for hoping to use pair production by photons in the pinch field to resolve the spatial structure of the current channel in the angstrom region. The spectral and angular distributions of the positrons carry information on the spatial structure of the current and can reveal, in principle, the presence of linear atoms in pinches, and can consequently ascertain whether the compression under the influence of the forces of collective interaction and radiation collapse[9,12-15] can reach the stage of electron degeneracy.

It must be noted, however, that despite the large cross section of the process ($\sim 10^{-13}$ cm^2), the characteristic time of pinch evolution in the state of maximum compression is estimated at $\lesssim 10^{-9}$ sec, so that realization of the proposed experiment is a major problem in high-energy experimental physics.

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Nonstationary Josephson effect in quasi-two-dimensional superconductors

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Equations are obtained for the Green's function describing the nonequilibrium phenomena in quasi-two-dimensional superconductors. The flow of a current in a direction perpendicular to the conducting layers is investigated by means of these equations in dirty quasi-two-dimensional superconductors ($\tau\varepsilon_1 \ll 1$, where τ is the mean free path along the layer, ε_1 is the width of the energy band corresponding to motion across the layers). For finite voltages V applied to the sample, and sufficiently weak coupling between the layers ($\tau\varepsilon_1^2 \ll \Delta$), Josephson oscillations occur in the system with a frequency $2eV/N$, where N is the number of layers in the system. In contrast to tunnel junctions, in which the electric field is localized in the dielectric and does not enter the superconductor, the field in a quasi-two-dimensional superconductor does not vanish at any point within the crystal. The energy distribution of the quasiparticles is not an equilibrium one and this results in an increase in the energy gap of the superconductor. The transverse conductivity of the system in the normal state has the form $\sigma_\perp \sim e^2 md\tau\varepsilon_1^2$.

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As is well known, a number of layered compounds become superconductors at helium temperatures. The most studied of these layered compounds are the dichalcogenides of the transition metals, for example, $NbSe_2$, TaS_2, $TaSe_2$. The properties of such materials are described, for example in the review of Bulaevskiĭ.[1] These highly anisotropic crystals consist of layers, within which the binding between the atoms is covalent, while the layers are bound to one another by weak van der Waals interaction. The anisotropy of these compounds can be significantly increased artificially by intercalation—the introduction of other atoms or molecules in the space between the layers. Thus, for example, the ratio of the longitudinal and transverse conductivities of $2H$-TaS_2 increases from 28 to 10^5 by intercalation of pyridine. The critical temperature of superconductors of such type is determined basically by the interaction within the layer and changes in intercalation only to the degree that the characteristics of the conducting layer change upon change in the distance between the layers. Thus, for example, in the intercalation of $2H$-TaS_2 the temperature T_c increases from 0.8–2 K to 2–4.5 K, and in the case of $2H$-$NbSe_2$ it decreases from 7 to 4 K.

There is interest in the problem of the transverse conductivity of the quasi-two-dimensional superconductors. If the anisotropy is sufficiently strong that the characteristic energy connected with motion between the layers is less than the value of the gap energy, then the superconductivity has a quasi-two-dimensional character.[1)] In this case, in the flow of near-

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critical current between the layers, the modulus of the order parameter remains practically unchanged and the crystal can be regarded as a system of series connected Josephson tunnel junctions with superconductors of atomic thickness. The stationary Josephson effect in quasi-two-dimensional superconductors has been investigated with the help of the Ginzburg-Landau equations.[1] The hypothesis has also been advanced[1,3] that in such crystals a nonstationary Josephson effect should also be observed, i.e., that oscillations of the current in the sample should develop upon application to the crystal of a constant voltage V perpendicular to the layers.

It is understood that the theory of the nonstationary Josephson effect in tunnel junctions cannot be carried over directly to layered superconductors. For example, it is assumed in this theory that the electric field arising in the tunnel junction does not penetrate into the superconductor because of the screening by the electrons in the metal. In the case of layered superconductors, this assumption is, of course, invalid, since the thickness of the conducting layer has atomic dimensions and cannot be greater than the screening length. There are no theoretical schemes which would describe the effects of the type of the nonstationary Josephson effect in quasi-two-dimensional superconductors.

In ordinary superconductors, nonequilibrium processes are completely described by the equations for the Green's function, integrated over the energy variable ξ.[4,5] The equations for the quasi-two-dimensional superconductor should be distinguished from the equations for the ordinary superconductors by the fact that since averaging should not be carried out in them over distances of the order of interatomic (i.e., of the order of the distance between layers) in the direction perpendicular to the layers, since these equations should describe the physical quantities and their differences in each layer. Such equations will be obtained in this work. With the help of a calculation based on them, it will be shown that the nonstationary Josephson effect should actually occur in quasi-two-dimensional superconductors. The characteristic frequency of the Josephson oscillations and the voltage per layer are of the order of the energy gap Δ (of the order of Δ^2/T near T_c). Correspondingly, the characteristic total voltage on the sample turns out to be N times as large. The electric field arising in the volume of the superconductor in this case is not screened. The energy distribution of the electrons becomes nonequilibrium, which leads to an increase in the energy gap of the superconductor. The mechanism of this phenomenon is analogous to the mechanism proposed by Éliashberg[4] for the stimulation of superconductivity by a microwave field in ordinary superconductors.

