Sudostroenie, 1966.
[21]Ya. B. Zel'dovich and Yu. P. Raĭzer, Fizika udarnykh voln i vysokotemperaturnykh gidrodinamicheskikh yavleniĭ (Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena), Fizmatgiz, 1966 [Academic, Chap. I only].
[22]E. I. Zababakhin, Prik. Mekh. Mat. 24, 1129 (1960).
[23]V. M. Goloviznin, V. F. Tishkin, and A. P. Favorskiĭ, Variational Approach to the Construction of Difference Schemes for Hydrodynamic Equations in Spherical Coordinates, Preprint, Inst. Appl. Math. USSR Acad. Sci., No. 16, 1977.
[24]V. A. Gasilov, V. M. Goloviznin, V. F. Tishkin, and A. P. Favorskiĭ, Numerical Solution of one Model Problem on Rayleigh-Taylor Instability, Preprint No. 119, Inst. of Appl. Math. USSR Acad. Sci., 1977.

Translated by J. G. Adashko

Propagation of electromagnetic solitons in nonequilibrium dispersive media

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(Submitted 31 January 1980)
Zh. Eksp. Teor. Fiz. 79, 472–477 (August 1980)

We establish the possibility of the existence of stationary solitary electromagnetic waves in nonequilibrium plasma-beam systems. We show that the propagation of wave packets in resonance with the plasma (the retarding medium) or the beam is not accompanied by a dispersive smearing of the plasma. We determine the conditions for the existence of solitons, find their characteristics, and give a physical interpretation of the solutions obtained. As a concrete example we consider a helicon soliton in a plasma penetrated by an ion beam.

PACS numbers: 52.35.Hr, 52.40.Db, 52.40.Mj

It is well known that small monochromatic electromagnetic perturbations are unstable in plasma-beam systems and grow exponentially with time.[1] The nonlinear stage of the interaction of mono-energetic beams with a plasma (the retarding medium) is accompanied with the appearance of oscillations in time of the field amplitude which are caused by the trapping of the beam particles by the field of the wave and their periodic shift from decelerating to accelerating phases.[2-8]

In the present paper we wish to call attention to the possibility of the existence of stationary non-linear waves in non-equilibrium media, which are retarding systems which are penetrated by charged particle beams. In comparison with the above cited papers[2-8] which assume the wave number fixed and the initial amplitude of the perturbation to be constant along the beam, we obtain a solution of a solitary-wave type for the field amplitude. Since the carrier frequency and wave number in this case satisfy resonance conditions, the wave energy in each point of space is replenished from the translational energy of the beam particles. At the same time, however, the energy of each beam particle remains unchanged after passing through the region where the field is a maximum due to the non-linear effect of getting out of phase with the wave which leads to a shift of the particle from a decelerating to an accelerating phase and an increase in its energy up to its initial value. This nature of the interaction between the beam particles and the field explains the possibility of the existence of resonance solitons in non-equilibrium plasma-beam systems.

As an example of an actual model of a non-equilibrium medium we consider an arbitrary dispersive retarding medium through which a charged particle beam propagates along a constant external magnetic field $\mathbf{H}_0$. Since practically the whole information about the conditions for the existence of non-linear waves of the kind

$$f(t, x) = \operatorname{Re} A(\xi) \exp(i\omega_0 t - ik_0 x) \tag{1}$$

($\xi = x - ut$, u is the group velocity) is contained in the linear dispersion equation and allowance for the nonlinearity only enables one to determine the maximum amplitude and to find the shape of the wave pulse, we consider first the problem in the linear approximation.

The dispersion equation connecting the frequency ω and the wave number k of an electromagnetic perturbation propagating along the magnetic field in a medium with refractive index $n(\omega)$ has the form[2,6]

$$\frac{c^2k^2}{\omega^2} = n^2(\omega) - \frac{\omega_b^2(\omega - kv_0)}{\omega^2(\omega - kv_0 + \omega_H)}, \tag{2}$$

where $\omega_b^2 = 4\pi e^2 \rho_b / M$, $\omega_H = eH_0/Mc$, while ω_b, v_0, and M are the density, initial velocity, and mass of the beam.

