with the one observed experimentally by Trufanov, Blinov, and Barnik.[4] The threshold characteristics U_c and k_c are close to those calculated above in order of magnitude and in their frequency dependence. The threshold of U_c sharply increases in experiment as $\sigma_{\|}/\sigma_{\perp}\to 1$ and with decreasing $\sigma_{\|}$. According to Ref. 4 the observed domains are oriented along the y axis, but the inclinations of the director and the perturbation of the velocity were registered in the xy plane. The latter circumstance suggests two possibilities: the presence of oblique orientation on the layer boundaries, or nonlinear deviations of the velocity and of the orientation above the threshold U_c.

In conclusion, the authors are deeply grateful to L. M. Blinov for his experimental results prior to publication and for a helpful discussion of the present results.

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Nonlinear effects at the collisionless absorption threshold: plasmon spectrum and damping

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We study the spectrum and the damping of longitudinal plasma waves in a degenerate electron plasma at the collisionless absorption threshold, $\omega = kv_F$. We find the electron distribution function and show that the modulation of the electron velocity by the wave field leads to a dynamic smearing of the threshold, to the vanishing of the electron susceptibility singularities, and to a radical change in the spectrum in the neighborhood of the threshold. If the dynamic smearing of the threshold exceeds the temperature and impurity smearing, the plasmon spectrum is bounded in ω and k with $k_{\max}$ given by (50). In the strongly nonlinear regime when the oscillation period of the trapped particles is larger than the electron collision time one can observe in a one-component degenerate plasma an acoustic plasmon with a phase velocity below v_F. The observed changes in the spectrum in the threshold region when we go from the linear to the nonlinear regime are also typical of other kinds of waves that propagate in a degenerate electron gas and in a Fermi liquid.

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I. INTRODUCTION

In a degenerate solid state plasma one can observe a number of strikingly expressed threshold effects. Firstly, the damping of the quasiparticles which interact with the electrons changes abruptly at the collisionless absorption threshold. Secondly, in the excitation spectrum either there occurs a Kohn-type singularity, or the spectrum rearranges itself more radically, i.e., there appear new excitation branches. A typical example of the excitations which exist near the collisionless absorption threshold are the dopplerons which have recently been observed in many metals (see, e.g., Refs. 1, 2). Thirdly, the collisionless absorption threshold determine the phenomena of the anomalous field penetration into a metal. The effects listed here are connected with the singularities of the real and imaginary parts of the susceptibility of the degenerate electron gas, while the nature of the singularities is determined by the geometry of the Fermi surface.

The spectrum and the damping of a wave in the threshold region can change appreciably when the propagating wave has a large amplitude and changes the trajectories of resonant particles. It is well known that in this case the collisionless damping decreases if the period of the oscillations of the trapped particles ω_0^{-1} is less than the electron collision time, i.e., $a=(\omega_0\tau)^{-1}\ll 1$. Apart from this, the modulation of the velocity of the resonance particles by the wave field must lead to a dynamic smearing of the threshold by an amount of the order of magnitude of the velocity in the oscillations of the trapped particles $\tilde{v}=(\varphi_0/m)^{1/2}$, where φ_0 is the wave amplitude and m the particle mass. The singularity of the real part of the susceptibility at the collisionless absorption threshold must thus be weakened. As a result the wave spectrum in the threshold region can change radically. The wave spectrum and the damping at the threshold change, clearly, only when the dynamic smearing is larger than the smearing due to the temperature and impurities, i.e., if the inequality

$$\tilde{v}\gg k_BT/p_F, \quad \tilde{v}\gg 1/k\tau$$

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is satisfied (k_B is the Boltzmann constant, p_F the Fermi momentum, and k the wave vector of the wave).[1)] The last inequality is equivalent to $a \ll 1$. Notwithstanding the appreciable rearrangement of the spectrum in the threshold region, the shape of the wave may change little. For this it is necessary that there be fewer resonant than non-resonant particles. We show below that the appropriate condition has the form

$$\ln (v_F/\tilde{v}) \gg 1.$$

We study in the present paper the spectrum and damping of large-amplitude plasmons in the degenerate electron gas in the threshold region. It is well known that for waves with a sufficiently long wavelength ($k \ll 2p_F$) the collisionless absorption threshold is determined by the condition

$$\omega = \mathbf{k}\mathbf{v}_F.$$

We find the electron distribution function and show that in the case where the phase velocity of the wave is close to v_F the collisionless damping threshold is smeared over the velocities by an amount of the order of $4\tilde{v}$. Due to the smearing of the threshold the singularity in the real part of the susceptibility is weakened and this leads to a rearrangement of the spectrum. In the sub-threshold region the plasmon spectrum is bounded in frequency and wave vector; the limiting value k_{max} is of the order $k_{FT} \ln^{1/2}(v_F/\tilde{v})$ [see (50)], where k_{FT} is the inverse of the Fermi-Thomas radius. We show also that due to the decrease of the collisionless damping with increasing amplitude it becomes possible to observe an acoustic plasmon in a one-component degenerate plasma-an excitation with a linear spectrum as $k \to 0$ and a phase velocity less than v_F. (One does usually not consider this solution in the linear theory, as the damping is large at threshold.) The damping of the plasmon at phase velocities $w \leqslant v_F - 2\tilde{v}$ is smaller by a factor a than in the linear case, while at $w \geqslant v_F + 2\tilde{v}$ it equals the linear collisional damping. The damping in the threshold region may be either larger or smaller than the linear damping.

Similar changes in the spectrum and in the damping in the threshold region must, clearly, also occur for other kinds of waves on going from the linear to the non-linear regime. The doppleron is apparently the most convenient one for the observation of such effects. The calculation of the distribution function and of the non-linear conductivity at the cyclotron absorption threshold is, however, more complicated than in the case considered by us, even though it does not contain any new aspects in principle.

2. DISTRIBUTION FUNCTION

We find the distribution function f of a degenerate electron plasma in the field of a monochromatic wave. The kinetic equation has the form

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \dot{\mathbf{p}}\frac{\partial f}{\partial \mathbf{p}} + I\{f\} = 0. \qquad (1)$$

Here $\dot{\mathbf{p}} = -\nabla\varphi$, $\varphi(x,t) = -\varphi_0 \cos k(x - wt)$ is the potential of a longitudinal wave of sufficiently large amplitude, and $I\{f\}$ is the collision integral.

We shall look for the solution of Eq. (1) in the form

$$f = f_0(\varepsilon) + F(x, t, \mathbf{p}), \qquad (2)$$

where $f_0(\varepsilon)$ is the equilibrium Fermi function and $F(x, t, \mathbf{p})$ is the correction to the distribution function. At low temperatures when the scattering is mainly by impurities the collision integral is linear F. It will be clear from what follows that the non-linear effects are basically determined by the distribution function of the resonance particles.

