Resonant electron-electron bremsstrahlung in the field of an electromagnetic wave

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Bremsstrahlung in electron-electron collisions in the field of a weak electromagnetic wave is considered in the resonant region corresponding to a transition of a virtual electron to the mass shell. The cross section of the process is obtained and its value is estimated relative to ordinary bremsstrahlung in the free case. It is shown that the considered mechanism can ensure amplification of the bremsstrahlung in the resonant region with simultaneous registration of an electron scattered through a large angle.

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1. INTRODUCTION

A characteristic feature of electrodynamic processes of higher order in the fine-structure constant α in external fields is the possibility of their resonant behavior. This is due to the fact that processes of lower order, such as spontaneous emission or single-photon production and annihilation of electron-positron pairs are allowed for a bound electron. Therefore in a certain region of energy and momentum values the particle in the intermediate state can go off to the mass shell, and the considered higher-order process reduces effectively to two successive lower-order processes. This is manifest in the resonant divergence of the cross section of the process calculated by perturbation theory. This divergence is eliminated by introducing radiative corrections to the corresponding Green's functions of the particles in an external field.[1]

The resonant effects connected with the interaction of an electron with plane-wave-field protons of two different frequencies without emission of an additional photon was investigated by Fedorov.[2] Lebedev[3] considered in the nonrelativistic approximation the bremsstrahlung of an electron on a nucleus in the field of an electromagnetic wave, particularly also in the resonance region. The bremsstrahlung produced when a relativistic electron moving in a plane-wave field collides with a nucleus was investigated earlier[4] by the present authors. It was shown that the cross section of the process has a resonant character, but the resonance region was not investigated in detail in Ref. 4.

In the present paper we consider bremsstrahlung in electron-electron collisions at high energies in the field of a weak electromagnetic wave in the resonance region.

2. FACTORIZATION OF THE SQUARE OF THE AMPLITUDE OF THE PROCESS

Assume that the diagram of a certain process can be represented in the form of two parts, A and B, joined by a single line of a virtual electron with four-momentum f (Fig. 1). In the presence of an external field, f is taken to mean the quasimomentum, i.e., a set of four quantum numbers that satisfy[5,6] for a real particle the condition $f^2 = m^{*2}$ (m^* is the effective mass of the particle in the external field).

We express the amplitude of the process in the form[1)]

$$M=\bar{B}_k\frac{(\hat{f}+m^*)_{ki}}{f^2-\mu^2}A_i, \tag{1}$$

where A_i and B_k are the amplitudes of the processes corresponding to the blocks A and B of Fig. 1 (i and k are the spinor indices), $\mu = m^* - i\Gamma$; the imaginary part of the effective part of the electron in an external field $\Gamma = \operatorname{Im}\mu$ is connected with the probability W of emission of a photon by an electron by the relation

$$\Gamma=\frac{f^0}{2m^*}W \tag{2}$$

(f^0 is the particle energy in the intermediate state).

We shall assume henceforth that the external field is a plane-wave field of low intensity. The characteristic parameter of the wave intensity, according to Nikishov and Ritus[5,6] (see also Ref. 7, p. 463 and Ref. 8, p. 326) is

$$\xi^2=-e^2\mathcal{A}^2/m^2, \tag{3}$$

where $\mathcal{A}$ is the amplitude of the four-potential of the wave field. In the case of a weak field, the necessary condition for the possibility of expanding the cross section in terms of the wave intensity, corresponding to the condition of applicability of perturbation theory, is the inequality $\xi^2 \ll 1$. This condition is usually satisfied for the emission of modern lasers.

Let $\xi^2 \ll 1$; then, neglecting the inessential shifts of the real part of the electron mass, we can put $m^* \approx m$ and retain in the denominator of (1) only the imaginary part of μ, putting $\mu = m - i\Gamma$. We calculate now the square of the matrix element (1) $|M|^2$ in the resonant approximation, assuming everywhere except in the denominator that the intermediate electron is real, i.e., $f^2 = m^2$.

We use the expansion

$$\hat{f}+m=\sum_s u^s\bar{u}^s, \tag{4}$$

where $s = \pm 1$ is the helicity of the electron, with $\hat{f}u^s = mu^s$. For electrodynamic processes in which elec-

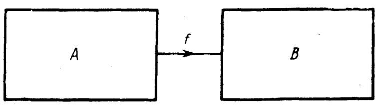

FIG. 1.