We look for a solution of (2) in the form $\omega = \omega_0 + \Delta\omega$ and $k = k_0 + \Delta k$, assuming that ω_0 and k_0 satisfy the resonance conditions:

$$ck_0 = \omega_0 n(\omega_0), \quad \omega_0 - kv_0 = -\omega_H. \tag{3}$$

The small corrections to the frequency and wave number are then connected by the relation

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$$c^2(2k_0\Delta k+\Delta k^2)=\frac{d}{d\omega_0}(\omega_0^2n^2)\Delta\omega+\frac{1}{2}\frac{d^2}{d\omega_0^2}(\omega_0^2n^2)\Delta\omega^2+\frac{\omega_b^2\omega_H}{\Delta\omega-v_0\Delta k}-\omega_b^2. \tag{4}$$

In the case of a monochromatic wave with a time-dependent amplitude which is excited in a dispersionless medium by a low-density beam we have[2,6]

$$\Delta\omega^2=-\frac{1}{2}\frac{\omega_b^2\omega_H}{\omega_0n^2}, \quad \Delta k=0. \tag{5}$$

Taking dispersion into account enables us to find the condition for the propagation of finite-width wave packets. Putting $\Delta\omega=u\Delta k$ in (4) we get

$$\Delta k^2=-\frac{v_g}{2nc}\frac{\omega_H}{\omega_0}\frac{\omega_b^2}{(u-v_g)(u-v_0)}, \tag{6}$$

$$1-\frac{u^2}{v_g^2}\left(1-\frac{k_0v_g'}{v_g}\right)+\left(\frac{\omega_b^2}{c\Delta k}\right)^2=0, \tag{7}$$

where

$$v_g=c\left[\frac{d}{d\omega_0}(\omega_0n)\right]^{-1}, \quad v_g'=\frac{dv_g}{dk_0}, \quad 1-\frac{k_0v_g'}{v_g}=\frac{v_g^2}{2c^2}\frac{d^2}{d\omega_0^2}(\omega_0^2n^2).$$

From Eq. (6), which generalizes Eq. (5), it follows that aperiodic perturbations corresponding in the non-linear approximation to soliton solutions exist when $u>v_g$, $u>v_0$ or when $u<v_g, u<v_0$. Relation (7) is obtained in the next approximation in the beam density and is the condition that there is no dispersive spreading of the wavepacket. One must note that when there is no beam such stationary solutions are not realized and wave packets in the medium with $v_g'\neq 0$ spread out due to dispersion, since Eq. (4) is equivalent to a parabolic equation.[9]

We find from Eqs. (6) and (7)

$$u=\left\{v_g+v_0\pm\left[\left(v_g-\frac{c}{n}\right)^2+\left(v_0^2-\frac{c^2}{n^2}\right)\frac{k_0v_g'}{v_g}\right]^{1/2}\right\}\Big/\left\{2+\frac{1}{v_g}\left(v_0-\frac{c}{n}\right)\left(1-\frac{k_0v_g'}{v_g}\right)\right\}. \tag{8}$$

We shall look for the solution of the non-linear problem in the form of a circularly polarized plane wave with an amplitude depending on $\xi=x-ut$, using the presence of the small parameter

$$q^2=\frac{1}{2}\frac{\omega_b^2}{\omega_H^2}\frac{v_g}{cn}\frac{v_0-u}{v_g-u}\ll 1. \tag{9}$$

