in Refs. 12 and 13 to describe the temperature dependence of the tunnel frequencies in the SNS spectrum.

We call attention in conclusion to the fact that the ITCF correlation times are not expressed in terms of the lifetimes on the levels. At low temperatures, the main contribution to τ_{EE}^{-1} and τ_{AE}^{-1} is due to phase relaxation.

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Influence of radiation on the motion of channeled particles

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A detailed investigation is made of the influence of radiation on the motion of channeled particles in a crystal. It is shown, in particular, that radiation can lead to an increase in the angular divergence of a particle beam in a channel. The analysis of the relaxation of the transverse energy via radiation takes into account the multiple scattering of the channeled particle in the crystal.

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INTRODUCTION

It is known (see Refs. 1–4 and the bibliography therein) that spontaneous radiative transitions between transverse-energy levels of a channeled particle lead to intense x and gamma radiation, accompanied by relaxation of the transverse energy. For light channeled particles, as noted by us earlier,[2] the process of radiatiave damping of the transverse energy can play a noticeable role alongside the nonradiative dechanneling processes that lead to an increase of the transverse energy. In this case, generally speaking, self-focusing of a beam of channeled particles is possible.

The effect of radiative focusing of a beam of channeled particles was considered also by others.[5-7] However, the results of Baryshevskiĭ and Dubovskaya,[5] as shown by us earlier,[8] turned out to be completely in error. Wedell's paper[7] also contains conclusions that

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are not fully justified. Correct estimates were obtained only by Bonch-Osmolovskiĭ and Podgoretskiĭ,[6] but their classical investigation of radiation deceleration in the case of channeling in a parabolic potential seems to us incomplete. We therefore present the results of a detailed theoretical investigation of the influence of radiation on the motion of a channeled particle. In particular, it will be shown that radiation can lead not only to focusing but also to an increase of the angular divergence of a beam of particles. We shall also indicate the reasons that have led the other workers to erroneous estimates of the effect of radiation focusing.

1. RADIATIVE ENERGY LOSSES

In a spontaneous radiative transition of a channeled particle to a lower energy level of transverse motion, a photon is radiated. The energy of the photon does not equal, generally speaking, the difference between the level energies, since the longitudinal motion of the particle leads to a Doppler shift of the radiation frequency. This circumstance, on the other hand, makes it possible to obtain a relation between the loss of the transverse energy ε and the loss of the total energy E of a channeled particle.

According to the results of Refs. 1–4, the energy of the radiated photon is connected with the difference of the energies of the transverse motion by the relation ($\hbar = m = c = 1$)

$$\omega = \frac{2[\varepsilon_i(E_\parallel) - \varepsilon_f(E_\parallel - \omega)]}{\theta^2 + E_\parallel^{-2}}. \quad (1)$$

Here $E_\parallel = (p_\parallel^2 + 1)^{1/2}$ is the energy of the longitudinal motion of the particle and practically coincides with its total energy. The photon energy and the emission angles are assumed to be relatively small ($\omega \ll E$, $\theta \ll 1$), and the particle is assumed to be ultrarelativistic ($E \gg 1$).

The transverse-energy levels are determined by an equation of the Schrödinger type. In the case of planar channeling, this equation takes the form

$$\left[-\frac{1}{2E}\frac{d^2}{dx^2} + U(x) \right] \Psi_i(x) = \varepsilon_i(E) \Psi_i(x). \quad (2)$$

The position of the levels in the potential well $U(x)$ of the transverse motion depends parametrically on the total energy of the particle.

Since the photon energy in (1) is assumed to be sufficiently small, we can use the approximate equality

$$\varepsilon_f(E - \omega) \approx \varepsilon_f(E) - \omega \partial\varepsilon_f/\partial E. \quad (3)$$

As a result, Eq. (1) takes the form

$$\omega = \frac{2\tilde{\omega}_{if}(E_\parallel)}{\theta^2 + E_\parallel^{-2} - 2\partial\varepsilon_f/\partial E}. \quad (4)$$

Here $\tilde{\omega}_{if} = \varepsilon_i(E_\parallel) - \varepsilon_f(E_\parallel)$ is the change of the transverse energy of the particle following the radiation. The denominator in the right-hand side of (4) can also be represented in the form

$$\theta^2 + E^{-2} - 2\partial\varepsilon_f/\partial E \approx 1 - v\cos\theta,$$

where $v = (v_\parallel^2 + v_\perp^2)^{1/2}$ is the total velocity, $v_\perp \approx (2\varepsilon/E)^{1/2}$ is the transverse particle velocity averaged over the period of the oscillations ($v_\perp \ll v_\parallel$). It is easily seen that relation (4) is indeed a consequence of the Doppler effect.

The earlier calculation[2-4] shows that the probability of energy loss ΔE by emitting a certain number of soft ($\omega \ll E$) photons with total energy ΔE is larger than the probability of loss by radiation of one or several harder photons with the same total energy. In addition, at relatively high energies of the particles ($E \sim 1$ GeV) the number of levels in the transverse-motion well is quite large (~100). Therefore the losses of both the total and transverse energy can in this case be regarded as quasicontinuous.