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trons and photons take part, the squares of the amplitudes A and B can be set respectively equal to

$$A\bar{A}=a_1I+a_2^\mu\gamma_\mu,\quad B\bar{B}=b_1I+b_2^\mu\gamma_\mu. \tag{5}$$

Here I is the unit matrix and γ^μ are Dirac matrices ($\mu=0,1,2,3$). There are no terms proportional to the matrices $\sigma^{\mu\nu}$, γ^5 and $\gamma^\mu\gamma^5$, since by definition the spinor satisfies the free equation $\hat{f}u^s=mu^s$ without an external field (i.e., $\sigma^{\mu\nu}F_{\mu\nu}=0$), while free interactions are not considered. We emphasize that the role of the external field in our approximation ($\xi^2\ll 1$) reduces to the onset of an imaginary part of the electron mass, which eliminates the resonant divergence of the amplitude (1) on the mass shell $f^2=m^2$.

Taking into account the equations

$$\bar{u}^s(p)u^{s'}(p)=2m\delta^{ss'},\quad \bar{u}^s(p)\gamma^\mu u^s(p)=2p^\mu\delta^{ss'} \tag{6}$$

we obtain for the square of the matrix elements, in which the corresponding averaging and summation over the polarization states is carried out,

$$|M|^2=\frac{8(a_1m+(a_2f))(b_1m+(b_2f))}{|f^2-\mu^2|^2}. \tag{7}$$

Introducing for the processes A and B the squared matrix elements $|M_A|^2$ and $|M_B|^2$, summed and averaged over the spints,

$$|M_A|^2=4(a_1m+(a_2f)),\quad |M_B|^2=2(mb_1+(fb_2)), \tag{8}$$

we write down the square of the amplitude (7) in an explicitly factorized form:

$$|M|^2=|M_A|^2|M_B|^2/|f^2-\mu^2|^2. \tag{9}$$

3. CROSS SECTION OF THE PROCESS IN THE RESONANCE APPROXIMATION

We consider the diagrams of processes with Compton scattering of a photon of a plane-wave field by the initial and final electron, as shown in Figs. 2a and 2b, respectively. Denoting by $M_c(p_1,k)$ the amplitude of the Compton scattering of the photon with momentum k by an initial electron p_1, and by $M_a^0(f,p_2)$ the amplitude of the principal process of collision of the electrons f and p_2 without Compton scattering, we write down in accordance with (9), in the resonance approximation, the square of the amplitude of process a in Fig. 2:

$$|M_a|^2=\frac{|M_c(p_1,k)|^2|M_a^0(f,p_2)|^2}{|f^2-\mu^2|^2}. \tag{10}$$

Taking into account this equation, the probability of the process a becomes factorized, and one of the factors is the probability dw_a^0 of the principal process, taken at

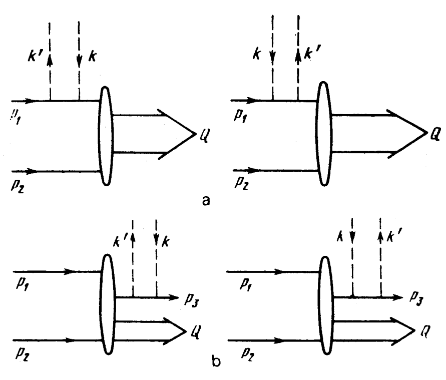

FIG. 2.

the momentum value $f=p_1+k-k'$:

$$dw_a=\frac{|M_c|^2}{|f^2-\mu^2|^2}\frac{f_0}{2\omega\varepsilon_1V}\frac{d^3k'}{(2\pi)^32\omega'}dw_a^0|_{f=p_1+k-k'}. \tag{11}$$

Here $\omega=k^0$, $\varepsilon_1=p_1^0$, $\omega'=k'^0$. We note that the possibility of considering of the processes a and b separately without allowance for their interference is based on the assumption that the colliding electrons have high energies: $\varepsilon_1\gg m$, $\varepsilon_2\gg m$ (and also $f^0\gg m$), that the electron scattering angle is relatively large $\theta=\measuredangle(\mathbf{p}_3,\mathbf{p}_1)\sim 1$, and that the photon emission angle is small, $\theta'\ll 1$, relative to $\mathbf{p}_1$, which is characteristic of the Compton process on relativistic electrons. The conditions of applicability of this approximation will be derived more rigorously below.

