Configuration mixing in a μ-mesic atom shell and possibility of observing neutral weak interaction between a muon and a nucleus

D. P. Grechukhin and A. A. Soldatov

I. V. Kurchatov Institute of Atomic Energy
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The intensities of the series of the discrete satellite lines $\hbar\omega_k$, which are shifted relative to the $\hbar\omega_0$ of the mesic transition from the mixed $|2s\,1/2\rangle$ states to the $|1s\,1/2\rangle$ orbit (this line carries information concerning the weak neutral interaction) are estimated for mesic atoms in the region $6 \le Z \le 17$. Numerical calculation reveals for a number of selected mesic-atom electron-shell configurations a dependence of the intensities of the strongly shifted and weakly shifted satellite lines on the character of occupation of the electron orbit. Also obtained are the probabilities of the possible competing decay channels for the mesic $|2s\,1/2\rangle$ state, which determine the relative intensity of the $\hbar\omega_0$ line for one act of meson capture into the $|2s\,1/2\rangle$ states. It is shown that capture of electrons on the $2s$ or $2p$ orbit, in the case $6 \le Z \le 11$, opens a channel for the conversion of the meson $E1$ transitions from the $|2s\,1/2\rangle$ state to the $|2p\,1/2\rangle$ and $|2p\,3/2\rangle$ orbits, so that the relative intensity of the $\hbar\omega_0$ line decreases to a level $\sim 10^{-5}$. In the case of heavier mesic atoms, $12 \le Z \le 17$, a channel is opened for the $E1$ conversion of these mesic transitions on the $1s$-shell electrons, and the relative intensity likewise remains at the 10^{-5} level. It is concluded from the analysis of the situation that it is possible to observe effects of weak neutral interaction between the muon and the nucleus in the mesic atoms μNe and μNa, provided experimental conditions are obtained wherein only the K orbit manages to become populated in the electron shell of the mesic atom and the mesic atom remains in this state for a time of the order of 10^{-10} sec. A complete determination of the μNa electron shell does not lead to a high intensity of the weakly shifted satellite lines, which cannot be discriminated from $\hbar\omega_0$, but in this case the relative yield of the $\hbar\omega_0$ quanta turns out to be of the order of 10^{-5}, whereas the observed quantity (circular polarization of $\hbar\omega_0$ quanta) is of the order of 10^{-3} [see G. Feinberg and M. Y. Chen, Phys. Rev. D **10**, 190 (1974)]. A method is indicated for an empirical search for situations wherein the mesic atom has not managed to capture even one electron in the L shell.

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1. INTRODUCTION

A weak neutral interaction of the muon and the nucleus intermixes mesic-atom states with opposite parities, for example $|2s1/2\rangle$ and $|2p1/2\rangle$. If the mesic-atom spin is polarized in the initial state, then the mixing of states of different parity manifests itself in the angular distribution of the emitted quanta $W(\theta) = 1 + \alpha\cos\theta$ relative to the polarization direction. On the other hand, if complete depolarization takes place during the mesic-atom transition cascade, then an effect linear in the weak neutral interaction takes place only in the circular polarization of the quanta emitted in the transition from the mixed state to a lower orbit, for example in the transition $|2s1/2\rangle \to |1s1/2\rangle$. These effects of weak neutral interaction were estimated in a number of studies[1-7] in fact for the variant of mixing of the mesic-atom states $|2s1/2\rangle$ and $|2p1/2\rangle$, and in Ref. 2 the calculation was made for the region $3 \leqslant Z \leqslant 82$, whereas the other studies dealt with light mesic atoms $1 \leqslant Z \leqslant 8$. The manifestation of weak neutral interaction between a muon and a nucleus in nonradiative transitions was investigated recently.[8] For heavy mesic atoms $Z = 55-60$ we have obtained[9] an estimate of the possible magnitude of the parity nonconservation effect following the mixing of "crossing" hyperfine-structure components of the mesic orbits $|3p3/2\rangle$ and $|3d5/2\rangle$ in the case of odd nuclei. We have shown that by virtue of the small penetration of the $|3p\rangle$ and $|3d\rangle$ orbits of the meson into the volume of the nucleus, the effect of the weak neutral interaction is smaller by several orders of magnitude than the effect of the Coulomb polarization mechanism of the transfer of parity nonconservation of nuclear states to mesic-atom states (see Ref. 9). It is natural to seek experimentally a more convenient situation in the mixing of the mesic orbits $|2s1/2\rangle$ and $|2p1/2\rangle$. In this case the coefficient α of the angular distribution and the circular polarization of the quanta of the $|2s1/2\rangle \to |1s1/2\rangle$ transition are determined by the amplitude $\delta_W(2s1/2, 2p1/2)$ of the admixture of the state $|2p1/2\rangle$ to the "initial" $|2s1/2\rangle$ state and by the ratio of the probabilities $W_\mu(E1; 2p \to 1s)$ and $W_\mu(M1; 2s \to 1s)$ of the radiative transitions of the meson to the $|1s1/2\rangle$ orbit, i.e., by the factor

$$\frac{[W_\mu(E1;2p\to1s)\,W_\mu(M1;2s\to1s)]^{1/2}\,\mathrm{Im}\,\delta_W(2s^1/_2,2p^1/_2)}{W_\mu(M1;2s\to1s)+|\delta_W(2s^1/_2,2p^1/_2)|^2 W_\mu(E1;2p\to1s)}. \tag{1}$$

In the case of light mesic atoms, where the effect of the finite dimensions of the nucleus still does not manifest itself in the energy of the $2s \to 1s$ transition of the meson, we have according to Refs. 1 and 2 for the probabilities of the $E1$ and $M1$ transitions

$$W_\mu(E1;2p\to1s)\approx1.29\cdot10^{11}Z^4\ [\mathrm{sec}^{-1}],$$
$$W_\mu(M1;2s\to1s)\approx5.16\cdot10^{-4}Z^{10}\ [\mathrm{sec}^{-1}]. \tag{2}$$

The competing ratio $W_\mu(E1)/W_\mu(M1)$ decreases rapidly with increasing nuclear charge Z, but the amplitude of the admixture $\delta_W(2s1/2, 2p1/2)$ in the interval $3 \leqslant Z \leqslant 82$, as follows from Ref. 2, remains a practically constant quantity of the order of $\sim 10^{-7}$.

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2. The difference $\Delta(2p1/2)=[\varepsilon(2s1/2)-\varepsilon(2p1/2)]$ of the mesic terms, which increases rapidly with Z, is offset by an equally rapid increase of the probability of penetration of the meson into the volume of the nucleus in the case of mixed orbits $|2s1/2\rangle$ and $|2p1/2\rangle$, and it is this which makes the amplitude constant. For this reason it is precisely the light μ-mesic atoms $1\leq Z\leq 8$ which were considered as the most suitable systems for the experimental investigation of the effect of weak neutral interaction between the muon and the nucleus, and the calculations reported in Refs. 1–7 predict a sufficiently large magnitude of the effect.