DERIVATION OF THE EQUATIONS DESCRIBING THE NONEQUILIBRIUM PROCESSES IN QUASI-TWO-DIMENSIONAL SUPERCONDUCTORS

The most powerful and convenient method of study of nonequilibrium processes in ordinary superconductors is provided by the equations for the Green's function, integrated over $\xi = p^2/2m - \varepsilon_F$.[4,5] In this section, we shall derive similar equations describing the quasi-two-dimensional superconductors.

We consider a quasi-two-dimensional metal. We choose the z axis of the system in a direction perpendicular to the layers. The Fermi surface of the layered metal with weak coupling between the layers has a shape that is close to a cylindrical surface with a generatrix parallel to the z axis. Let the single-electron spectrum of the metal in the normal state have the form

$$\varepsilon = \frac{p_x^2 + p_y^2}{2m} + \varepsilon_z(p_z),$$

where $\varepsilon_z \ll \varepsilon_F$. In the strong coupling approximation, $\varepsilon_z = \varepsilon_1 \cos a p_z$ for the motion of the electron along the z axis, in which the interaction only between neighboring layers is taken into account. We shall not make specific the form of the interaction between the layers, since account of any sort of interaction, for example, correlation,[6] leads only to a renormalization of the parameters of the spectrum. For our purposes, it is only important that the spectrum exist and have a quasi-two-dimensional form, i.e., $\varepsilon_z \ll \varepsilon_F$. We assume that there are no other energy bands near the Fermi surface, and shall take only one band into account.

We write down the Gor'kov equations for the Green's function in the Keldysh technique,[7] generalized to the case of a superconductor as is done in Refs. 5 and 8:

$$\left\{ i\sigma_z \frac{\partial}{\partial t} + \frac{1}{2m}\left(\frac{\partial}{\partial \mathbf{r}} - ie\mathbf{A}\sigma_z\right)^2 + \check{\Delta} - e\varphi + \varepsilon_F - U(z) - \check{\Sigma} \right\} \check{G} = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}'). \tag{1}$$

Here

$$\check{G} = \begin{pmatrix} \hat{G}^R & \hat{G} \\ 0 & \hat{G}^A \end{pmatrix} \qquad \hat{G} = \begin{pmatrix} g_1 & F_1 \\ -F_2 & g_2 \end{pmatrix}, \qquad \check{\Delta} = \begin{pmatrix} \hat{\Delta} & 0 \\ 0 & \hat{\Delta} \end{pmatrix},$$

$$\hat{\Delta} = \begin{pmatrix} 0 & \Delta_1 \\ -\Delta_2 & 0 \end{pmatrix}, \qquad \check{\Sigma} = \begin{pmatrix} \hat{\Sigma}^R & \hat{\Sigma} \\ 0 & \hat{\Sigma}^A \end{pmatrix}, \qquad \check{\sigma}_z = \begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix};$$

σ_z is a Pauli matrix; g and F are the ordinary and the Gor'kov Green's functions; $\hat{\Sigma}$ is the mass operator which describes the scattering of electrons by impurities and phonons. The expressions for $\hat{\Delta}$ and $\hat{\Sigma}$, and also the relations between the components of the matrix G, can be found in Refs. 5 and 8.

Further, as in the derivation of the equations for ordinary superconductors, we subtract from Eq. (1), which has the form $G_0^{-1}G = 1$, the conjugate equation $GG_0^{-1} = 1$ and transform to the momentum representation in coordinates along the layer. Just as in isotropic superconductors, making use of the fact that Σ changes at energies that are large in comparison with the characteristic energies of the superconductor, we can integrate the obtained equation over $\xi_\parallel = (p_x^2 + p_y^2)/2m - \varepsilon_F$. We transform to the nodal representation of Wannier in the coordinates z and z'. As a result, we obtain

$$\frac{\mathbf{p}_\parallel}{m}\frac{\partial \check{G}_{nn'}}{\partial \mathbf{R}} + \sum_l (\varepsilon_{nl}\check{G}_{ln'} - \check{G}_{nl}\varepsilon_{ln'}) - \check{\sigma}_z\varepsilon\check{G}_{nn'}$$
$$+ \check{G}_{nn'}\check{\sigma}_z\varepsilon' + \sum_l (H_{nl}\check{G}_{ln'} - \check{G}_{nl}H_{ln'})$$
$$+ {}^1/_2 i \sum_{m,l,k} (\nu_{nmlk}\check{G}_{ml}\check{G}_{kn'} - \nu_{kmln'}\check{G}_{nk}\check{G}_{ml}) = \check{I}_{ph},$$
$$H_{nl} = -\check{\Delta}_{nl} + e\varphi_{nl}, \tag{2}$$

where by the product of functions, we mean the convolution with respect to the internal frequency variable, for example,

$$\int \breve{G}(\varepsilon,\varepsilon_1)\breve{G}(\varepsilon_1,\varepsilon')\frac{d\varepsilon_1}{2\pi},$$

where

$$\breve{G}_{nn'}=\int \breve{G}(z,z')w_n(z)w_{n'}(z')\,dz\,dz', \tag{3}$$

where the integrand contains the function $\breve{G}$, integrated over $\xi_\parallel$:

$$\varphi_{nn'}=\int \varphi(z)w_n(z)w_{n'}(z)\,dz, \tag{4}$$

$$\varepsilon_{nm}=\sum_k \exp[ik(R_n-R_m)]\varepsilon(k). \tag{5}$$

The terms with the vector potential were not written out in Eq. (2), since it is more convenient for the solution of our problem to include the potential A in the phase of an order parameter, $\chi\to\chi+(e/c)\psi$, where $\nabla\psi=\mathbf{A}$. Moreover, it has been taken into account in expressions (3)–(5) that the Wannier functions $w_n(z)$ are real for bands that do not intersect with other bands.