We take into account the fact that the effective angles of the emission by relativistic particles are determined by the relation $\theta_{eff} \approx E^{-1}$. In addition, for a number of important models of the continuous potential of the planes and axes of the crystal, the transverse energy depends on the total energy in power-law fashion: $\varepsilon \propto E^\alpha$. Thus, for example, for a square well with infinitely tall wells $\alpha = -1$, for a parabolic well $\alpha = -1/2$, and for a potential in the form $U(\rho) \propto 1/\rho$ (ρ is the distance to the crystal axis) we have $\alpha = 1$. As a result, the differential equation that connects the losses of the total and transverse energies takes the form

$$dE = \frac{E^2}{b - \alpha\varepsilon E} d\varepsilon. \quad (5)$$

These elementary estimates do not make it possible, however to determine the constant b in (5). The exact calculation that follows will show that this constant is equal to unity. We denote the values of the transverse and longitudinal energies at the instant of the entry of the particle into crystal by ε_0 and E_0, respectively. Then the solution of the differential equation (5) takes the form

$$\varepsilon = \left(\frac{E_0}{E}\right)^\alpha \left\{ \varepsilon_0 + \frac{1}{(\alpha-1)E_0} \left[\left(\frac{E}{E_0}\right)^{\alpha-1} - 1 \right] \right\}, \quad \alpha \neq 1, \quad (6)$$

$$\varepsilon = \frac{E_0}{E} \left[\varepsilon_0 + \frac{1}{E_0} \ln \frac{E}{E_0} \right], \quad \alpha = 1. \quad (6')$$

We assume that the crystal is long enough to permit the transverse energy to relax to zero (to $U_0 < 0$ in the case $\alpha = 1$). According to (6), the total energy of the particle then becomes

$$E_\infty = E_0[1 - (\alpha - 1)\varepsilon_0 E_0]^{1/(\alpha-1)}. \quad (7)$$

For a parabolic well $\alpha = -1/2$ and expression (7) coincides with the corresponding result (23) obtained by another method by Bonch-Osmolovskiĭ and Podgoretskiĭ.[6] In the case (6′) we obtain for E_∞ the transcendental equation

$$\ln(E_\infty/E_0) = -\varepsilon_0 E_0 + U_0 E_\infty. \quad (7')$$

According to (7) and (7′), at relatively small initial particle energy $|\varepsilon_0| E_0 \ll 1$, the loss of its total energy during the time of relaxation of the transverse energy can be neglected. On the contrary, at relatively high initial energy ($\varepsilon_0 E_0 \gg 1$) the relaxation of the transverse energy is accompanied by loss of a large part of the total energy of the particle. We note that the equation $\varepsilon_0 E_0 = 1$ corresponds to energies $E \approx 1$ GeV and E

≈10 GeV for axial and planar channeling of electrons and positrons.

According to our calculations (7) and (7′), the energy of the channeled particle E_∞ increases together with its initial energy E_0 in accordance with a law that is determined by the form of the channeling potential. The statement made by Baryshevskiĭ *et al.*[9] that there exists a certain limiting energy E_{cr} for the value E_∞ is in error. The point is that the result of Pomeranchuk (see, e.g., Ref. 10, Sec. 76), to which the authors of Ref. 9 refer, was obtained for cases when the transverse component of the radiation-friction force can be neglected. This condition, however, is not satisfied for channeled particles (see also Ref. 6 concerning this question).

The square of the angle ψ at which the channeled particle moves relative to the axes or planes of the crystal is determined by the ratio of the transverse and longitudinal momenta of the particle

$$\psi^2=(p_\perp/p_\parallel)^2.$$

For ultrarelativistic particles $p_\parallel=E$, and the transverse motion is nonrelativistic [see (2)], therefore $p_\perp^2=2E\varepsilon$, and

$$d\psi^2=\frac{2}{E}d\varepsilon-\frac{2\varepsilon}{E^2}dE. \tag{8}$$

Thus, according to (8), the losses of the transverse energy upon radiation lead to a decrease of the angle ψ, while the corresponding losses of the total energy lead to an increase of this angle. We shall use expression (5) (where $b=1$) for the increment of the square of the angle in terms of the increment of the transverse energy. As a result we get

$$d\psi^2=\frac{2}{E}\frac{1-(\alpha+1)\varepsilon E}{1-\alpha\varepsilon E}d\varepsilon. \tag{9}$$

In planar channeling of positively charged particles, a parabolic well is a sufficiently good model for the channeling potential. In this case $\alpha=-\frac{1}{2}$ and, according to (9), under the condition $\varepsilon E>2$ the radiation leads not to a decrease but, conversely, to an increase of the deflection angle of the particle. This does not lead, however, to dechanneling, since the Lindhard critical angle also increases with decreasing total energy of the particle. Radiation focusing of a beam is possible only under the condition $\varepsilon E<2$. For positrons and characteristic values of the transverse energy $\varepsilon=10$ eV, the total energy should be less than $E\approx 52$ GeV.

