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New type of high-frequency instability in nematic liquid crystals

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The presence of inertia effects gives rise to the appearance of a new high-frequency electrohydrodynamic instability in nematic liquid crystals. The fundamental difference between this new instability and those previously known is considered theoretically. A qualitatively new behavior of the threshold characteristics of the instability is obtained analytically and numerically as a function of the field frequency, of the anisotropic parameters of the medium, and of the thickness of the liquid-crystal layer.

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INTRODUCTION

It is known that there are two regimes of electrohydrodynamic (EHD) instability of nematic liquid crystals (NLC), namely the low-frequency conduction regime and the high-frequency dielectric regime.[1] In the conduction regime, which is realized at frequencies ω of the external electric field E lower than the reciprocal τ_e^{-1} of the space-charge relaxation time, the orientation of the director n and the flow velocity v of the NLC hardly change with time, and the volume electric charge Q oscillates almost in phase with the electric field $E(t)$. In the dielectric regime at $\omega \gg \tau_e^{-1}$ the situation is reversed: the space charge oscillates weakly about a certain mean value, and the orientation and velocity vary with the frequency of the external field. These regimes differ substantially in the frequency dependence of the threshold characteristics—the voltage $U_c(\omega)$ and the wave numbers of the produced modulated structure $k_c(\omega)$. At $\omega \ll \tau_e^{-1}$ we have $U_c(\omega)=\text{const}$ and $k_c(\omega)\sim \pi d^{-1}$, where d is the thickness of the NLC layer; at $\omega \gg \tau_e^{-1}$ we have $U_c(\omega)\sim d\omega^{1/2}$ and $k_c(\omega)\sim\omega^{1/2}$. We emphasize that in the dielectric regime the instability threshold depends strongly on the thickness d, while the wave number is independent of the latter.

These EHD instability regimes of NLC are described by the system of linear equations of nematodynamics,[1] in which one usually neglects the inertial terms connected with the derivative dv/dt in the Navier-Stokes equations. In fact, this term is small at sufficiently low field frequency ω and layer thickness d.[2] The inertial effects were taken into account by us approximately in an earlier paper.[2] A solution branch was obtained, corresponding to oscillations of the space-charge density, of the flow velocity, and of the director orientation.

In the present paper we describe analytically and numerically a new EHD instability regime, which arises at $\omega \gg \tau_e^{-1}$ as a result of the presence of inertial effects. The fundamental difference between this regime and those mentioned earlier is that in the present case the space charge oscillates almost in counterphase with the electric field, whereas the orientation of the director and the flow velocity vary little about their mean values. A physical consequence of these solutions is a qualitatively new behavior of the threshold characteristics as functions of the field frequency, of the material parameters, and of the thickness of the NLC layer.

ANALYTIC SOLUTION

We write down the complete system of equations of nematodynamics. We introduce a coordinate system (x, y, z) with the x axis directed along the preferred orientation of the molecules in the initial state in the substrate plane, the z axis perpendicular to the substrate, and the y axis along the direction of the domain structure. Assuming that the deviations of the director and the motion of the liquid occur in the xz plane, we obtain in the linear approximation the following system of equations[2]:

$$\begin{gathered}\frac{dv_z}{ds}+\frac{1}{\tau_v}v_z+\varkappa\psi+\delta E(t)Q=0,\\ \frac{d\psi}{ds}+\Gamma\psi+\Omega v_z+\frac{1}{\eta}E(t)Q=0,\\ \frac{dQ}{ds}+\frac{1}{\tau_e}Q+\Sigma E(t)\psi=0,\end{gathered}\tag{1}$$

where $\psi=\partial n_z/\partial x$, Q is the space charge, $q=k_z/k_x$ is the ratio of the wave vectors of the deformation along the z and x axes (here, as in Ref. 2, the boundary conditions are taken into account approximately, and it is assumed