The last term on the left side of Eq. (2) represents the integral of the collisions with the impurities, obtained with the help of the "cross technique" of averaging over the impurities with neglect of the intersecting diagrams.[9] A bar over a function indicates averaging over the angle φ in the p_xp_y plane:

$$\nu_{nmlk}=\frac{m}{2\pi}\int\left\langle \sum_a u(z_1-z_a)u(z_2-z_a)\right\rangle w_n(z_1)$$
$$\times\, w_m(z_1)w_l(z_2)w_k(z_2)\,dz_1\,dz_2; \tag{6}$$

$u(z_1-z_a)$ is the Fourier component in the x and y coordinates of the potential of the impurity atom at $\xi_\parallel=0$, we assume the scattering in the p_xp_y plane to be isotropic, z_a are the coordinates of the impurity atoms, and the angular brackets indicate averaging over the impurity locations.

We note that the Hamiltonian of electron-phonon interaction used in our research is of the form

$$H_{e\text{-}ph}=\int n(z)\varphi(z')K(z,z')\,dz\,dz',$$

where $n(z)$ is the electron density operator, φ is the phonon operator, and $K(z,z')$ is the interaction function. In ordinary superconductors the $K(z,z')$ are replaced by δ functions because K falls off at distances $|z-z'|$ that are small in comparison with the characteristic dimensions of the problem. In layered superconductors, the fall-off radius of K is not small in comparison with the distance between the layers and therefore $K(z,z')$ cannot be replaced by a δ function.

As a result, arguing in the same way as in the case of ordinary superconductors, i.e., assuming that the characteristic phonon frequency $\omega_D\gg\Delta$, T and $\omega_D\ll\varepsilon_F$, we obtain a self-consistent equation which describes Δ in terms of F. This equation differs from the corresponding equation for ordinary superconductors by the fact that Δ depends on two coordinates. For the matrix elements we get

$$\Delta_{nn'}=\sum_{n_1n_2}\lambda_{n_1n_2}^{nn'}\int\frac{d\varepsilon}{2\pi}F_{n_1n_2}, \tag{7}$$

where

$$\lambda_{n_1n_2}^{nn'}=\int dz_1\,dz_2\lambda(z_1z_2)w_n(z_1)w_{n_1}(z_1)w_{n'}(z_2)w_{n_2}(z_2),$$

$\lambda(z_1z_2)$ falls off over the same distances $|z_1-z_2|$ as the function K.

The Green's functions for ordinary superconductors satisfy the orthogonality condition,[5] greatly simplifying the solution of the equations. A similar relation can be obtained also for layered superconductors:

$$\sum_m \breve{G}_{nm}\breve{G}_{mn'}=1. \tag{8}$$

We also need a formula for the z component of the current in terms of the Green's function $\hat{G}_{nm}$. This formula is easily obtained from the general expression for the current in terms of the Green's function[9]; it has the form

$$j=\frac{e}{32}\operatorname{Sp}\sigma_z\int\frac{d\varepsilon}{2\pi}\frac{d\varphi}{2\pi}\sum_{n,m}(\hat{G}_{nm}-\hat{G}_{mn})\left(\frac{\partial w_n}{\partial z}w_m-w_n\frac{\partial w_m}{\partial z}\right). \tag{9}$$

The charge density is determined by the expression

$$\rho=\frac{e^2m}{2}\int\operatorname{Sp}\sum_{n,m}\delta(\hat{G}_{nm}+\hat{G}_{mn})w_n(z)w_m(z)\frac{d\varepsilon}{2\pi}\frac{d\varphi}{2\pi}, \tag{10}$$

where $\delta G_{nm}=G_{nm}-G_{nm}^0$, and G_{nm}^0 is the Green's function in the absence of currents and potentials. Thus, the current is determined by a combination of the Green's functions that is antisymmetric in the number of layers, and the charges and consequently the potentials by a symmetric combination.

EQUATION FOR THE DISTRIBUTION FUNCTION; THE JOSEPHSON RELATION

We consider a layered superconductor, through which flows a current in the direction perpendicular to the layers. We assume the size of the sample in the direction along the layers to be small in comparison with the penetration depth of the magnetic field (which is much greater than the Meissner length in ordinary superconductors[1]), so that the current is distributed uniformly over the cross section of the sample and the quantities entering into the equation do not depend on the x and y coordinates. In the general case, it is impossible to find a solution of Eq. (2), and we shall limit ourselves to the case in which the interaction between the layers is so small that we can restrict ourselves to the approximation of strong coupling of the electrons in the layers and to take into account the overlap of the Wannier functions only in the neighboring layers. In this approximation, we omit ε_{mn} with $|m-n|>1$ in Eqs. (2) and retain only $\varepsilon_{nn+1}\equiv\varepsilon_1$, which is equal to the band width in the p_z direction. Moreover, we discard in Eqs. (2)–(7) the nondiagonal matrix elements, which contain the overlap integrals, and leave only the largest diagonal elements

