Transverse runaway of hot electrons

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The possibility is indicated of a new type of runaway of hot electrons in a transverse magnetic field–transverse runaway. An analysis of the condition for the onset of runaway in a magnetic field shows that for a certain combination of scattering mechanisms, the runaway momenta and energies connected with the so-called restricted constraining scattering mechanisms, are not realized physically. Only transverse runaway is realistic for these combinations. The question whether the transverse runaway predicted by us and the observed effect called transverse breakdown are identical is discussed.

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1. INTRODUCTION

The concept of carrier runaway in semiconductors in a strong electric field was introduced by Levinson in Ref. 1. The same paper, starting from the asymptotic behavior of the warming function that characterizes the difference between the nonequilibrium and equilibrium distribution functions, presents a classification of the possible runaway types. Bass[2] has shown that the runaway classification given in Ref. 1 is not all-exhaustive, and introduced a new type of runaway due to the boundary conditions (experimental conditions). Bass attributed these runaways to the presence of a strong transverse magnetic field. It was noted in Refs. 3 and 4, however, that similar runaway occurs in any nonzero magnetic field. It is thus obvious that the types of runaway noted in Refs. 2-4 are simply particular cases of a more general effect realized for certain scattering mechanisms in any nonzero magnetic field in the given-current regime.[5]

It is known that in quasi-elastic scattering of hot electrons by various types of phonons, and of the momentum by various crystal defects in crossed electric (E) and nonquantizing magnetic (H) fields, the distribution function can be represented in the form

$$f_0\propto\exp\left(-\int^x\frac{dx}{1+E^2\Theta_{ik}{}^H(x)}\right),$$

where the warming function (see e.g., the review[6]) is

$$\Theta_{ik}{}^H(x)=\frac{e^2l_i{}^0\tilde{l}_k{}^0}{3(k_0T)^2}\frac{x^{(t+t_0)/2}}{1+\eta_ix^t}=\frac{1}{(E_{ik}{}^0)^2}\frac{x^{(t+t_0)/2}}{1+\eta_ix^t};\qquad(1)$$

l_i^0 and $\tilde{l}_k^0$ are energy independent mean-free-path momentum and energy factors, respectively; t and t_0 are the exponents of the energy dependences of l_i and $\tilde{l}_k$:

$$x=\frac{\varepsilon}{k_0T}\qquad \eta_i=\frac{(eHl_i{}^0)^2}{2mc^2k_0T}=\left(\frac{H}{H_i{}^0}\right)^2.$$

The values of t and t_0 for all the known scattering mechanisms are given in Ref. 6. The remaining symbols are standard.

Analytically lucid kinetic coefficients are usually obtained in the strong ($\eta_i\bar{x}^t\gg1$) and weak ($\eta_i\bar{x}^t\ll1$, $\bar{x}$ is the average energy) magnetic-field approximations. These approximations lead automatically to the following runaway conditions: a) for nonconstricting scattering mechanisms: $t_0\mp t>2$; b) for limited constricting scattering mechanisms: $t_0\mp t=2$. In the latter case the warming function increases linearly with energy at infinity and the runaway sets in when a certain critical electric field is reached, with a value that depends on the energy dependence of the averaged quantities.[1]

However, when the asymptotic behavior of the warming function is considered, the question of the strength of the magnetic field must be approached with certain caution. At any rate, it can be verified that some of the conditions a) and b) are the results of an incorrect asymptotic form of the warming function obtained in the weak- and strong-field approximations. The point is that in the weak-magnetic field approximation, at $t>0$, η_ix^t is discarded in (1) compared with unity, which shows immediately that it is impossible to obtain correct information on the asymptotic behavior of the warming function. For $t=-|t|<0$, an analogous situation arises in a strong magnetic field. In fact, in this case

$$\Theta_{ik}{}^H(x)\propto x^{(t_0+|t|)/2}/(x^{|t|}+\eta_i),$$

and by discarding $x^{|t|}$ compared with η_i we become unable to obtain the true asymptotic form of the warming function.