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that $k_z = \pi d^{-1}$);

$$\tau_v = \frac{\rho\omega(1+q^2)}{[\zeta-(\alpha_3 q^2-\alpha_2)^2\gamma_1^{-1}]k_x^2},$$

$$\Gamma = \left[k_x^2(K_{11}q^2+K_{33}) - \frac{\varepsilon_a\varepsilon_\perp}{4\pi\nu}(1+q^2)E^2(t)\right]\gamma_1^{-1}\omega^{-1},$$

$$\tau_e = \omega\left[\frac{4\pi\mu}{\nu}+D_\parallel k_x^2+D_\perp k_z^2\right]^{-1},$$

$$\zeta = \frac{1}{2}(\alpha_4+\alpha_5-\alpha_2)+(\alpha_1+\alpha_3+\alpha_4+\alpha_5)q^2+\frac{1}{2}(\alpha_3+\alpha_4+\alpha_6)q^4, \tag{2}$$

$$\delta = \frac{\varepsilon_a\xi\nu^{-1}-1}{\rho\omega(1+q^2)}, \quad \eta = -\gamma_1\nu\varepsilon_a^{-1}\omega, \quad \varepsilon_a = \varepsilon_\parallel-\varepsilon_\perp,$$

$$\Sigma = (1+q^2)(\sigma_\parallel\varepsilon_\perp-\sigma_\perp\varepsilon_\parallel)\nu^{-1}\omega^{-1}, \quad \nu = \varepsilon_\parallel+\varepsilon_\perp q^2, \quad \mu = \sigma_\parallel+\sigma_\perp q^2,$$

$$\Omega = \frac{\xi k_x^2}{\omega}, \quad \xi = \frac{\alpha_3 q^2-\alpha_2}{\gamma_1}, \quad \varkappa = \frac{(\alpha_2-\alpha_3 q^2)\Gamma}{\rho(1+q^2)}, \quad \gamma_1 = \alpha_3-\alpha_2.$$

Here $\varepsilon_\parallel, \varepsilon_\perp$ and $\sigma_\parallel, \sigma_\perp$ are respectively the dielectric constants and the conductivity, measured in directions parallel and perpendicular to the director, ω is the frequency of the external sinusoidal field $E(t) = E_0 \sin(\omega t)$, $s = \omega t$ is the relative time, K_{ii} are the elasticity coefficients, α_i are the Leslie viscosity coefficients, and ρ is the density of the liquid crystal. The system (1) differs from that of the earlier paper[2] in that an inertial term dv_z/ds connected with the change of velocity with time has been added.

The quantities $\tau_v, \Gamma = \tau_0^{-1}, \tau_e$ in Eqs. (1) can be approximately replaced, without qualitatively changing any essential property of the considered system, by the expressions

$$\frac{1}{\tau_v} \approx \frac{\bar\gamma}{\rho}\frac{(k_x^2+k_z^2)^2}{\omega k_x^2}, \quad \frac{1}{\tau_0} \approx \frac{\bar K}{\omega\bar\gamma}(k_x^2+k_z^2), \quad \frac{1}{\tau_e} \approx \frac{4\pi\bar\sigma}{\omega\bar\varepsilon}, \tag{3}$$

where $\bar\gamma, \bar K, \bar\sigma, \bar\varepsilon$ are the mean values of the material parameters; we can also put $\varepsilon_a = 0$ and use the inequality $\rho\bar K \ll \bar\gamma^2$.

At high frequencies ($1 \gg \tau_e^{-1}, \tau_v^{-1}$), Eqs. (1) have a solution branch characterized by the fact that the function $\psi(t)$ and $v_z(t)$ are almost stationary, and the function $Q(t)$ varies in counterphase with change of the field $E(t)$:

$$\begin{gathered} v_z = v_0 + A_v \sin 2\omega t + B_v \cos 2\omega t + \ldots, \\ \psi = \psi_0 + A_\psi \sin 2\omega t + B_\psi \cos 2\omega t + \ldots, \\ Q = A_1 \sin \omega t + B_1 \cos \omega t + A_3 \sin 3\omega t + B_3 \cos 3\omega t + \ldots, \\ |B_1| \gg |A_1|, \quad |v_z^0| \gg (|B_v|, |A_v|), \\ |\psi_0| \gg (|B_\psi|, |A_\psi|), \quad |B_1| \gg (|B_3|, |A_3|). \end{gathered} \tag{4}$$