The calculations necessary to derive the set of slow equations are then in principle not different from those given in Ref. 6 for the amplitudes and phases which depend only on the time. However, now the equations depend on the parameter u which we can determine by retaining in the equation for the field the next terms in the expansion in the small parameter (9) as compared to the equations in Ref. 6:

$$(v_g-u)\frac{dE}{d\xi}=-2\pi e\rho_b\frac{v_g}{cn}\left[1+\frac{v_0n}{c}\frac{(c-nu)(v_0-v_\parallel)}{(c-nv_0)(v_0-u)}\right]v-i\frac{2\pi e\rho_bv_g}{k_0c^2}\left\{\frac{v_gc}{2n(v_g-u)}\left[1-\frac{u^2}{v_g^2}\left(1-\frac{k_0v_g'}{v_g}\right)\right]-\frac{c(v_0-u)}{c-nv_0}\right\}\frac{dv}{d\xi}. \tag{10}$$

One sees easily that the expression in braces vanishes if the group velocity u satisfies Eqs. (6) and (7). The coefficient of v_g-u in the principle term in (10) is thereby determined and the correction to unity turns out to be small and can be dropped.

In the dimensionless parameters

$$v_\perp=cae^{i\vartheta}, \quad E_\perp=H_0\varepsilon e^{i\varphi}, \quad b=v_\parallel n/c-1,$$
$$\zeta=\omega_H\xi/(v_0-u), \quad \Omega=\omega_0/\omega_H, \quad \eta=\vartheta-\varphi$$

the set of non-linear equations takes the form

$$da/d\zeta=-b\varepsilon\cos\eta, \quad db/d\zeta=n^2a\varepsilon\cos\eta,$$
$$d\varepsilon/d\zeta=-q^2a\cos\eta, \quad d\eta/d\zeta=1-\Omega b-\operatorname{tg}\eta\,(d/d\zeta)\ln\varepsilon a \tag{11}$$

and is the same as the analogous set in Ref. 6, if we change in the latter to the non-relativistic limit. However, this agreement is formal, as we are considering a totally different kind of solution.

Using the integrals of motion of Eqs. (11)

$$n^2a^2+b^2=\Omega^{-2}, \quad \varepsilon^2-\varepsilon_0^2=2\frac{q^2}{n^2}(\Omega^{-1}-b), \quad \sin\eta=\frac{(1-\Omega b)^2}{2a\varepsilon\Omega n^2} \tag{12}$$

and changing to the new variables $n\Omega a=\sin\psi$ and $\Omega b=\cos\psi$, we get the following equation:

$$\frac{d\psi}{d\zeta}=\left[\varepsilon_0^2n^2+\frac{q^2}{\Omega}\psi^2-\left(\frac{\psi}{2}\right)^6\right]^{1/2}, \quad |\psi|\ll 1. \tag{13}$$

If there is no external field, $\varepsilon_0=0$, the solution of (13) has the form of a solitary wave:

$$\psi(\zeta)=\left(\frac{8q}{\Omega^{1/2}}\right)^{1/2}\operatorname{ch}^{-1/2}\left(\frac{2q}{\Omega^{1/2}}\zeta\right). \tag{14}$$

In dimensional variables we obtain for the field amplitude the expression

$$|E_\perp|=E_m\operatorname{ch}^{-1/2}(2|\Delta k|\xi), \quad E_m=\frac{H_0}{n}(2q)^{3/2}\left(\frac{nv_0}{c}-1\right)^{3/4}, \tag{15}$$

where the parameter Δk which determines the width of the pulse is given by Eq. (6) of the linear theory.

The physical reason for the existence of a stationary wave in a non-equilibrium medium can be understood by considering the process of the interaction of resonance particles with the wave in the soliton frame of reference where the beam moves with the speed $v_\parallel-u$. Since the frequency of the field is in resonance, a particle incident on the trailing edge of the wave will then start to slow down in the longitudinal direction, and its transverse energy increases. This process, accompanied by the transformation of the energy of the longitudinal motion into wave energy, stops when the particle reaches the maximum of the wave amplitude, since at that point the difference in phases between the transverse velocity and the electrical field vectors, due to the detuning of the anomalous Doppler resonance as the result of the longitudinal retardation, reaches the value $|\eta|=\pi/2$. After this the particle reaches a retarding phase, loses transverse energy, and accelerates along the magnetic field to reach its initial velocity.