$$\lambda_{nn'}^{nn'}\equiv\lambda_{nn'},\quad \varphi_{nn},\quad \nu_{nnmm}.$$

Let the sample contain only such impurities for which the decay radius of the potential $u(z)$ is small in comparison with the thickness of the conducting layer, i.e., with the fall-off radius of the Wannier functions. We can then neglect also the quantities ν_{nmmm} with $n\neq m$ and retain only the quantity

$$1/\tau \equiv \nu_{nnnn} = \sum_i \frac{mn_{2i}}{2\pi} \int |u_i(z)|^2 w^i(z)\,dz,$$

which has the meaning of the free path time in motion along the layer, and in which n_{2i} is the concentration of the ith type impurities in a single layer.

Thus we take into account the nondiagonal matrix elements ε_1 only for an atomic potential having a characteristic value of a few electron volts, and shall not take into consideration the nondiagonal matrix elements of quantities of the type of the potential φ, whose characteristic value is of the order of a milli-electron volt.

We now make one more simplifying assumption, that the superconductor is dirty, i.e.,

$$1/\tau \gg \varepsilon_1. \tag{11}$$

Since ε_1 characterizes the probability of transition of the electron from one layer to another, the condition (11) means that the electron manages to be scattered many times from the impurities before undergoing transition to the neighboring layer. In this case, the functions $G_{nn\pm k} \sim (\tau\varepsilon_1)^k G_{nn}$ which are nondiagonal in terms of the number of the layer turn out to be small, which greatly simplifies the solutions of Eqs. (2). In particular, thanks to the relation (7), the gap components that are nondiagonal in the layer number also turn out to be small. With the help of direct calculation, we can establish the fact that

$$\Delta_{nn\pm k}/\Delta_{nn} \sim (\tau\varepsilon_1)^k \lambda.$$

For this reason, we shall take into consideration only the diagonal elements Δ_n. Thus, we shall solve the equation

$$\varepsilon_1(\check{G}_{n-1\,n'} + \check{G}_{n+1\,n'} - \check{G}_{nn'+1} - \check{G}_{nn'-1}) - \varepsilon\check{\sigma}_z\check{G}_{nn'} + \check{G}_{nn'}\check{\sigma}_z\varepsilon' - \check{\Delta}_n\check{G}_{nn'} + \check{G}_{nn'}\check{\Delta}_{n'} + e\varphi_n\check{G}_{nn'} - \check{G}_{nn'}e\varphi_{n'} - \frac{i}{2\tau}(\check{G}_{nn}\check{G}_{nn'} - \check{G}_{nn'}\check{G}_{n'n'}) = \check{I}_{ph} \tag{12}$$

which is much simpler than (2).

In the equilibrium case, in the absence of current, under the condition that the time of energy relaxation by the phonons τ_ε is large, $\Delta\tau_\varepsilon \gg 1$, the solution of the matrix equation (12) is given by the functions

$$\hat{g}_{nn'}^{R(A)} = \frac{\varepsilon\sigma_z + i\Delta\sigma_y}{\xi_\varepsilon^{R(A)}}\delta(\varepsilon-\varepsilon')\delta_{nn'}, \qquad \xi^{R(A)} = [(\varepsilon \pm i0)^2 - \Delta^2]^{1/2}(\xi_{\varepsilon>\Delta}^{R(A)} \gtrless 0);$$

$$\hat{g}_{nn'} = (\hat{g}^R_{nn'} - \hat{g}^A_{nn'})\,\mathrm{th}\frac{\varepsilon}{2T} \tag{13}$$

and by the functions obtained from (13) with the help of the transformation $\check{S}g\check{S}^+$, where $\check{S}$ has the form (in the time representation)

$$\check{S} = \begin{pmatrix} \hat{S} & 0 \\ 0 & \hat{S} \end{pmatrix}, \qquad \hat{S} = \begin{pmatrix} \exp\{-i\chi(t)/2\} & 0 \\ 0 & \exp\{i\chi(t)/2\} \end{pmatrix}.$$

This transformation corresponds to a change in the phase of the order parameter by an amount $\chi(t)$.

Upon neglect of the transitions between layers, i.e., at $\varepsilon_1 = 0$, the solutions of Eq. (12) would also be the solutions obtained from (13) by a shift of the phase $\chi_n(t)$ by a different amount in each later. At $\varepsilon_1 \neq 0$, solutions of the type $S_n g S_n^+$ do not satisfy the equation (12). For a solution of (12), we separate the phase factors χ_n of the order parameter in layer n and transform from the functions $\check{G}$ to the functions

$$\check{g}_{nn'} = \hat{S}_n^+ \hat{G}_{nn'} \hat{S}_{n'}. \tag{14}$$

We obtain the following equation for $\check{g}_{nn'}$:

