[23]A. M. Semenov and E. E. Shpil'rain, in: Uravnenie sostoyaniya gazov i zhidkostei (Equation of State of Gases and Liquids), ed. by I. I. Novikova, Nauka, 1975, p. 77.
[24]D. A. Kirzhnits, Yu. E. Lozovik, and G. V. Shpatakovskaya, Usp. Fiz. Nauk **117**, 3 (1975) [Sov. Phys. Usp. **18**, 649 (1975)].
[25]J. M. Ziman, Calculation of Bloch Functions (Russ. transl.), Mir, 1977.
[26]J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird, Molecular Theory of Gases and Liquids, Wiley, 1954.
[27]B. V. Zelener, Teplofiz. Vys. Temp. **15**, 893 (1977).
[28]E. A. Mason and T. H. Spurling, The Virial Equation of State, Pergamon, 1970.
[29]N. B. Vargaftik, Spravochnik po teplofizicheskim svoistvam gazov i zhidkostei (Handbook of Thermophysical Properties of Gases and Liquids), Nauka, 1972.
[30]G. M. Harris, J. Trulio, and J. Roberts, Phys. Rev. **119**, 1832 (1960).
[31]A. Sabukinas and A. Z. Chizhyunas, Lit. Fiz. Sb. **14**, 73 (1974).
[32]D. R. Hartree, Calculation of Atomic Structures (Russ. transl.), IIL, 1960.
[33]B. J. Alder and T. E. Wainraiht, J. Chem. Phys. **33**, 1439 (1960).
[34]H. C. Graboske, D. J. Harwood, and F. J. Rogers, Phys. Rev. **186**, 210 (1969).
[35]N. W. Ashcroft and J. Lekner, Phys. Rev. **145**, 83 (1966).
[36]W. G. Hoover, G. Stell, E. Goldmark, and G. D. Degani, J. Chem. Phys. **63**, 5434 (1975).
[37]B. V. Zelener, G. E. Norman, and V. S. Filinov, Teplofiz. Vys. Temp. No. 10, 1160 (1972).
[38]V. K. Gryaznov and I. L. Iosilevskiĭ, Chislennye metody mekhaniki sploshnoĭ sredy **4**, 166 (1973); in: Teplofizicheskie svoistva nizkotemperaturnoi plazmy (Thermophysical Properties of Low-Temperature Plasma), ed. by V. M. Ievleva, Nauka, 1976, p. 25.
[39]A. E. Glauberman and I. P. Yukhnovskiĭ, Zh. Eksp. Teor. Fiz. **22**, 562 (1952).
[40]F. H. Stillinger and R. Lowett, J. Chem. Phys. **49**, 1991 (1968).
[41]I. L. Iosilevskiĭ, Teplofiz. Vysok. temperature **18**, No. 2, 1980.
[42]A. A. Vedenov and A. I. Larkin, Zh. Eksp. Teor. Fiz. **36**, 1133 (1959) [Sov. Phys. JETP **9**, 806 (1959)]; A. I. Larkin, *ibid.* **38**, 1896 (1960) [**11**, 1363 (1960)].
[43]F. Rogers and H. De Witt, Phys. Rev. **8**, 1061 (1973).
[44]V. E. Fortov, B. N. Lomakin, and Yu. G. Krasnikov, Teplofiz. Vys. Temp. **9**, 869 (1971).
[45]Yu. G. Krasnikov, Zh. Eksp. Teor. Fiz. **73**, 516 (1977) [Sov. Phys. JETP **46**, 270 (1977)].
[46]Yu. G. Krasnikov and V. I. Kucherenko, Teplofiz. Vys. Temp. **16**, 43 (1978).

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The nonlinear theory of the current instability of short-wavelength drift oscillations

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We study the current instability of an inhomogeneous plasma, which leads to the excitation of short-wavelength drift oscillations with a frequency close to the lower-hybrid resonance. We show that the saturation of the instability is connected with the spectral transfer of the oscillations into the short-wavelength ragion, which is due to the modulational instability, and we determine the maximum amplitudes of the electrical fields of the oscillations. We evaluate the effective electron collision frequency due to the current instability and we show that the Parker-Sweet diffusion model for the reconnection of the magnetic field, modified to allow for the anomalous resistivity mechanism studied in the present paper, gives for the width of the magneto-pause an estimate that agrees satisfactorily with experiment.

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§1. INTRODUCTION

The instabilities of the currents flowing across a magnetic field are important both for laboratory plasmas (shock waves, theta pinch, turbulent heating) and for the plasma in the magnetosphere (magnetic field reconnection, anomalous resistivity in the boundary layers of the magnetosphere, and so on). One of these instabilities—the so-called "tearing" instability—is of an electromagnetic type.[1] It leads to the generation of a transverse magnetic field component both in thermonuclear magnetic bottles[2] and in the magnetosphere primarily in its tail part.[3]

At the same time, other kinds of instability are basically responsible for the occurrence of the anomalous resistivity; they lead to the excitation of potential or close to potential oscillations. The lowest threshold for excitation of them corresponds to the current instability for short-wavelength drift oscillations which are polarized in the plane at right angles to the magnetic field. Mikhailovskiĭ and Timofeev were the first[4] to study the linear theory of this instability and in Ref. 5 the fact that the oscillations may be non-potential, which in important for a plasma with a finite β (ratio of the gas-kinetic to the magnetic pressure), was taken into account.

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The first time that attention was focused on the importance of this instability for the Earth's magnetosphere was in Ref. 6, where a qualitative analysis was given of the anomalous resistivity mechanism, based on it, in the tail part of the magnetosphere. When estimating the possibility of the occurrence of an instability of the short-wavelength drift oscillations one must bear in mind that all conditions which are fundamental for it (threshold current velocity less than the thermal ion speed, hot ions $T_i \gg T_e$, and a large value of β) are realized in the first place in the frontal part of the magnetosphere—the magneto-pause. Moreover, according to recent satellite measurements,[7] strong electrical field oscillations are observed in the magneto-pause region with frequencies up to the lower-hybrid one $\sim(\omega_{Hi}\omega_{He})^{1/2}$, which agrees with the theoretical estimate of the spectrum of the oscillations excited when there is an instability.

This all gives us grounds to assume that the instability considered exists in the vicinity of the magneto-pause and may be responsible for the anomalous resistivity and the magnetic-field diffusion in that region.

The aim of the present paper is the construction of a non-linear theory of the instability. We consider the mechanism of its stabilization to be the spectral transfer of energy into the region of "oblique" oscillations, $k_z \neq 0$, where the resonance absorption of the oscillations by the electrons becomes important. The transfer arises as a result of the modulational instability of the short-wavelength drift mode excited by the current. We evaluate for that mechanism the maximum amplitude of the electrical field of the drift oscillations and the effective electron-collision frequency caused by the instability.