Thus, in the analysis of the onset of runaway in a magnetic field, the latter must be regarded arbitrary but different from zero. It is then possible to conclude the following from the asymptotic form of the function $\Theta_{ik}^H(x)$:

1) At negative t, runaway occurs only at $t_0-|t|\geqslant2$. At $t_0+|t|\geqslant2$ ther is no runaway.[7]

2) At positive t, no runaway of type a) or b) occurs at $t_0+t\geqslant2$. This runaway develops only at $t_0-t\geqslant2$.

However, under the conditions $t>0$ and $t_0+t=2$, a new type of runaway appears in the given-current regimes (particular cases of this runaway were noted in Refs. 2-4), which we shall name transverse runaway. It can be shown to be physically connected with the fact that the nondissipative part of the current tends to infinity while its dissipative part tends simultaneously to zero. The abrupt growth of the current under conditions of transverse runaway at certain critical values of the applied field (E_x) calls for an investigation of the relation between the breakdown and transverse-runaway effects.

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The present paper is devoted to an investigation of the kinetic coefficients and of certain experimental consequences under conditions of transverse runaway.

2. GALVANOMAGNETIC CHARACTERISTICS AT A CONSTANT FREE-CARRIER DENSITY

Under conditions of transverse runaway, the solution of the equation that connects the warming field with the applied field yields under strong heating conditions ($E^2\Theta_{ik}^H(x)\gg 1$)

$$E^2=\frac{E_x^2}{1-(E_x/E_0)^2},\quad E_0=\Gamma\left(\frac{t+3}{2}\right)\Gamma^{-1}\left(\frac{2t+3}{2t}\right)\frac{E_{ik}^0}{t^{1/2}},\tag{2}$$

where $\Gamma(x)$ is the gamma function. As $E_x\to E_0$ the warming field increases strongly. The value of the critical field (E_0) is determined by the energy and momentum scattering mechanisms.

Expressing the galvanomagnetic characteristics in terms of the applied field, we obtain for the Hall angle, for the magnetoresistance, and for the Hall constant

$$\operatorname{tg}\theta_i^k=\frac{E_x}{E_0}\left[1-\left(\frac{E_x}{E_0}\right)^2\right]^{-1/2},\tag{3}$$

$$\frac{\rho_{j,i}^k}{\rho_0^i}=D(t)\frac{H}{H_i^0}\frac{E_x}{E_0}\left[1-\left(\frac{E_x}{E_0}\right)^2\right]^{1/2},\tag{4}$$

$$\frac{R_i^k}{R_0}=D(t)r_0^i\frac{H}{H_i^0}\left(\frac{E_x}{E_0}\right)^2,\tag{5}$$

where

$$r_0^i=c/\mu_0^i,\quad R_0=-(enc)^{-1},\quad \rho_0^i=-(en\mu_0^i)^{-1}.$$

The current-voltage characteristic (CVC) is given by

$$\frac{j_x}{j_0}=D^{-1}(t)\Gamma\left(\frac{3+t}{2t}\right)\Gamma^{-1}\left(\frac{3+2t}{2t}\right)t^{-1/2}\frac{H_i^0}{H}\left[1-\left(\frac{E_x}{E_0}\right)^2\right]^{-1/2},\tag{6}$$

where

$$D(t)=\Gamma\left(\frac{3}{2t}\right)\Gamma\left(\frac{t+5}{2}\right)\Gamma^{-1}\left(\frac{3}{2}\right)\Gamma^{-1}\left(\frac{3+2t}{2t}\right)t^{-1},$$

$$j_0=-en\mu_0^iE_{ik}^0.$$

In the field region $E_x\ll E_0$ the Hall angle and the magnetoresistance increase linearly with the field. In the same electric-field region, the current is saturated. With increasing electric field, the Hall angle tends to $\pi/2$. At the same value of the applied field, an S-shaped section appears on the CVC, and the current tends to infinity, the magnetoresistance, on the other hand, after reaching its maximum at $E_x=E_0/\sqrt{2}$, tends to zero.