It can be shown that the condition for the existence of a solution (4), i.e., the smallest of the next terms of the expansion in the Hamiltonian is the following inequality:

$$\tau_v \gg \tau_e. \tag{5}$$

The coefficients of the expansion (4) are connected with one another by the relations

$$A_1 \approx -\frac{(\sigma_\parallel-\sigma_\perp)E_0\psi_0}{\omega\tau_e}, \quad B_1 \approx \frac{(\sigma_\parallel-\sigma_\perp)E_0}{\omega}\psi_0, \quad \psi_0 \approx -\frac{k_x^2\tau_0 v_0}{\omega},$$

$$A_v \sim \frac{B_v}{\tau_v}, \quad B_v \sim \frac{\tau_e}{\tau_v}v_0, \quad A_\psi \sim \frac{\tau_e}{\tau_0\tau_v}\psi_0,$$

$$B_\psi \sim \max\{A_\psi/\tau_e, \quad A_\psi/\tau_v\} \ll A_\psi,$$

$$A_3 \sim \frac{\tau_e}{\tau_0\tau_v}B_1, \quad B_3 \sim \frac{\tau_e}{\tau_0\tau_v}\max\left\{\frac{B_1}{\tau_v}, A_1\right\} \ll A_3.$$

When the condition (5) is satisfied, the threshold characteristics are obtained from the condition that the amplitudes B_1, v_z^0, ψ_0, not be trivial. This condition states that

$$\frac{(\sigma_\parallel-\sigma_\perp)E_0^2\tau_0\tau_v}{d^2\omega\,\tau_e} \sim 1. \tag{6}$$

Substitution of expressions (3) in (6) and minimization of the function $E_0^2(k_x^2)$ yield the threshold values

$$U_c = E_c d \sim \omega\left[\frac{\pi\bar\varepsilon\bar K}{\bar\sigma(\sigma_\parallel-\sigma_\perp)}\right]^{1/2}, \quad k_c \sim \frac{\pi}{d}. \tag{7}$$

At $\bar K \sim 10^{-6}$, $\bar\varepsilon \sim 10$, $\bar\sigma \sim 10^4$, $\omega \sim 10^5$, $(\sigma_\parallel - \sigma_\perp) \sim \bar\sigma$ cgs esu we obtain the threshold value $U_c \sim 20$ V.

Expressions (7) show that the new regime of the high-frequency EHD instability should have a voltage threshold. The spatial period of the produced structure is comparable with the thickness of the layer and does not depend on the frequency, whereas the threshold voltage is proportional to the frequency. The threshold U_c should decrease with increasing average value of the electric conductivity and with decreasing dielectric constant, and should also increase sharply with decreasing anisotropy of the electric conductivity.

Since this effect exists only at high frequencies, where dielectric relaxation and a corresponding dependence of the electric conductivity can take place, the real $U_c(\omega)$ dependence can deviate noticably from linearity. For example, taking into account the dispersion of the dielectric constant

$$\varepsilon_\parallel(\omega) = \varepsilon_\infty + \frac{\varepsilon_\parallel(\omega=0)-\varepsilon_\infty}{1+\omega^2\tau_D^2},$$

where τ_D is the Debye relaxation time, we obtain the known expression for the electric-conductivity component

$$\sigma_\parallel(\omega) = \sigma_\parallel(\omega=0) + \frac{(\varepsilon_\parallel(\omega=0)-\varepsilon_\infty)\omega^2\tau_D}{1+\omega^2\tau_D^2}.$$

It follows therefore that in the intermediate frequency region

$$\left[\frac{\sigma_\parallel(\omega=0)}{(\varepsilon_\parallel(\omega=0)-\varepsilon_\infty)\tau_D}\right]^{1/2} < \omega < \tau_D^{-1}$$

the product $\bar\sigma(\sigma_\parallel - \sigma_\perp)$ in (7) depends strongly on the frequency ω, increasing with increasing ω, and by the same token weakens strongly the frequency dependence of the threshold voltage. In particular, it is possible in principle that the threshold U_c decreases with increasing frequency ω.