The results are used to elucidate the reconnection of the magnetic field lines in the magneto-pause region. The reconnection is described in the framework of the Parker-Sweet diffusion model,[8] modified to take into account the mechanism for the anomalous resistivity considered in the present paper. We show that this model gives for the width of the magneto-pause an estimate of the order of a few ion Larmor radii, which agrees satisfactorily with experiments.[9]

§2. LINEAR THEORY OF THE INSTABILITY. EFFECTIVE COLLISION FREQUENCY

We consider the instability of a plasma with an electron current flowing across the magnetic field (see Fig. 1). The current maintains a field gradient determined from the equation

$$\frac{dH_z}{dx}=\frac{4\pi e n_0}{c}u_{ey}, \tag{1}$$

u_{ey} is the electron current velocity, and we choose a frame of reference in which the ions are at rest. From the condition for a balance between the magnetic and the gas-kinetic pressures

$$\frac{d}{dx}\frac{H_z^2}{8\pi}+(T_e+T_i)\frac{dn_0}{dx}=0 \tag{2}$$

we find that

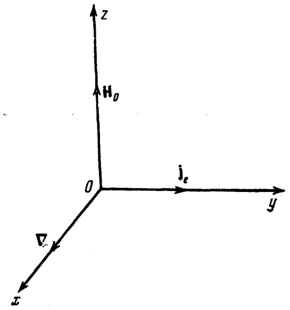

FIG. 1.

$$u_{ey}=-\frac{c(T_e+T_i)}{eH_0}\varkappa,\quad \varkappa=\frac{1}{n_0}\frac{dn_0}{dx}. \tag{2'}$$

The condition for the balance of the pressures is violated in non-stationary processes, and the electron current velocity is thus an arbitary parameter about which we shall assume only that $u_{ey} \ll v_{Ti}$.

The electron current leads to an excitation of oscillations propagating in the plane perpendicular to the magnetic field ($\mathbf{k}\perp\mathbf{H}_0$). We look for the electrical field of the oscillations in the form

$$E\sim\exp\left[i\left(\int k_x dx+k_y y-\omega t\right)\right].$$

The frequency of the oscillations lies in the range $\omega_{Hi} \ll \omega \ll \omega_{He}$ ($\omega_{H\alpha}=eH_0/m_\alpha c$ is the cyclotron frequency of the charged particles) while the wavelength satisfies the condition for the applicability of geometric optics

$$\left|\frac{1}{k_x}\frac{d}{dx}\ln n_0\right| \ll 1.$$

For the oscillations which are built up by the electron current, the resonance condition $\omega \approx k_y u_{ey} \ll k v_{Ti}$ is satisfied. In the limiting case considered (unmagnetized "hot" ions) we have for the perturbation of the ion density in the oscillations the standard kinetic-theory formula (see, e.g., Ref.10):

$$n_i'=-\frac{\operatorname{div}\mathbf{E}}{4\pi e}\left[\frac{1}{k^2 r_{Di}^2}-\pi i\frac{\omega_{pi}^2}{k^2 n_0}\frac{\partial f_{0i}}{\partial v}\left(\frac{\omega}{k}\right)\right], \tag{3}$$

$\omega_{p\alpha}=(4\pi e^2 n_0/m_\alpha)^{1/2}$ is the Langmuir frequency, $r_{D\alpha}=(T_\alpha/4\pi e^2 n_0)^{1/2}$ is the Debye radius, and $\alpha=e$ or i.

At the same time in the studied oscillations the electrons are magnetized and their velocities are determined from the drift theory.

We restrict ourselves to the terms of second order in the small parameter $|\omega, k_y u_{ey}|/\omega_{He} \ll 1$. Moreover, bearing first of all in mind the application of the results to the magneto-pause, we shall consider in what follows a plasma with hot ions, $T_i \gg T_e$. Neglecting in the small terms $\propto\omega_{He}^{-2}$ the thermal motion of the electrons (zero Larmor radius approximation) we get the following relations for the components of the electron velocity:

$$\begin{aligned} v_{ex}&=\frac{c}{H_0}E_y+\frac{i(\omega-k_y u_{ey})}{\omega_{He}}\left[\frac{c}{H_0}E_x\left(1-\frac{k_y u_{ey}}{\omega}\right)+\frac{c}{H_0}E_y\frac{k_x u_{ey}}{\omega}\right]+ik_y\frac{cT_e}{eH_0}\frac{n_e'}{n_0}, \\ v_{ey}&=-\frac{c}{H_0}\left[E_x\left(1-\frac{k_y u_{ey}}{\omega}\right)+\frac{k_x u_{ey}}{\omega}E_y\right]+\frac{i(\omega-k_y u_{ey})}{\omega_{He}}\frac{c}{H_0}E_y-ik_x\frac{cT_e}{eH_0}\frac{n_e'}{n_0}. \end{aligned} \tag{4}$$

When obtaining these formulae we used the relation for the magnetic field of the wave

$$H_z'=c(k_xE_y-k_yE_x)/\omega, \tag{5}$$

which follows from the Maxwell equations.

From the equation of continuity for the electrons

$$i(k_yu_{ey}-\omega)n_e'+v_{ex}dn_0/dx+n_0\operatorname{div}\mathbf{v}_e=0$$

we get, using Eq. (4) for the perturbation of the electron density

$$n_e'(\omega-k_yu_{ey}+k_yu_{dH})=\frac{c}{H_0}n_0(k_xE_y-k_yE_x)\left(1-\frac{k_yu_{ey}}{\omega}\right)$$
$$-icE_y\frac{d}{dx}\left(\frac{n_0}{H_0}\right)+i\frac{\omega-k_yu_{ey}}{\omega_{He}}\frac{cn_0}{H_0}\operatorname{div}\mathbf{E}. \tag{6}$$

In this formula

$$u_{dH}=\frac{cT_e}{eH_0^2}\frac{dH_0}{dx} \tag{7}$$

is the electron drift velocity in an inhomogeneous magnetic field. Restricting ourselves in what follow to considering "cold" electrons we shall neglect this drift; the condition for this neglect is

$$k_yu_{dH}\ll|\omega-k_yu_{ey}|. \tag{7'}$$

The main term in the formula for the electron density [the first term on the right-hand side of (6)] vanishes for purely potential oscillations. For a plasma with finite β the deviation from potentiality becomes important. The degree of non-potentiality of the oscillations can be determined using the Maxwell equation

$$ik_yH_z'=-4\pi en_0v_{ex}/c,$$

substituting in it H_z' from (5) and v_{ex} from (4). As a result we get the equation

$$k_xE_y-k_yE_x=i\frac{\omega_{pe}^2}{\omega_{He}}\frac{\omega}{k_yc^2}E_y. \tag{8}$$

We shall consider below sufficiently short-wave oscillations with $kr_L*\gtrsim 1$; here $r_L*=(T_i/m_e)^{1/2}\omega_{He}^{-1}$ is the electron Larmor radius evaluated with respect to the ion temperature. In that case the parameter $\omega_{pe}^2/k^2c^2\sim\beta$ where we have written: $\beta=4\pi n_0T_i/H_0^2$. It follows from Eq. (8) that even when $\beta\gtrsim 1$ the oscillations considered are close to potential, as the rotational part of the electrical field is small compared to the potential part in the ratio ω/ω_{He}.