However, all these calculations are valid only for mesic ions having no electron shell at all and isolated from the action of the external medium during the time of the radiative transition from the mixed state. Under real conditions it is hardly possible to satisfy such stringent requirements, since the stopping of the meson and the subsequent cascade of transitions over the orbit and the population of the $|2s1/2\rangle$ state take place in the presence of atoms of the target medium, which leads to partial or complete restoration of the mesic-atom electron shell that it could lose in the cascade of Auger processes upon population of the "initial" $|2s1/2\rangle$ orbit.

We have previously considered[10] a number of possible effects which are due to the possession of an electron shell by a mesic atom or to the presence of the atoms of the medium, and hinder substantially the observation of the effects of weak neutral interaction of the muon and the nucleus in the case $Z\leq 6$. It seems to us that for mesic atoms with $Z\geq 3$ the most dangerous is the configuration mixing in the electron shell, since it leads to the appearance of intense satellite lines, which provide no information, and have energies $\hbar\omega_k=\hbar\omega_0-\delta\hbar\omega_k$, close to the energy $\hbar\omega_0$ of the informative mesic-atom transition $|2s1/2\rangle\to|1s1/2\rangle$ from the mixed $|2s1/2\rangle$ state. In the case of metallic targets, there is also configuration mixing due to the Coulomb interaction of the meson with the electrons from the conduction band. In this case a continuous spectrum of satellite lines is produced and is immediately adjacent to the $\hbar\omega_0$ line.

All these effects were considered in Ref. 10, but quantitative calculations were made only for the mesic atoms μLi, μBe, μB, and μC. However, the relative role of the mechanism of the configuration mixing changes both with increasing charge of the nucleus and when the initial configuration of the electron shell and of the mesic atom is varied. We have therefore continued to study this mechanism, which hinders the experiment, for heavier mesic atoms with $6\leq Z\leq 17$, so as to ascertain the regions and situations which are most suitable for the performance of experiments aimed at observing effects of weak neutral interaction of the muon and the nucleus. It is necessary to search for regions in which the parity non-conservation effect is yet too small, the satellite lines can either be distinguished from $\hbar\omega_0$, or are not too intense compared with the information-containing line. In the experiment it is also of interest to estimate the yield of the $\hbar\omega_0$ quanta per meson capture in the $|2s1/2\rangle$ state. For this purpose we must determine the probabilities of all the competing decay channels of the $|2s1/2\rangle$ level of the mesic atom. For some radiative transitions, such calculations were performed in Ref. 2, but if the mesic atom has a residual electron shell with the electron $2s$ or $2p$ orbits filled, the decisive lifetime of the mesic atom on the $|2s1/2\rangle$ orbit becomes the conversion $E1$ transition $|2s1/2\rangle\to|2pj_k\rangle$ ($j_k=\frac{1}{2}$ and $\frac{3}{2}$), which determines in fact the relative yield of the informative quantum $\hbar\omega_0$. At $Z\geq 12$ and at a definite population of the shell, the channel of $E1$ conversion of the mesic transition $|2s1/2\rangle\to|2pj_k\rangle$ on the K-shell electrons is opened. This channel becomes the principal one at $Z\geq 15$. The results of the calculation of the probabilities of the conversion $E1$ transitions $|2s1/2\rangle\to|2pj_k\rangle$ of the meson are also given below for the particular case of a neutral atomic configuration of the shell.

3. The quantum detector has a finite resolution $\Delta\hbar\omega$; for example, in Ref. 11, in measurements of the energy of the quanta of the $2p\to 1s$ transitions in light mesic atoms $5\leq Z\leq 17$ the resolution $\Delta\hbar\omega$ amounted to ~0.15 keV. Let the intensity of the satellite lines $\hbar\omega_k$ that fall in the interval $\Delta\hbar\omega$, be $\{\sum_k W(\hbar\omega_k)\}_{\Delta\hbar\omega}$, then the parity nonconservation effects observed in the experiment (the coefficient α or the circular polarization of the quanta) become attenuated compared with the case of a "bare" mesic atom by a factor

$$W(\hbar\omega_0)\left[W(\hbar\omega_0)+\left\{\sum_k W(\hbar\omega_k)\right\}_{\Delta\hbar\omega}\right]^{-1}. \tag{3}$$

Our purpose is to find the mesic-atom region where the factor (3) is not too small. For this purpose we must calculate the intensities of the possible mesic-atom satellite lines due to configuration mixing in the electron shell as a result of the dipole–dipole Coulomb interaction of the meson and electrons:

$$\hat{H}_{\mu e}(\mathbf{r}_i\mathbf{r}_\mu)=e^2\frac{4\pi}{3}\sum_{i=1}^{Ne}\sum_m\frac{r_\mu}{r_i^2}Y_{1m}^*(\mathbf{r}_i)Y_{1m}(\mathbf{r}_\mu); \tag{4}$$

here $\mathbf{r}_\mu$ is the radius vector of the muon, and $\mathbf{r}_i$ is the radius vector of the i-th electron of the shell. Only the terms $r_\mu\leq r$ are retained in (4), since the region $r_\mu>r_i$ can be neglected.

The procedure of calculating the intensities of the mesic-atom satellite lines was calculated in considerable detail in our preceding paper,[10] to which we refer the reader; we present below only the quantitative results of the calculation for the mesic-atom region $6\leq Z\leq 17$.

We use the notation of Ref. 10. The complete system (meson + electron shell) is described in the basis of the states

$$|\{k\}J_k;j_k;Ff\rangle=\sum_{\mu_k\nu_k}(j_kJ_k\mu_k\nu_k|Ff)\,|n_kl_kj_k\mu_k\rangle\,|\{k\}J_k\nu_k\rangle \tag{5}$$

with fixed total angular momentum $\mathbf{F}=\mathbf{J}_k+\mathbf{j}_k$ (f-projection), where j_k is the angular momentum of the meson on the orbit $|n_kl_kj_k\mu_k\rangle$, J_k is the angular momentum of the electron shell in the state $|\{k\}J_k\nu_k\rangle$, belonging to the configuration $\{k\}$, which is specified, for example, by the population numbers of the electron orbits $N_kL_kI_k$ in the mean-atomic-field scheme; the initial configuration of the shell is designated $\{0\}J_0$.

2. SOME ASPECTS OF THE NUMERICAL CALCULATION OF THE SATELLITE LINE INTENSITIES

1. In our preceding paper[10] we obtained Eqs. (23) and (26) for the probability of emission of the quantum $\hbar\omega_0$ of the informative line $W(\hbar\omega_0; F_1 \to F_2\{0\}J_0)$ and of the quanta $\hbar\omega_k$ of the satellite lines $W(\hbar\omega_k; F_1 \to F_2\{k\}J_k)$ in mesic-atom transitions from the superposition $\Psi_{F_1 f_1}$ over the basic component $|\{0\}J_0; (2s1/2); F_1 f_1\rangle$ with the meson on the $|2s1/2\rangle$ orbit. Since the splitting of the terms F_2 on the configuration of the system is much less than the resolution $\Delta\hbar\omega$, it is natural to sum the probabilities over the permissible final values of the angular momentum F_2. We do not know the starting configurations in which the initially stripped electron shell of the mesic atom manages to be restored during the lifetime of the meson on the $|2s1/2\rangle$ orbit. We have therefore considered a number of conceivable possible configurations $\{0\}J_0$ of the shells of light mesic atoms, so as to trace the variation of the effect of the configuration mixing with variation of the population of the electron orbit.