NUMERICAL SOLUTION

For a numerical solution of the system (1) we can propropose the following algorithm, which is a generalization of the method proposed in Ref. 2 for the case of three variables: $v_z = y_1, \psi_2, Q = y_3$. Assuming $\mathbf{Y} = (y_1, y_2, y_3)$, we integrate numerically Eq. (1) from $s = 0$ to $s = 2\pi$ at three initial values of the vector $\mathbf{Y}$:

$$\mathbf{Y}_1^{in} = (\varepsilon,0,0), \quad \mathbf{Y}_2^{in} = (0,\varepsilon,0), \quad \mathbf{Y}_3^{in} = (0,0,\varepsilon),$$

where $\varepsilon = \text{const} \neq 0$ [the vector $\mathbf{Y}_i(s)$ ranges from the valvalue $\mathbf{Y}_i$ at $s = 0$ to the value $\mathbf{Y}_i$ at $s = 2\pi$]. In this case the general solution of the system (1) is

$$\sum_{i=1}^{3} c_i \mathbf{Y}_i$$

and does not depend on the choice of ε. The condition

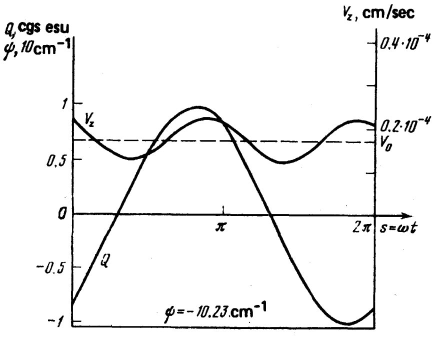

FIG. 1. Changes of the volume charge Q, of the velocity v_z, and of the curvature ψ during one period of action of the voltage $U=dE=U_0\ \sin\omega t,\ 0\leqslant s=\omega t\leqslant 2\pi$. The maximum value of the function $Q(s)$ is set equal to unity. The usual[2] material parameters of MBBA were used for the numerical calculation: $\varepsilon_a=0,\ \varepsilon_\perp=5.25,\ \sigma_\parallel=4\cdot 10^{-8}\ \Omega^{-1}\mathrm{cm}^{-1},\ \sigma_\parallel/\sigma_\perp=1.1,\ \omega=20\cdot 10^3\ \mathrm{sec}^{-1}$. The thickness of the liquid crystal is $d=200\ \mu$m.

that the solution be periodic then reduces to the condition that the constant c_i, at which the solution obtained by us takes on the same values at the start and at the end of the period of variation of $E(t)$, (at $s=0$ and at $s=2\pi$) be nontrivial. This condition can be written in the form

$$\det\{Y_1^{\mathrm{fin}}-Y_1^{\mathrm{in}},\ Y_2^{\mathrm{fin}}-Y_2^{\mathrm{in}},\ Y_3^{\mathrm{fin}}-Y_3^{\mathrm{in}}\}=0. \qquad (8)$$

The threshold value of the electric field intensity E_c and the wave vector of the deformation are determined by the minimum of the $E(k_x)$ dependence:

$$E_c=E(k_c)=\min E(k_x),\quad k_x\geqslant 0.$$

A numerical solution of the problem (1), (2) has confirmed the qualitative theoretical calculation considered above. The changes of the quantities v_z and Q during one period of the external voltage are shown in Fig. 1. Assuming that the amplitude of the charge oscillations to be equal to unity, we find that Q varies in time in accordance with a nearly harmonic law $Q\propto\sin(\omega t+\varphi)$, where φ is the initial phase. The velocity v_z fluctuates about a certain equilibrium position $v_0\sim 0.17\cdot 10^{-4}$ cm/sec, the curvature ψ remains constant at $\psi\sim -10$ (the angle of inclination of the director does not vary with time).