$$\varepsilon_1(\check{A}_{n\,n-1}\check{g}_{n-1\,n'} + \check{A}_{n\,n+1}\check{g}_{n+1\,n'} - \check{g}_{n\,n'+1}\check{A}_{n'+1\,n'} - \check{g}_{n\,n'-1}\check{A}_{n'-1\,n'}) - \varepsilon\check{\sigma}_z\check{g}_{nn'} + \check{g}_{nn'}\check{\sigma}_z\varepsilon' - i\check{\sigma}_y\Delta_n\check{g}_{nn'} + \check{g}_{nn'}i\check{\sigma}_y\Delta_{n'} + \mu_n\check{g}_{nn'} - \check{g}_{nn'}\mu_{n'} = \frac{i}{2\tau}(\check{g}_{nn}\check{g}_{nn'} - \check{g}_{nn'}\check{g}_{n'n'}) + \check{I}_{ph}, \tag{15}$$

where

$$\mu_n = e\varphi_n + \frac{1}{2}\frac{\partial\chi_n}{\partial t}, \qquad \check{A}_{nn'} = \begin{pmatrix} \hat{A}_{nn'} & 0 \\ 0 & \hat{A}_{nn'} \end{pmatrix};$$

in the time representation

$$\hat{A}_{nn'} = \begin{pmatrix} \exp\{i(\chi_n - \chi_{n'})/2\} & 0 \\ 0 & \exp\{-i(\chi_n - \chi_{n'})/2\} \end{pmatrix}. \tag{16}$$

As has already been noted, under condition (11), the components of $\check{g}$ that are nondiagonal in n and that determine the current are less than the diagonal ones, so that there is an analogy with an ordinary superconductor, in which the characteristic $l\partial/\partial x \ll 1$ (where l is the free path length). The role of the anisotropic part of the function $\check{g}$ in an ordinary superconductor is played in our problem by $\check{g}_{nn\pm1}$, while the role of the isotropic part is played by $\check{g}_{nn}$. Correspondingly, our method of solving Eq. (15) is similar to the method used in the case of ordinary superconductors: from Eq. (15) at $n' = n \pm 1$, we express $\check{g}_{nn\pm1}$ in terms of $\check{g}_{nn}$, and then substitute the found $\check{g}_{nn\pm1}$ in Eq. (15) at $n' = n$ and, as a result, obtain an equation for $\check{g}_{nn}$.

For brevity, we make still another simplifying assumption; we shall assume that $\Delta\tau$, $T\tau$, $\dot{\chi}\tau \ll 1$. This assumption is not one of significance, and the problem is rather easily solved even for arbitrary $\Delta\tau$ and $T\tau$ (see for example, the review, Ref. 10). We also take it into account that we are solving a spatially homogeneous problem (if, of course we disregard the microscopic inhomogeneity); therefore all the functions should depend only on the differences between the indices, but not on the indices themselves. Therefore we omit the indices and introduce the notation:

$$\check{g}_{nn} \equiv \check{g}, \qquad \check{g}_{n\,n\pm1} \equiv \check{g}_\pm, \qquad \check{A}_{n\,n\pm1} = \check{A}_\pm$$

and so forth. The equations obtained for $\check{g}_\pm$ and $\check{g}$ have the form

$$\check{g}_\pm = i\tau\varepsilon_1(\check{A}_\pm - \check{g}\check{A}_\pm\check{g}), \tag{17}$$

$$-\varepsilon\check{\sigma}_z\check{g} + \check{g}\check{\sigma}_z\varepsilon' - i\check{\sigma}_y\Delta\check{g} + \check{g}\Delta i\check{\sigma}_y + \mu\check{g} - \check{g}\mu + i\tau\varepsilon_1^2(\check{g}\check{A}_+\check{g}\check{A}_- + \check{g}\check{A}_-\check{g}\check{A}_+ - \check{A}_+\check{g}\check{A}_-\check{g} - \check{A}_-\check{g}\check{A}_+\check{g}) = \check{I}_{ph}, \tag{18}$$

where, as in Eqs. (2) and (15), convolution over the internal frequency variable is implied.

The terms of Eq. (18) describing $\hat{g}^R$ and $\hat{g}^A$ correspond to the upper and lower diagonal blocks of (18). The equations for $\hat{g}^R$ and $\hat{g}^A$ have the same form as in (18), only $\hat{g}^R$ or $\hat{g}^A$ replaces $\hat{g}$. It is seen from these equations that the corrections to the solutions (13) for $\hat{g}^R$ and $\hat{g}^A$, necessitated by the current flow, have the order of magnitude of $\tau\varepsilon_1^2/\Delta$. Consequently, at sufficiently weak coupling between the layers, when $\tau\varepsilon_1^2 \ll \Delta$, the density of states of the superconductor, which is determined by the functions $g^{R(A)}$, changes little when current flows. Thus, at

$$\tau\varepsilon_1^2 \ll \Delta \tag{19}$$

we can speak of quasi-two-dimensional superconductivity with Josephson interaction of the layers (see Ref. 1). In what follows, we shall assume the condition (19) to be satisfied.

We now consider the term of Eq. (18) corresponding to the equation for $\hat{g}$ (a 2×2 block in the upper right corner of the matrix). This equation describes the perturbation of the distribution functions and has the same structure as the equation describing the stimulation of superconductivity by microwave radiation, considered by Éliashberg and co-workers.[4,11] The Éliashberg equation differs from ours in that his contains $\sigma_z Q_\omega$ in place of $A_\pm$, where Q_ω is the superfluid momentum in the ordinary superconductor. As in Ref. 11, the solution of the equation for $\hat{g}$ has the form

$$g=(g^R-g^A)(1-2n), \tag{20}$$

where n is a nonequilibrium distribution function subject to determination. The part of the Green's function g_μ that is even in ε, such that $g_\mu = Sp\hat{g} \neq 0$, which determines the nonequilibrium character of the distribution of the quasiparticles over the branches of the excitation spectrum,[10] is equal to zero in our case.