The actual calculations were performed for sequences of ions and atoms with configurations $\{0\}$:

$$\begin{array}{l} 6\leqslant Z\leqslant 11, \text{ ions of the series } \{(1s)^2(2s)^2\}; \\ 6\leqslant Z\leqslant 11, \text{ neutral atoms } \{(1s)^2(2s)^2(2p)^n\}; \\ 12\leqslant Z\leqslant 19, \text{ ions of the series } \{(1s)^2(2s)^2(2p)^6\}; \\ 12\leqslant Z\leqslant 19, \text{ neutral atoms with configuration} \\ \{(1s)^2(2s)^2(2p)^6(2s)^n(3p)^m\}. \end{array} \tag{6}$$

In these initial configurations, the dipole–dipole interaction (4) excites several different series of satellite lines due to $E1$ transitions of the electron from the orbit belonging to the configuration $\{0\}J_0$, with principal quantum number N_0, orbital angular momentum L_0, and total angular momentum $I_0 (I_0 = L \pm \frac{1}{2})$, to the orbit $N_k L_k I_k$ in the shell configuration $\{k\}J_k$. Since both the spin-orbit splitting of the electron orbits and the splitting between the states of fixed shell configuration are much less than the resolution of the quantum detector $\Delta\hbar\omega$ (~100 eV), it is natural to combine the satellite lines belonging to one electron $E1$ transition $N_k L_k \to N_0 L_0$, but having different values of I_0, I_k, J_k, into a single satellite line with the summary intensity. To be able to disregard at the present stage the details of the problem of population of the states $(2s1/2); F_1 f_1\rangle$ of the initial configuration, we confine ourselves to a variant that is the simplest [but widely encountered in the selected configurations (6)] of the transitions $N_k L_k \to N_0 L_0$ in the shells of mesic atoms with even numbers of electrons populating completely the series of orbits $N_0 L_0 I_0$ of the initial configuration. In this variant, the angular momentum of the shell J_0 is equal to zero, therefore $F_1 = j_0 = \frac{1}{2}$. Accordingly we obtain for the probability of emission of the quantum $\hbar\omega_0$ of the informative line

$$W(\hbar\omega_0) = W_\mu(M1; 2s\to 1s) + |\delta_W(2s^1/_2, 2p^1/_2)|^2 W_\mu(E1; 2p\to 1s), \tag{7}$$

where $W_\mu(M1)$ and $W_\mu(E1)$ are defined by Eq. (2). For the intensity of the combined satellite line $\hbar\omega_k$, referred to the electronic transition $N_k L_k \to N_0 L_0$, we obtain by carrying out the summation in Eq. (26) of Ref. 10 over the allowed (at fixed L_k and L_0) numbers of the angular momenta F_2, I_0, I_k, and J_k,

$$W(\hbar\omega_k) = |a(N_k L_k \to N_0 L_0)|^2 W_\mu(E1; 2p\to 1s). \tag{8}$$

We have introduced here the summary probability of the configuration admixture in accordance with the definition

$$|a(N_k L_k \to N_0 L_0)|^2 = \sum_{F_2}\sum_{J_k}\sum_{I_k}\sum_{I_0}\Big|\sum_{(2pj_k)} a(\{k\}J_k; 2pj_k; {}^1/_2)\, u({}^1/_2 J_k 1 {}^1/_2; j_k F_2)\Big|^2, \tag{9}$$

where $u(abcd;ef)$ is a Racah function, tabulated by Jahn in Ref. 12; $(2pj_k)$ stands for $(2p1/2)$ and $2p3/2)$. The amplitude $a\{k\}J_k; 2pj_k; F_1)$ is defined by Eq. (7) of Ref. 10, and is taken here at a value $F_1 = \frac{1}{2}$, since we have assumed $J_0 = 0$. In the case of complete population of the electron orbits NLI, we obtain an estimate of the intensity of the combined satellite lines by introducing the population factors of these orbits, but rough tentative values can be obtained also by graphic interpolation from results for neighboring atoms.

2. In the actual calculation of $W(\hbar\omega_k)$ for the chosen configurations (6) we obtain within the framework of the relativistic Hartree–Fock–Slater (HFS) approach the mean field of the atom (at an effective nuclear charge $Z_{eff} = Z - 1$), and obtain next the wave functions of the electron orbits NLI and the energies $E(NLI)$ of these orbits. Accordingly, in the amplitude of the admixture $a(\{k\}J_k; 2pj_k; F_1)$ [see Eq. (7) of Ref. 10] the difference between the energy terms of the entire electron shell $E(\{0\}J_0) - E(\{k\}J_k)$ is replaced by the difference of the terms of the electron orbits $E(N_0 L_0 I_0) - E(N_k L_k I_k)$. The matrix element of the operator $H_{int}(\mathbf{r}_\mu \cdot \mathbf{r}_i)$ is expressed in terms of the radial single-electron element

$$\langle N_k L_k I_k | x^{-2} | N_0 L_0 I_0\rangle$$

$(x = r/a_0, a_0 = 5.29 \cdot 10^{-9}$ cm), and the factor

$$\xi_{F_1}(\{k\}J_k L_k I_k j_k;\ \{0\}J_0 L_0 I_0 j_0)$$

is exactly separated (see Eq. (15) of Ref. 10); this factor is connected with the concrete structure of the states of the electron shell $|\{0\}J_0\rangle$ and $|\{k\}J_k\rangle$. As a result we have for the amplitude of the configuration admixture

$$a(\{k\}J_k;\ 2pj_k;\ F_1) = \beta(2pj_k;\ N_0L_0;\ N_kL_k)\,\xi_{F_1}(\{k\}J_k L_k I_k j_k;\ \{0\}J_0 L_0 I_0 j_0), \tag{10}$$

where we have separated a quantity that determines mainly the scale of the configuration admixture:

$$\beta(2pj_k; N_0L_0; N_kL_k) \equiv \beta(2pj_k) = -\frac{e^2}{a_0} 3\sqrt{3}\frac{m}{m_\mu Z} \cdot \frac{\langle N_k L_k I_k | x^{-2} | N_0 L_0 I_0\rangle}{[\varepsilon(2s^1/_2) - \varepsilon(2pj_k)] - [E(N_k L_k I_k) - E(N_0 L_0 I_0)]}; \tag{11}$$

m_μ is the mass of the meson ($m_\mu \approx 206.8\ m$) and $a_0 \approx 5.29 \cdot 10^{-9}$ cm).