To evaluate this function we can write the collision integral in (1) in the relaxation time approximation:

$$I\{F\} = F/\tau, \qquad (3)$$

where τ is the departure time for an electron from the region of the effective interaction. Indeed, in the case of elastic scattering by impurities the collision integral has the form

$$I\{f\} = \int d\mathbf{p}'\, W(\mathbf{p}, \mathbf{p}')\,(f(\mathbf{p}) - f(\mathbf{p}')). \qquad (4)$$

As we can separate in the distribution function the resonance and the smooth part in velocity space, the arrival term will, as in the theory of the anomalous skin effect,[3] be smaller than the departure term which equals $F(\mathbf{p})\int d\mathbf{p}' W(\mathbf{p}, \mathbf{p}')$. The resonance part of the distribution function will thus be determined by the departure term and one can easily show that this is larger by a factor $v_F/\tilde{v}$ than the arrival term.[2)]

The smooth part of the distribution function which describes the non-resonant electrons is practically the same as the distribution function found in the linear approximation and as $\tau \to \infty$ it is independent of the shape of the collision integral. We shall in what follows evaluate the wave spectrum just in that approximation; the damping, however, is determined only by the resonance part of F.

Changing in (1) to dimensionless variables

$$s = (v_x - w)/\tilde{v}, \quad \xi = k(x - wt), \quad a = (\omega_0\tau)^{-1}, \quad \omega_0 = k\tilde{v}, \quad \varphi' = \varphi(x,t)/\varphi_0$$

and taking (3) into account we get

$$s\frac{\partial F}{\partial \xi} - \sin\xi \frac{\partial F}{\partial s} + aF = \frac{\varphi_0 w}{\tilde{v}} \frac{\partial f_0(\varepsilon_\perp, s, \xi)}{\partial \varepsilon_\perp} \sin\xi \left[1 + \frac{\tilde{v}}{w}s\right]. \qquad (5)$$

Here $\varepsilon_\perp$ is the energy of the motion in the plane perpendicular to the direction of k.

As (5) is linear in F it is convenient to look for the solution in the form

$$F = F_1 + F_2, \qquad (6)$$

where F_1 and F_2 satisfy the equations

$$s\frac{\partial F_1}{\partial \xi} - \sin\xi\frac{\partial F_1}{\partial s} + aF_1 = \frac{\varphi_0 w}{\tilde{v}} f_0' \sin\xi, \qquad (7)$$

$$s\frac{\partial F_2}{\partial \xi} - \sin\xi\frac{\partial F_2}{\partial s} + aF_2 = \varphi_0 f_0' s \sin\xi. \qquad (8)$$

We shall show below that the function F_1 in the resonance region is of the order of $\varphi_0 w f_0'/\tilde{v}$ while far from resonance it is of the order of $\varphi_0 w f_0'/v_F$. The function

F_2 is in the whole range of velocities $\sim\varphi_0 f_0'$. In the resonance region the function F_1 is thus for $w/\tilde{v}\gg 1$ much larger than F_2 and it is just this function which determines the non-linear effects. In the non-resonance region the sum F_1+F_2 is the same as the linear distribution function and equal to

$$F_L=\varphi_0 f_0'\left[-\cos\xi+\frac{1}{k\tau}\frac{w\sin\xi}{1/(\tau k)^2+(w-v_x)^2}+\frac{w(w-v_x)\cos\xi}{1/(k\tau)^2+(w-v_x)^2}\right]. \quad (9)$$

We turn to the solution of Eq. (7). In the system of coordinates fixed to the wave the particle trajectories are determined by the potential $\varphi'=-\cos\xi$, and

$$s^2/2-\cos\xi=2/\varkappa^2-1 \quad (10)$$

is an integral of motion. One sees easily that the values $|\varkappa|>1$ correspond to trapped particles and $|\varkappa|<1$ to untrapped ones.

The distribution function of the untrapped particles must satisfy the periodicity condition

$$F_{1ut}(s,\xi,\varepsilon_\perp)=F_{1ut}(s,\xi+2\pi,\varepsilon_\perp),$$

while the trapped particle distribution function must satisfy the condition for specular reflection of the particles from the walls of the well:

$$F_{1t}(s\to+0,\xi_0,\varepsilon_\perp)=F_{1t}(s\to-0,\xi_0,\varepsilon_\perp),\quad \cos\xi_0=2/\varkappa^2-1.$$

The solution of (7) satisfying the given boundary conditions can be found by the method of characteristics:

$$F_1(t,\varkappa,\varepsilon_\perp)=\frac{\varphi_0 w}{\tilde{v}}\int_{-\infty}^{t}e^{a(\tau-t)}\frac{\partial f_0(\varepsilon_\perp,s,\xi)}{\partial\varepsilon_\perp}\sin\xi(\varkappa,\tau)\,d\tau. \quad (11)$$

Here t is the time of the motion along the trajectory

$$t=\int_0^{\xi(\varkappa,t)}d\xi/s(\xi,\varkappa). \quad (12)$$

We shall in what follows by interested in the distribution function integrated over the energy $\varepsilon_\perp$:

$$g_1(t,\varkappa)=\int_0^\infty d\varepsilon_\perp F_1(t,\varkappa,\varepsilon_\perp). \quad (13)$$

As we assumed that the spreading out of the threshold in the field of a strong wave exceeded both the temperature and the impurity spreading out we have for a degenerate electron gas

$$\frac{\partial f_0(\varepsilon_\perp,s,\xi)}{\partial\varepsilon_\perp}=-\delta\left(\varepsilon_\perp+\frac{m(s\tilde{v}+w)^2}{2}-\mu\right) \quad (14)$$

($\mu=mv_F^2/2$ is the chemical potential). Substituting (14) into (11) we further find from (13)[3)]

$$g_1(t,\varkappa)=-\frac{\varphi_0 w}{\tilde{v}}\int_{-\infty}^{t}e^{a(\tau-t)}\theta\left(\mu-\frac{m(s\tilde{v}+w)^2}{2}\right)\sin\xi(\varkappa,\tau)\,d\tau, \quad (15)$$

where

$$\theta(x)=\begin{cases}1, & x>0\\ 0, & x<0\end{cases}.$$

For the evaluation of the integral (15) it is necessary to expand beforehand the function $\sin\xi(\tau,\varkappa)$ in a Fourier series in τ, after which one can evaluate the integral exactly. We give the corresponding calculations in the Appendix.

