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Translated by R. T. Beyer

Instabilities of two-dimensional plasma waves

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(Submitted 28 December 1979)

Zh. Eksp. Teor. Fiz. 79, 555-560 (August 1980)

The main types of plasma instabilities in two-dimensional electron systems are investigated from the point of view of amplifying two-dimensional plasma waves. Structures consisting of two plasma layers or of a plasma layer above a conducting half-space are considered. The conditions for the onset of two-stream, kinetic, and dissipative instabilities are found. Under certain conditions the instability criteria differ qualitatively from their three-dimensional analogs. The critical drift velocities and oscillation growth factors are calculated.

PACS numbers: 52.35.Py

Experimental studies of plasma waves in two-dimensional electron systems carried through during the last three years¹⁻³ have confirmed the principal theoretical conclusions concerning the dispersion law for such oscillations. Their report⁴ includes a detailed review of these studies. Certain specific characteristics of two-dimensional plasmons—their gapless spectrum, complicated dispersion law, and relatively low group velocity—make them very attractive objects for physical research and open up prospects for interesting applications. From this point of view it would certainly be desirable to work with two-dimensional plasmons as with “ordinary” traveling waves (e.g. ultrasonic waves), i.e., to modulate them, amplify them, etc. We note that the experimental results now available relate to the case of standing plasma waves, whose presence was detected either by a change in the Q factor of a resonator (in the case of electrons above a liquid helium surface) or by the resonant absorption of radiation in the far infrared (in the case of the inversion layer in a metal insulator-semiconductor structure).

In this paper we examine the principal types of plasma instabilities in two-dimensional systems as they relate to the problem of amplifying two-dimensional plasma waves. As in the three-dimensional problem, a wave may become unstable as a result of the drift of one part of the plasma with respect to another (see, e.g., Ref. 5).

A specific feature of the case we are considering is that the two parts of the plasma are spatially separated: for example, they may be two parallel thin plasma layers, or a plasma layer above a conductive half-space. In such systems coupled waves arise and amplification can be achieved at a certain drift velocity. The

coupling coefficient depends on the distance between the layers and on the plasmon momentum; this considerably complicates the dispersion law for the waves. In addition, the criterion for instability may differ substantially from its three-dimensional analog. In particular, it turns out that the beam instability is characterized by a threshold drift velocity that depends on the distance between the two plasma layers. At below-threshold drift velocities, instability can arise only for plasmons whose wave number k lies within a certain interval: $k_{\min} < k < k_{\max}$ ($k_{\min} = 0$ in the three-dimensional case).

What was said above is valid for the beam instability of a cold plasma. When the thermal motion of the particles is taken into account, wave amplification as a result of kinetic instability becomes possible. The most favorable case for the development of this instability is realized when the effective masses of the particles of the moving and stationary plasmas differ greatly. Finally, when electrons are strongly scattered in one of the plasmas and the drift velocity is low enough the situation is reminiscent of the amplification of sound by an electric current in a piezoelectric medium.

Estimates show that in the case of electrons above a helium film on a conductive backing, amplification begins at comparatively low drift velocities of the carriers in the backing. At present, such a system is the most promising for obtaining amplification of two-dimensional plasma waves.

The problem of oscillations in spatially nonuniform plasma streams has been discussed in the literature. For example, Mikhaïlovskii and Pashitskii⁶ investigated the stability of two neighboring electron streams separa-

ted by a thin transition layer of thickness δ . They assumed that $k\delta \ll 1$ and that the wave frequency was small as compared with the cyclotron frequency (the case of a strongly magnetized plasma). The dispersion equation for waves in a plasma layer with sharp boundaries has also been found for the same limiting case of a strong magnetic field.⁷ The present paper is concerned with a system of plasma layers that are very thin compared with the distance Δ between them, but for which the parameter $k\Delta$ can assume arbitrary values. We also treat plasma (Langmuir) waves in the absence of a magnetic field. Thus, the results obtained below do not overlap the results of the papers cited above since they are actually obtained for an opposite limiting case. We also note that the structures we discuss are precisely those that either have already been realized experimentally or can be easily produced by current techniques. As far as we know, however, no experiments have been done up to now in which one plasma has been caused to drift relatively to another, so that no observations of the amplification of two-dimensional waves have been reported.

