Mass formula for mesons and baryons

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The number of independent parameters in the semiempirical formula for meson and baryon masses proposed by Ya. B. Zel'dovich and the author is reduced by applying chromodynamics considerations. The results are compared with experiment. A summary of the new predictions is given.

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Ya. B. Zel'dovich and the author proposed a semi-empirical formula which gives a unified description of the masses of the mesons and baryons with the wave function of the quarks in the lowest s state (Ref. 1).[1)] For mesons we shall write the formula with the minimum number of independent parameters as

$$M=\delta_M+m_1+m_2+b\frac{m_0^2}{m_1m_2}\sigma_1\sigma_2 \tag{1}$$

and for baryons

$$M=\delta_B+m_1+m_2+m_3+\frac{b}{3}\left(\frac{m_0^2}{m_1m_2}\sigma_1\sigma_2+\frac{m_0^2}{m_2m_3}\sigma_2\sigma_3+\frac{m_0^2}{m_3m_1}\sigma_3\sigma_1\right); \tag{2}$$

here δ_M, δ_B, and b are parameters with the dimensions of mass and m_i are the masses of the quarks. Below we use the following notation: m_0 is the mass of the ordinary quark u or d; m_s and m_c are the masses of the strange and charmed quarks. Altogether there are six parameters.

The last term in Eqs. (1) and (2) is the spin-spin interaction of the quarks; $\sigma_i\cdot\sigma_j$ is the scalar product of the quark spin vectors; $\sigma_1\cdot\sigma_2=-3/4, +1/4$ respectively for pseudoscalar and vector mesons. With total baryon spin $J=3/2$ we have

$$\sigma_1\sigma_2=\sigma_2\sigma_3=\sigma_3\sigma_1={}^1/_4.$$

With total baryon spin $J=1/2$, if the baryon contains two identical quarks q_2 and q_3 we have

$$\sigma_2\sigma_3={}^1/_4,\quad \sigma_1\sigma_2=\sigma_1\sigma_3=-{}^1/_2.$$

However, if the baryon contains three different quarks, then the eigenvalues of the spin-spin interaction operator

$$H_{\sigma\sigma}=A\sigma_1\sigma_2+B\sigma_2\sigma_3+C\sigma_3\sigma_1$$

are

$$h_{1/2}=-{}^1/_4(A+B+C)\pm2^{-1/2}((A-B)^2+(B-C)^2+(C-A)^2)^{1/2},$$
$$h_{3/2}=+{}^1/_4(A+B+C). \tag{3}$$

These eigenvalues can be found by direct solution of the secular equation of eighth order. We previously[2] pointed out a simple procedure for separation of the second-order secular equation and for finding $h_{1/2}$. We shall consider a two-dimensional space corresponding to spin 1/2 and a definite projection. The two-dimensional operators $\sigma_i\cdot\sigma_j$ have eigenvalues $-1/4\pm1/2$ and are obtained from each other by rotation of the basis by 120° (from symmetry considerations). If one of them is diagonal in a certain basis,

$$\alpha=-\frac{1}{4}+\frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix},$$

then the other two in the same basis are:

$$\beta=-\frac{1}{4}+\frac{1}{2}\begin{pmatrix}-\frac{1}{2}&\frac{\sqrt{3}}{2}\\\frac{\sqrt{3}}{2}&\frac{1}{2}\end{pmatrix},\quad \gamma=-\frac{1}{4}+\frac{1}{2}\begin{pmatrix}-\frac{1}{2}&-\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&\frac{1}{2}\end{pmatrix}.$$

The quantities $h_{1/2}$ are the eigenvalues of the matrix

$$H_{\sigma\sigma}'=\alpha A+\beta B+\gamma C.$$

Equation (3) describes the masses of the isodoublet baryons of composition (o, c, s), where o is an ordinary quark, and also (for $C=B$) the masses of the doublets Σ, Λ and Σ_c, Λ_c. We note that in Ref. 2 in the corresponding formula we erroneously emitted the factor 1/2 in front of the radical, but the numerical values have been calculated correctly.

The spin-spin interaction in Eqs. (1) and (2) are interpreted as the interaction of magnetic-like gluon moments which are proportional to $g/2m$, by analogy with the ordinary Dirac magnetic moment; g is the interaction constant. Of course, the large value of the radiation and vacuum corrections makes this interpretation somewhat arbitrary (compare Ref. 3). The empirical factor 1/3 in Eq. (2) can be explained in the following way. The spin-spin interaction is proportional to $(g_1g_2)/V$, where V is the effective volume of the hadron and (g_1g_2) is the scalar part of the color charge vectors in two-dimensional charge space. We assume that the effective volumes of the baryons and meson are related as 3 : 2 in correspondence with the number of quarks. In the case of a meson the scalar product of the charge vectors of the quark and antiquark is equal to $-g^2$. The angles between the charge vectors of the quarks of three different colors entering into the composition of the baryon in charge space are equal to 120°. The scalar products are equal to $g^2\cos120°=-g^2/2$. Thus, the scalar products of the charges for the baryon and meson are related as (1/2) : 1. Collecting the factors, we have $(2/3)\cdot(1/2)=1/3$.