In the case of most interest where the non-linearity is strong ($a\ll 1$) it follows from Eqs. (A.6) to (A.9) that the trapped particle distribution function has for $|\varkappa|\leqslant 2/A$ $(A=(w-v_F)/\tilde{v}>0)$ the form

$$g_{1t}(t,\varkappa)=-\frac{\varphi_0 w}{\tilde{v}}\times\begin{cases}-(s+s_0)+a(\xi+s_0(t+T/2)), & -T/2\leqslant t\leqslant-\tau_A & (16a)\\ -s_0+A+a(s_0-A)t, & -\tau_A\leqslant t\leqslant\tau_A. & (16b)\\ -(s+s_0)+a(\xi+s_0(t-T/2)), & \tau_A\leqslant t\leqslant T/2 & (16c)\end{cases}$$

Here $s=s(t,\varkappa)$, $\xi=\xi(t,\varkappa)$,

$$s_0=s_0(\varkappa)=(\xi_A+A\tau_A)/2K(\varkappa^{-1}),\quad \xi_A=2\arcsin(\varkappa^{-2}-A^2/4)^{1/2};$$

τ_A is defined by (A.5) and K is the complete elliptical integral of the first kind.

One can similarly show that when $w\leqslant v_F$ (i.e., $A\leqslant 0$) the formulae for $g_{1t}(t,\varkappa)$ have the form (16a to c), but with

$$\tau_A=F(\arcsin(1-A^2\varkappa^2/4)^{1/2},\varkappa^{-1}).$$

Moreover, in the region $|\varkappa|\geqslant 2/A$ $(A>0)$ the function $g_{1t}(t,\varkappa)=0$, and for $|\varkappa|\geqslant-2/A$ $(A<0)$

$$g_{1t}(t,\varkappa)=-\frac{\varphi_0 w}{\tilde{v}}(-s+a\xi). \quad (17)$$

Using the results (A.13) to (A.16) of the calculations of the function of the untrapped particles g_{1ut} we find for $a\ll 1$ and $A>0$:

In the region $\varkappa\geqslant 0$

$$g_{1ut}(t,\varkappa)=0.$$

When $\varkappa<0$ everywhere where $-B\leqslant s(t,\varkappa)\leqslant-A$ which is equivalent to

$$2/B\leqslant|\varkappa|\leqslant(1+A^2/4)^{-1/2}\quad (B=(w+v_F)/\tilde{v}),$$

we have

$$g_{1ut}(t,\varkappa)=-\frac{\varphi_0 w}{\tilde{v}}[-(s-\bar{s})+a(\xi-\bar{\xi})], \quad (18)$$

where $s=s(t,\varkappa)$ and $\xi=\xi(t,\varkappa)$ are the velocity and coordinate of the untrapped particles with "energy" $\varkappa$, $\bar{s}=\pi/\varkappa K(\varkappa)$, $\bar{\xi}=\pi t/\varkappa K(\varkappa)$.

When $\varkappa<0$, in the region where $s(\xi=0,\varkappa)\leqslant-A\leqslant s(\xi=\pi,\varkappa)$, which corresponds to "energies"

$$\left(1+\frac{A^2}{4}\right)^{-1/2}\leqslant|\varkappa|\leqslant\beta(A),\quad \beta(A)=\begin{cases}1, & A\leqslant 2\\ 2/A, & A\geqslant 2\end{cases}$$

we shall have

$$g_{1ut}(t,\varkappa)=-\frac{\varphi_0 w}{\tilde{v}}\times\begin{cases}-s_1+A+a(s_1-A)(t+T/2), & -T/2\leqslant t\leqslant-\tilde{\tau}_A & (19a)\\ -(s+s_1)+a(\xi+s_1 t), & -\tilde{\tau}_A\leqslant t\leqslant\tilde{\tau}_A, & (19b)\\ -s_1+A+a(s_1-A)(t-T/2), & \tilde{\tau}_A\leqslant t\leqslant T/2 & (19c)\end{cases}$$

where $s_1=[-\xi_A+A\tilde{\tau}_A+A\varkappa K(\varkappa)]/\varkappa K(\varkappa)$ and $\tilde{\tau}_A$ is defined by (A.14).

In the region $\varkappa<0$, $|\varkappa|\leqslant 2/B$ we have

$$g_{1ut}(t,\varkappa)=0.$$

It is easy to generalize these formulae to the case $A<0$.

We now consider the function F_2 defined by Eq. (8). The procedure for solving (8) is analogous to the solution of (7) given here. We have then in the non-resonance region when there are no collisions ($\tau \to \infty$) from (8) after integration over $\varepsilon_\perp$

$$g_2 = -\varphi_0 \theta(\mu - {}^1/_2 m(s\tilde{v} + w)^2) \cos\xi. \tag{20}$$

To obtain this solution it is sufficient to neglect the second term on the left-hand side of (8) (which is equivalent to the linear approximation) and also to let a tend to zero. In the resonance region g_2 is of the order $\varphi_0 \psi(s, \xi)$, where $\psi(s, \xi) \sim 1$. We shall not give here the explicit form of the function $\psi(s, \xi)$ as in the resonance region $g_2 \ll g_1$.

We show in Fig. 1 the regions of phase space in the variables (s, ξ) where the function g_1 is determined [see (16) to (19)]. In region 1 inside the separatrices the function g_{1t} is given by Eq. (16b). Similarly in region 2 it is given by (16a) and (16c). The untrapped particles correspond to the regions 3 to 5.

We note that in the linear theory in the collisionless regime the distribution function $g_L(s, \xi)$ has a singularity of the form $1/s$ as $s \to 0$. In the field of a strong wave (as follows from the formulae given above) which appreciably changes the particle trajectories the singularity is smeared out.

3. DAMPING COEFFICIENT AND NON-LINEAR FREQUENCY SHIFT

Using the distribution functions for the trapped and untrapped particles found in the preceding section we can evaluate the reaction of the Fermi gas to a strong monochromatic wave with a phase velocity close to the Fermi velocity. After that we can determine the non-linear frequency shift of that wave and the non-linear damping coefficient. Our calculation will be performed for the case of a longitudinal plasma wave in a one-component degenerate plasma.

We turn to the Poisson equation:

$$\nabla^2 \varphi = 4\pi e^2 \int \delta f \frac{d\mathbf{p}}{4(\pi\hbar)^3}, \tag{21}$$

where $\delta f = f - f_0(\varepsilon)$ is the non-equilibrium correction to the distribution function. We can solve Eq. (21) in the next approximation. We shall assume that all particles are split into two groups. One group of particles (the non-resonant particles for which $|v_x - w| \gtrsim 2\tilde{v}$) move along weakly distorted trajectories. The contribution of these particles to the non-equilibrium concentration is, as in the linear theory, for $w \sim v_F$ proportional to $\varphi_0 \cos k(x - wt) \ln(v_F/\tilde{v})$. This is connected with the fact that the distribution function of the untrapped particles fast approaches the linear distribution function (9) when the difference $|w - v_x|$ becomes larger than $2\tilde{v}$. Particles in the second group (resonant particles) move along strongly distorted trajectories. Their contribution to the nonequilibrium concentration contains, apart from a term proportional to $\varphi(x, t)$, higher harmonics of the potential. The contribution from these and others to the concentration does not contain a large factor such as $\ln(v_F/\tilde{v})$. If the inequality $\ln(v_F/\tilde{v}) \gg 1$ is satisfied the wave is basically formed by the non-resonant particles and one can neglect the higher harmonics on the right-hand side of (21) as they do not lead to a strong change in the potential in the absence of a space-time synchronism.