We now calculate the distribution function n in the case in which the phase difference of corresponding neighboring levels increases linearly with the time:

$$\dot{\chi}_{n+1}-\dot{\chi}_n=2ev. \tag{21}$$

It will be shown below that, as in Josephson junctions, the condition (21) corresponds to a constant voltage $V = vN$ applied to the sample, where N is the number of layers in the sample. We shall assume that $v \sim \Delta \gg 1/\tau_\varepsilon$. In this case, as in Ref. 11, it turns out that

$$g(\varepsilon, \varepsilon+v) \sim \frac{1}{v\tau_\varepsilon} \breve{g}(\varepsilon,\varepsilon) \ll \breve{g}(\varepsilon,\varepsilon),$$

and the time average of the distribution function can change markedly in the system.

Substituting the explicit forms of $\hat{g}^R$ and $\hat{g}^A$ (13) and using (20), we get the following equation for the function $n(\varepsilon)$:

$$2\tau\varepsilon_1^2\left\{\frac{\varepsilon+v}{\xi_{\varepsilon+v}}\theta(|\varepsilon+v|-\Delta)[n(\varepsilon+v)-n(\varepsilon)] + \frac{\varepsilon-v}{\xi_{\varepsilon-v}}\theta(|\varepsilon-v|-\Delta)[n(\varepsilon-v)-n(\varepsilon)]\right\}=-I_{ph}(n). \tag{22}$$

It is seen from this equation that the distribution function differs little from its equilibrium form at $\varepsilon_1^2\tau\tau_\varepsilon \ll 1$ and can differ strongly from the equilibrium form at $\varepsilon_1^2\tau\tau_\varepsilon \gg 1$.

In the first case, n is easily found by perturbation theory, by considering the case of a temperature close to critical, when the region of significant energies $\varepsilon \sim \Delta$, $v \ll T$ and the τ approximation can be used:

$$I_{ph}=\frac{1}{\tau_\varepsilon}(n-n_0),$$

where n_0 is the equilibrium distribution function,

$$n_1=-\tau\tau_\varepsilon\varepsilon_1^2\left[\frac{\varepsilon+v}{\xi_{\varepsilon+v}}\theta(|\varepsilon+v|-\Delta)\left(\text{th}\frac{\varepsilon+v}{2T}-\text{th}\frac{\varepsilon}{2T}\right) + \frac{\varepsilon-v}{\xi_{\varepsilon-v}}\theta(|\varepsilon-v|-\Delta)\left(\text{th}\frac{\varepsilon-v}{2T}-\text{th}\frac{\varepsilon}{2T}\right)\right]. \tag{23}$$

At $\tau\tau_\varepsilon\varepsilon_1^2 \gg 1$ analysis of Eq. (20) is complicated; a similar equation was solved under this condition in Ref. 11; we shall not consider this case.

We note that it follows from (23) that the disequilibrium increases the energy gap. In particular, the nonequilibrium increment to the Ginzburg-Landau equation introduced in Ref. 11 has the form

$$-\frac{v}{\Delta}\theta(2\Delta-v)F\left(\arcsin\left(\frac{\Delta}{v+2\Delta}\right)^{1/2}, \frac{(4\Delta^2-v^2)^{1/2}}{2\Delta}\right) +2\theta(v-2\Delta)\left[\Pi\left(\frac{\pi}{2}, \frac{v-2\Delta}{v}, \frac{(v^2-4\Delta^2)^{1/2}}{v}\right) -\left(1+\frac{v}{\Delta}\right)F\left(\frac{\pi}{2}, \frac{(v^2-4\Delta^2)^{1/2}}{v}\right) +F\left(\arcsin\left(\frac{v}{v+2\Delta}\right)^{1/2}, \frac{(v^2-4\Delta^2)^{1/2}}{v}\right)\right],$$

where F and Π are elliptical integrals of the first and third kind, respectively.

We shall now show that the voltage on the sample is determined by the relation $V=Nv$, where $v=(\dot{\chi}_{n+1}-\dot{\chi}_n)/2e$, i.e., for the potential averaged over the space, the Josephson relation is valid. In order to calculate the distribution of the potential $\varphi(z)$, we must solve the Poisson equation

$$d^2\varphi/dz^2=4\pi\rho, \tag{24}$$

where ρ is defined by Eq. (10).

The function $\hat{g}$ of the form (20) satisfies the condition $Sp\hat{g}=0$, but at the same time $\int g d\varepsilon$ diverges at high energies; therefore, we much approach the calculation of ρ by formula (10) with great care. We must use the expression for the function G not integrated over $\xi_\parallel$ (it can be calculated in the zeroth approximation in ε_1) and subtract from it the function G^0 in the absence of a potential. As a result, we obtain converging integrals that do not depend on the order of the integration and the calculation of the trace. Such a procedure is applied in the calculation of the charge density in ordinary superconductors.[4] As a result, we obtain from (24)

$$d^2\varphi/dz^2=k^2d\sum_n(\varphi_n-\chi_n/2e)w_n^2(z), \tag{25}$$

where $k^2=8e^2m/d$ has the meaning of the square of the reciporcal screening radius, and differs from the k^2 of Tomas-Fermi for an isotropic metal by the fact that in place of the Fermi wavelength there appears the characteristic scale of the fall-off of the Wannier function.