We thus find that, on the one hand, the beam constantly replenishes the energy of the wave, and, on the other hand, there is no exchange of energy of the beam particles with the wave over a time which is larger than the time during which the particle passes through the field pulse. As a result, the energy density of a resonance wave pulse propagating in a non-equilibrium medium remains unchanged.

We note that the small parameter (9) in the case considered differs from the analogous parameter of Ref. 6 by the factor $1-u/v_g$ in the denominator. The energy density of a soliton with a group velocity u close to the

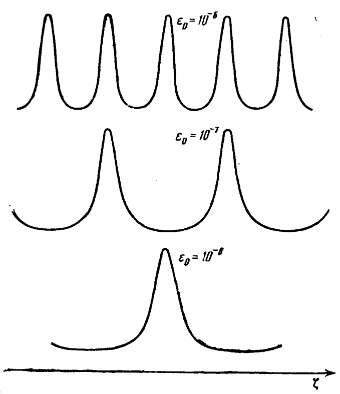

FIG. 1.

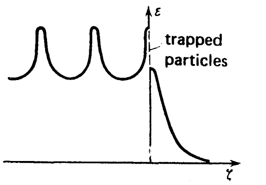

FIG. 2.

group velocity v_g of the medium is thus appreciably larger than the analogous quantity for monochromatic field perturbations.[6]

The nature of the solution changes, according to Eq. (13), when $\varepsilon_0 \neq 0$; this corresponds to the propagation or wave packets which are extrinsic to the system, whereas the solutions (14) and (15) are eigenoscillations of the non-equilibrium medium. It follows from Fig. 1 that there is a modulation of the wave amplitude due to the interaction with the beam particles (the ordinates represent $\varepsilon/\varepsilon_{max}$). When the parameter ε_0 decreases the width or each peak increases and, as $\varepsilon_0 \to 0$ the periodic structure is transformed into a soliton. It is interesting to note that Eq. (13) describes only waves which are unbounded in space. If, however, the field amplitude is given, for instance, in a half-space, there occurs a discontinuity or the function $\varepsilon(\zeta)$ in the point $\zeta = 0$ (see Fig. 2). Hence it follows that this kind of solution can exist only when there are trapped particles present. The possibility of the propagation of shock waves without discontinuities in a finite-temperature plasma[10] is, apparently, caused by the presence of resonance particles with velocities close to the group velocity of the wave, which smooth out these discontinuities.

In a strong electrical field we find from Eq. (13)

$$\frac{dy}{d\zeta} = \frac{1}{\Lambda}(1-y^6)^{1/2}, \quad (n\varepsilon_0)^{2/3} \gg \frac{q}{\Omega^{1/2}},$$
$$y=\psi/\psi_m, \quad \psi_m = 2(n\varepsilon_0)^{1/3}, \quad \Lambda = 2(n\varepsilon_0)^{-2/3}. \tag{16}$$

The parameters ψ_m and Λ determine the depth of the modulation of the wavepacket and the low-frequency period of the change of the field.

In concluding this paper we consider a helicon soliton in a plasma when there is an ion beam present.[1)] The refractive index of the plasma is in this case equal to

$$n^2(\omega) = \omega_p^2/\omega\omega_{He} \gg 1, \quad \omega_{Hi} \ll \omega \ll \omega_{He} \tag{17}$$

(ω_p is the plasma frequency, ω_{Hi} and ω_{He} are the ion and electron gyrofrequencies of the plasma). The frequencies $\omega_\pm$ satisfying the resonance conditions (3)

$$\frac{\omega_\pm}{\omega_{Hi}} = \left[\frac{v_0}{2C_A} \pm \left(\frac{v_0^2}{4C_A^2} - 1\right)^{1/2}\right]^2, \tag{18}$$

determine two values for the soliton group velocity:[2)]

$$u_+ = v_0 + C_A(\omega_+/\omega_{Hi})^{1/2}, \quad u_- = C_A(\omega_-/\omega_{Hi})^{1/2} \tag{19}$$

[$C_A = H_0(4\pi\rho_p M)^{1/2}$ is the Alfvén velocity and ρ_p the plasma density].