The relations connecting the mass differences of the baryons and mesons follow from Eqs. (1) and (2). They are adequately satisfied experimentally (see Refs. 4–6):

$$\frac{\Delta-N}{\rho-\pi}=\frac{1}{2}\quad\text{(experimentally 0.46)}, \tag{4a}$$

$$\frac{\Sigma^*-\Sigma}{K^*-K}=\frac{1}{2}\quad\text{(experimentally 0.48)}, \tag{4b}$$

$$\frac{K^*-K}{\rho-\pi}+\frac{3}{2}\frac{\Sigma-\Lambda}{\Delta-N}=1 \quad \text{(experimentally 1.02)}, \tag{4c}$$

$$\frac{D^*-D}{\rho-\pi}+\frac{3}{2}\frac{\Sigma_c-\Lambda_c}{\Delta-N}=1 \quad \text{(experimentally 1.08)}, \tag{4d}$$

$$m_s-m_o={}^1/_4(3K^*+K)-{}^1/_4(3\rho+\pi)=\Lambda-N \quad \text{(experimentally } 179\approx177), \tag{4e}$$

$$m_c-m_o={}^1/_4(3D^*+D)-{}^1/_4(3\rho+\pi)=\Lambda_c-N \quad \text{(experimentally } 1356\approx1318). \tag{4f}$$

The presence of the factor 1/2 in the expression for the spin-spin interaction of the quarks in the baryon can be checked without use of the ratio of the hadron volumes if we turn to the electromagnetic mass differences (see a forthcoming article[7]). Making the substitution for mesons $-g^2 \to -g^2+e_1e_2$ and for baryons $-g^2/2 \to -g^2/2+e_ie_j$, we have

$$\frac{D_0^*-D_0}{D_+^*-D_+}=\frac{\Sigma_+^*-\Sigma_+}{\Sigma_-^*-\Sigma_-}=\frac{m_d}{m_u}+\frac{2}{3}\frac{e^2}{g^2},$$

and experimentally

$$1+(1.79\pm1.45)\cdot10^{-2}\approx1+(2.03\pm0.67)^{-2}. \tag{4g}$$

Taking for the parameters of Eqs. (1) and (2) the values

$$m_o=285,\ m_s=463,\ m_c=1621;$$
$$\delta_M=40,\ \delta_B=230,\ b=615,$$

we find the hadron masses, which are given below together with their experimental values. For the mesons

	π	ρ	ω	K	K^*	φ	D	D^*	ψ
Formula	149	764	764	504	882,5	1024	1865	1973	3287
Experiment	138	773	783	494	892	1020	1865	2005	3105

and for the baryons

	N	Δ	Σ	Λ	Σ^*	Ξ	Ξ^*	Ω	Σ_c	Λ_c
Formula	931	1239	1188	1109	1377	1335	1524	1677	2436	2267
Experiment	939	1232	1193	1116	1385	1318	1533	1672	2425	2257

In Ref. 2 we used formulas with a larger number of parameters than in Eqs. (1) and (2), namely

$$M=a+\sum(m_s-m_o)|s|+\sum(m_c-m_o)|c|+b\sum\xi_i\xi_j\sigma_i\sigma_j \tag{5}$$

with different parameters for the mesons and baryons. The mass differences $D^*-D=132$ MeV and $\Sigma_c-\Lambda_c=161.5$ MeV predicted in Ref. 2 have been confirmed experimentally: respectively 140 and 168 MeV (see Refs. 5 and 6). However, the absolute mass values turned out to be somewhat higher than those predicted.

The best description of the masses of the D, D^*, and Λ_c (but not of the ψ) requires some change of the constants in Ref. 2, namely, for the mesons

$$a=614,\quad b=635,\quad m_s-m_o=179,\quad m_c-m_o=1356$$
$$\xi_s=0.626,\quad \xi_c=0.22,$$

and for the baryons

$$a=1085.5,\quad b=195.5,\quad m_s-m_o=177,$$
$$m_c-m_o=1318,\quad \xi_s=0.605,\quad \xi_c=0.178$$

[ξ_c for the baryons is determined as $m_0/m_c=285/(285+1318)$].

With these constants the expected masses of the $s\bar{c}$ and $c\bar{s}$ mesons are 2083 for $J=0$ and 2171 for $J=1$. The $\psi-\eta''$ mass difference (for the vector and psuedoscalar particles of composition $c\bar{c}$) without allowance for the mixing with η and η', is 31 MeV.

The masses of the baryons turn out to be equal to the following values:

	$ooc(\Lambda_c\Sigma_c)$	$osc(\Xi_c)$	$ssc(\Omega_c)$	$occ(N_{cc})$	$scc(\Omega_{cc})$	$ccc(\Omega_{ccc})$
$J={}^1/_2$:	2257/2417.5	2491.5/2582.5	2754	3688.5	3879	–
$J={}^3/_2$:	2469.5	2624	2786	3740.5	3910.5	5044

The experimental value of the Σ_c mass is $\Sigma_c=2425$ MeV. If we define

$$\xi_c=1-\frac{3}{2}\frac{\Sigma_c-\Lambda_c}{\Delta-N}=0.14,$$

then the calculated $\Xi_c''-\Xi_c'$ mass difference increases somewhat:

$$\Xi_c''-\Xi_c'=\frac{b}{2^{1/2}}[(\xi_s-\xi_c)^2+(\xi_s-\xi_s\xi_c)^2+(\xi_c-\xi_c\xi_s)^2]^{1/2}$$
$$=\begin{cases}91\text{ MeV} & \text{for}\quad \xi_c=0.178\\ 96.7\text{ MeV} & \text{for}\quad \xi_c=0.14\end{cases}$$

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[1)] In Ref. 1 in addition to a linear mass formula we discuss also the model of construction of a baryon from four quarks and an antiquark, which does not present interest here.

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