We emphasize that for mesic atoms of the region $6 \leqslant Z \leqslant 19$ the quantities $\beta(2pj_k)$ are practically independent of the total angular momentum I_0 and I_k, but are determined by the numbers $N_0 L_0$ and $N_k L_k$ of the orbits. The radial electronic element depends little on the spin–orbit coupling of the electron, with the exception of the cases when it is close to zero, i.e., the sign of the element is reversed on going to larger Z or when the configuration of the shell changes. The electron-term spin–orbit splitting itself does not influence as a rule the value of $\beta(2pj_k)$, with the exception of the possible case of "passage" of the denominator of (11) through zero. But in this case it is necessary to use a more accurate description of the states and of the energy levels of the electron shell, and forgo perturbation theory, since it is necessary to consider a strongly mixed meson-electron state. In our calculations for the region $6 \leqslant Z \leqslant 19$ we did not observe this extreme situation, but the accuracy with which the terms were determined within the framework of the HFS method is low, so that such a situation cannot be regarded as completely excluded for some configurations of the mesic atoms μMg and μAl, where "crossing" takes place of the differences of the electronic terms $E(Np) - E(1s)$ with the mesic-atom differences $\Delta(2pj_k) \equiv \varepsilon(2s1/2) - \varepsilon(2pj_k)$ for the $|2p1/2\rangle$ and $|2p3/2\rangle$ orbit (see Fig. 1). In addition, such a situation is not completely excluded also for light mesic atoms μB, μC and μN, where "crossing" takes place of the differences $\Delta(2pj_k)$ and $E(Np) - E(2s)$.

3. The differences of the mesic-atom terms $\Delta(2p^{1/2})$ and $\Delta(2p^{3/2})$ are determined by the effect of the finite dimensions of the nucleus and by the effect of polarization of the vacuum. The quantities $\Delta(2pj_k)$ which are used by us below were obtained by graphic interpolation from the values of $\epsilon(2s^{1/2}) - \epsilon(2p^{1/2})$ calculated in Ref. 2 for the mesic atoms μC, μO, μNa and μCl. We have used here the spin–orbit splitting of the meson terms in the field of the pointlike nucleus.

$$\varepsilon(2p^{3}/_{2}) - \varepsilon(2p^{1}/_{2}) \approx \frac{1}{32} m_\mu c^2 \left(Z \frac{e^2}{\hbar c} \right)^4 \approx 0.94 \cdot Z^4 \text{ eV}. \tag{12}$$

The quantities $\Delta(2p^{1/2})$ and $\Delta(2p^{3/2})$ selected in this manner are listed in Table 1, which gives also the values of the binding energy of the $1s$ electron and the minimum quantum energies $\hbar\omega_{\min}$ for the $Np \to 1s$ electron transition, obtained within the framework of the HFS method in the mean field generated by the configurations of the ions $\{(1s)^2(2s)^2\}$, $\{(1s)^2(2s)^2(2p)^6\}$ and of the neutral atoms. For the sake of argument we present here the values $\hbar\omega_\mu$ of the quantum energy of the mesic $(2p \to 1s)$ transition, which we obtained experimentally in Ref. 11.

Of course, graphic interpolation yields the values of $\Delta(2pj_k)$ with considerable errors, so that the picture shown in Fig. 2, of the "crossing" of the mesic and electronic differences that determine the denominator $\beta(2pj_k)$ can be substantially different, i.e., a closer "crossing" of the differences is also possible. However, even in this rough estimate the mesic atoms μMg and μAl differ in having a steeper growth of the intensity of the satellite lines of the series $Np \to 1s$; it is most likely that these mesic atoms are not suitable for experimentation.

4. For both the summary satellite lines of the $N_k L_k \to N_0 L_0$ transitions and of the more specialized transitions $N_k L_k I_k \to N_0 L_0 I_0$ or $N_k L_k \to N_0 L_0 I_0$ and $N_k L_k I_k \to N_0 L_0$, where the summation is carried out only over the left-out numbers I_0 or I_k, the amplitude of the configuration admixture can be represented in the form

$$|a(N_k L_k I_k \to N_0 L_0 I_0)|^2 = A(L_k I_k \to L_0 I_0)\{|\beta(2p^{1}/_{2})|^2 + 2|\beta(2p^{3}/_{2})|^2\}. \tag{13}$$

In the case of an initial configuration $\{0\}$ with angular momentum $J_0 = 0$ we have for $E1$ transitions of an electron from fully occupied orbits $N_0 L_0 I_0$ to completely empty orbits $N_k L_k I_k$ the following system of the values of the coefficients A:

$$\begin{gathered} A(p^{1}/_{2} \to s^{1}/_{2}) = {}^{2}/_{27},\ A(s^{1}/_{2} \to p^{1}/_{2}) = {}^{2}/_{27},\ A(d \to p^{1}/_{2}) = {}^{4}/_{27}, \\ A(p^{3}/_{2} \to s^{1}/_{2}) = {}^{4}/_{27},\ A(s^{1}/_{2} \to p^{3}/_{2}) = {}^{4}/_{27},\ A(d \to p^{3}/_{2}) = {}^{8}/_{27}; \\ A(p \to s) = {}^{2}/_{9},\ A(s \to p) = {}^{2}/_{9},\ A(d \to p) = {}^{4}/_{9}. \end{gathered} \tag{14}$$

Equations (2), (8), (11), (13), and (14) solve practical-

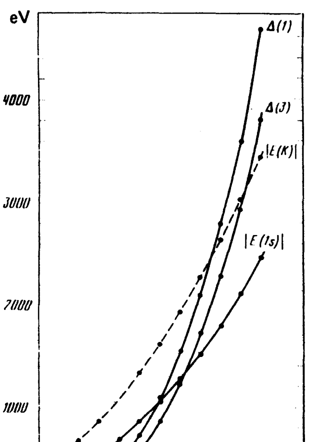

FIG. 1. "Crossing" of the differences of the mesic-atom terms $\Delta(2p1/2) \equiv \Delta(1)$ and $\Delta(2p3/2) \equiv \Delta(3)$ and of the binding energies of the K electron for two extreme configurations of the mesic-atom shell: $|E(1s)|$ in the case of the ordinary configuration of the neutral atom and $|E(K)|$ in the case of a hydrogen-like ion with only one electron in the K shell.

TABLE I.

Z	$\hbar\omega_\mu$ $(2p \to 1s)$, keV [11]	$\Delta(2p^{1/2})$, eV	$\Delta(2p^{3/2})$, eV	Neutral-atom configuration: $\|E(1s)\|$, eV	Neutral-atom configuration: $\hbar\omega_{\min}$, eV	Configuration of ions: $\|E(1s)\|$, eV	Configuration of ions: $\hbar\omega_{\min}$, eV
						$\{(1s)^2(2s)^2\}$	
6	75.25±0.15	33	21	196	189	210	192
7	102.29±0.15	90	67	291	282	329	292
8	133.56±0.15	162	124	405	393	475	411
9	168.45±0.15	280	218	537	523	649	551
10	–	460	366	688	671	850	712
11	250.21±0.15	727	588	858	856	1079	892
						$\{(1s)^2(2s)^2(2p)^6\}$	
11	250.21±0.15	727	588	858	856	858	856
12	296.55±0.15	1100	905	1064	1061	1074	1064
13	346.82±0.15	1500	1230	1295	1291	1317	1296
14	400.22±0.15	2100	1740	1550	1545	1588	1554
15	456.54±0.15	2800	2320	1828	1822	1887	1836
16	516.24±0.25	3600	2980	2130	2122	2214	2142
17	578.56±0.30	4640	3860	2456	2445	2569	2473

Note. The energies $\hbar\omega_\mu$ $(2p \to 1s)$ are given in accordance with Ref. 11, the differences $\Delta(2pj_k) = \varepsilon(2s1/2) - \varepsilon(2pj_k)$ between the terms of the mesic atoms were obtained by interpolation of the values μC, μO, μNa, and μCl from Ref. 2; the binding energies of the $1s$ electron $E(1s)$ and the frequencies of the lower transitions $Np \to 1s$ $(N = 2$ or $3)$ for the configurations of the ions and of the neutral atom were obtained by the Hartree-Fock-Slater method[13] for an atom with an effective nuclear charge $Z_{\text{eff}} = Z - 1$.