3. CRITERIA FOR CORRECTNESS OF THE CALCULATIONS

Assuming that in the calculation of the macrocharacteristics the essential region of integration is $x\approx\bar{x}$, the strong-heating condition can be expressed in the form

$$\frac{E_x}{E_0}\gg\frac{\Phi_1(t)H/H_i^0}{[1+\Phi_1^2(t)(H/H_i^0)^2]^{1/2}},\tag{7}$$

$$\Phi_1(t)=t^{(2t-1)/2t}\left[\Gamma\left(\frac{5}{2t}\right)\Gamma^{-1}\left(\frac{3}{2t}\right)\right]^{t(t-1)/2}\Gamma\left(\frac{2t+3}{2t}\right)\Gamma^{-1}\left(\frac{t+3}{2t}\right).\tag{7a}$$

It is obvious from (2) that the values of the applied field have an upper bound

$$E_x<E_0.\tag{8}$$

The condition that (7) and (8) be compatible imposes an upper bound on the possible values of the magnetic field:

$$H/H_i^0\ll\Phi_1^{-1}(t)\tag{9}$$

The stronger the inequality (9), the wider the interval of variation of the applied electric field in which the calculated effects can be observed. However, the magnetic field can not be arbitrarily small, since its values are bounded from below by additional conditions. These conditions are connected with the fact that the calculated results are valid only for rigorously defined energy and momentum scattering mechanisms. In the field region $E\lesssim E_0$, the average energy increases strongly and it is obvious that the conditions that ensure predominance of the necessary scattering mechanisms can be violated. Consequently, requiring that the corresponding momentum and energy scattering mechanisms predominate, we obtain

$$H/H_i^0\gg\Phi_1'(t)\,(l_i^0/l_{i'}^0)^{t/(t-t')},\tag{10}$$

$$H/H_i^0\gg\Phi_1'(t)\,(\tilde{l}_k^0/\tilde{l}_{k'}^0)^{2/(2-t-t_0')},\tag{11}$$

where

$$\Phi_1'(t)=\Gamma\left(\frac{t+3}{2t}\right)\Gamma^{-1}\left(\frac{2t+3}{2t}\right)\left[\Gamma\left(\frac{5}{2t}\right)\Gamma^{-1}\left(\frac{3}{2t}\right)\right]^{t/2}.\tag{11a}$$

By i' and k' are denoted the momentum and energy scattering mechanisms that compete with the main scattering mechanisms (i,k): $l_{i'}^0$ and $l_{k'}^0$ are factors independent of energy, while t' and t_0' are the exponents of the energy dependences of the mean free paths, and correspond to the scattering mechanisms i' and k'. It is obvious that in the general case $t'\neq t$ and $t_0'\neq 2-t$. If a change in only one scattering mechanism is expected, then the corresponding inequality in (10) or (11) is left.

The conditions (9), (10), and (11) are satisfied if

$$\left(\frac{\tilde{l}_k^0}{\tilde{l}_{k'}^0}\right)^{t/(2-t-t_0')},\quad\left(\frac{l_i^0}{l_{i'}^0}\right)^{t/(t-t')}\ll\left[\Gamma\left(\frac{3}{2t}\right)\Gamma^{-1}\left(\frac{5}{2t}\right)\right]^{t^2/2}t^{(1-2t)/2t}.\tag{12}$$

It should be noted that the conditions (7) and (9)-(11) are quite stringent. In practice, it suffices to satisfy these inequalities with a two- or threefold margin.