The maximum values of the magnetic field amplitude $(H_\pm)_m$ of each of the waves are equal to

$$(H_\pm)_m = H_0\left(4\frac{\rho_b}{\rho_p}\right)^{1/4} \begin{cases} (\omega_+/\omega_{Hi})^{3/4} \\ (\omega_{Hi}/\omega_-)^{3/4} \end{cases}. \tag{20}$$

The characteristic sizes of the pulses are then of the same magnitude:

$$|\Delta k_\pm| = \left(\frac{\rho_b}{\rho_p}\right)^{1/2} \frac{\omega_{Hi}}{C_A}, \quad k_\pm = \frac{\omega_{Hi}}{C_A}\left(\frac{\omega_\pm}{\omega_{Hi}}\right)^{1/2}. \tag{21}$$

We note that the ion beam excites in the plasma short-wavelength plasma oscillations, if its velocity exceeds the thermal velocity v_{Te} of the electrons in the plasma. The condition for the applicability of Eqs. (17) to (21) is thus the inequality

$$v_{Te} > v_0 \geqslant 2C_A. \tag{22}$$

The results obtained are of interest in connection with the possibility to generate electromagnetic pulses in the circumterrestrial plasma by solar corpuscular currents (plasma of the solar wind), and also for the transmission of weakly damped regular electromagnetic signals in a plasma along charged particle beams. It is necessary in the latter case that the time growth rate $|\Delta\omega| = u|\Delta k|$ exceeds the damping of the wave in the plasma.

The authors are grateful to V. I. Karpman for a discussion of the results of this paper and for a useful comment.

[1)]We note that the electrical field vector rotates in step with the resonance ions in the beam. In contrast to the paper by Istomin and Karpman[10] we therefore consider an ion helicon, while the electrons in the plasma guarantee the decrease in the phase velocity of the wave.

[2)]Compared with the asymptotic Eqs. (4), (6) to (8), which are accurate up to terms $\propto \Delta\omega^3$, these expressions are exact.

[1]Ya. B. Faĭnberg, At. Energ. 11, 313 (1961).

[2]V. B. Krasovitskiĭ and V. I. Kurilko, Zh. Eksp. Teor. Fiz. 49, 1831 (1965) [Sov. Phys. JETP 22, 1252 (1966)].

[3]V. B. Krasovitskiĭ, Radiofiz. 13, 1902 (1970); Pis'ma Zh. Eksp. Teor. Fiz. 15, 346 (1972) [Radiophys. Qu. Electr. 13, 1468 (1973); JETP Lett. 15, 244 (1972)].

[4]N. Matsiborko, I. N. Onishchenko, Ya. B. Faĭnberg, V. D. Shapiro, and V. I. Shevchenko, Zh. Eksp. Teor. Fiz. 63, 874 (1972) [Sov. Phys. JETP 36, 460 (1973)].

[5]A. A. Ivanov, V. V. Parail, and T. K. Soboleva, Zh. Eksp. Teor. Fiz. 63, 1678 (1972) [Sov. Phys. JETP 36, 887 (1973)].

[6]V. B. Krasovitskiĭ, Zh. Eksp. Teor. Fiz. 66, 154 (1974) [Sov. Phys. JETP 39, 71 (1974)].

[7]A. B. Kitsenko, I. M. Pankratov, and K. N. Stepanov, Zh. Eksp. Teor. Fiz. 66, 166 (1974) [Sov. Phys. JETP 39, 77 (1974)].