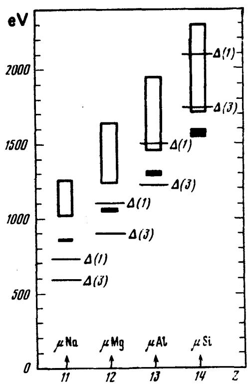

FIG. 2. Details of the picture of the "crossing" of the differences of the mesic-atom $\Delta(2p1/2)$ [index $\Delta(1)$] and $\Delta(2p3/2)$ [index $\Delta(3)$] and of the electronic $|E(Np)-E(1s)|$ terms. The intervals in which the electronic differences of the terms fall in the case of ion configuration $\{(1s)^2(2s)^2(2p)^6\}$ and the neutral atomic configuration are shown black. The light rectangles contain the intervals of $|E(Np)-E(1s)|$ for a hydrogenlike ion with one electron.

ly completely our problem of estimating the relative intensities of the satellite lines.

3. RESULTS OF NUMERICAL CALCULATION

1. For the mesic atoms with $6 \leqslant Z \leqslant 19$, the matrix elements H_{int} and the electronic terms of the orbits NLl up to $L=5$ were calculated with the electron functions in the unified mean field of the atom ($Z_{eff}=Z-1$), which within the framework of the HFS method is specified by the initial configuration $\{0\}$. The mean field was calculated using the programs of the RAINE complex.[13] The contribution of the discrete states with $N_k \geqslant 6$ at fixed N_0 ($N_0=1,2,3$) was estimated from the asymptotic behavior of the radial element of the dipole interaction

$$|\langle N_kL_kl_k|x^{-2}|N_0L_0l_0\rangle|^2 \infty N_k^{-3}. \tag{15}$$

As a result we get for the summary contribution of the transitions $N_kL_kl_k \to N_0L_0l_0$ at $N_k \geqslant 6$ to the intensity of a satellite line of this series

$$\sum_{N_k=6}^{\infty} W(N_kL_kl_k \to N_0L_0l_0)=2W(5L_kl_k \to N_0L_0l_0). \tag{16}$$

Of course, this estimate is most reliable for the satellites with $N_0=1$ and $N_0=2$, i.e., in the series $Np \to 1s$ and $Np \to 2s$, but it is these which determine mainly the spectrum of the satellite lines.

2. The $\hbar\omega_k$ satellite lines radiated by the mesic atom and connected with the electronic transition $N_kL_k \to N_0L_0$, are shifted relative to $\hbar\omega_0$ by an amount $\delta\hbar\omega_k$:

$$\hbar\omega_k=\hbar\omega_0-\delta\hbar\omega_k, \tag{17}$$

where $\delta\hbar\omega_k$ is determined, with sufficient accuracy for our purpose, by the difference between the electronic terms of the HFS method

$$\delta\hbar\omega_k \cong E(N_kL_k)-E(N_0L_0). \tag{18}$$

It is natural to break up all the series of the satellites into two essentially different groups:

A. The group of strongly shifted satellite lines connected with the electron transitions $Np \to 1s$. The minimal value of the shift $\delta\hbar\omega_{min}$ is limited here by the term difference

$$\delta\hbar\omega_{min}=\begin{cases} E(2p)-E(1s) & 6 \leqslant Z \leqslant 11 \\ E(3p)-E(1s) & 12 \leqslant Z \leqslant 19 \end{cases}. \tag{19}$$

For the discrete satellite lines $\delta\hbar\omega_{max}=|E(1s)|$. We note that $\delta\hbar\omega_{min}$ becomes somewhat larger than $\Delta\hbar\omega=150$ eV already at $Z=7$, and increase rapidly with increasing Z. This makes it possible to distinguish these satellite lines of group A from the information-carrying quantum $\hbar\omega_0$.

B. The group of weakly shifted satellites, due in the mesic atoms with $6 \leqslant Z \leqslant 11$ to the electron-transition series

$$Np \to 2s,\ Ns \to 2p,\ Nd \to 2p\ (N \geqslant 3),$$

to which there are added in the region $12 \leqslant Z \leqslant 19$ the electron transitions

$$Np \to 3s,\ Ns \to 3p,\ Nd \to 3p\ (N \geqslant 4).$$

For all these satellites, the values of $\delta\hbar\omega_k$, even at $Z=17$, turn out to be only of the order of $\Delta\hbar\omega$, and at smaller Z they are much smaller than $\Delta\hbar\omega$. We shall therefore add the contributions of all the weakly displaced satellites together to form a single quantity $W(B)$–the probability of a summary satellite line $\hbar\omega_B$ that is indistinguishable from $\hbar\omega_0$.

Accordingly, Table II below lists for the chosen configurations (6) these summary intensities $W(A)$ and $W(B)$ of the groups of strongly and weakly displaced satellites, as well as the displacement intervals $\delta\hbar\omega$ specified for the discrete spectrum of the satellites by the quantities $\delta\hbar\omega_{min}$ and $\delta\hbar\omega_{max}$. All the values of $\delta\hbar\omega$ were obtained as differences of the electronic terms calculated within the framework of the HFS method for the configurations $\{0\}$. For weakly displaced lines this calculation yields correctly only the order of magnitude