1. BEAM INSTABILITY

Let us consider two thin parallel plasma layers separated by a dielectric gap, and let us assume that one of them moves with respect to the other with the drift velocity u . One can think of several ways to realize such a system experimentally. First, the system could consist of two thin semiconducting or semimetallic films deposited on opposite sides of a dielectric plate. Second, having produced an inversion layer in a sufficiently thin semiconducting plate, one can obtain an enrichment layer on the opposite side of the plate; in this case we have electron- and hole-type plasma layers separated by a depletion region. Finally, by using modern molecular epitaxy techniques one can produce layered structures in which a two-dimensional electron gas forms at the boundary of heterojunctions as a result of loss of electrons from donor levels or from the valence band of one component of the structure to the conduction band of another.^{8,9}

For simplicity we shall assume the dielectric constant ϵ to be the same in the regions inside and outside the gap between the plasma layers. Since the thickness of the layers is assumed to be much smaller than the wavelength of the oscillations, the equation for the electrostatic potential φ takes the form

$$\Delta\varphi = -\frac{4\pi e}{\epsilon} \sum_i \tilde{N}_i \delta(z-z_i), \quad i=1,2. \quad (1)$$

Here z_i is the z coordinate of the i -th layer (both layers being perpendicular to the z axis) and \tilde{N}_i is the non-equilibrium addition to the surface density of the plasma N_{S_i} in the i -th layer.

The quantity \tilde{N}_i can be obtained from a self-consistent solution of the kinetic equation for the electron distribution function in which the field $\nabla\varphi$ of the plasma wave is taken into account. If we consider only a cold collision plasma, however, it is sufficient to use the equation of motion

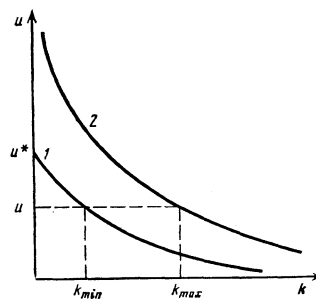


FIG. 1. Curves 1 and 2 are graphs of the lower and upper limits in inequality (3), respectively. The instability region is the region between the curves.

$$m_i \ddot{\xi} = -e \nabla \varphi,$$

in which $\xi(r, t)$ is the displacement of the electron liquid from the equilibrium position. Then we obviously have $\tilde{N}_1 = -N_{S1} \operatorname{div} \xi$, and the set of equations is thus closed.

Let us assume that the drift velocity in the first layer is directed along the x axis. Then if we assume that all the quantities are proportional to $\exp i(kx - \omega t)$ we easily obtain the dispersion equation

$$[(\omega - ku)^2 - \omega_1^2](\omega^2 - \omega_2^2) = e^{-2k\Delta} \omega_1^2 \omega_2^2, \quad (2)$$

$$\omega_i^2 = 2\pi e^2 N_{S_i} k / m_i e,$$

in which m_i is the effective mass of a particle in the i -th layer and Δ is the distance between the layers.

Instability arises when a pair of complex conjugate roots of Eq. (2) appears. The case $\omega_1 = \omega_2 = \omega_0$ can be analyzed simply and completely since the equation reduces to a biquadratic one. The following double inequality must be satisfied for instability to appear:

$$\frac{2\omega_0}{k}(1 - e^{-k\Delta})^{1/2} < u < \frac{2\omega_0}{k}(1 + e^{-k\Delta})^{1/2}. \quad (3)$$

As $k \rightarrow 0$, the left-hand side of inequality (3) tends to the constant value $u^* = 2(2\pi N_{S_1} e^2 \Delta / m_1 \epsilon)^{1/2}$. Thus, if $u > u^*$, waves of arbitrarily long wavelength will be unstable (as in the three-dimensional case⁵). If $u < u^*$, however, the wave number k must lie in the interval $[k_{\min}, k_{\max}]$ if instability is to develop (see Fig. 1).

Equation (2) can be analyzed simply in the case in which $\omega_2 \gg \omega_1$ and $k\Delta \ll 1$. Then $k_{\min} = 0$ if $u > 2(\pi N_{S_2} e^2 \Delta / m_2 \epsilon)^{1/2}$ and $k_{\min} > 0$ if $u < 2(\pi N_{S_2} e^2 \Delta / m_2 \epsilon)^{1/2}$. The upper limit k_{\max} obviously exists in the general case: as a function of ω , the left hand side of Eq. (2) has four real roots, whereas the right-hand side is independent of ω and assumes arbitrarily small positive values as k increases. Thus, a value k_{\max} can always be found such that all four solutions of (2) are real $k > k_{\max}$. We note that the real part ω' of the root that leads to instability is given by $\omega' = ku/2$ when $\omega_1 = \omega_2$, and by $\omega' = ku$ when $\omega_2 \gg \omega_1$.