In the phase-volume element $d^3k'=\omega'^2d\omega'\sin\theta'd\theta'd\varphi'$ we change from the variables ω' and θ' to the variables

$$u=kk'/kf\approx\omega'/(\varepsilon_1-\omega'),\quad \delta=\theta'\varepsilon_1/m. \tag{12}$$

For the resonant denominator in (11) we obtain in terms of these variables

$$|f^2-\mu^2|^2=\left(\frac{m^2u}{u+1}\right)^2\{(a-\delta^2)^2+b^2\}, \tag{13}$$

where

$$a=\frac{\varkappa}{u}-1+\frac{u+1}{u}\frac{\Gamma^2}{m^2},\quad b=2\frac{u+1}{u}\frac{\Gamma}{m}; \tag{14}$$

the invariant parameter $\varkappa$ is equal to

$$\varkappa=2(kp_1)/m^2. \tag{15}$$

Introducing now the Compton-effect probability

$$dw_c=\frac{1}{8\pi}\frac{|M_c|^2}{4\omega\varepsilon_1V}\frac{du}{(u+1)^2}, \tag{16}$$

where

$$|M_c|^2=32\pi^2e^4\left[2+\frac{u^2}{u+1}-4\frac{u}{\varkappa}\left(1-\frac{u}{\varkappa}\right)\right], \tag{17}$$

we write down, after integrating with respect to the angle φ', Eq. (11) in the form

$$dw_a=\frac{1}{\pi}\frac{d\delta^2}{(a-\delta^2)^2+b^2}\frac{f^0(u+1)}{m^2u}dw_cdw_a^0. \tag{18}$$

We consider the process in the c.m.s. (or opposing beams), when

$$\mathbf{p}_1+\mathbf{p}_2=0,\quad \varepsilon_1=\varepsilon_2=\varepsilon\approx|\mathbf{p}|\gg m.$$

Changing now to cross sections and taking into account the value of the damping constant in the case of the absorption of a single photon,

$$\Gamma=\sigma_c(\varkappa_f)\frac{\varkappa_fm}{4\omega V}, \tag{19}$$

where, $\varkappa_f=2(kf)/m^2=\varkappa/(u+1)$, and $\sigma_c(\varkappa_f)$ is the total cross section of the Compton effect on the intermediate electron f, we obtain ultimately

$$d\sigma_a=\eta(\varkappa,u)\frac{d\sigma_c(\varkappa,u)}{\sigma_c(\varkappa_f)}d\sigma_a^0|_{f=p_1+k-k'}. \tag{20}$$

In this formula, the function

$$\eta(\varkappa,u)=\frac{b}{\pi}\int_0^\infty\frac{d\delta^2}{(a-\delta^2)^2+b^2}=\frac{1}{\pi}\left[\frac{\pi}{2}+\text{arctg}\frac{a}{b}\right] \tag{21}$$

constitutes a smoothed step function, which assumes in regions far from the resonance point, $|\varkappa-u|\gg 2(u+1)\Gamma/m$, and at resonance, $\varkappa=u$, the limiting values

$$\eta(\varkappa, u)=\begin{cases}1, & u<\varkappa \\ 1/2, & u=\varkappa \\ ub/\pi(u-\varkappa), & u>\varkappa\end{cases}. \tag{22}$$

We consider similarly the process described by the diagrams on Fig. 2b in the resonance approximation $(p_3+k'-k)^2=m^2$. In this case the resonant denominator is of the form

$$|f^2-\mu^2|^2=m^4v^2[(a'-\delta'^2)^2+b'^2], \tag{23}$$

where

$$v=\frac{kk'}{kp_3}\approx\frac{\omega'}{f^0-\omega'},\quad \delta'=\theta'\frac{\varepsilon_3}{m},\quad a'=\frac{\varkappa_f}{v}-1-\frac{\Gamma^2}{m^2v},\quad b'=\frac{2\Gamma}{mv},\quad \varkappa_f=\frac{2(kf)}{m^2}. \tag{24}$$

The angle θ' of the emission of the photon k' is reckoned from the direction of the momentum $\mathbf{p}_3$ and is small compared with the electron scattering angle $\theta=\measuredangle(\mathbf{p}_1, \mathbf{p}_3)$. In the c.m.s., $\mathbf{p}_1+\mathbf{p}_2=0$, the cross section of the process in the resonant approximation turns out to be

$$d\sigma_b=\eta(\varkappa_f, v)(v+1)^2\frac{d\sigma_c(\varkappa_f, v)}{\sigma_c(\varkappa_f)}\frac{d^3\sigma_b^0}{d^3f}\bigg|_{f=p_3+k'-k}d^3p_3, \tag{25}$$

where the function $\eta(\varkappa_f, v)$ is determined by formulas (21) and (22) in which we make the substitutions $a\to a'$ and $b\to b'$.