TABLE II. Summary intensities of groups of satellite lines in mesic atoms.*

Mesic atom	Strongly displaced line group (A)			Weakly displaced line group (B)			$W_\mu(M1)$, sec^{-1}
	$W(A)$, sec^{-1}	$\delta\hbar\omega_{min}$, eV	$\delta\hbar\omega_{max}$, eV	$W(B)$, sec^{-1}	$\delta\hbar\omega_{min}$, eV	$\delta\hbar\omega_{max}$, eV	
	Electron-shell configuration $\{(1s)^2(2s)^2\}$						
μC	$2.9\cdot10^8$	192	210	$4.1\cdot10^8$	6.2	23.6	$3.12\cdot10^4$
μN	$7.3\cdot10^8$	292	329	$5.0\cdot10^7$	8.6	45.9	$1.46\cdot10^5$
μO	$1.4\cdot10^9$	411	475	$4.4\cdot10^7$	11.0	75.2	$5.54\cdot10^5$
μF	$2.7\cdot10^9$	551	649	$3.6\cdot10^7$	13.4	111.3	$1.80\cdot10^6$
μNe	$6.0\cdot10^9$	712	850	$2.8\cdot10^7$	15.7	154.3	$5.16\cdot10^6$
μNa	$2.0\cdot10^{10}$	892	1079	$2.2\cdot10^7$	18.0	204.2	$1.2\cdot10^7$
	Electron-shell configuration $\{(1s)^2(2s)^2(2p)^6\}$						
μNa	$4.2\cdot10^8$	856	858	$1.6\cdot10^5$	15.5	43	$1.2\cdot10^7$
μMg	$2.6\cdot10^{10}$	1064	1074	$3.6\cdot10^5$	31.5	73.5	$3.2\cdot10^7$
μAl	$3.7\cdot10^{12}$	1296	1317	$1.3\cdot10^6$	51.5	111	$7.1\cdot10^7$
μSi	$1.3\cdot10^{10}$	1554	1588	$2.8\cdot10^6$	75.5	156	$1.5\cdot10^8$
μP	$3.8\cdot10^9$	1836	1887	$4.7\cdot10^6$	103.5	207.5	$2.6\cdot10^8$
μS	$2.4\cdot10^9$	2142	2214	$6.8\cdot10^6$	134.5	266.5	$4.5\cdot10^8$
μCl	$1.5\cdot10^9$	2473	2569	$8.4\cdot10^6$	169.5	332	$7.56\cdot10^8$
	Neutral electron configuration $\{(1s)^2(2s)^2(2p)^n(3s)^m(3p)^k\}$						
μN	$2.8\cdot10^8$	282	291	$3.1\cdot10^7$	5.6	17.6	$1.46\cdot10^5$
μF	$4.9\cdot10^8$	523	537	$6.7\cdot10^6$	10.1	29.2	$1.80\cdot10^6$
μNa	$4.2\cdot10^8$	856	858	$1.6\cdot10^5$	15.4	43.2	$1.2\cdot10^7$
μAl	$9.4\cdot10^8$	1291	1295	$8.6\cdot10^4$	3.3	89.5	$7.1\cdot10^7$
μP	$6.4\cdot10^8$	1822	1828	$1.1\cdot10^5$	3.6	151.5	$2.6\cdot10^8$
μCl	$1.4\cdot10^8$	2445	2456	$9.4\cdot10^4$	6.9	226	$7.56\cdot10^8$

*For comparison the table lists the probability $W_\mu(M1) \equiv W(M1;\ 2s \to 1s)$ of the mesic transition.

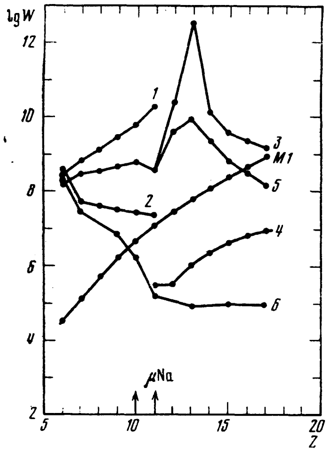

FIG. 3. Logarithms (to base 10) of the summary probabilities of the groups of strongly displaced satellites ($W(A)$) and weakly displaced satellites ($W(B)$) for a series of configurations of the mesic-atom cell: index 1—$W(A)$, 2—$W(B)$ for $\{(1s)^2(2s)^2\}$; index 3—$W(A)$, 4—$W(B)$ for $\{(1s)^2(2s)^2(2p)^6\}$; index 5—$W(A)$, 6—$W(B)$—for neutral atomic configuration of the shell; $M1$—probability of radiative transition $|2s1/2\rangle\to|1s1/2\rangle$ of the meson.

of $\delta\hbar\omega$, and deviations of the order of several electron volts are realistic here, but this is immaterial since $\Delta\hbar\omega\approx150$ eV.

The summary picture of the ratio of the intensities of the information-carrying line $\hbar\omega_0$ of the meson transition $|2s1/2\rangle\to|1s1/2\rangle$ and of the group of strongly and weakly displaced lines is shown in Fig. 3. The correction to the intensity of the $\hbar\omega_0$ line, necessitated by the quantity $|\delta_W(2s1/2, 2p1/2)|^2$, is negligible throughout in this case. In the data of Fig. 3 we used the estimates for configurations with incomplete population of the orbits. The contribution of the indistinguishable line $\hbar\omega_B$ depends radically on the initial configuration $\{0\}$ of the mesic-atom shell. Filling of the orbits to the configuration of the neutral atom lowers the intensity of the weakly displaced group of satellites, this being due to the decrease of the radial element of $\hat{H}_{\rm int}(\mathbf{r}_\mu\cdot\mathbf{r}_i)$ when the screening effect becomes stronger with increasing number of electrons in the shell.

3. The possible objects for experiments aimed at observing the effect of the weak neutral interaction of the muon and the nucleus can be the mesic atoms μP, μS, and μCl, but particular interest attaches to μNe and μNa, where the expected parity nonconservation effect is larger than in the heavier ones.

On the basis of the data of Fig. 3 it might seem convenient to restore the neutral configuration $\{(1s)^2(2s)^2(2p)^6\}$, in the electron shell of the μNa mesic atom, since the intensity of the weakly displaced satellites is much less here than for the configuration $\{(1s)^2(2s^2\}$ of the mesic atom $(\mu\mathrm{Na})^{+6}$. Such a restoration is more probable when a metallic target is used to stop the meson. This, however, raises the question of the possible competition between the information transition $|2s1/2\rangle\to|1s1/2\rangle$ of the meson and other processes of de-excitation of the mesic $|2s1/2\rangle$ state.

There are quite a few competing processes in this case:

a) Radiative $E1$ transition $|2s1/2\rangle\to|2pj_k\rangle$ of the meson. At a fixed term difference $\Delta(2pj_k)$, the summary probability

$$W_\gamma(E1)=\sum_{(2pj_k)}W_\gamma(E1; 2s^1/_2\to 2pj_k) \tag{20}$$

can be easily obtained with the wave functions in the field of a point nucleus. In the calculation of $W_\gamma(E1)$ we used the quantities $\Delta(2pj_k)$ given in Table I.

b) Two-quantum radiative meson transition $|2s1/2\rangle\to|1s1/2\rangle$. The probability of this process was estimated in Ref. 2; $W_{\gamma\gamma}=2.6\cdot10^9$ sec^{-1} in the case of the mesic atom μNa.

c) Conversion $E0$ transition $|2s1/2\rangle\to|1s1/2\rangle$ of a meson with ejection of a K electron. According to an earlier nonrelativistic Born calculation (see Refs. 2 and 10), we have for the $E0$ conversion on a filled K shell

$$W_\mu^{\rm Born}(E0; 2s\to1s; [K]^2)\approx2.2\cdot10^9\left(\frac{Z-1}{Z}\right)^3\ \mathrm{sec}^{-1}. \tag{21}$$

In the case of μNa our calculation of the probability of $E0$ conversion on a filled K shell, performed with the relativistic electron functions of the HFS method, yielded the value

$$W_\mu(E0;\ 2s\to1s;\ [K]^2)=3.09\cdot10^9\ \mathrm{sec}^{-1}, \tag{22}$$

which is ~1.87 times larger than the Born nonrelativistic estimate. It was also established that the probability of the $E0$ conversion depends very little on the shell configuration.