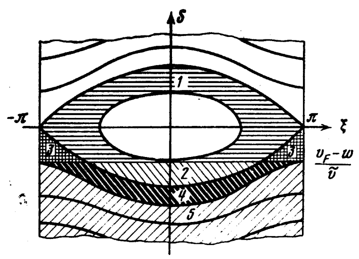

FIG. 1. Phase trajectories of particles in the field of a wave moving with a velocity $w > v_F$.

The contribution from the resonance particles which, as we shall show below, is proportional to $\varphi_0 \cos k(x - wt)$ determines the correction to the linear dispersion law, i.e., the frequency shift. In this approximation, i.e., when the condition $\ln(v_F/\tilde{v}) \gg 1$ is satisfied, the Poisson equation can be solved by the method of slowly varying amplitudes and phases. Performing the appropriate calculations one obtains easily the equation which determines the change in the frequency and the damping coefficient:

$$k^2 = -4\pi e^2 \Pi_N(w, \varphi_0), \tag{22}$$

where

$$\Pi_N(w, \varphi_0) = \frac{p_F m}{\pi^2 \hbar^3} + \frac{m^2 \tilde{v}}{2(\pi\hbar)^3 \varphi_0} \int_{-\pi}^{\pi} e^{i\xi} d\xi \int g(s, \xi) ds, \tag{23}$$
$$g = g_1 + g_2.$$

In the linear theory Eq. (22) is well known to have the form[4]

$$k^2 = -4\pi e^2 \Pi_L(w); \tag{24}$$

$$\Pi_L(w) = \frac{p_F m}{\pi^2 \hbar^3} \left[1 - \frac{w}{2v_F} \ln \left| \frac{w + v_F}{w - v_F} \right| + \frac{i\pi w}{2v_F} \theta(v_F - w) \right]. \tag{25}$$

One finds easily from (22) to (25) the small correction to the frequency of the wave and the non-linear damping coefficient:

$$\delta\omega = [\operatorname{Re} \Pi_L(w_L) - \operatorname{Re} \Pi_N(w_L)] k \Big/ \frac{\partial \operatorname{Re} \Pi_N}{\partial w} \Big|_{w_L}, \tag{26}$$

$$\Gamma_N = \operatorname{Im} \Pi_N(w_L) k \Big/ \frac{\partial \operatorname{Re} \Pi_N}{\partial w} \Big|_{w_L} \frac{\partial \omega}{\partial k}, \tag{27}$$

where

$$w = w_L + \frac{\delta\omega}{k}, \quad \Gamma_N = -\frac{1}{\varphi_0} \frac{d\varphi_0}{dx}.$$

It is necessary to note that our calculation is valid only in the case when the damping is small along a wavelength and along the mean free path of the particles.

We turn to the evaluation of the function $\Pi_N(w, \varphi_0)$. First we find the imaginary part. Separating in (23) the imaginary part and changing from the variables ξ, s to the variables $\xi, \varkappa$ we get

$$\operatorname{Im} \Pi_N(w, \varphi_0) = \frac{m^2 \tilde{v}}{(\pi\hbar)^3 \varphi_0} \int_{-\pi}^{\pi} \sin\xi \, d\xi \int_{-\varkappa(\xi)}^{\varkappa(\xi)} \frac{d\varkappa}{\varkappa^2} \frac{g(\varkappa, \xi)}{[1 - \varkappa^2 \sin^2(\xi/2)]^{1/2}}, \tag{28}$$

where $\varkappa(\xi)=1/|\sin\xi/2|$. When $a\ll 1$ the main contribution to $\mathrm{Im}\Pi_N(w,\varphi_0)$ comes from that part of the function $g_1(t,\varkappa)$ which is linear in a.

It is convenient for the evaluation of the double integral in (28) to change to the variables $t,\varkappa$. For the trapped particles

$$\mathrm{Im}\,\Pi_N{}^t(w,\varphi_0)=\frac{2m^2\tilde v}{(\pi\hbar)^3\varphi_0}\left\{\int\limits_1^\infty\frac{d\varkappa}{\varkappa^3}\int\limits_{\substack{0\le t\le T/4\\3T/4\le t\le T}} g_{1t}(t,\varkappa)\sin\xi(t,\varkappa)\,dt+\int\limits_{-1}^{-\infty}\frac{d\varkappa}{\varkappa^3}\int\limits_{T/4}^{3T/4}g_{1t}(t,\varkappa)\sin\xi(t,\varkappa)\,dt\right\},\quad T=4K(\varkappa^{-1}).\qquad(29)$$

For the untrapped particles

$$\mathrm{Im}\,\Pi_N{}^{ut}(w,\varphi_0)=\frac{2m^2\tilde v}{(\pi\hbar)^3\varphi_0}\int\limits_{-1}^{1}\frac{d\varkappa}{\varkappa^2}\int\limits_{-K(\varkappa)}^{K(\varkappa)}g_{1ut}(t,\varkappa)\sin\xi(u,\varkappa)\,du.\qquad(30)$$

Here $u=t/\varkappa$, $\sin\xi(u,\varkappa)=2\,\mathrm{sn}(u,\varkappa)\,\mathrm{cn}(u,\varkappa)$; sn and cn are the Jacobian elliptic functions. The functions $g_{1t}(\varkappa,t)$ and $g_{1ut}(\varkappa,t)$, linear in $a\ll 1$, are determined as functions of the values of t and $\varkappa$ by Eqs. (16) to (19). One can show, however, that only the functions

$$g_{1t}(t,\varkappa)=-\frac{\varphi_0 w}{\tilde v}a\xi,\qquad g_{1ut}(t,\varkappa)=-\frac{\varphi_0 w}{\tilde v}a(\xi-\bar\xi),$$

give a contribution to $\mathrm{Im}\Pi_N(w)$ where the integration in (29) and (30) must be taken over the $(t,\varkappa)$ region bounded by the condition

$$-B\le s(t,\varkappa)\le -A.\qquad(31)$$

We can put the results of the appropriate calculations for trapped particles in the form

$$\mathrm{Im}\,\Pi_N{}^t(w,\varphi_0)=-\frac{m^2w}{2(\pi\hbar)^3}a\begin{cases}\gamma_t(A), & A\ge 0\\ {}^{128}/_9-\gamma_t(-A), & A\le 0\end{cases},\qquad(32)$$

where

$$\gamma_t(A)=\left(-4A+\frac{2}{3}A^3\right)\arcsin\left(1-\frac{A^2}{4}\right)^{1/2}-\left(\frac{10}{9}A^2-\frac{64}{9}\right)\left(1-\frac{A^2}{4}\right)^{1/2}A\le 2,$$
$$\gamma_t(A)=0,\quad A\ge 2.\qquad(33)$$