However, the deviation from the potentiality turns out to be important in Eq. (6) for the electron density, as the main term—the first one on the right-hand side contributes only to the rotational field.

Using this, the formula for the electron density takes its final form:

$$n_e'=\frac{ic}{H_0}\operatorname{div}\mathbf{E}\left[\frac{1}{k_yu_{ey}-\omega}\frac{k_y}{k^2}\frac{dn_0}{dx}\left(1-\beta\frac{u_{ey}}{u_{di}}\right)+\frac{n_0}{\omega_{He}}\left(1+\frac{\omega_{pe}^2}{k^2c^2}\right)\right]. \tag{9}$$

When the condition $\omega_{pe}\gg\omega_{He}$ is satisfied we may assume that the oscillations considered are quasi-neutral. Comparing (3) and (9) we then get the following dispersion equation:

$$\frac{1}{k^2r_{Di}^2}+\frac{\omega_{pe}^2}{\omega_{He}^2}\left(1+\frac{\omega_{pe}^2}{k^2c^2}\right)-\frac{\omega_{pe}^2}{\omega_{He}}\frac{k_y\varkappa}{k^2(\omega-k_yu_{ey})}\left(1-\beta\frac{u_{ey}}{u_{di}}\right)$$
$$-\pi i\frac{\omega_{pi}^2}{k^2n_0}\frac{\partial f_{0i}}{\partial v}\left(\frac{\omega}{k}\right)=0. \tag{10}$$

From Eq. (10) we get the following formulae for the frequency and the growth rate:

$$\omega=k_yu_{ey}+k_yu_{di}\frac{1-\beta u_{ey}/u_{di}}{1+\beta+k^2r_{L*}^2}$$
$$\gamma=\pi k_yu_{di}\left(1-\frac{\beta u_{ey}}{u_{di}}\right)\frac{1}{(1+\beta+k^2r_{L*}^2)^2}\frac{T_i}{n_0m_i}\frac{\partial f_{0i}}{\partial v}\left(\frac{\omega}{k}\right) \tag{11}$$
$$u_{di}=(cT_i/eH_0)\varkappa.$$

The condition for the occurrence of the instability can be written in the form

$$k_y\varkappa>0,\quad u_{ey}>|u_{di}|\frac{1}{1+k^2r_{L*}^2}=\frac{v_{Ti}|\varkappa|r_{Li}}{1+k^2r_{L*}^2}. \tag{11'}$$

One sees easily that when the last of the conditions (11′) is satisfied the first term in the formula for the frequency becomes the main one, the frequency $\omega>0$, and, if $k_y\varkappa<0$, the instability arises when

$$\frac{\partial f_{0i}}{\partial v}\left(\frac{\omega}{k}\right)<0,$$

i.e., for a plasma with a Maxwellian ion velocity distribution. Under the conditions of the magneto-pause one finds the electron current velocity from the pressure balance condition, i.e., it is given by Eq. (2′). Even when $T_i\gg T_e$ this velocity is higher than the instability threshold (11′), provided that we consider sufficiently short-wavelength oscillations with $kr_L*\gtrsim 1$.

Well above the instability threshold, the formulae for the frequency and the growth rate are transformed to the following simple form:

$$\omega\approx\omega_{LH}\frac{u_{ey}}{v_{Ti}}k_yr_{L*}\frac{1-\beta u_{ey}/u_{di}}{1+\beta+k^2r_{L*}^2},$$
$$\gamma=\left(\frac{\pi}{2}\right)^{1/2}\omega\frac{k_y}{k}\frac{|\varkappa|r_{Li}(1-\beta u_{ey}/u_{di})}{(1+\beta+k^2r_{L*}^2)^2}. \tag{12}$$

The maximum growth rate is reached when $kr_L*\approx[(1+\beta)/5]^{1/2}$ and equals

$$\gamma_{\max}=0{,}32\omega_{LH}\frac{u_{ey}}{v_{Ti}}\frac{|\varkappa|r_{Li}}{(1+\beta)^{1/2}}, \tag{12'}$$

$\omega_{LH}=(\omega_{Hi}\omega_{He})^{1/2}$ is the frequency of the lower-hybrid resonance in a dense plasma, $\omega_{pe}\gg\omega_{He}$.

The instability described by the dispersion equation (10) was obtained in Ref. 10 as the high-frequency limit of the drift-cyclotron instability (high harmonics of ω_{Hi} when a large number of resonances become important at once). The condition for neglecting the effect of the magnetic field on the ions has, according to Ref. 10, the form $\gamma\gg\omega_{Hi}$ and can, by the use of (12′), be reduced to the following form:

$$|\varkappa|^2r_{Li}^2\gg\left[\frac{m_e}{m_i}(1+\beta)\right]^{1/2} \tag{13}$$

[we substituted u_{ey} from (2′)]. For the magneto-pause where the dimension of the transition layer $1/\varkappa$ is of the order of 2 to 3 ion Larmor radii this condition is amply satisfied.

Finally, condition (7′) for the neglect of the electron magnetic drift can with the aid of Eq. (11) be rewritten in the form

$$\frac{T_e}{T_i}\beta \ll \frac{1+\beta}{1+\beta+k^2r_L.^2}\qquad (14)$$

and can also be satisfied in a plasma with finite β provided the electron temperature is sufficiently small.