4. ALLOWANCE FOR THE FIELD DEPENDENCE OF THE FREE-CARRIER DENSITY

We consider now the case when the carrier lifetime is controlled by unlike charged trapping centers. From the condition that the free-electron density be stationary, and with allowance for the thermal and impact ionization on the one hand, and of the thermal trapping on the other, and using the approximation

$$\frac{4A_T(N_D-N_A)(A_I+B_T)}{[A_T+B_TN_A-(N_D-N_A)A_I]^2}\ll 1\tag{13}$$

we obtain for the field dependence of the free-electron density

$$\frac{n}{n_0}=B_T^0n_1(N_D-N_A)\Bigg\{B_T^0n_0n_1+(N_D-N_A)\left[\frac{c_0B_0}{1-c_0}\left(\frac{H}{H_i^0}\right)^{(2r-1)/t}\left(\frac{[1-(E_x/E_0)^2]^{1/2}}{E_x/E_0}\right)^{(2r-1)/t}-A_0\left(\frac{H_i^0}{H}\right)^{1/t}\left(\frac{E_x/E_0}{[1-(E_x/E_0)^2]^{1/2}}\right)^{1/t}\right]\Bigg\}^{-1}.\tag{14}$$

Here $A_T(E,H)$ and $A_I(E,H)$ are the respective coefficients of the thermal and impact ionization, n_0 is the equilibrium density, T_T^0 and $B_T(E,H)$ are the coefficients of thermal trapping in thermodynamic equilibrium and in external fields, $n_1 \equiv N_{D0}^+/(N_D - N_{D0}^+)$ is the equilibrium constant, N_{D0}^+ is the density of the ionized donors at thermal equilibrium, $c_0 \equiv N_A/N_D$ is the degree of compensation,

$$B_0=\sigma_T^0\left(\frac{2k_0T}{m}\right)^{1/2}\Gamma\left(\frac{2-r}{t}\right)\Gamma^{-1}\left(\frac{3}{2}\right)\left[\Gamma\left(\frac{2t+3}{2t}\right)\Gamma^{-1}\left(\frac{t+3}{2t}\right)\right]^{(2r-1)/t} \quad (15)$$

$$A_0=\sigma_I^0\left(\frac{2k_0T}{m}\right)^{1/2}\left[\Gamma\left(\frac{t+3}{2t}\right)\Gamma^{-1}\left(\frac{2t+3}{2t}\right)\right]^{1/t}. \quad (16)$$

The coefficient B_t was calculated in the approximation of the "power-law" recombination (trapping cross section $\sigma_T = \sigma_T^0 x^{-r}, r > 0$), and in the calculation of A_i it was assumed that $\sigma_I = \sigma_I^0 = \text{const}$.[6]

As $E_x \to E_0$ the first term in the brackets of the denominator tends to zero, and the second to infinity. For values $E_x = E_x^{cr} < E_0$, however, these terms are comparable—the density increases rapidly and saturates. The difference $E_0 - E_x^{cr}$ depends on c_0 and H, when the latter increases the difference decreases and tends asymptotically to zero. For the critical value of the applied electric field we have:

$$\frac{E_x^{cr}}{E_0}=\frac{\Phi_2(c_0,t)H/H_i^0}{[1+\Phi_2^2(c_0,t)(H/H_i^0)^2]^{1/2}}, \quad (17)$$

$$\Phi_2(c_0,t)=\left[\frac{\sigma_T^0}{\sigma_I^0}\Gamma\left(\frac{2-r}{t}\right)\Gamma^{-1}\left(\frac{3}{2t}\right)\frac{c_0}{1-c_0}\right]^{t/2r} \times\Gamma\left(\frac{2t+3}{2t}\right)\Gamma^{-1}\left(\frac{t+3}{2t}\right). \quad (17a)$$

By definition, E_x^{cr} is the breakdown field. The inequality (13) takes for $E_x = E_x^{cr}$ (under the worst conditions) the form

$$4N_DB_T \ll n_0 n_1 B_T^0, \quad (18)$$

from which it is seen that it can be fully satisfied.