For untrapped particles

$$\mathrm{Im}\,\Pi_N{}^{ut}(w,\varphi_0)=\frac{-m^2w}{2(\pi\hbar)^3}a\begin{cases}\gamma_{ut}(A), & A\ge 0\\ C-\gamma_{ut}(-A), & A\le 0\end{cases},\qquad(34)$$

where

$$C=32\int\limits_0^1\frac{dx}{x^4}\left[2E(x)-\frac{\pi^2}{2K(x)}\right],\qquad(35)$$

$$\gamma_{ut}(A)=32\int\limits_0^{\alpha(A)}\frac{dx}{x^4}\left[E(x)-\frac{\pi^2}{4K(x)}\right]+16\int\limits_{\beta(A)}^{\alpha(A)}\frac{dx}{x^3}\left[\frac{A}{2}\xi_A(x)-\frac{2}{x}E\left(\frac{\xi_A}{2},x\right)-\frac{\pi A}{2K(x)}F\left(\frac{\xi_A}{2},x\right)+\frac{\pi\xi_A(x)}{2xK(x)}\right].\qquad(36)$$

Here

$$\xi_A(x)=2\arcsin(\varkappa^{-2}-A^2/4)^{1/2},$$
$$\alpha(A)=(1+A^2/4)^{-1/2},$$
$$\beta(A)=\begin{cases}1, & 0\le A\le 2\\ 2/A, & 2\le A\le\infty\end{cases},\qquad(37)$$

$E(x)$ and $E(\xi_A/2,x)$ are the complete and incomplete elliptic integrals of the second kind.

The evaluation of $\gamma_{ut}(A)$, using Eq. (36) was performed on a computer. As there is no damping ($\mathrm{Im}\Pi_N\sim a$) in the non-linear regime when there are no collisions it makes sense to compare our result with $\mathrm{Im}\Pi_L$ when there are electron collisions present:

$$\mathrm{Im}\,\Pi_L=\frac{p_Fm}{2\pi^2\hbar^3}\left(\frac{1}{2}-\frac{1}{\pi}\,\mathrm{arctg}\,\frac{w-v_F}{1/k\tau}\right).\qquad(38)$$

We show in Fig. 2 the w-dependence of $\mathrm{Im}\Pi_N$ (curves 2, 3) and of $\mathrm{Im}\Pi_L$ (curve 1). Curve 1 is drawn for $(k\tau)^{-1}=0.01v_F$, curve 2 for $\tilde v=0.1v_F$, $a=0.1$, and curve 3 for $\tilde v=0.05v_F$, $a=0.2$. It is clear from the graph that the imaginary parts of the susceptibility differ only for $w\le v_F+2\tilde v$. Outside that region the curves are practically the same, since there are no resonant particles for $w\ge v_F+2\tilde v$. It also follows from the graph that in the non-linear regime the threshold is smeared over a region of the order of $2\tilde v$ (such a smearing out occurs also in the above-threshold region) where the damping can be larger than the linear one. At the threshold $w=v_F$ $(A=0)$

$$\mathrm{Im}\,\Pi_N(w,\varphi_0)=2a\,\mathrm{Im}\,\Pi_L(w=v_F)\qquad(39)$$

beyond the threshold outside the region of dynamic smearing out an analogous relation $\mathrm{Im}\Pi_N/\mathrm{Im}\Pi_L=2a$ is satisfied.[5]

The real part of $\Pi_N(w,\varphi_0)$ is determined from (23):

$$\mathrm{Re}\,\Pi_N(w,\varphi_0)=\frac{m^2\tilde v}{2(\pi\hbar)^3\varphi_0}\int\limits_{-\pi}^{\pi}\cos\xi\,d\xi\int g(s,\xi)\,ds.\qquad(40)$$

In contrast to $\mathrm{Im}\Pi_N$ in $\mathrm{Re}\Pi_N$ there is a contribution from the function $g(s,\xi)$ of zeroth order in a. Using (20) we can write $\mathrm{Re}\Pi_N$ in the form

$$\mathrm{Re}\,\Pi_N=p_Fm/\pi^2\hbar^3+\mathrm{Re}\,\Pi_N{}^1,\qquad(41)$$

where $\mathrm{Re}\Pi_N^1$ is determined solely by the function $g_1(t,\varkappa)$. Furthermore, separating in (16) and (17) the terms of zeroth order in a, substituting them into (40), and then changing to the variables $\varkappa,t$ we get for trapped particles

$$\mathrm{Re}\,\Pi_N{}^{1t}(w,\varphi_0)=\frac{m^2w}{2(\pi\hbar)^3}\begin{cases}R_t(A), & A\ge 0\\ R_t(-A), & A\le 0\end{cases},\qquad(42)$$

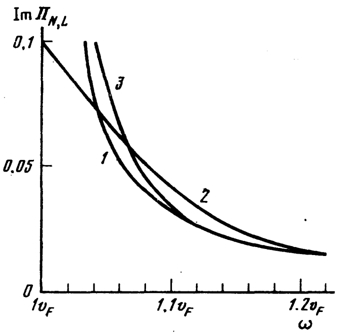

FIG. 2. The function $\mathrm{Im}\Pi_L(w)$—curve 1—drawn for $1/k\tau=10^{-2}\,v_F$. The curve 2 corresponds to $\mathrm{Im}\Pi_N(w,\varphi_0)$ for $a=10^{-1}$, $\tilde v/v_F=10^{-1}$, the curve 3 to $\mathrm{Im}\Pi_N(w,\varphi_0)$ for $a=0.2$, $\tilde v/v_F=5\times10^{-2}$.

where

$$R_t(A)=A\left(\frac{A^2}{2}-3\right)\left(1-\frac{A^2}{4}\right)^{1/2}+2(A^2-3)\arcsin\left(1-\frac{A^2}{4}\right)^{1/2}$$
$$+16\int_{A/2}^{1}\frac{x\,dx}{K(x)}\left\{2E(x)\arcsin\left(x^2-\frac{A^2}{4}\right)^{1/2}-AE(x)\right.$$
$$\left.\times F\left(\arcsin\left(1-\frac{A^2}{4}\right)^{1/2},x\right)+AK(x)E\left(\arcsin\left(1-\frac{A^2}{4x^2}\right)^{1/2},x\right)\right\} \quad (43)$$

[$F(\varphi, k)$ is the incomplete elliptical integral of the first kind].