The instability considered by us is the instability of a negative energy wave. Indeed, according to Ref. 11 the energy of a wave in a medium with permittivity tensor ε_{ik} is given by the equation

$$W=\frac{\partial}{\partial\omega}(\omega\varepsilon_{ik})\frac{E_i^*E_k}{8\pi}+\frac{c^2}{\omega^2}\cdot\frac{[\mathbf{k}\times\mathbf{E}]^2}{8\pi}.$$

Using the standard formulae for the components ε_{ik} (see, e.g., Ref. 10), and recognizing that the deviation from potentiality of the wave is important only in the terms $\propto\varepsilon_{12}$, ε_{21}, we are led after straightforward but cumbersome calculations to the following formula for W:

$$W=\frac{\omega_{pe}^2\omega(1-\beta u_{ey}/u_{di})}{\omega_{He}(\omega-k_yu_{ey})^2}\frac{k_y\varkappa}{k^2}\frac{E^2}{8\pi}.\qquad (15)$$

When the first of conditions (11′), which is necessary for the occurrence of the instability, is satisfied the energy of the wave is negative. The energy dissipation connected with the Landau damping on the ions thus leads to the instability.[12]

We note that in a plasma with hot ions, $T_i\gg T_e$, yet another current instability is possible—the electron-acoustic instability leading to a pumping of "oblique" ($k_z\ll k$) oscillations.[13] This instability can be obtained, if we take into account in the dispersion equation the term due to the longitudinal motion of the electrons. The corresponding term has the form

$$-\frac{k_z^2}{k^2+\omega_{pe}^2/c^2}\frac{\omega_{pe}^2}{(\omega-k_yu_{ey})^2},$$

it becomes dominating when

$$\frac{k_z^2}{\varkappa^2}\gg k^2r_L.^2(1+\beta)^2\frac{1+\omega_{pe}^2/k^2c^2}{1+\beta+k^2r_L.^2}.\qquad (16)$$

(The condition given here can easily be obtained from a comparison in the dispersion equation of the terms describing the perturbation of the density due to the longitudinal motion of the electrons and to their drift across the magnetic field.) When condition (16) is satisfied the electron-acoustic instability develops with a frequency and growth rate given by the relations

$$\omega=k_yu_{ey}-k_z\left(\frac{T_i}{m_e}\right)^{1/2}\left[\left(1+\frac{\omega_{pe}^2}{k^2c^2}\right)(1+\beta+k^2r_L.^2)\right]^{-1/2},$$
$$\gamma=\left(\frac{\pi}{8}\right)^{1/2}\frac{\omega}{kv_{Ti}}(k_yu_{ey}-\omega)\frac{1}{1+\beta+k^2r_L.^2}.\qquad (17)$$

The threshold value of the current velocity is

$$u_{ey}=\frac{k_z}{k}\left(\frac{T_i}{m_e}\right)^{1/2}\left[\left(1+\frac{\omega_{pe}^2}{k^2c^2}\right)(1+\beta+k^2r_L.^2)\right]^{-1/2},\qquad (18)$$

which by virtue of (16) is appreciably higher than the threshold velocity for the mode with $k_z=0$, which is given by condition (11′).

In the conditions of the magneto-pause the current velocity $u_{ey}=v_{Ti}|\varkappa|r_{Li}$ lies, when we use condition (16), below the threshold (18) so that it is not possible to excite directly oscillations with $k_z\neq0$ as a result of the electron-acoustic instability.

The main macroscopic consequence of the excitation of the oscillations considered above is the occurrence of an anomalous resistivity, i.e., losses in electron momentum transferred to the ions participating in the oscillations. Following Galeev and Sagdeev[14] we write the loss in electron momentum in the form $n_0m_eu_{ey}\nu_{\text{eff}}$ and, using the momentum conservation law, we write

$$n_0m_eu_{ey}\nu_{eff}=2\sum_{\mathbf{k}}\frac{k_y}{\omega_{\mathbf{k}}}\gamma_{e\mathbf{k}}\tilde{W}_{\mathbf{k}}.\qquad (19)$$

The right-hand side of this equation is the transfer of momentum from the electrons to the oscillations, γ_{ek} is the electron contribution to the growth rate, and $\tilde{W}_k$ the spectral density of the oscillation energy. The $v_z\approx\omega/k_z$ transfer of momentum to the oscillations arises from the group of electrons and at resonance with the oscillations, of velocity. The appearance of these electrons is connected with the transfer of energy to the region of "oblique" oscillations, $k_z\neq0$. The mechanism of spectral transfer is based upon the modulational instability of the short-wavelength drift oscillations, and will be considered in the next section. In accordance with what we have said, $\tilde{W}_k$ is the spectral density of the oscillation energy in that region of the spectrum where $k_z\neq0$ and where the resonance interaction with electrons is important.

The resonance absorption by the electrons must lead to the establishment of quasi-stationary turbulence. In such turbulence the energy transferred by the ions to the oscillations (we remind ourselves that we consider oscillations with a negative energy) is transferred to large k_z and in final reckoning is absorbed by the electrons. The balance condition can then be written in the form

$$\sum_{\mathbf{k}}\gamma_i(\mathbf{k},\omega)W_{\mathbf{k}}+\sum_{\mathbf{k}}\gamma_e(\mathbf{k},\omega)\tilde{W}_{\mathbf{k}}=0.$$

Using this condition we write for ν_{eff}:

$$\nu_{eff}\approx-\frac{2}{m_en_0u_{ey}}\sum_{\mathbf{k}}\gamma_{i\mathbf{k}}W_{\mathbf{k}}\frac{k_y}{\omega_{\mathbf{k}}}.$$

Substituting into that formula the wave energy W from (15), the maximum growth rate of the instability from (12′), and the wavelength of the most unstable mode $kr_L*\approx[(1+\beta)/5]^{1/2}$, we are led to the following final formula for ν_{eff}:

$$\nu_{eff}=(10\pi)^{1/2}\frac{m_i}{m_e}\frac{W_E}{n_0T_i}\frac{\omega_{pe}^2}{\omega_{He}^2}\frac{\omega_{LH}}{(1+\beta)^{1/2}},$$
$$W_E=\sum_{\mathbf{k}}\frac{|E_{\mathbf{k}}|^2}{8\pi}\qquad (20)$$

is the energy of the electrical field of the short-wavelength drift oscillations with $k_z=0$, which are excited owing to the current instability studied in the present section.

The excitation of the oscillations is accompanied with a heating of the electrons and ions. The rate of the heating of the ions can be found from the energy conservation law (see Ref. 14):

$$\frac{3}{2}n_0\frac{dT_i}{dt}=2\sum_{\mathbf{k}}\gamma_{i\mathbf{k}}|W_{\mathbf{k}}|,$$

and by analogy with the evaluation of ν_{eff} we have

$$\frac{dT_i}{dt}=\frac{2}{3}(10\pi)^{1/2}\omega_{LH}\frac{\omega_{pe}^2u_{ey}^2}{\omega_{He}^2v_{Ti}^2}\frac{W_E}{n_0}. \tag{21}$$

According to Ref. 14 the ratio of the rates of heating of the electrons and ions is determined from the equation

$$\frac{dT_e}{dT_i}\approx\frac{k_yu_{ey}-\omega}{\omega}. \tag{22}$$

Sufficiently far from the threshold of the instability the ratio on the right-hand side is appreciably smaller than unity, i.e., the oscillations lead to a preferential heating of the hotter ion component.