2. KINETIC INSTABILITY

Now let us take the thermal motions of the plasma particles into account and let us assume that at a given temperature the thermal velocities of the particles in the two plasmas differ greatly ($v_{T2} \ll v_{T1}$) as a result

of a large difference between the effective masses in them ($m_2 \gg m_1$). Further, let us consider a frequency region in which $\omega \sim \omega_2 \gg kv_{T2}$ (a cold plasma). We shall assume that the velocity distribution function for the plasma particles in layer 1 is a Boltzmann function with its center shifted by the quantity u . We shall use the equation that determines the plasma oscillations of a two-dimensional nondegenerate gas [see from Eq. (6) of Ref. 10]. Then the dispersion equation for the coupled oscillations has the form

$$\omega^2 = \omega_2^2 \left\{ 1 - e^{-2k\Delta} \left[1 + i\pi^{1/2} \frac{\omega - ku}{kv_{T1}} W \left(\frac{\omega - ku}{kv_{T1}} \right) \right] / \left[1 + kr_0 + i\pi^{1/2} \frac{\omega - ku}{kv_{T1}} W \left(\frac{\omega - ku}{kv_{T1}} \right) \right] \right\}, \quad (4)$$

$$W(z) = e^{-z^2} \left(1 + \frac{2i}{\pi^{1/2}} \int_0^z e^{t^2} dt \right),$$

where r_0 is the two-dimensional screening radius in layer 1.

The inequality $|\omega - ku| \ll kv_{T1}$ is satisfied near the instability threshold, and the solution of Eq. (4) can be easily found:

$$\text{Re } \omega = \omega' = \omega_2 \left[1 - \frac{e^{-2k\Delta}}{1 + kr_0} \right]^{1/2}, \quad (5)$$

$$\text{Im } \omega = \omega'' = \frac{\pi^{1/2}}{2} e^{-2k\Delta} \frac{\omega_2^2}{\omega'} \frac{kr_0}{(1 + kr_0)^2} \frac{ku - \omega'}{kv_{T1}}.$$

As is evident from (5), instability is most likely to develop when $kr_0 \sim 1$.

3. DISSIPATIVE INSTABILITY

In this section we shall consider the situation in which the plasma in one of the layers (say the first one) is highly collisional so that there are no plasma oscillations in that layer. We shall describe the electron gas in that layer by the two-dimensional conductivity σ which, generally speaking, depends on the tensile field and on the diffusion constant D . Then we easily obtain the following dispersion equation:

$$\omega^2 = \omega_2^2 \left[1 - \frac{e^{-2k\Delta}}{1 + \varepsilon kD/2\pi\sigma - i\varepsilon(\omega - ku)/2\pi k\sigma} \right]. \quad (6)$$

It is not difficult to see from Eq. (6) that when

$$u = u_0 = \frac{\omega_2}{k} \left[1 - \frac{e^{-2k\Delta}}{1 + \varepsilon kD/2\pi\sigma} \right]^{1/2} \quad (7)$$

the imaginary part ω'' of the frequency changes sign while the real part is equal to ku_0 .

Near the threshold the growth factor is given by

$$\omega'' = \frac{\varepsilon e^{-2k\Delta} \omega_2^2}{4\pi\sigma k} \frac{u - u_0}{u_0}.$$

As $u \rightarrow \infty$, the solution to Eq. (6) obviously becomes

$$\omega = \omega_2 \left(1 + i \frac{\pi e^{-2k\Delta} \sigma}{\varepsilon u} \right).$$

Thus, the growth factor for waves with a fixed value of k passes through a maximum as the drift velocity increases from the threshold value.

It is easy to discern an analogy between this case and the amplification of ultrasound in a piezoelectric semiconductor. The part played by the velocity of sound is played here by the phase velocity u_0 of a plasmon in

layer 2, whose dispersion law differs from the simple relation $\omega \propto k^{1/2}$ because of the interaction with the carriers in the first layer. We note that in the easily realized limiting case $k\Delta \ll 1$ the coupling between the layers does not become small, whereas in the amplification of ultrasound the interaction of the electron stream with the lattice is small, as a rule, because the electromechanical coupling constant itself is small.

4. A PLASMA LAYER ABOVE A CONDUCTING HALF-SPACE

All the instabilities of two-dimensional plasma waves investigated here can also occur in the case (which is evidently easier to realize experimentally) in which one of the plasma layers is replaced by a bulky conductor. It is obvious, for example, that beam instability will arise because of the coupling of a surface plasmon of the massive conductor with a two-dimensional plasmon in the thin film. The formulas of Sec. 1 remain valid provided one of the plasma frequencies ω_1 is replaced by the frequency $\omega_p/\sqrt{2}$ of a surface plasmon, where ω_p is the "three-dimensional" plasma frequency.¹⁾

The phase velocity of two-dimensional plasmons in inversion layers of semiconductors is equal in order of magnitude to 10^7 – 10^8 cm/sec (see Ref. 1). Great difficulties would be encountered in attempting to achieve such drift velocities in a solid. However, one might propose the following experimental setup: an electron beam moves in vacuo along the axis of a hollow dielectric cylinder onto the outer surface of which a thin semiconductor or semimetal film has been deposited. In the most realistic case, in which $kR \gg 1$ (R is the radius of the beam), the problem obviously reduces to the two-dimensional case treated above. We note that the development of two-stream instability is the most practicable in such a system, since the beam velocity can be made considerably higher than in a solid-state plasma.