Equations (42) and (43) are valid for $|A| \leqslant 2$; in the region $|A| \geqslant 2$ the contribution of the trapped particles to $\Pi_N^1(w, \varphi_0)$ vanishes.

Similarly, substituting the zeroth order terms from (18) and (19) into (40) we can find the contribution from the untrapped particles when $|A| \leqslant 2$:

$$\operatorname{Re}\Pi_N^{1ut}(w,\varphi_0)=\frac{m^2w}{2(\pi\hbar)^3}\begin{cases}R_{ut}(A,B), & A\geqslant 0\\ R_{ut}(-A,B), & A\leqslant 0\end{cases}, \quad (44)$$

where

$$R_{ut}(A,B)=-\pi\left(\frac{3}{2}+A^2\right)+\left(3-\frac{A^4}{2}\right)\arcsin\left(1-\frac{A^2}{4}\right)^{1/2}$$
$$+\left(\frac{A^3}{4}+\frac{3A}{2}\right)\left(1-\frac{A^2}{4}\right)^{1/2}+16\int_1^{\alpha(A)}f_1(x,A)\,dx+8\pi\int_{2/B}^{\alpha(A)}f_2(x)\,dx. \quad (45)$$

The functions $f_1(x, A)$ and $f_2(x)$ are defined by the expressions

$$f_1(x,A)=\frac{1}{x^3K(x)}\left[-E(x)\xi_A+AE(x)xF\left(\frac{\xi_A}{2},x\right)\right.$$
$$\left.-AK(x)xE\left(\frac{\xi_A}{2},x\right)\right], \quad (46)$$

$$f_2(x)=\frac{1}{x^3K(x)}[2E(x)-2K(x)+x^2K(x)], \quad (47)$$

[$\xi_A(x)$, $\alpha(A)$ are defined earlier by Eqs. (37)]. When $|A| \geqslant 2$

$$\operatorname{Re}\Pi_N^{1ut}(w,\varphi_0)=\frac{m^2w}{2(\pi\hbar)^3}R_{ut}^1(A,B), \quad A\geqslant 0,$$
$$\operatorname{Re}\Pi_N^{1ut}(w,\varphi_0)=\frac{m^2w}{2(\pi\hbar)^3}\left[16\pi\int_{-2/A}^{1}f_2(x)\,dx+R_{ut}^1(-A,B)\right], \quad A\leqslant 0, \quad (48)$$

where

$$R_{ut}^1(A,B)=-{}^3/_2\pi-A^2\pi+16\int_{2/A}^{\alpha(A)}f_1(x,A)\,dx+8\pi\int_{2/B}^{\alpha(A)}f_2(x)\,dx. \quad (49)$$

We have constructed in Fig. 3 the graphs of the functions

$$\operatorname{Re}\Pi_N(w,\varphi_0)=p_Fm/\pi^2\hbar^3+\operatorname{Re}\Pi_N^{1t}+\operatorname{Re}\Pi_N^{1ut}$$

for different values of $\tilde{v}/v_F$. We give in the same figure the graph of the function $\operatorname{Re}\Pi_L(w)$ which has a logarithmic singularity at the threshold. It is clear from the figure that this singularity disappears for a finite wave amplitude. The function $\operatorname{Re}\Pi_N(w, \varphi_0)$ differs appreciably from $\operatorname{Re}\Pi_L(w)$ in the interval $|w - v_F| \sim 4\tilde{v}$. The maximum of the function $\operatorname{Re}\Pi_N$ is shifted to the region of velocities larger than v_F and it decreases with increasing amplitude. Such changes in the electron susceptibility lead to a considerable rearrangement of the plasmon spectrum near the threshold. We consider

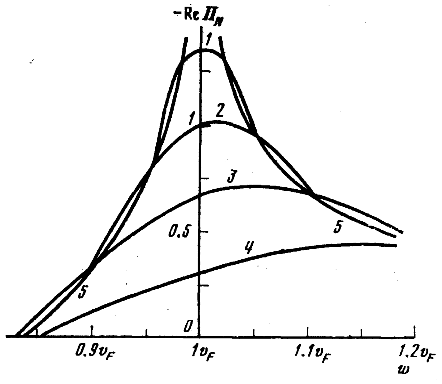

FIG. 3. The function $\operatorname{Re}\Pi_N(w, \varphi_0)$ for different values of $\tilde{v}/v_F$: curve 1 for $\tilde{v}/v_F = 0.025$, curve 2 for $\tilde{v}/v_F = 0.05$, curve 3 for $\tilde{v}/v_F = 0.1$, and curve 4 for $\tilde{v}/v_F = 0.2$. Curve 5 corresponds to $\operatorname{Re}\Pi_L(w)$.

this problem in the next section.

4. DISCUSSION OF THE RESULTS

It follows from the results obtained that due to the modulation of the electron velocity in the field of a strong wave the collisionless absorption threshold is smeared out over a region of velocities of the order of $4\tilde{v}$ near v_F while the logarithmic singularity of the real part of the susceptibility of a degenerate electron gas disappears.

We discuss the changes in the plasmon spectrum. We show in Fig. 4 the spectrum of a large amplitude plasmon in a one-component degenerate plasma. The dispersion curve is constructed by means of the graphs shown in Fig. 3. In practice one must use Fig. 3 to solve graphically the equation

$$k^2=-4\pi e^2\operatorname{Re}\Pi_N(w,\varphi_0)$$

to construct the dispersion curve and then to change from $w(k)$ to $\omega(k)$. As $\operatorname{Re}\Pi_N(w, \varphi_0)$ has no singularities, a plasmon exists in the non-linear regime only when $k \leqslant k_{max}$, where

$$k_{max}^2=k_{FT}^2\ln(v_F/\tilde{v}) \quad (50)$$

[$k_{FT} = (4p_Fme^2/\pi\hbar^3)^{1/2}$ is the universe of the Fermi-Thomas radius]. The plasmon spectrum in the region where there is a solution does not strongly differ from the spectrum of a small amplitude plasmon. In the threshold region the correction to the plasmon frequency $\delta\omega(k)$ is of the order of $k\tilde{v} \ll \omega_p$.