§3. MODULATIONAL INSTABILITY OF DRIFT OSCILLATIONS. MAXIMUM AMPLITUDES OF THE ELECTRICAL FIELDS

To determine the level of W_E which occurs in Eq. (20) for ν_{eff} it is necessary to construct a non-linear theory based upon some concrete mechanism for saturating the current instability. As we have already noted above, we shall assume that such a saturation is connected with the spectral transfer to the region of large k_z in which the resonance absorption of the oscillation energy by the electrons becomes important. Usually induced scattering is adduced as the mechanism for the spectral transfer in the theory of the anomalous resistivity (see Ref. 14). However, in our case the spectral transfer caused by the modulational instability of the drift oscillations turns out to be much more important. One can easily understand the mechanism of such a transfer by analogy with Langmuir oscillations (see, e.g., Ref. 15). Small fluctuations in the intensity of the high-frequency oscillations $\delta W_E(z)$ under the action of the high-frequency pressure leads to a modulation of the plasma density $\delta n(z)$. In the density wells formed in the regions where the high-frequency field is localized, additional portions of high-frequency quanta are trapped. This leads to an increase in the depth of the modulation δW_E and as a consequence to a spectral transfer of energy of the drift oscillations, which were initially uniform in z, to the region of large k_z.

We shall assume that the slow plasma motions which arise under the action of the high-frequency pressure force are quasi-neutral and that their characteristic frequency Ω satisfies the conditions

$$\Omega\ll kv_{Ti},\quad k_zv_{Te}. \tag{23}$$

For the dispersion law of the high-frequency oscillations given by Eq. (10), the averaging over the fast time-scale in the equations for the low-frequency motions is, when $\omega\approx k_yu_{ey}$, equivalent to averaging over the y-coordinate. We shall therefore assume in what follows that all quantities characterizing the low-frequency mode depend only on the x,z-coordinates and on the slow time t, so that the electron drift with current velocity u_{ey} is unimportant for that mode.

When conditions (23) are satisfied we get from the equations for the motion of the electrons and ions along the z-axis the following formula when the density varies slowly:

$$\frac{T_e+T_i}{n_0}\frac{\partial\delta n}{\partial z}=-\frac{m_ec}{H_0}\left\langle\left(E_y\frac{\partial}{\partial x}-E_x\frac{\partial}{\partial y}\right)v_{ez}\right\rangle+\frac{e}{c}u_{ey}\delta H_x. \tag{24}$$

On the right-hand side of Eq. (24) we have retained the main non-linear term which arises when we take into account the term $m_e\langle(\mathbf{v}_{e\perp}\nabla_\perp)v_{ez}\rangle$ in the electron equation of motion. The brackets in the non-linear term correspond to averaging over the fast time-scale. Assuming that the frequency of the slow motions satisfies the additional condition $\Omega\ll kv_A$ we find that the density and magnetic field variations in the low-frequency mode are connected by the simple relation

$$\delta H_z=-4\pi\delta nT_i/H_z. \tag{25}$$

If we use the equation $\operatorname{div}\delta\mathbf{H}=0$ to determine the transverse magnetic field component δH_x, we can show easily that the last term on the right-hand side of Eq. (24) is small in the ratio $(m_e/m_i)^{1/2}\omega_{pe}^2/k^2c^2$, and we neglect it in what follows. The quantity v_{ez} in that equation is the longitudinal component of the high-frequency electron velocity which is given by the equation

$$\left(\frac{\partial}{\partial t}+u_{ey}\frac{\partial}{\partial y}\right)v_{ez}=-\frac{eE_z}{m_e}+\frac{cH_x'}{m_ec}u_{ey}. \tag{26}$$

To find the field E_z we use the equation

$$\frac{\partial}{\partial z}\operatorname{div}\mathbf{E}_\perp-\nabla_\perp^2E_z=\frac{4\pi en_0}{c^2}\frac{\partial v_{ez}}{\partial t}. \tag{27}$$

It follows from Eq. (8) that the transverse components of the electrical field in the high-frequency motions are close to being potential and we can in Eqs. (24) and (27) substitute approximately

$$E_x=-\partial\varphi/\partial x,\quad E_y=-\partial\varphi/\partial y.$$

If we then use Eq. (27) to eliminate E_z from (26) and eliminate H_z' by using the equation

$$-\frac{1}{c}\frac{\partial H_x'}{\partial t}=\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z},$$

we are finally led to the following equation for v_{ez}:

$$\left(\frac{\omega_{pe}^2}{c^2}-\nabla^2\right)\left(\frac{\partial}{\partial t}+u_{ey}\frac{\partial}{\partial y}\right)v_{ez}=-\frac{e}{m_e}\frac{\partial}{\partial z}\nabla^2\varphi. \tag{28}$$

This equation together with the equation for δn:

$$\frac{\partial}{\partial z}\delta n=\frac{en_0}{T_i\omega_{He}}\langle[\nabla v_{ez}\nabla\varphi]_z\rangle, \tag{29}$$

gives us the starting set of equations for describing the low-frequency mode.

The equation for the high-frequency motions is obtained by analogy with the derivation of the dispersion equation of the linear theory given in the second section. Now, however, we must in the continuity equation for the electrons take additionally into account the non-linearity arising from the modulation of the plasma density and the magnetic field of the low-frequency mode, and also the density perturbation caused by the longitudinal motion of the electrons of velocity v_{ez}, which is given by Eq. (28). The variation of the ion density is found from E:. (3). Assuming as before the high-frequency oscillations to be quasi-neutral and dropping for the sake of simplicity the terms describing the pumping of the oscillations by the ions and their damping on the electrons, we are led to the following equation for the "potential" φ of the high-frequency mode:

$$\left(\nabla^2-\frac{\omega_{pe}^2}{c^2}\right)\left(\frac{\partial}{\partial t}+u_{ey}\frac{\partial}{\partial y}\right)\left[(1-r_{Le}^2\nabla^2+\beta)\left(\frac{\partial}{\partial t}+u_{ey}\frac{\partial}{\partial y}\right)\varphi\right.$$
$$\left.+u_{di}\left(1-\beta\frac{u_{ey}}{u_{di}}\right)\frac{\partial\varphi}{\partial y}+\frac{cT_i}{eH_0}\frac{1+\beta}{n_0}\frac{\partial\delta n}{\partial x}\frac{\partial\varphi}{\partial y}\right]-\frac{T_i}{m_e}\frac{\partial^2}{\partial z^2}\nabla^2\varphi=0. \qquad (30)$$