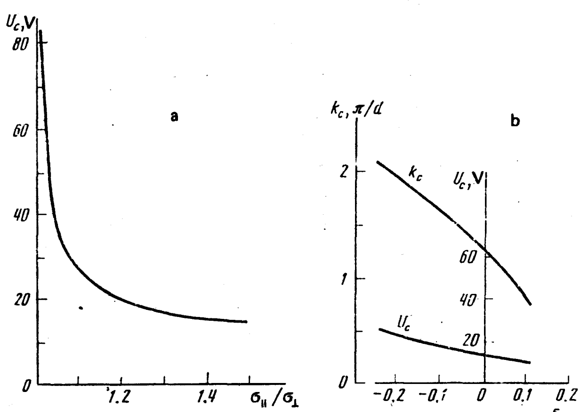

FIG. 2. a) Dependence of the threshold voltage U_c on the anisotropy of the conductivity $\sigma_\parallel/\sigma_\perp$ at $\varepsilon_\parallel=\varepsilon_\perp=5.25$. b) Dependence of the threshold voltage U_c and of the wave number k_c of the instability on the dielectric anisotropy $\varepsilon_a(\varepsilon_\perp=5.25)$ at $\sigma_\parallel/\sigma_\perp=1.5$. The remaining parameters used for the calculation are the same as for Fig. 1.

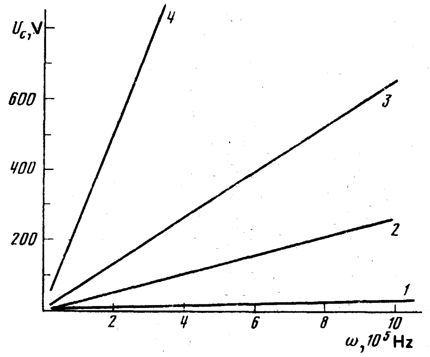

FIG. 3. Frequency dependences of the threshold voltage $U_c(\omega)$ at $\varepsilon_a=0,\ \varepsilon_\perp=5.25$, and $\sigma_\parallel/\sigma_\perp=1.5$. The electric conductivity $\sigma_\parallel$ takes on the following values: 10^{-6} (curve 1), 10^{-7} (2), $4\cdot 10^{-8}$ (3), 10^{-8} (4) $\Omega^{-1}\mathrm{cm}^{-1}$. The remaining parameters used for the calculations are the same as for Fig. 1.

Figure 2 shows the dependences of the threshold voltage on the anisotropy of the electric conductivity and of the dielectric constant. It is seen from Fig. 2a that the mechanism at which the given type of EHD instability sets in is determined essentially by the anisotropy $\sigma_\parallel-\sigma_\perp$, with $U_c\to\infty$ as $\sigma_\parallel/\sigma_\perp\to 1$. The anisotropy ε_a does not play a significant role in the considered phenomenon, although it does influence the wave number of the orientational perturbation k_c (Fig. 2b).

Figure 3 shows the numerical $U_c(\omega)$ dependence, which is linear if no account is taken of the dispersion of the dielectric constant and of the electric conductivity, as would correspond to Eq. (7). As seen from Fig. 3, the slopes of the straight lines depend strongly on $\sigma_\parallel$, and $U_c(\omega)\approx$ const at large values of the electric conductivity $\sigma_\parallel>10^{-6}\ \Omega^{-1}\ \mathrm{cm}^{-1}$. The numerical calculation has also confirmed that the threshold voltage does not depend on the thickness of the NLC layer (within the range of variation of d from 22 to 200 μm).

Thus, the results show that the high-frequency EHD instability of NLC can be characterized in principle by a low value of the threshold U_c and by a spatial period of the modulated structure π/k_c of the order of the layer d, if the electric conductivity of the NLC is high enough. The presence of the dispersions $\varepsilon_\parallel(\omega)$ and $\sigma_\parallel(\omega)$ greatly weakens the frequency dependence of $U_c(\omega)$. At the same time, at such high frequencies the anisotropy of the electric conductivity plays as before a decisive role, thereby radically distinguishing this type of instability of the NLC from all the previously known ones.