We discuss the acoustic section of the spectrum. In

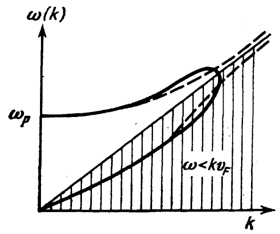

FIG. 4. The spectrum of a large amplitude plasmon in a degenerate electron gas. The dashed curve shows $\operatorname{Re}\omega(k)$ in the linear regime.

the linear theory the acoustic plasmon is strongly damped (Imω ~ Reω). When the inequality $\omega_0\tau \gg 1$ is satisfied the damping is reduced and it may become possible to observe the acoustic plasmon. Assuming that the phase velocity of the acoustic plasmon in the small k region differs appreciably from the Fermi velocity we get from (27)

$$\Gamma_N \approx 0.4ak. \tag{51}$$

The phase velocity of an acoustic plasmon in the small k region depends weakly on the wave amplitude and is approximately equal to $0.83v_F$, if the difference $w - v_F > 2\tilde{v}$. Figure 3 confirms this. One checks easily that the acoustic plasmon spectrum is formed by two groups of electrons on the Fermi surface which move in antiphase so that as $k \to 0$ the acoustic plasmon is not accompanied by oscillations of the electron density. A small group of resonance particles, which do not affect the spectrum, are responsible for the damping.

Because $\delta\omega$ depends on the amplitude φ_0 there may develop in the resonance region of the phase velocities a modulational instability (MI) of the plasma wave which leads to a modulation of the wave amplitude and a splitting of the wave into packets. The problem of the MI of a plasmon in a Maxwellian plasma was considered in Ref. 6. It is well known (see, e.g., Ref. 7) that this instability occurs when the condition $\delta\omega\partial^2\omega_L/\partial k^2 < 0$ is satisfied. One easily determines, using Fig. 4, the sign of $\partial^2\omega_L/\partial k^2$ and $\delta\omega$ in the whole range of resonance velocities and checks that the MI condition is satisfied only in a narrow region near v_F. Estimates made by us show that the MI growth rate is small under realistic experimental conditions, i.e., along the length of a sample the amplitude cannot change appreciably.

We give the values of the electric field strength necessary in order that the inequalities

$$a \ll 1, \quad \ln(v_F/\tilde{v}) \gg 1, \quad \tilde{v} \gg k_B T/p_F$$

are satisfied. For parameters corresponding to bismuth ($m^* = 10^{-29}$ g, $v_F = 10^8$ cm/s, $\tau = 10^{-9}$ s, $\omega_p = 10^{13}\,\mathrm{s}^{-1}$) and $T = 1$ K these inequalities are satisfied, if $E \sim 0.1$ V/cm. We note that the inequality $a \ll 1$ is satisfied already in appreciably weaker fields: $E \sim 10^{-4}$ V/cm.

In conclusion we note that the dynamic smearing of the threshold must affect also the spectrum of acoustic plasmons in a two-component plasma: if the permittivity singularity that forms an acoustic plasmon is strongly smeared, the corresponding solution of the dispersion equation is not present. Similar effects must be observed also for dopplerons in metals in a magnetic field.

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APPENDIX

A. Trapped particles

For the calculation of the distribution function determined by the integral (15) we transform the argument of the θ-function to the form

$$\theta(\mu - m(s\tilde{v}+w)^2/2) = \theta(-A - s(\tau, \varkappa)), \tag{A.1}$$

where $A = (w - v_F)/\tilde{v}$. Equation (A.1) is valid for trapped particles for which $|s(t,\varkappa)| \leq 2$. To fix the ideas we put $A \geq 0$, i.e., the propagation speed of the wave is larger than v_F. We split the integration region in (15) into intervals:

$$\int_{t-T}^{t} + \int_{t-2T}^{t-T} + \ldots,$$

where $T(\varkappa)$ is the period of the oscillations of the trapped particles in the well [$T(\varkappa) = 4K(\varkappa^{-1})$, with $K(\varkappa^{-1})$ the complete elliptical integral of the first kind]. After the obvious change of variables in each of such integrals we can write (15) in the form

$$g_{1t}(t,\varkappa) = -\frac{\varphi_0 w}{\tilde{v}}\frac{1}{e^{aT}-1}\int_t^{t+T}\theta(-A-s(t,\varkappa))e^{a(\tau-t)}\sin\xi(\tau,\varkappa)\,d\tau. \tag{A.2}$$

We can expand $\sin\xi(\tau,\varkappa)$ in a Fourier series in τ using the equation of motion:

$$\sin\xi(\tau,\varkappa) = -ds/d\tau = \sum_{n=1}^{\infty} b_n\omega_n \sin\omega_n\tau, \tag{A.3}$$

$$b_n = \frac{4\pi}{K(\varkappa^{-1})}\frac{q^{n-1/2}(\varkappa^{-1})}{1+q^{2n-1}(\varkappa^{-1})}, \qquad q(x) = \exp\left(-\frac{\pi K(x')}{K(x)}\right),$$
$$\omega_n = \frac{(2n-1)\pi}{2K(\varkappa^{-1})}, \qquad x' = (1-x^2)^{1/2}. \tag{A.4}$$

As $g_1(t,\varkappa) = g_1(t+T,\varkappa)$ we can consider, without loss of generality, t in (A.2) to be in the interval $-T/2 \leq t \leq T/2$. The following cases are then possible (Fig. 5):

a) $2/|\varkappa| = s_{max} \leq A$, i.e., $|\varkappa| \geq 2/A$. Everywhere in the integral in (A.2) we have then $\theta(-A-s) = 0$ and, hence, $g_{1t}(t,\varkappa) = 0$. We note that for $A \geq 2$ this condition holds for all $|\varkappa| \geq 1$;

b) $2/|\varkappa| \geq A$ and $\theta(-A-s) = 1$ for $-T/2 \leq \tau \leq -\tau_A$ and $\tau_A \leq \tau \leq T/2$ (in the period), where

$$\tau_A = 2K(\varkappa^{-1}) - F(\arcsin(1-\varkappa^2A^2/4)^{1/2}, \varkappa^{-1}) \tag{A.5}$$

[$F(\varphi,k)$ is the incomplete elliptic integral of the first kind]. The function $g_{1t}(t,\varkappa)$ will be different in three intervals.

1. Let $-T/2 \leq t \leq -\tau_A$. We then have from (A.2)

$$g_{1t} = -\frac{\varphi_0 w}{\tilde{v}}\frac{1}{e^{aT}-1}\left[\int_t^{-\tau_A} e^{a(\tau-t)}\sin\xi\,d\tau + \int_{\tau_A}^{t+T} e^{a(\tau-t)}\sin\xi\,d\tau\right]$$

Substituting here (A.3) we find the trapped particle distribution function:

$$g_{1t}(t,\varkappa) = -\frac{\varphi_0 w}{\tilde{v}}\left\{\sum_{n=1}^{\infty} b_n\omega_n g_n(t) - \frac{2e^{-at}}{e^{aT}-1}\sum_{n=1}^{\infty}\frac{b_n\omega_n}{a^2+\omega_n^2}\right.$$
$$\left.\times[-a\sin\omega_n\tau_A\,\mathrm{ch}\,a\tau_A + \omega_n\cos\omega_n\tau\,\mathrm{sh}\,a\tau_A]\right\}, \tag{A.6}$$

$$g_n(t) = \frac{a\sin\omega_n t - \omega_n\cos\omega_n t}{a^2+\omega_n^2}. \tag{A.7}$$