In the general case the set of Eqs. (28) to (30) is very complicated and we restrict our investigation with its aid to the linear theory of the modulational instability that leads to the creation from a monochromatic wave which is uniform along z of satellites with $k_z\neq 0$, and we determine the amplitude of the main wave for which the satellites produced fall in the region of an effective absorption by electrons. We perform our investigation in two limiting cases of long and short wavelengths of the high-frequency oscillations

$$k_\perp r_{Le}\ll(1+\beta)^{1/2},\quad k_\perp r_{Le}\gg(1+\beta)^{1/2}.$$

We first study the case of long wavelengths. We can then easily split off from the total expression for the potential of the high-frequency oscillations the time- and y-coordinate-dependence of all the waves in the form

$$\varphi={}^1/_2\varphi(t,x,z)\exp[i(k_{y0}y-k_{y0}(u_{ey}+u_{di}\Gamma)t)]+\text{c.c.},$$

$\varphi(t,x,z)$ is the complex amplitude of the potential, $\Gamma=(1-\beta u_{ey}/u_{di})(1+\beta)^{-1}$. We consider the instability in a four-wave system. For the main wave the complex amplitude of the potential equals

$$\varphi=\varphi_0\exp\left[i\left(\int k_{x0}\,dx-\delta_0 t\right)\right],$$

$\delta_0=k_{y0}u_{di}k_0^2r_{Le}^2\Gamma/(1+\beta)$ is the dispersive correction to the frequency ω determined from (10). The instability leads to the pumping of two high-frequency satellites

$$\varphi(t,x,z)=\exp(-i\delta_0 t)\left\{\varphi_+\exp\left(i\left(\int(k_{x0}+k_x)\,dx+k_zz-\Omega t\right)\right)\right.$$
$$\left.+\varphi_-\exp\left[i\left(\int(k_{x0}-k_x)\,dx-k_zz+\Omega t\right)\right]\right\}$$

and of a low-frequency wave

$$\delta n(t,x,z)=\frac{1}{2}\delta n\exp\left[i\left(\int k_x dx+k_zz-\Omega t\right)\right]+\text{c.c.},$$

One can easily find the dispersion equation for the modulational instability from (28) to (30), using standard procedures; it has the form

$$1=\frac{e^2|\varphi_0|^2}{4m_e^2}\frac{k_z^2k_{y0}}{\omega_{He}^2u_{di}\Gamma}\left[\frac{1}{A_-(\Omega+\Delta_-/(1+\beta))}-\frac{1}{A_+(\Omega-\Delta_+/(1+\beta))}\right] \qquad (31)$$

where we used the notation

$$\Delta_\pm=k_{y0}u_{di}\frac{\Gamma}{1+\beta}(k_0^2r_{Le}^2-k_\pm^2r_{Le}^2)+\frac{k_z^2T_i}{m_e}\frac{1}{k_{y0}u_{di}\Gamma A_\pm}$$

for the frequency difference between the main and the test waves, the wave vector $\mathbf{k}_\pm=\mathbf{k}_0\pm\mathbf{k}$. The quantity $A_\pm=1+\omega_{pe}^2/k_\pm^2c^2$.

When solving the dispersion equation we shall assume that $k\ll k_0$ and correspondingly $A_+\approx A_-$. The solution of the dispersion equation then has the form

$$\Omega=\frac{\Delta_+-\Delta_-}{2(1+\beta)}\pm\left[\frac{(\Delta_++\Delta_-)^2}{4(1+\beta)^2}-\frac{e^2|\varphi_0|^2k_z^2k_{y0}}{4m_e^2\omega_{He}^2u_{di}\Gamma}\frac{\Delta_++\Delta_-}{A(1+\beta)}\right]^{1/2}. \qquad (32)$$

Using the fact that $k_{y0}u_{di}<0$, this solution corresponds to the following condition for the occurrence of an instability:

$$\frac{e^2|\varphi_0|^2}{Am_e^2\omega_{He}^2}\frac{k_z^2k_{y0}}{u_{di}}<\frac{\Delta_++\Delta_-}{1+\beta}<0. \qquad (33)$$

It follows from (33) that as a result of the modulational instability oscillations with $k_z\neq 0$ are, indeed, excited. The region of resonance absorption of such oscillations by electrons corresponds to those k_z for which the following condition is satisfied:

$$k_z^2\frac{T_e}{m_e}\approx\frac{1}{\alpha^2}(\omega-k_{y0}u_{ey})^2\approx\frac{k_{y0}^2u_{di}^2}{\alpha^2}.$$

The parameter $\alpha>1$ due to the fact that the oscillations are absorbed by the "tail" of the electron distribution function; a more exact value of α will be found below by using the balance equation (19). From the condition (33) for the occurrence of the modulational instability it follows that such large values of k_z are reached for pumping amplitudes φ_0:

$$\frac{e^2|\varphi_0|^2}{4m_e^2}\approx\frac{T_i}{T_e\alpha^2}\frac{\omega_{He}^2u_{di}^2}{k_z^2}\frac{1}{1+\beta}. \qquad (34)$$

This estimate is obtained for the instability of a monochromatic wave, but one can show that it remains valid also, as to order of magnitude, for a not too wide ($\Delta k/k\lesssim 1$) packet of oscillations. The relation (34) corresponds to the following electric field energy level of the short-wavelength drift mode:

$$W_E=\sum_{\mathbf{k}}\frac{|E_{\mathbf{k}}|^2}{8\pi}=n_0T_i\frac{m_e}{m_i}\frac{\omega_{He}^2}{\omega_{pe}^2}(\varkappa r_{Li})^2\frac{T_i}{T_e\alpha^2(1+\beta)}. \qquad (35)$$

We note that the mechanism considered by us of the spectral transfer due to the modulational instability is the most effective one. Estimates show that the spectral transfer caused by induced scattering on electrons becomes appreciable at levels W_E which are approximately m_i/m_e larger than the one determined by Eq. (35).