d) Conversion of $E1$ transitions $|2s1/2\rangle\to|2pj_k\rangle$ of a meson on the K and L shell of a mesic atom. The probability of this process depends radically on the rate of reconstruction of the shell of the mesic atom in the medium. Thus, for the region $6\leqslant Z\leqslant11$ this process is energywise allowed only on the $2s$ and $2p$ electron orbits. In the case of μNa, our calculation of the conversion for different possible shell configurations within the framework of the relativistic HFS method yields the following values for the summary probability of the conversion $E1$ transitions of the meson to the $|2p1/2\rangle$ and $|2p3/2\rangle$ levels:

the configuration $\{(1s)^2(2s^2\}$:

$$W_\mu(eE1;\ 2s\to2p;\ [2s]^2)=1.16\cdot10^{12}\ \mathrm{sec}^{-1}; \tag{23}$$

the configuration $\{(1s)^2(2s)^2(2p)^6\}$:

$$\begin{aligned}&W_\mu(eE1;\ 2s\to2p;\ [2s]^2)=0.70\cdot10^{12}\ \mathrm{sec}^{-1},\\&W_\mu(eE1;\ 2s\to2p;\ [2p]^6)=0.71\cdot10^{12}\ \mathrm{sec}^{-1}.\end{aligned} \tag{24}$$

From this we get for the total probability of the $E1$ conversion in this configuration

$$W_\mu(eE1)=1.41\cdot10^{12}\ \mathrm{sec}^{-1}. \tag{25}$$

The subscript e will be used hereafter to label a conversion transition and distinguish it from radiative $E1$ transitions.

The conversion of an $E1$ meson transition on the K

shell becomes possible for the atomic configuration of μMg; with increasing nuclear charge Z, the number of electrons in the shell needed to open this channel becomes increasingly smaller, and finally, in the case of μP the process is possible even in the hydrogenlike situation with one electron on the K shell.

4. To gain a quantitative idea of the ratio of the probabilities of the different decay processes of the mesic $|2s1/2\rangle$ state, Table III lists the probabilities $W_\gamma(E1)$, $W_\mu(E0)$ and $W_\mu(eE1)$, calculated for different subshells of the normal atomic configuration of the mesic atom

$$\{(1s)^2(2s)^2(2p)^n(3s)^m(3p)^k\}$$

in the region $6 \leqslant Z \leqslant 19$. Although the probability of conversion on the $[NL]$ orbit, $W_\mu(eE1[NL]^x)$, depends on the configuration of the entire electron shell, whose screening field forms the electron wave functions, and consequently it is necessary to carry out a separate calculation for each mesic-atom configuration, the example of μNa shows that the order of magnitude of the conversion probability per electron of the $[NL]$ orbit is roughly preserved. Thus, from the data of Table III we can estimate with reasonable accuracy the rates of the conversion $E1$ transitions of the meson from $|2s1/2\rangle$ orbits to $|2pj_k\rangle$ orbits at different occupations of the mesic-atom shell.

4. CONCLUSION

The resultant picture of the competition of different decay channels of the mesic-atomic state $|2s1/2\rangle$ is shown in Fig. 4. It is seen from it, in particular, that the relative intensity of the line $\hbar\omega_0$ is determined by the competition of the conversion $E1$ transitions of the meson. In the case of the most suitable mesic atoms μNe and μNa, in order for the intensity of the $\hbar\omega_0$ line be at the level $\sim 10^{-3}$ per captured meson in the $|2s1/2\rangle$ state, it is necessary to permit in experiment only the population of the $1s$ orbit of the electron shell. Capture of an electron in the L shell immediately lowers the yield of the $\hbar\omega_0$ quanta to a value 10^{-5}. When only the $1s$ orbits of the electron are filled in the ions $(\mu\text{Ne})^{+5}$ and $(\mu\text{Na})^{+6}$, there are no weakly displaced satellite lines at all, and the strongly displaced lines of the $Np \to 1s$ series can be distinguished from the $\hbar\omega_0$ line. Consequently, the problem of experimentally observing parity and conservation effects in light mesic atoms reduces to a search for μNe or μNa mesic-atom formation conditions such that during the lifetime of the meson on the $|2s1/2\rangle$ orbit ($\tau \sim 10^{-10}$ sec) only the electron K orbit can be filled. The appearance of an electron on the L shell immediately opens (see Fig. 4) the $E1$ conversion channel, i.e., the chain of transitions $|2s1/2\rangle \xrightarrow{e} |2pj_k\rangle \xrightarrow{\gamma} |1s1/2\rangle$, in which a hard quantum is radiated

$$\hbar\omega = \hbar\omega_0 - \Delta(2pj_k).$$

Therefore an experimental indication of satisfaction of these conditions can be the observation of the ratio $\eta(e/\gamma)$ of the number of hard electrons with kinetic energies $E_{kin} \lesssim \hbar\omega_0$ to the number of hard quanta with energies $\hbar\omega \lesssim \hbar\omega_0$, radiative in the decay of the $|2s1/2\rangle$ state of the mesic atom $\hbar\omega_0 \approx 2.1Z^2$ [keV]). If there are still no L electrons in the shell of the μNa mesic atom and only the K orbit is populated, then we get for the ratio η the value

$$\eta \approx \frac{W_\mu(E0)}{W(\text{A}) + W_\gamma(E1) + W_\mu(M1)} \approx 10^{-1}. \quad (26)$$

The appearance of at least one L electron adds to the denominator of the ratio η the quantity $W_\mu(eE1)$, thereby lowering η to the level $\sim 10^{-3}$. This abrupt jump of the ratio η can in fact be used for an empirical selection of the conditions for organizing an experiment on parity nonconservation in the mesic atoms μNe and μNa.

TABLE III. Probabilities of $E1$ transitions* of the mesic atom from the $|2s1/2\rangle$ state to the orbits $|2p1/2\rangle$ and $|2p3/2\rangle$.

Mesic atom	$W_\gamma(E1)$, sec^{-1}	$W_\mu(eE1[1s]^2)$, 10^{12} sec^{-1}	$W_\mu(eE1[2s]^2)$, sec^{-1}	$W_\mu(eE1[2p]^n)$, 10^{11} sec^{-1}	$W_\mu(eE1[3s]^m)$, 10^{10} sec^{-1}	$W_\mu(eE1[3p]^k)$, 10^{10} sec^{-1}	Total $W_\mu(eE1)$, sec^{-1}
μC	$3.35\cdot10^5$	–	$6.29\cdot10^9$	0.80	–	–	$8.63\cdot10^{10}$
μN	$6.14\cdot10^6$	–	$1.14\cdot10^{11}$	2.09	–	–	$3.23\cdot10^{11}$
μF	$1.2\cdot10^8$	–	$3.72\cdot10^{11}$	4.91	–	–	$8.63\cdot10^{11}$
μNa	$1.47\cdot10^9$	–	$6.97\cdot10^{11}$	7.15	–	–	$1.41\cdot10^{12}$
μAl	$9.42\cdot10^9$	7.61	$1.06\cdot10^{12}$	8.22	5.26	–	$9.54\cdot10^{12}$
μP	$4.68\cdot10^{10}$	25.5	$1.45\cdot10^{12}$	8.32	12.2	1.49	$2.79\cdot10^{13}$
μCl	$1.65\cdot10^{11}$	28.4	$1.77\cdot10^{12}$	8.31	17.8	4.10	$3.12\cdot10^{13}$

*The conversion $E1$ transitions are considered for the electron-shell configuration $\{(1s)^2(2s)^2(2p)^n(3s)^m(3p)^k\}$ corresponding to a neutral atom with an effective nuclear charge $Z_{eff} = Z - 1$. We used in the calculation the values of $\Delta(2p1/2)$ and $\Delta(2p3/2)$, listed in Table 1.