4. CONDITIONS OF APPLICABILITY OF THE METHOD AND DISCUSSION OF THE RESULTS

The applicability of the results is based on the possibility of considering separately the processes described by diagrams a and b of Fig. 2. Obviously, diagram a will predominate if

$$|(p_1+k-k')^2-\mu^2|^2\ll|(p_3+k'-k)^2-\mu^2|^2. \tag{26}$$

Considering large-angle scattering ($\sin^2(\theta/2)\gg\omega/\omega'$, ω/ε) and assuming the frequency ω to be small ($\omega\ll\omega'$, $\omega\ll\varepsilon'$, $\omega\ll\varepsilon-\omega'$), we can rewrite (26) in the form

$$4(p_3k')^2\gg|(p_1+k-k')^2-\mu^2|^2 \tag{27}$$

or, using (13),

$$4\omega'^2\varepsilon_3^2(1-\cos\theta)^2\gg\left(\frac{m^2u}{u+1}\right)^2\{(a-\delta^2)^2+b^2\}. \tag{28}$$

From this it follows, in particular, that far from resonance, i.e., at

$$\left|\frac{\varkappa}{u}-1-\delta^2\right|\gg2\frac{u+1}{u}\frac{\Gamma}{m}, \tag{29}$$

the scattering angle θ should satisfy the condition

$$\sin^2\frac{\theta}{2}\gg\frac{m^2}{4\varepsilon_1\varepsilon_3}\left(1+\delta^2-\frac{\varkappa}{u}\right). \tag{30}$$

At resonance, i.e., at $\varkappa/u=1+\delta^2$, we get in place of (30)

$$\sin^2\frac{\theta}{2}\gg\frac{m^2}{2\varepsilon_3\omega'}\frac{\Gamma}{m}. \tag{31}$$

Diagrams b of Fig. 2 predominate in the case when the inequality inverse to (26) is satisfied, i.e.,

$$4(p_1k')^2\gg|(p_3+k'-k)^2-\mu^2|^2, \tag{32}$$

from which it follows in accordance with (23) that

$$4\omega'^2\varepsilon_1^2(1-\cos\theta)^2\gg m^4v^2[(a'-\delta'^2)^2+b'^2]. \tag{33}$$

Therefore far from resonance at

$$|\varkappa_f/v-1-\delta'^2|\gg2\Gamma/mv \tag{34}$$

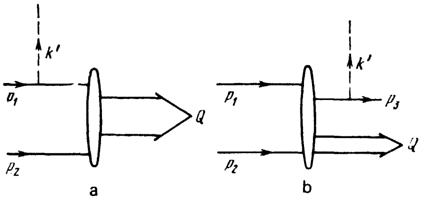

FIG. 3.

we obtain the condition

$$\sin^2\frac{\theta}{2}\gg\frac{m^2}{4\varepsilon_1^2}(v+1)\left(1+\delta'^2-\frac{\varkappa_f}{v}\right), \tag{35}$$

whereas at the resonance $\varkappa_f/v=1+\delta'^2$ we obtain the condition

$$\sin^2\frac{\theta}{2}\gg\frac{m^2}{2\varepsilon_1\omega'}\left(\frac{\Gamma}{m}\right). \tag{36}$$

If the conditions (28) and (33) are not satisfied, for example when the cones of the scattering angles θ and of the emission angles θ' intersect, it is no longer possible to neglect the interference of M_a and M_b, and the factorized formulas for the cross section, (20) and (25), no longer hold.