[8]Kh. D. Aburdzhaniya, I. M. Pankratov, and A. B. Kitsenko, Fiz. Plazmy 4, 227 (1978) [Sov. J. Plasma Phys. 4, 130 (1978)].
[9]B. B. Kadomtsev, Kollektivnye yavleniya v plazme (Collective phenomena in a plasma) Nauka, 1976, p. 91 [English translation published by Pergamon Press].
[10]Ya. N. Istomin and V. I. Karpman, Zh. Eksp. Teor. Fiz. 63, 1698 (1972) [Sov. Phys. JETP 36, 897 (1973)].

Translated by D. ter Haar

Self-pumping of gas in pulsed-periodic energy supply

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(Submitted 18 February 1980)
Zh. Eksp. Teor. Fiz. 79, 478–480 (August 1980)

Self-pumping of gas in a closed loop by pulsed-periodic energy supply was realized in experiment. Some questions involved in the investigation of the possibilities of self-pumping in periodic-action pulsed lasers are considered.

PACS numbers: 42.55. – f

The feasibility of self-pumping of a gas mixture in a closed loop without the use of special pumping devices, and its use in periodic-action pulsed lasers (PAPL), was first indicated by Gubarev, Drobyazko and Yakushev.[1] The interest in this problem is due to the fact that by dispensing with the compressor it would be possible to increase the efficiency of the PAPL, as well as to ensure good hermetic sealing of the loop when the laser operates in a closed cycle.

The energy released in a pulsed discharge leads to rapid increase of a pressure in the discharge region and to the onset of shock waves propagating away from the discharge zone in the gas channel. The thermodynamic cycle that characterizes the operation of the gas in such a process is the Lenoir cycle[1,2] which includes: 1) isochoric energy input; 2) adiabatic expansion described by the Poisson formula; 3) isobaric cooling of the gas. The thermal efficiency of this cycle, when the pressure is doubled at the instant of the discharge, is ~10% for air. From the calulation of the cycle it follows that to increase its efficiency the energy supply must be made rapid and as large as possible. The estimated energy of a self-pumping system is optimistic. The necessary pumping rate can be calculated by specifying the gas temperature at the input, using the condition that the average power input to the discharge is carried away by the gas stream.

The dominant factor in our case was the thermal resistance, and the power loss necessary to surmount it is easy to calculate. In this case the ratio of the power needed to ensure the gas flow to the average power input into the discharge turns out to be 6×10^{-5}. Assuming a thermodynamic-cycle efficiency 10^{-1}, it can be concluded that the only problem is the conversion of the work of the waves into the work necessary to pump the gas mixture.

In our experiments, self-pumping of the gas was realized by using a system with an aerodynamic valve (Fig. 1). The electrode system consisted of a solid aluminum anode 1 and a sectionalized cathode 2 located near the closed end of a quarter-wave acoustic resonator 20 cm long. The length of the input channel was 12 cm. The inductive decouplings of the cathode sections and their small geometrical dimensions ensured low sensitivity of the electrode system to the gas dynamic inhomogeneities. The discharge volume was 3.2 cm^3 at an interelectrode distance 1.5 cm. The energy input to the discharge in one pulse was 0.55 J.

The choice of the construction of the input channel 4 is governed by the desire to effectively absorb and scatter the waves that enter the input channel from the acoustic resonator. The ratio of the cross section areas of the acoustic resonator and of the exit valve was chosen equal to three. The stream velocity in the acoustic resonator was determined from the deflection of a test body secured with the aid of a cathetometer. The chamber was placed in a Mach-Zahnder interferometer; the displacement of the interference fringes at a chosen point of the channel was registered with a photomultiplier. The signal from the photomultiplier was used to monitor the resonant excitation of the natural frequencies of the quarter-wave resonator.

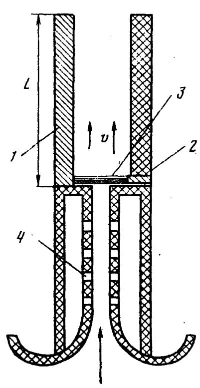

FIG. 1. Diagram of discharge chamber. 1—Anode, 2—cathode, 3—discharge region, 4—input channel.

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