In correspondence with the condition (21), we shall assume that $\chi_n = 2evn$ (the part of the phase that is independent of n does not have a value; to it corresponds a change in the potential φ by a constant amount). We shall seek a solution of (25) in the form

$$\varphi(z)=Ez+\varphi^p(z), \tag{26}$$

where $\varphi^p(z)$ is a periodic function with period a. It follows from (26) that

$$\varphi_n=aEn+\tilde{\varphi}+E\tilde{z}, \tag{27}$$

where $\tilde{\varphi}$ and $\tilde{z}$ are the diagonal matrix elements of the function φ^p and are linearly periodic functions with period a; they do not depend on the number n. The first term in (26) drops out of the left side of (25); this term is determined by equating to zero of the term under the summation sign in the right hand side of (25), a term that is increasing linearly with z:

$$aE=v. \tag{28}$$

Thus, the fall-off of the potential aE in a single period is equal to v. For the periodic part of the potential, we obtain the following equation from (25)–(28):

$$d^2\varphi^p/dz^2=k^2d(\tilde{\varphi}+E\tilde{z})\sum_n w^2(z-an),$$

the solution of which has the form

$$\varphi^p=\frac{E_0\tilde{z}}{1+(\alpha k)^2}\sum_n U(z-an), \tag{29}$$

where U is a function satisfying the conditions $U''(z)=w^2(z)$ and $\lim U(z)=0$, $z\to\infty$, while

$$\alpha^2=d\int_{-\infty}^{+\infty}[U'(z)]^2\,dz.$$

In order of magnitude, $\alpha\sim d$ and $z\sim d$. However, if the function $w^2(z)$ is even, then $\tilde{z}=0$ and, according to (29), $\varphi^p=0$. If the distribution of quasiparticles over the branches of the spectrum were also perturbed, then the numerator in (29) would have the form

$$E\tilde{z}+\mathrm{Sp}\int\frac{d\varepsilon}{2\pi}g_\mu.$$

CALCULATION OF THE TRANSVERSE CURRENT

For calculation of the current, we need to separate in (17) the component corresponding to g and, carrying out a transformation that is the inverse to (14), we substitute it in Eq. (9). The results can be reduced to the following form:

$$j=\frac{e}{32}\int\frac{d\varepsilon}{2\pi}\tau\varepsilon_1\,\mathrm{Sp}\,\sigma_z\{[(\hat{g}^R\hat{A}_+\hat{g}+\hat{g}\hat{A}_+\hat{g}^A)A_-]_+ -[\hat{A}_+(\hat{g}^R\hat{A}_-\hat{g}+\hat{g}\hat{A}_-\hat{g}^A)]_+\}J, \tag{30}$$

where $[\ldots]_+$ denotes the anticommutator. The quantity

$$J=\sum_n(w_n'w_{n-1}-w_{n-1}'w_n)$$

does not depend on z, as can easily be shown. A rough estimate gives the order-of-magnitude value $J\sim\varepsilon_1 md/a$. Calculation of (30) under the condition that $V=0$ and the phase difference between neighboring layers $\delta\chi$ does not depend on time gives the Josephson relation between j and $\delta\chi$:

$$j=j_c\sin\delta\chi$$

—such a result was, obtained earlier with the help of the Ginzburg-Landau equation.[1]

Now let the phase difference increase linearly in time according to (21). As was shown in the preceding section, this means that a constant potential $V=Nv$ is applied to the sample. In this case we can obtain from (30) the following expression, which determines the current:

$$j=\frac{\sigma_\perp}{4a}\int_{-\infty}^{+\infty}d\varepsilon\{[g^R(\varepsilon)-g^A(\varepsilon)][g^R(\varepsilon+v)-g^A(\varepsilon+v)][n(\varepsilon+v)-n(\varepsilon)]$$
$$+2i[f^R(\varepsilon)-f^A(\varepsilon)][f^R(\varepsilon+v)+f^A(\varepsilon+v)]n(\varepsilon)\sin 2evt$$
$$+[f^R(\varepsilon)-f^A(\varepsilon)][f^R(\varepsilon+v)-f^A(\varepsilon-v)][n(\varepsilon+v)-n(\varepsilon)]\cos 2evt, \tag{31}$$

where

$$\sigma_\perp=e^2\tau\varepsilon_1 aJ,\qquad g^{R(A)}(\varepsilon)=\frac{\varepsilon}{\xi_\varepsilon^{R(A)}}=\frac{\varepsilon}{\Delta}f^{R(A)}(\varepsilon).$$

The quantity $\sigma_\perp$ characterizes the transverse conductivity of the layered metal in the normal state, the order of magnitude of which is $\sigma_\perp\sim e^2\tau\varepsilon_1^2dm$, where $\tau\varepsilon_1^2$ has the meaning of the reciprocal time of hopping between neighboring layers.

Thus the transverse conductivity is proportional to τ, just as the longitudinal. We note that in dirty quasi-two-dimensional conductors ($\tau\varepsilon_1\ll 1$), we also have $\tau_1^{-1}\sim\tau\varepsilon_1^2$; this result was obtained experimentally on TTF-TCNQ,[12] and a physical interpretation was given to it.