It is easy to verify that when (14) is taken into account, the magnetoresistance (and the Hall constant) decrease sharply at $E_x = E_x^{cr}$, and reach minimum values. In the region of the magnetic fields that satisfy the conditions

$$H/H_i^0 \ll \Phi_2^{-1}(c_0, t), \quad (19)$$

the minimum value (relative to the electric field) of ρ increases quadratically with increasing magnetic field. The interval $E_0 - E_x^{cr}$ is in this case wide enough for the formation of the minimum indicated above. At $E_x^{cr} < E_x < E_0$ the magnetoresistance ρ begins to increase linearly with increasing E_x and behaves subsequently in the manner indicated in Sec. 2. The current density at $E_x = E_x^{cr}$ increases sharply and saturates in the field interval $E_x^{cr} < E_x < E_0$.

5. DISCUSSION OF RESULTS AND COMPARISON WITH EXPERIMENT

The following among the known energy and momentum scattering mechanisms satisfy the conditions $t > 0$ and $t_0 + t = 2$:

a) $t = 3$ and $t_0 = -1$—the momentum is scattered by the impurity ions, and the energy by the deformation potential of the acoustic phonons, both in the high-temperature and in the low-temperature approximations.

b) $t = +1$ and $t_0 = +1$—the momentum is scattered by dipole centers or by the polarized potential of the acoustic phonons in the high-temperature approximation, or by the polarization potential of the optical phonons, the energy is scattered in this case by the polarization potential of the acoustic phonons in the high- or low-energy approximation, or else by the deformation potential of the optical phonons.

We shall obtain estimates for $t = 3$ and $t_0 = -1$ in the case of n-Ge. Using the general connection between $l_e(x)$ and $l_a(x)$, which is important in energy scattering by deformation acoustic phonons in the high-temperature approximation, we replace l_a^0 in the expression for E_0 by l_a^0. We ultimately obtain

$$E_0 \approx 2\times10^{-7} z n_I^{2/3} \text{ [V/cm]}, \quad (20)$$

where n_I is the ion density z is the ionization multiplicity of the impurity center. We see that for the indicated x scattering mechanisms E_0 does not depend on temperature or magnetic field.[1)]

Taking into account, on the one hand, the inequality (9), and on the other the inequality (10),[2)] we obtain for the magnetic field interval

$$1.3\times10^2\left(\frac{n_I}{10^{15}}\right)^{1/4}\left(\frac{T}{10}\right)^{3/4}\text{[Oe]} \ll H \ll 2\times10^4\left(\frac{n_I}{10^{15}}\right)\left(\frac{T}{10}\right)^{-3/2}\text{[Oe]}. \quad (21)$$

These inequalities are compatible at

$$n_I/T^3 \gg 3.2\times10^9. \quad (22)$$

We now make a tentative comparison with experiment.

We know of several experimental studies of nonlinear phenomena in n-Ge in strong electric and magnetic fields,[8,9] whose results recall the effects calculated by us and consist in the following.

1. The CVC show three basic regions: a region of linearity of $j(E_x)$ in weak electric field, a saturation region, and a region of rapid rise in the current. The first and second CVC regions are separated by sections with faster growth of the current with the field than in the initial linear region.

2. Corresponding to the indicated CVC sections are, on the magnetoresistance curves, a saturation region, a region of linearity of $\rho(E_x)$, and a region of a fast decrease of ρ. The faster increase of the current between the first and second regions on the CVC correspond on the $\rho(E_x)$ curves to sections where the magnetoresistance decreases with increasing electric field. It is obvious from the foregoing that ahead of the start of the linear $\rho(E_x)$ dependence the magnetoresistance ρ has a minimum, and a maximum at the end of the linear region.

3. The Hall field has approximately the same dependence on E_x as the CVC.

4. In the regions where the current saturates and increases sharply, the CVC assume N and S shapes, respectively. The N-shape section on the CVC, however, is not directly observable, but is implied by the presence of voltage oscillations on the potential probes.