We note that the first term in (A.6) is the same as the trapped particle distribution function found under the condition $w \ll v_F$.[4,5]

Similarly we have for $-\tau_A \leq t \leq \tau_A$

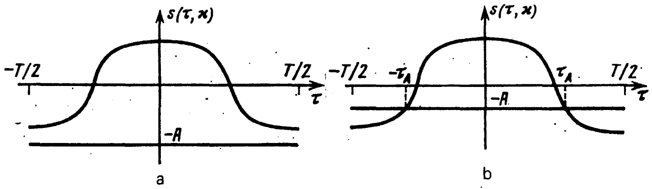

FIG. 5.

$$g_{1t}(t,\varkappa)=-\frac{\varphi_0 w}{\tilde v}\frac{2e^{-at+aT/2}}{e^{aT}-1}\sum_{n=1}^{\infty}\frac{b_n\omega_n}{a^2+\omega_n^2}\Big[-a\sin\omega_n\tau_A\operatorname{ch}\Big(\tau_A-\frac{T}{2}\Big)+\omega_n\cos\omega_n\tau_A\operatorname{sh}\Big(\tau_A-\frac{T}{2}\Big)\Big] \quad (A.8)$$

3. For $\tau_A\leqslant t\leqslant T/2$

$$g_{1t}(t,\varkappa)=-\frac{\varphi_0 w}{\tilde v}\Big\{\sum_{n=1} b_n\omega_n g_n(t)-2e^{a(T-t)}\sum_{n=1}\frac{b_n\omega_n}{a^2+\omega_n^2}\times[-a\sin\omega_n\tau_A\operatorname{ch}a\tau_A+\omega_n\cos\omega_n\tau_A\operatorname{sh}a\tau_A]\Big\}. \quad (A.9)$$

B. Untrapped particles

When $w>v_F$ the untrapped particle distribution function is non-vanishing only when $\varkappa<0$ and when

$$\frac{2}{B}\leqslant|\varkappa|\leqslant 1\Big/\Big(1+\frac{A^2}{4}\Big)^{1/2},\quad A=\frac{w-v_F}{\tilde v},\quad B=\frac{w+v_F}{\tilde v}$$

we have

$$g_{1ut}(t,\varkappa)=-\frac{\varphi_0 w}{\tilde v}\sum_{n=1}\tilde b_n\tilde\omega_n\tilde g_n(t), \quad (A.10)$$

$$\tilde b_n=\frac{4\pi}{\varkappa K(\varkappa)}\frac{q^n(\varkappa)}{1+q^{2n}(\varkappa)},\quad \tilde\omega_n=\frac{\pi n}{\varkappa K(\varkappa)}, \quad (A.11)$$

$$\tilde g_n(t)=\frac{a\sin\tilde\omega_n t-\tilde\omega_n\cos\tilde\omega_n t}{a^2+\tilde\omega_n^2}, \quad (A.12)$$

where the function $g_{1ut}(t,\varkappa)$ has the same form as when $w\ll v_F$.[4,5] In the case

$$1\Big/\Big(1+\frac{A^2}{4}\Big)^{1/2}\leqslant|\varkappa|\leqslant\beta(A),\quad \beta(A)=\begin{cases}1, & A\leqslant 2\\ 2/A, & A\geqslant 2\end{cases}$$

the untrapped particle distribution function has a different form for different t and fixed $\varkappa$. Because of the periodicity it is sufficient to consider $-\tilde T/2\leqslant t\leqslant\tilde T/2$, where $\tilde T=2|\varkappa|K(\varkappa)$.

1. In the interval $-\tilde T/2\leqslant t\leqslant-\tilde\tau_A$ we have

$$g_{1ut}(t,\varkappa)=-\frac{\varphi_0 w}{\tilde v}\frac{2e^{-at}}{1-e^{a\tilde T}}\times\sum_{n=1}^{\infty}\frac{\tilde b_n\tilde\omega_n}{a^2+\tilde\omega_n^2}[-a\sin\tilde\omega_n\tilde\tau_A\operatorname{ch}a\tilde\tau_A+\tilde\omega_n\cos\tilde\omega_n\tilde\tau_A\operatorname{sh}a\tilde\tau_A], \quad (A.13)$$

where

$$\tilde\tau_A=|\varkappa|F(\arcsin(\varkappa^{-2}-A^2/4)^{1/2},\ \varkappa). \quad (A.14)$$

2. When $-\tilde\tau_A\leqslant t\leqslant\tilde\tau_A$

$$g_{1ut}(t,\varkappa)=-\frac{\varphi_0 w}{\tilde v}\Big\{\sum_{n=1}^{\infty}\tilde b_n\tilde\omega_n\tilde g_n(t)+\frac{2\exp(-at+a\tilde T/2)}{\exp(a\tilde T)-1}\times\sum_{n=1}^{\infty}\frac{\tilde b_n\tilde\omega_n}{a^2+\tilde\omega_n^2}\Big[a\sin\tilde\omega_n\tilde\tau_A\operatorname{ch}a(-\tilde\tau_A+\tilde T/2)+\tilde\omega_n\cos\tilde\omega_n\tilde\tau_A\times\operatorname{sh}a\Big(-\tilde\tau_A+\frac{\tilde T}{2}\Big)\Big]\Big\}. \quad (A.15)$$

3. In the interval $\tilde\tau_A\leqslant t\leqslant\tilde T/2$

$$g_{1ut}(t,\varkappa)=-\frac{\varphi_0 w}{\tilde v}\frac{2\exp(-at+a\tilde T)}{1-\exp(a\tilde T)}\times\sum_{n=1}^{\infty}\frac{\tilde b_n\tilde\omega_n}{a^2+\tilde\omega_n^2}[-a\sin\tilde\omega_n\tilde\tau_A\operatorname{ch}a\tilde\tau_A+\tilde\omega_n\cos\tilde\omega_n\tilde\tau_A\operatorname{sh}a\tilde\tau_A]. \quad (A.16)$$

1) In order that a classical discussion be valid it is necessary that the condition $2k\tilde v\gg k^2/2m$ be satisfied.

2) In the theory of the anomalous skin-effect the corresponding small factor is equal to $1/k\tau v_F$.

3) In papers devoted to the non-linear theory of wave absorption[4,5] the trapped and untrapped particle distribution functions are found under the conditions $w\ll v_F$, $\tilde v\ll v_F$. The θ-function in (15) can then in the whole integration domain be replaced by unity.

4) The function $\Pi_L(w)$ determines the dielectric permittivity of the electron gas:

$$\varepsilon_L(\omega,k)=1+\frac{4\pi e^2}{k^2}\Pi_L\Big(\frac{\omega}{k}\Big).$$

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