When the spectral transfer along k_z caused by the modulational instability is present the condition for the energy balance in the source region ($k_z=0$) can be written in the form

$$\sum_{\mathbf{k}}\gamma_i(\mathbf{k},\omega)W_{\mathbf{k}}=\sum_{\mathbf{k}}\gamma_{\text{mod}}(\mathbf{k},\omega)\tilde{W}_{\mathbf{k}} \qquad (36)$$

(the energy influx into the turbulence with $k_z=0$ due to the current instability with a growth rate $\gamma_i(\mathbf{k},\omega)$ is compensated by the spectral transfer caused by the modulational instability). In Eq. (36) $\gamma_{\text{mod}}(\mathbf{k},\omega)$ is the modulational instability growth rate. When the amplitude of the main wave is given by Eq. (34) we have the following estimate for the growth rate:

$$\gamma_{\text{mod}}\approx\omega_{LH}|\varkappa|r_{Li}k_\perp r_{Le}\frac{T_i}{T_e\alpha^2}\frac{1}{(1+\beta)^2}, \qquad (37)$$

and it follows from (36) that we then have the following approximate relation for the oscillation energy in the absorption region:

$$\tilde{W}_E=W_E\frac{\gamma_i}{\gamma_{\text{mod}}}=W_E\frac{u_{ey}}{v_{Ti}}\frac{T_e\alpha^2(1+\beta)}{T_i}. \qquad (38)$$

Apart from (36), the balance condition (19) must also be satisfied—the energy transferred to large k_z is in final reckoning absorbed by the electrons. The damping rate of the resonance absorption by the magnetized electrons is equal to

$$\gamma_e=\pi\frac{\omega_{He}}{k_{y0}dn_0/dx}\frac{(\omega-k_{y0}u_{ey})^2}{1+\beta}\left[\frac{\partial f_{0e}}{\partial v_z}-\right.$$
$$\left.\left.-\frac{k_{y0}}{k_z}\frac{1}{\omega_{He}}\frac{\partial f_{0e}}{\partial x}\right]\right|_{v_z\approx\omega/k_z}$$
$$\approx-\left(\frac{\pi}{2}\right)^{1/2}\alpha\omega_{LH}k_{y0}r_{L*}|\varkappa|r_{Li}\left(1\right.$$
$$\left.+\frac{T_i}{T_e}\frac{1+\beta}{1+\beta+k_\perp^2r_{L*}^2}\right)\frac{\exp(-\alpha^2/2)}{1+\beta+k_\perp^2r_{L*}^2}. \quad (39)$$

In obtaining this last relation for γ_e we assumed that the electron distribution function is Maxwellian and we substituted for ω from the dispersion equation of the linear theory, (10). Using the balance condition we then easily get a simple equation for α:

$$\alpha^3e^{-\alpha^2/2}=(2/\pi)^{1/2}[(1+\beta)(1+6T_e/5T_i)]^{-1}. \quad (40)$$

We can consider the case of short wavelengths of the high-frequency mode, $k_0^2r_{L*}^2\gg 1+\beta$ in a completely analogous manner. In that case we can also split off the fast time-dependence in the potential φ:

$$\varphi={}^1/_2\varphi(t,x,z)\exp[ik_{y0}(y-u_{ey}t)]+\text{c.c.}.$$

As before, the complex amplitude $\varphi(t,x,z)$ is in the form of a superposition of the main wave and test waves (satellites), where in the present case $\delta_0=k_{y0}u_{di}\Gamma(1+\beta)/k_0^2r_{L*}^2$. We then still have for the determination of Ω the same dispersion Eq. (31), with the only difference that now

$$\Delta_\pm=k_{y0}u_{di}(1+\beta)\left(\frac{1}{k_\pm^2r_{L*}^2}-\frac{1}{k_0^2r_{L*}^2}\right)\Gamma+k_z^2\frac{T_i}{m_e}\frac{1}{k_{y0}u_{di}(1+\beta)}.$$

The regions of absorption in the case considered correspond to k_z given by the formula

$$k_z^2\frac{T_e}{m_e}\approx\frac{1}{\alpha^2}\frac{k_{y0}^2u_{di}^2}{k_0^4r_{L*}^4}(1+\beta)^2. \quad (41)$$

Transfer to such k_z becomes possible at a level W_E:

$$W_E=n_0T_i\frac{m_e}{m_i}\frac{\omega_{He}^2}{\omega_{pe}^2}\varkappa^2r_{Li}^2\frac{T_i(1+\beta)}{T_e\alpha^2k_0^4r_{L*}^4}. \quad (42)$$

At $k_0^2r_{L*}^2\sim 1+\beta$ this level is of the same order of magnitude as the estimate (35).

In the conditions of the magneto-pause ($n_0=10$ cm^{-3}, $H=(4\text{ to }5)\times10^{-4}$ Oe, $T_i\approx300$ eV, $T_e\approx$eV, $\varkappa r_{Li}\approx1/3$), Eq. (35) gives for the mean square of the electrical field of the lower-hybrid oscillations the estimate $\langle E^2\rangle\sim10^{-6}\ V^2/m^2$. We note that satellite measurements performed by Gurnett *et al.*[7] indicate the existence in the neighborhood of the magneto-pause of a maximum in the spectrum of the electrical field oscillations at frequencies close to the lower-hybrid one ($f\sim30$ to 50 Hz) and the experimental value of $\langle E^2\rangle\sim10^{-6}$ to $10^{-7}\ V^2/m^2$.

Substituting the estimate obtained for W_E into Eq. (20) for ν_{eff} we find that the effective collision frequency for electrons caused by the current instability of the drift oscillations is equal to

$$\nu_{eff}=(10\pi)^{1/2}\omega_{LH}(\varkappa r_{Li})^2\frac{1}{(1+\beta)^{3/2}}\frac{T_i}{T_e\alpha^2}. \quad (43)$$

4. MAGNETIC FIELD RECONNECTION IN THE FRONTAL PART OF THE MAGNETOSPHERE. ESTIMATE OF THE MAGNETO-PAUSE

We have already noted in the Introduction that the reconnection of the magnetic field lines in the frontal part of the magnetosphere can be described in the framework of the Parker-Sweet diffusion model.[8] In this model one considers the reconnection as the result of the mutual diffusion of oppositely oriented magnetic fields at the boundary of the magnetosphere and the solar wind (see Fig. 2). We then get for the width of the transition layer the following formula:

$$d\approx\frac{c}{\omega_{pe}}\left(\frac{L\nu_{eff}}{v_A}\right)^{1/2}. \quad (44)$$

In this formula L is the dimension of the inhomogeneity along the transition layer of the magneto-sphere, i.e., a quantity of the order of 5 to 10 Earth's radii, v_A is the Alfvén velocity.

Basic for the diffusion model is the determination of the effective collision frequency ν_{eff} in the transition layer. It is most obvious to connect the anomalous resistivity in the transition layer with the instability of the short-wavelength drift oscillations considered above. The reasons for this are the following: the current velocity of the electrons in the layer is given by Eq. (2′), i.e., it is above the threshold for the occurrence of the instability, the plasma in the boundary layer is non-isothermal: $T_i\gg T_e$, and, finally, the instability considered is possible also for large values of the parameter β.