The main characteristics due to these non-ohmic phenomena are the following:

1) the effect is not observed at high temperatures, only below the liquid-nitrogen temperature;

2) the effect does not depend on the dimensions of the sample;

3) it is not observed when the magnetic field is parallel to the direction of the current in the sample;

4) the effect is observed at $T = 20\,K$ in the impurity-density region $10^{14}\,cm^{-3} \lesssim n_I \lesssim 10^{16}\,cm^{-3}$;

5) there is no effect when the Hall contacts are shorted.

Similar effects were observed by a number of workers in n-InSb.[10-12] They attempted to explain the decrease of the magnetoresistance with the field by resorting to the results of Ref. 13. This, however, is wrong, as is correctly indicated in Ref. 9, inasmuch as the calculations in Ref. 13 were made without allowance for the Hall field, whereas in Refs. 10-12 the measurements were made in the given-current regime.

In Ref. 8, to interpret the results, the idea of "transverse" breakdown (proposed in Ref. 14) is invoked. According to this idea, the Hall field in transverse field can be much stronger than the applied field, and after reaching a certain value it can cause a "transverse" breakdown. As shown in Ref. 9, the increase of the current in the third region of the CVC cannot be attributed to an increase of the density.

Our investigation explains in the main the nonlinear effects described in Refs. 8 and 9. According to theory and experiment, the $j(E_x)$ and $\rho(E_x)$ curves shift in parallel with increasing magnetic field towards stronger applied electric field and towards a higher magnetoresistance. The steeper increase of the CVC at the end of the linear region is due to impurity breakdown (14); at $E_x \approx E_0$ an S-shaped section appears on the CVC. However, theory does not yield saturation of the Hall field as a function of the applied field, nor an N-shaped section in the saturation region of the CVC.

The following explanations of the cited main characteristics are also obvious:

1) the calculated effect should be observed only at low temperatures, so as to ensure the needed momentum and energy scattering mechanism;

2) it is a volume effect, independent of the sample size;

3) the effect should be observed only in transverse fields;

4) at $T = 20\,K$, according to (22), it can be observed at ion densities $n_I \gg 2.6\times10^{13}\,cm^{-3}$, while on the other hand, at $n > 10^{16}\,cm^{-3}$, there appear new more effective energy relaxation mechanisms (inelastic scattering of the hot electrons by the neutral impurity atoms,[15] and electron-electron scattering);

5) the effect does not appear in the given-field regime, only in the given-current regime.

The magnetic fields used in the experiments of Refs. 8 and 9 were quite strong, in the range 30–125 kOe, whereas according to our theory the fields required are not very strong. In fact, substituting in (21) $n_I = 6.2\times10^{14}\,cm^{-3}$ and $T = 20\,K$,[6] we obtain

$$0.2\ \text{kOe} \ll H < 4.5\ \text{kOe}.$$

It must be noted here, however, that to observe the calculated effects an upper bound on the magnetic field is not a principal requirement, since it is necessitated only by the strong-heating requirement, which was considered by us only to make the analytic calculation possible. As to the lower bound of the magnetic field, it is important, since it is connected with the condition that ensures preservation of the required scattering mechanisms.

For the same data, we obtain from (20) $E_0 \approx 5V/cm$, whereas in experiment, when the magnetic field is varied in the interval indicated above, E_0 changes from 20 to 40 V/cm. Taking into account the foregoing and the fact that we have neglected the logarithmic dependence of E_0 on the temperature and on the magnetic field, the agreement can apparently be regarded as satisfactory.

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1) Disregarding the weak logarithmic dependence.

2) Inasmuch as in this case the scattering of the momentum by the ions can give way, with increasing average energy, to scattering by acoustic phonons ($t' = -1$), whereas the energy-scattering mechanism does not change.

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