It is seen from (31) that at constant voltage on the sample, in addition to the constant component, there is an alternating one in the current, with frequency $2ev$. The component corresponding to the constant current has a simple physical meaning—the integral contains the product of the state densities in different layers and the difference between their distribution functions. If we use the equilibrium distribution for the distribution function in (31), the expression for the current (31) will differ from the similar expression for the Josephson tunnel junction only in the value of the resistance. This type of voltage dependence of the current has been analyzed in many researches (see, for example, Ref. 13). In particular, at high voltages ($v=V/N\gg\Delta, T$),

$$j=\frac{\sigma_\perp}{a}\left(v+\frac{\pi\Delta^2}{ve^2}\sin 2evt-\frac{2\Delta^2}{ve^2}\ln\frac{2ve}{\Delta}\cos 2evt\right),$$

and the value of the critical current at $ev\ll\Delta$ is

$$j_c=\frac{\pi}{2}\frac{\Delta}{eR}\,\mathrm{th}\frac{\Delta}{2T}.$$

However, the distribution function n differs from the equilibrium one. At $\tau\tau_\varepsilon\varepsilon_1^2\ll 1$ the nonequilibrium correction is proportional to the small parameter $\tau\tau_\varepsilon\varepsilon_1^2$ but on the other hand it contains a divergence at $\varepsilon\approx\Delta$, $\Delta+v$ connected with the divergence of the density of states. But, since the distribution function in Eq. (31) is multiplied by the density of states, an unintegrable divergence is obtained under the integral. In fact, the divergent expression is of course valid only at $|\varepsilon-\Delta|$, $|\varepsilon-v-\Delta|>1/\tau$ and is cut off inside these intervals. The part of the direct current connected with the nonequilibrium nature of the distribution function at $\tau\tau_\varepsilon\varepsilon_1^2\ll 1$, $1/\tau_\varepsilon\ll v$, $\Delta\ll T$, $|v-2\Delta|\gg 1/\tau_\varepsilon$, calculated with logarithmic accuracy, has the form

$$\delta j=\frac{\sigma_\perp}{a}\frac{\tau\tau_\varepsilon\varepsilon_1^2}{2T}v^{1/2}\Delta\Big[-\frac{v+\Delta}{(v+2\Delta)^{1/2}}\ln v\tau_\varepsilon +\theta(v-2\Delta)\frac{v-\Delta}{(v-2\Delta)^{1/2}}\ln(v-2\Delta)\tau_\varepsilon\Big].$$

Thus, in this case the difference of the shape of the volt-ampere characteristics of the Josephson tunnel junction and the quasi-two-dimensional superconductor is small.

At $\tau\tau_\varepsilon\,\varepsilon_1^2 \gg 1$ the difference of the distribution function from the equilibrium value is not small, while the dependence of the current on the voltage in the quasi-two-dimensional superconductor differs greatly from the dependence in the Josephson tunnel junction. We shall not calculate the current in this case.

We have calculated the current in the case of a given voltage on the sample. In the specified current regime, the volt-ampere characteristic of a quasi-two-dimensional superconductor will obviously be similar to the characteristic of a superconducting point contact.

CONCLUSION

Microscopic equations have been obtained in the present work, describing nonequilibrium processes in quasi-two-dimensional superconductors. These equations are similar to the equations for the nonequilibrium Green's functions, integrated over ξ, which describe the kinetics of ordinary three-dimensional superconductors. The obtained equations are used for the calculation of the current arising in a quasi-two-dimensional superconductor under the action of a constant voltage applied transverse to the layers. It is shown that at a sufficiently weak interaction between the layers in the sample, in addition to the constant current, an alternating current also develops, i. e., a nonstationary Josephson effect takes place. The frequency of the oscillations and the voltage developed across a single layer, as in tunnel junctions, are connected by the Josephson relation. The electric field is not equal to zero over the entire volume of the layered superconductor. The energy distribution of the quasiparticles can deviate significantly from equilibrium, this nonequilibrium character leads to an increase in the energy gap of the superconductor.

We have considered the case in which the periodicity of the crystal in the transverse direction is not violated. In the solutions of the kinetic equations obtained by us for a periodic and homogeneous crystal, the fields and the currents in the different layers are identical, i. e., the oscillations in them are synchronous. However, in real crystals, there can be significant departures from periodicity, for example, those connected with intercalation. Such a crystal should correspond to a system of Josephson junctions that differ from one another. In such systems, as is well known, the oscillations in the different transitions can become synchronized even in the case of different junctions, if their parameters are close together (see, for example, Ref. 14). Therefore, it should be expected that synchronized oscillations will also arise in layered crystals with small departures from periodicity.

According to the estimates given in the review,[1] the parameters of several quasi-two-dimensional superconductors satisfy the necessary requirements and it would undoubtedly be of interest to observe this effect experimentally. Observation of the nonstationary Josephson effect in layered crystals can obviously be of interest from the point of view of applications, since the characteristic voltage on the sample is N times greater than in the case of a tunnel junction, and consequently the power of the Josephson radiation from junctions should increase also.

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1)As Dzyaloshinskiĭ and Kats have shown,[2] even a weak interaction between the layers suppresses the phase fluctuations of the order parameter, disrupting the superconducting long-range order in purely two-dimensional systems.

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