(1-\tilde{d}^2)^{1/2} \equiv \xi(\tilde{d}), \tag{5.2}$$

$$2\nu_0(\tilde{d})-1 \approx -1+(1-\tilde{d}^2)^{1/2} \equiv 1-\xi. \tag{5.3}$$

The function $f(d)$ turns out to be quite close to the value of the factor B in the diffusion limit, i.e., to $B(1,d)$. This allows us to count on a good accuracy of the interpolation of the total factor B in the entire region of z and $\tilde{d}$ with the aid of the relation

$$B(z,\tilde{d}) \approx \xi z^{1-\xi}. \tag{5.4}$$

The accuracy of the approximation (5.4) turns out to be not worse than 15%. The use of (5.4) to calculate the effective emission (4.3) yields

$$S(\tilde{d}) \approx 3\frac{\xi}{4-\xi}. \tag{5.5}$$

The function $Q(\beta,\tilde{d})$, which characterizes the emissivity [(4.5), (4.7)], is approximated for values $\tilde{d} \leqslant 0.9$, with accuracy 5–10%, by the formula

$$Q(\beta,\tilde{d}) \approx 2\xi\pi^{-1/2}e^{-\beta}[1+\beta^{\xi}\Gamma^2(2-\xi/2)]^{1/\xi}. \tag{5.6}$$

In conclusion, we summarize the principal results of the article. Their gist is most clearly represented by Fig. 6, which shows the mean difference between the exact and Born results. It is seen that at a given emission frequency (at a given β) this difference increases rapidly when the dipole moment approaches its critical value. Next, the deviation from the Born approximation increases monotonically with increasing frequency (with increasing β), and the rate of this increase is larger the larger d. We note finally that all the indicated results pertain only to bremsstrahlung in the case of potential (nonresonant) scattering.

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1) The term "threshold" is used here in a somewhat broader sense to describe the abrupt break in the bremsstrahlung spectrum in the region of high frequencies $\omega \approx \omega_{\max} = \varepsilon/\hbar$.
2) A similar problem was solved in broadening theory.[12]

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Photon emission in collisions of a proton or positron with an atom

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Photon emission produced upon collision of a proton and a positron with a hydrogen atom is considered. It is shown that the emission cross section contains contributions from collisions at which the state of the atom remains unchanged (pure bremsstrahlung) as well as from collisions at which excitation of the atom takes place simultaneously with the emission of the proton. The cross section of the bremsstrahlung is calculated in the Born approximation in the characteristic frequency band in which the photon energy is much higher than the atom ionization energy. The results differ substantially in a wide range of emission frequencies from the known results of bremsstrahlung theory. The difference is due only to the more accurate formulation of the problem: in the present paper the bremsstrahlung is regarded as the emission of an "atom plus incident particle" system, so that the role of the atomic electron does not reduce merely to static screening of the nucleus, in contrast to earlier assumptions. A consistent analysis leads to the appearance of a new effect—emission of a photon by a proton (positron) with simultaneous excitation of the atom into the discrete or continuous spectrum state. The emission of the proton with ionization of the atom greatly exceeds the "pure bremmstrahlung" in a definite frequency interval.

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1. INTRODUCTION

The bremsstrahlung produced when a charged particle is scattered by an atom or an ion is customarily calculated in the given-field approximation.[1] This approximation means that the electron of the target atom is regarded as a static charge that screens the nucleus, so that the bremsstrahlung of the incident particle takes place in the given electrostatic field of the nucleus and of the electron ψ-cloud. When the particle is scattered in a given external field, both the quantum and the classical electrodynamics lead to a bremsstrahlung cross section $\propto(e/m)^2$, where e and m are the charge and mass of the incident particle.[1] This leads obviously to the conventional notions concerning the bremsstrahlung of a proton or positron, namely that the proton bremsstrahlung cross section is negligibly small compared with the electron bremsstrahlung cross section and that the positron and electron bremsstrahlung cross sections are equal in the first Born approximation.

It is shown in the present paper that these two conclusions are the consequence of the approximate formulation of the bremsstrahlung problem, in which the role of the atomic electron reduces only to static screening of the nuclear field. If the given-field method in the bremsstrahlung problem is replaced by an exact multiparticle formulation it turns out that, in a definite particle region, the integral cross sections of the proton and electron bremsstrahlung are comparable at equal particle velocities relative to the target atom, and the positron and electron bremsstrahlung cross sections differ substantially even in the first Born approximation.

Bremsstrahlung scattering of a charge particle by an atom, from the point of view of the quantum mechanics is a several-particle problem: the Hamiltonian of the system should take into account on a par all the particles (the nucleus of the atom, the atomic electrons, the incident particle), and allowance must be made for all the kinetic energies of the particles, all the interactions between them, and the interaction of each particle with the electromagnetic field. In particular, the proton bremsstrahlung cross section turns out to be comparable with the electron bremsstrahlung cross section because the exact Hamiltonian of the system takes into account not only the interaction of the proton with the electromagnetic field $\propto 1/m_p$, but also the interaction of the atomic electron with the electromagnetic field, $\propto 1/m$. If we use for the problem this formulation, which is certainly more accurate than the given-field approximation, and calculate the first nonvanishing term in the expansion of the transition amplitude in terms of the interactions of the incident particle with the atom and of all the particles with the electromagnetic field, new formulas are obtained for the proton

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