The instability is eliminated in the region of small magnetic fields (neutral layer) where the condition that the electrons be magnetized, $\omega\ll\omega_{He}$ is violated.

Since $\omega=$ const in the propagation in an inhomogeneous plasma, we find, substituting ω and $H=H_m\varkappa x$, that the size of the layer where there is no instability is $l_0\approx r_{L*}$ (the Larmor radius is evaluated for the maximum field H_m), i.e., it is appreciably less than the wavelengths in the neutral layer region. The oscillations drift to the region where there is no instability and are damped by interaction with resonance ions. The damping length is

$$l_{\text{damp}}\sim\frac{1}{\text{Im}\,k}\sim\frac{v_{Ti}}{\omega}\sim\frac{v_{Ti}}{\omega_{LH}}\frac{1}{|\varkappa|r_{Li}}\gg r_{L*},$$

i.e., there is no appreciable damping in the neutral layer region. Under those conditions the presence of a neutral layer can in no way appreciably affect the magnitude of the anomalous resistivity.

Substituting in Eq. (44) for the width of the magneto-pause ν_{eff} from (43) and assuming that the characteristic length of the plasma inhomogeneity $|\varkappa|^{-1}$ is of the order of the magneto-pause width d, we get a final formula for d:

$$d\approx\left[\frac{6}{(1+\beta)^{3/2}}\frac{T_i}{T_e\alpha^2}\frac{c}{\omega_{pe}}Lr_{Li}^2\right]^{1/4}. \quad (45)$$

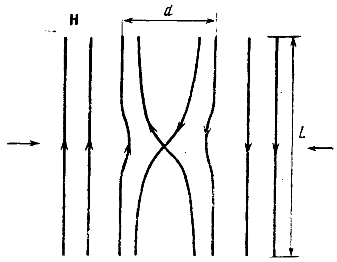

FIG. 2.

In the conditions of the Earth's magnetosphere $d \sim (2$ to $3) \times 10^7$ cm, which agrees with the observational results.

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[1]B. Coppi, G. Laval, and R. Pellat, Phys. Rev. Lett. 16, 1208 (1966).
[2]B. B. Kadomtsev, Fiz. Plazmy 1, 710 (1975) [Sov. J. Plasma Phys. 1, 389 (1975)].
[3]A. A. Galeev and L. M. Zeleny in Theoretical and Computational Plasma Physics, IAEA, Vienna, 1978, p. 93.
[4]A. B. Mikhaĭlovskiĭ and A. V. Timofeev, Zh. Eksp. Teor. Fiz. 44, 919 (1963) [Sov. Phys. JETP 17, 626 (1963)].
[5]I. P. Gudblud, A. I. Pyatak, and V. L. Sizonenko, Zh. Eksp. Teor. Fiz. 64, 2084 (1973) [Sov. Phys. JETP 37, 1051 (1973)].
[6]J. D. Huba, N. T. Gladd, and K. Papadopoulos, Geophys. Res. Lett. 4, 125 (1977).
[7]D. A. Gurnett, R. R. Anderson, B. T. Truruntani, E. J. Smith, G. Paschman, G. Haerendel, S. J. Bame, and C. T. Russell, Preprint Univ. Iowa, 79-7, 1979.
[8]E. N. Parker, J. Geophys. Res. 62, 509 (1957); P. A. Sweet, Proc. IAU Symp. No. 7, 1958, p. 123.
[9]C. T. Russell and R. C. Elphic, Space Sc. Rev. 22, 681 (1978).
[10]A. B. Mikhaĭlovskiĭ, Teoriya plazmennykh neustoĭchivosteĭ (Theory of plasma instabilities) Vol. 2, Atomizdat, 1976 [English translation published by Plenum Press, New York].
[11]V. D. Shafranov, Voprosy Teor. Plazmy 3, 3 (1963) [Rev. Plasma Phys. 3, 1 (1967)].
[12]B. B. Kadomtsev, A. B. Mikhaĭlovskiĭ, and A. V. Timofeev, Zh. Eksp. Teor. Fiz. 47, 2264 (1964) [Sov. Phys. JETP 20, 1517 (1965)].
[13]A. B. Mikhaĭlovskiĭ, Nucl. Fusion 6, 125 (1965).
[14]A. A. Galeev and R. Z. Sagdeev, Voprosy Teor. Plazmy 7, 3 (1973) [Rev. Plasma Phys. 7, 1 (1979)].
[15]V. E. Zakharov, Zh. Eksp. Teor. Fiz. 62, 1745 (1972) [Sov. Phys. JETP 35, 908 (1972)].

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Photon coalescence in a dispersive medium when scattered by impurity centers without a change in its state

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The probability of scattering multimode light by impurity centers, with absorption of an arbitrary number of photons and production of a single photon (photon-coalescence probability) is calculated for an isotropic medium with frequency dispersion but not with spatial dispersion of the dielectric constant in the transparency region. It is shown that the probability depends not only on the total number of centers but also on their distribution in space. The following cases are considered; 1) uniform concentration of the centers, 2) specified coordinate dependence of the concentration, 3) center concentration randomly fluctuating in space. The previously derived equations for the coalescence probability of two or three photons (uniform concentration of the scattering centers) differ from those obtained in the present paper, which makes use of consistent quantization of the field in a dispersive medium [S. I. Pekar, Sov. Phys. JETP 41, 430 (1975)].

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Multiphoton processes in a dispersive medium must be treated by quantizing the electromagnetic field in the medium. This quantization was considered in a number of papers on the basis of crystal microtheory.[1-3] In these papers they used not the complete system of the crystal basis functions, but only its excitonic excitations. The results of these papers are therefore valid only in a narrow spectral region near the exciton resonance. When photons coalesce, however, the frequency of the light wave changes severalfold, and to analyze the coalescence we must be able to quantize the field in a wide spectral region. This is why the results of Refs. 1–3 are not used in the theory of photon coalescence, and in particular in the present paper.

The coalescence of photons (the generation of multiple harmonics) on molecules of the host substance was previously considered a number of times (see, e.g., Refs. 4–8). In these studies the field was quantized in a wide spectral interval, the light waves were considered macroscopically, and the dielectric constant of the crystal $\varepsilon(\omega)$ was introduced phenomenologically. The field quantization, however, was not consistent: the electromagnetic-field energy operator was postulated in the form

$$\sum_k \hbar\omega_k a_k^+ a_k,$$

but the form of the operators a_k^+ and a_k was not derived