We emphasize that in the considered case (planar boundary conditions) the orientation of the director and the flow velocity undergo perturbations in the xz plane, which is perpendicular to the domain direction (the y axis). In the case of oblique orientation on the boundaries of the NLC layer the corresponding perturbations can occur in the xy plane (the components v_y and n_y).[3] Similar perturbations, as a secondary phenomenon, are possible also in the planar situation if the primary perturbations v_x and n_x are large enough.[3]

The described EHD instability has some similarity with the one observed experimentally by Trufanov, Blinov, and Barnik.[4] The threshold characteristics U_c and k_c are close to those calculated above in order of magnitude and in their frequency dependence. The threshold of U_c sharply increases in experiment as $\sigma_{\parallel}/\sigma_{\perp}\to 1$ and with decreasing $\sigma_{\parallel}$. According to Ref. 4 the observed domains are oriented along the y axis, but the inclinations of the director and the perturbation of the velocity were registered in the xy plane. The latter circumstance suggests two possibilities: the presence of oblique orientation on the layer boundaries, or nonlinear deviations of the velocity and of the orientation above the threshold U_c.

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Nonlinear effects at the collisionless absorption threshold: plasmon spectrum and damping

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We study the spectrum and the damping of longitudinal plasma waves in a degenerate electron plasma at the collisionless absorption threshold, $\omega = kv_F$. We find the electron distribution function and show that the modulation of the electron velocity by the wave field leads to a dynamic smearing of the threshold, to the vanishing of the electron susceptibility singularities, and to a radical change in the spectrum in the neighborhood of the threshold. If the dynamic smearing of the threshold exceeds the temperature and impurity smearing, the plasmon spectrum is bounded in ω and k with $k_{\max}$ given by (50). In the strongly nonlinear regime when the oscillation period of the trapped particles is larger than the electron collision time one can observe in a one-component degenerate plasma an acoustic plasmon with a phase velocity below v_F. The observed changes in the spectrum in the threshold region when we go from the linear to the nonlinear regime are also typical of other kinds of waves that propagate in a degenerate electron gas and in a Fermi liquid.

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I. INTRODUCTION

In a degenerate solid state plasma one can observe a number of strikingly expressed threshold effects. Firstly, the damping of the quasiparticles which interact with the electrons changes abruptly at the collisionless absorption threshold. Secondly, in the excitation spectrum either there occurs a Kohn-type singularity, or the spectrum rearranges itself more radically, i.e., there appear new excitation branches. A typical example of the excitations which exist near the collisionless absorption threshold are the dopplerons which have recently been observed in many metals (see, e.g., Refs. 1, 2). Thirdly, the collisionless absorption threshold determine the phenomena of the anomalous field penetration into a metal. The effects listed here are connected with the singularities of the real and imaginary parts of the susceptibility of the degenerate electron gas, while the nature of the singularities is determined by the geometry of the Fermi surface.

The spectrum and the damping of a wave in the threshold region can change appreciably when the propagating wave has a large amplitude and changes the trajectories of resonant particles. It is well known that in this case the collisionless damping decreases if the period of the oscillations of the trapped particles ω_0^{-1} is less than the electron collision time, i.e., $a=(\omega_0\tau)^{-1}\ll 1$. Apart from this, the modulation of the velocity of the resonance particles by the wave field must lead to a dynamic smearing of the threshold by an amount of the order of magnitude of the velocity in the oscillations of the trapped particles $\tilde{v}=(\varphi_0/m)^{1/2}$, where φ_0 is the wave amplitude and m the particle mass. The singularity of the real part of the susceptibility at the collisionless absorption threshold must thus be weakened. As a result the wave spectrum in the threshold region can change radically. The wave spectrum and the damping at the threshold change, clearly, only when the dynamic smearing is larger than the smearing due to the temperature and impurities, i.e., if the inequality

$$\tilde{v}\gg k_BT/p_F,\quad \tilde{v}\gg 1/k\tau$$

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