Dissipative instability can be obtained in a layer of electrons above a liquid-helium film deposited on a bulky conducting substrate. In this system the surface charge density may be of the order of 10^8 – 10^9 cm⁻² and then the necessary carrier drift velocity in the substrate is much lower than in solid body structures. The corresponding formulas are similar to those presented in Section 3. For example, the threshold drift velocity is

$$u_0 = \frac{\omega_2}{k} \left[1 - \frac{e^{-2k\Delta}}{1 + (\varepsilon + 1)k^2 D_0 / 4\pi\sigma_0} \right]^{1/2}, \quad (8)$$

where Δ is the thickness of the helium film and ε , D_0 , and σ_0 are the substrate dielectric constant, diffusion constant, and conductivity, respectively (the dielectric constant of helium is taken as unity).

If the plasmon wavelength is much greater than Δ while $k^2 D_0 / \sigma_0 \sim k^2 r_D^2 \ll k\Delta$ (r_D is the Debye radius in the substrate), u_0 will be independent of k . We obtain the following expressions for the real and imaginary parts of the frequency near the amplification threshold:

$$\omega' = k u_0, \quad \omega'' = \frac{(\varepsilon + 1) \omega_2^2}{8\pi\sigma_0} \frac{u - u_0}{u_0}. \quad (9)$$

The inequality $k^2 r_D^2 \ll k\Delta$ is easily compatible with the inequality $k\Delta \ll 1$ over a wide range of substrate carrier densities and temperatures. For $N_S \sim 10^8 \text{ cm}^{-2}$ and $\Delta \sim 10^{-5} \text{ cm}$, we obtain $u_0 \sim 10^6 \text{ cm/sec}$ for virtually all values of k (for which, of course, $k < N_S^{1/2}$). In connection with this estimate we note that the drift velocities in some semiconductors are much higher (e.g., $5 \times 10^7 \text{ cm/sec}$ in InSb).

¹As before, we assume the dielectric constant to be the same in all of space. No real difficulty is encountered in taking the differences in the values of ε in various regions into account, but the resulting formulas, although they are very cumbersome, do not alter the qualitative picture.

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Translated by E. Brunner

Absorption of surface electromagnetic waves by thin oxide films on metal surfaces

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(Submitted 4 January 1980)

Zh. Eksp. Teor. Fiz. **79**, 561–574 (August 1980)

The effect of thin silicon oxide films on the propagation of infrared surface electromagnetic waves (SEW) over copper is investigated. The SEW spectra are used to determine the optical constants of the films and substrates. The SEW spectra of natural oxide films on aluminum and on molybdenum are obtained.

PACS numbers: 78.20.Dj, 78.65.Jd, 73.40.Ns

Surface IR electromagnetic waves (SEW), which can propagate over the surface of a well-conducting metal to distances up to several centimeters,^{1,2} are very sensitive to the surface state of the metal and to the presence of thin films on the surface. The increase of the SEW absorption following the deposition of a film, with the absorption increasing near the film oscillation frequencies, permits the development of a new effective spectroscopic method—the spectroscopy of surface electromagnetic waves.^{2,3}

This method was used for an experimental investigation of silicon monoxide films on copper, of apatite on silver,⁴ and of films of cellulose acetate and benzene on copper.⁵ Investigation of the absorption of the SEW has made it possible to obtain a dependable spectrum of a monomolecular Langmuir film of siloxane acid on the surface of copper.⁶ The spectrum obtained by us earlier⁴ for the SEW of a silicon monoxide film on a copper surface differs noticeably from that calculated by using the published data on the optical constants of silicon monoxide.⁷ In the present paper we have used the SEW spectra to determine the optical constants of silicon-oxide films on copper surface and of the natural oxide

film on aluminum and molybdenum in the 10- μm region of the spectrum (CO₂ laser).

1. THEORY

Surface optical excitations exist on the interface of two media with dielectric constants ε that have opposite signs. Agranovich and Ginzburg² have developed the crystal optics of surface waves and have shown that, in analogy to three-dimensional crystal optics, the propagation of SEW can be described by introducing the SEW refractive index (which we designate by κx), which connects the frequency ν and the excitation wave vector k_x .¹⁻³ When damping is taken into account, the refractive index of SEW becomes complex, and its imaginary part defines the SEW absorption coefficient α . The reciprocal of the absorption coefficient is called the path length L (distance over which the SEW intensity attenuates by a factor $e = 2.72$).

Consider an air–metal interface. Air has $\varepsilon_1 = 1$; using for the metal the Drude formula for the dielectric constant, we obtain²

$$\alpha = 1/L \approx 2\pi\nu^2\nu_0/\nu_p^2, \quad (1)$$