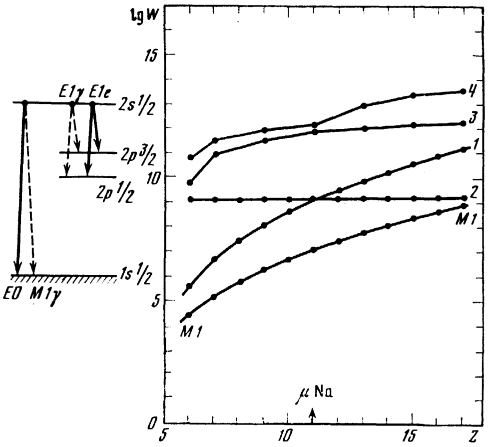

FIG. 4. Logarithms (to base 10) of the probabilities of the competing channels of the decay of the mesic-atom level $|2s1/2\rangle$ in the case of neutral atomic configuration of the shell: 1—summary probability of the radiative $E1$ transitions $2s1/2 \to 2pj_k$ of the meson; 2—$E0$ conversion on the electrons of the $(1s)^2$ configuration in the transition $2s \to 1s$ of the meson, Born estimate; 3—summary probability of conversion on the electrons of the $2s$ shell in $E1$ transitions $2s1/2 \to 2pj_k$ of the meson; 4—summary probability of conversion on all the electron subshells in $E1$ transitions $2s1/2 \to 2pj_k$ of the meson. $M1$—probability of radiative transition $2s \to 1s$ of the meson with emission of an $\hbar\omega_0$ quantum.

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Mean electromagnetic field in a randomly inhomogeneous medium

Yu. A. Ryzhov

Gor'kiĭ Radiophysics Research Institute
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The mean electromagnetic field in a medium with large random fluctuations of the permittivity is considered. The problem of finding the mean field is equivalent to the determination of the effective permittivity of such a medium. This latter has been sufficiently well studied in the case of weak fluctuations, i.e., at $\sigma_\varepsilon/\langle\varepsilon\rangle \ll 1(\sigma_\varepsilon = \langle \varepsilon - \langle\varepsilon\rangle)^2\rangle^{1/2}$ and $\langle\varepsilon\rangle$ is the mean permittivity of the medium. The limiting case $\langle\varepsilon\rangle \to 0$ corresponding to the case of large relative fluctuations, is studied. As an illustration, we have considered the problem of the effective permittivity of a cold plasma with fluctuations of the electron density.

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1. INTRODUCTION

One of the fundamental problems of electrodynamics of randomly inhomogeneous media is the problem of finding the regular component (mean value) of the field of sources immersed in such a medium. In linear electrodynamics, this problem reduces to the calculation of the effective permittivity of the random medium. This permittivity of a randomly inhomogeneous medium is determined from the relation between the mean values of the electric field and the induction:

$$\langle D_i \rangle = \hat{\varepsilon}_{ij}^{eff} \langle E_j \rangle. \tag{1}$$

Averaging is carried out over the ensemble of realizations of the random medium.

In an unbounded statistically homogeneous and an isotropic medium, the operator $\hat{\varepsilon}_{ij}^{eff}$ for harmonic fields ($e^{-i\omega t}$) is a linear integral operator with a difference kernel:

$$\langle D_i(\mathbf{r}) \rangle = \int \varepsilon_{ij}^{eff}(\mathbf{r}-\mathbf{r}_1) \langle E_j(\mathbf{r}_1) \rangle d\mathbf{r}_1. \tag{2}$$

The Fourier transform of the kernel $\varepsilon_{ij}^{eff}(\mathbf{r}, \omega)$

$$\varepsilon_{ij}^{eff}(\omega, \mathbf{k}) = \int \varepsilon_{ij}(\mathbf{r}, \omega) e^{i\mathbf{k}\mathbf{r}} d\mathbf{r} \tag{3}$$

has the form

$$\varepsilon_{ij}^{eff}(\omega, \mathbf{k}) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \varepsilon^{tr}(\omega, k) + \frac{k_i k_j}{k^2} \varepsilon^{l}(\omega, k). \tag{4}$$

The problem of the calculation of $\varepsilon_{ij}^{eff}(\omega, k)$ was considered earlier in the theory of permittivity of gases[1,3] and the electrostatics of mixtures.[2,3] Similar problems are widely discussed at the present time in the theory of percolation and phase transitions.[4,5] In the theory of wave propagation in randomly inhomogeneous media, the effective permittivity has been considered in Refs. 6 and 7. In particular, for the case of a weakly inhomogeneous dielectric ($\sigma^2/\varepsilon_0^2 \ll 1$, $\sigma_\varepsilon^2 = \langle(\varepsilon - \varepsilon_0)^2\rangle$, $\langle\varepsilon\rangle = \varepsilon_0$, $\varepsilon(r) = \varepsilon_0 + \Delta\varepsilon(r)$) formulas were obtained in Ref. 7 for $\varepsilon^{eff}(\omega, 0)$ (without account of spatial dispersion due to the inhomogeneity of the medium). More general expressions for $\varepsilon^{eff}(\omega, \mathbf{k})$ were obtained in Refs. 8 and 9 by the self-consistent field method. In particular, the case of large fluctuations ($\sigma^2/\varepsilon_0^2 \gg 1$) was considered in Ref. 9. This case is physically very interesting, since it describes real situations that arise in the electrodynamics of strongly inhomogeneous mixtures and so on. In particular, an important example is an inhomogeneous plasma with macroscopic fluctuations of the electron density in which $\varepsilon_0(\omega) \to 0$ and $\sigma_\varepsilon/\varepsilon_0 \to \infty$ at the Langmuir frequency.

The following approximate expression was obtained earlier[9] for an inhomogeneous cold plasma:

$$\varepsilon^{eff}(\omega, k=0) = 0.56\varepsilon_0(\omega) + 0.5i\sigma_N/\langle N \rangle, \tag{5}$$

where $\sigma_N^2 = \langle \Delta N^2 \rangle$ is the variance of the electron density fluctuations, which is assumed to be small ($\sigma_N/\langle N \rangle \ll 1$);

$$\varepsilon_0(\omega) = 1 - \omega_0^2/\omega^2, \quad \omega_0^2 = 4\pi e^2 \langle N \rangle / m.$$

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