The process considered by us, which proceeds with absorption of the incident photon ω, is of higher order with respect to the fine-structure constant than the ordinary bremsstrahlung process in the collision of electrons, described by diagrams a and b in Fig. 3. In the same kinematic region of large scattering angles and relativistic matrix of the colliding electrons ($\varepsilon_1, \varepsilon_2, \varepsilon-\omega'\gg m$) the last process was considered by Baier, Fadin, and Khoze,[9] and it was shown by them that the amplitudes of the processes a and b in Fig. 3 have poles in different regions of emission angles, and consequently do not interfere. The cross section here also factorizes[9]:

$$d\sigma_a'=dW_{p_1}(k')d\sigma_a'^0|_{f=p_1-k'},\quad d\sigma_b'=dW_{p_3+k'}(k')d\sigma_b'^0|_{f=p_3+k'}, \tag{37}$$

where $W_p(k')$ is the probability of emission of a photon k' by an electron with momentum p.

We obtain now the ratios of the cross sections (37) of Ref. 9 to the cross sections (20) and (25) obtained by us:

$$\frac{d\sigma_a'}{d\sigma_a}=\frac{\sigma_c(\varkappa_f)dW_{p_1}(k')}{\eta(\varkappa, u)d\sigma_c(\varkappa, u)},\quad \frac{d\sigma_b'}{d\sigma_b}=\frac{\sigma_c(\varkappa_f)dW_f(k')}{\eta(\varkappa_f, v)(1+v)^2d\sigma_c(\varkappa_f, v)}. \tag{38}$$

We assume as an estimate that the parameter $\varkappa$ is small ($\varkappa\ll1$), and then $\omega'\ll\varepsilon$ and the electron energy is high, so that in the logarithmic approximation ($\ln(\varepsilon/m)\gg1$) we have for dW_{p_i}

$$dW_{p_1}(k')=2\frac{\alpha}{\pi}\frac{d\omega'}{\omega'}\ln\left(\frac{\varepsilon}{m}\right)^2, \tag{39}$$

The cross section σ_c, on the other hand, is determined by the Thomson formula[7,8]

$$\sigma_c=\frac{8\pi}{3}\left(\frac{\alpha}{m}\right)^2,\quad d\sigma_c=\frac{2\pi\alpha^2}{(kp_1)}\left[1-2\frac{u}{\varkappa}\left(1-\frac{u}{\varkappa}\right)\right]du. \tag{40}$$

Substituting (39) and (40) in (38) we obtain at resonance

$$\frac{d\sigma_a'}{d\sigma_a}=\frac{8}{3}\frac{\alpha}{\pi}\ln\left(\frac{\varepsilon}{m}\right)^2,\quad \frac{d\sigma_b'}{d\sigma_b}=\frac{8}{3}\frac{\alpha}{\pi}\ln\left(\frac{\varepsilon_f}{m}\right)^2. \tag{41}$$

As to the conditions for the applicability of (31) and (36), the value of Γ in them should be taken to be

$$\Gamma=\xi^2 m^3 \frac{\varkappa_f \sigma_c(\varkappa_f)}{16\pi\alpha}, \tag{42}$$

where ξ^2 from (3) replaces the quantity $4\pi\alpha/(\omega V m^2)$ in Eq. (19), taking into account by the same token the density of the number of photons of the incident-wave field. At $\varkappa \ll 1$, with (40) taken into account, the condition (31) reduces then to the inequality

$$\sin^2(\theta/2) \gg \xi^2 \alpha\omega/\omega'. \tag{43}$$

which is certainly satisfied by virtue of the conditions $\sin^2(\theta/2) \gg \omega/\omega'$ and $\xi^2 \ll 1$. The same inequality (43) is obtained for diagrams b of Fig. 2 if $\varkappa \ll 1$.

We estimate now the ratios (38) in the case of collision of laser photons of energy $\omega = 1$ eV with transverse-beam electrons of energy $\varepsilon = 5$ GeV. In the case of head-on collision of the photons with one of the electron beams, the parameter $\varkappa = 4\varepsilon\omega/m^2 = 0.08 \ll 1$. For elastic scattering $\varepsilon = \varepsilon_f$ and formulas (41) yield the same number 0.1, i.e., ordinary bremsstrahlung amounts to approximately 10% of the resonant process considered by us.

Thus, the irradiation of colliding opposing beams by laser light can increase the cross section of emission of a bremsstrahlung quantum in the resonance region, with simultaneous registration of an electron scattered through a large angle.

[1)]We use a system of units in which $c = \hbar = 1$, and the fine-structure constant $\alpha = e^2 = 1/137$. The employed metric has a signature $+---$, so that $a^\mu b_\mu = ab = a^0 b^0 - \mathbf{ab}$.

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