Similar calculations for axial channeling of negatively charged particles in a potential $U(\rho)\sim 1/\rho$, when $\alpha=1$, leads to the following result. The radiation focusing can take place only under the condition $|\varepsilon|E<1$. The critical energy for characteristic values $|\varepsilon|=100$ eV is 2.6 GeV.

In planar channeling of electrons, the potential differs substantially from parabolic. In this case we can use for estimates the square-well approximation $(\alpha=-1)$. According to (9), in a square well the radiation always leads to a decrease of the deflection angle. However, the relaxation rate begins to increase substantially when the parameters εE becomes larger than unity.[1)]

2. DEPENDENCE OF THE DEFLECTION ANGLE ON THE PATH OF THE PARTICLE IN THE CRYSTAL

We proceed now to calculate the dependences of the transverse and total energy on the path of the particle in the crystal. We consider first the case of relatively low energies, when $\varepsilon E\ll 1$. This makes it possible, on the one hand, to neglect the influence of the loss of the total energy on the particle deflection angle compared with the influence of the loss of transverse energy [see (8) and (9)]. On the other hand, the condition $\varepsilon E\ll 1$ (Ref. 2) enables us to use the dipole approximation for the calculation of the probability $dw_{if}/d\omega$ of the spontaneous transition per unit time from the state $|i\rangle$ to the state $|f\rangle$ of transverse motion. This probability takes in the dipole approximation the form[1-3]

$$dw_{if}/d\omega=|d_{if}|^2\tilde\omega_{if}^2(1-2\omega/\omega_m+2\omega^2/\omega_m^2)\eta(\omega_m-\omega). \tag{10}$$

Here d_{if} is the matrix element of the dipole moment,

$$\tilde\omega_{if}=\varepsilon_i(E)-\varepsilon_f(E),\quad \omega_m=2\tilde\omega_{if}\gamma^2,\quad \gamma=(1-v_z)^{-1/2},$$

v_z is the longitudinal velocity of the particle, and $\eta(\omega)$ is the Heaviside unit step function.

The average loss of the transverse energy per unit time is obtained by integrating (10) with respect to the radiation frequencies, followed by multiplication by the difference $\tilde\omega_{if}$ of the level energies and by summation over all the final states of the transverse motion. As a result we get

$$\frac{d\varepsilon}{dt}=\frac{4\gamma^2}{3}\sum_{f<i}\tilde\omega_{if}^3|d_{if}|^2. \tag{11}$$

We now multiply $dw_{if}/d\omega$ by the photon energy ω and integrate with respect to the frequencies in the sum over the final states. We then obtain the loss of total energy per unit time

$$\frac{dE}{dt}=\frac{4\gamma^4}{3}\sum_{f<i}\tilde\omega_{if}^3|d_{if}|^2. \tag{12}$$

Thus, in the dipole approximation according to (11) and (12), the losses of the total and transverse energy are connected by the relation

$$dE/dt=\gamma^2 d\varepsilon/dt. \tag{13}$$

As a result of a comparison of this expression with Eq. (5) we get the value of the constant $b=1$.

Wedell [see Ref. 7, relation (1) at $\langle\theta^2\rangle=0$] used to calculate the transverse half energy loss in the dipole approximation a relation different from (13)

$$\frac{d\varepsilon}{dt}=-\frac{\varepsilon}{E}\left|\frac{dE}{dt}\right|.$$

This relation follows from the general equation for the distribution function, obtained by Beloshitskiĭ and Kumakhov[11] for cases when the transverse component of the friction can be neglected. As shown by the presented calculation, in the case of radiative deceleration the transverse component of the friction force plays a substantial role. Therefore Wedell's results[7] should be regarded as generally incorrect. This inaccuracy was corrected in a subsequent paper by Wedell and Kumakhov.[12]

We shall continue the calculations for two concrete models of the plane potential. Let the plane potential be a well of width a with infinitely tall walls. Then the wave functions of the transverse motion $\Psi_n(x)$ and the energies corresponding to them take the form (see, e.g., Ref. 13, p. 88 of original)

$$\Psi_n(x)=\left(\frac{2}{a}\right)^{1/2}\sin\frac{\pi n}{a}x,\quad \varepsilon_n=\frac{\pi^2 n^2\hbar^2}{2m^* a^2},\tag{14}$$

where $m^*=\gamma m$ is the relativistic mass of the particle (we use ordinary units here). The matrix element of the dipole moment then takes the form

$$|d_{if}|=\frac{2^3 ea}{\pi^2}\frac{if}{(i^2-f^2)^2}\frac{1-(-1)^{i-f}}{2},\tag{15}$$

where e is the charge of the particle. We now use (14) and (15), followed by summation over the final states in (11). As a result we get for the differential losses of the transverse energy per unit crystal length dz the following expression:

$$\frac{d\varepsilon}{dz}=-\frac{2^{11/2}}{9\pi}\frac{e^2}{\hbar c}\frac{\varepsilon^{5/2}E^{1/2}}{a(mc^2)^2}.\tag{16}$$

A similar calculation for a parabolic plane potential $U(x)=(4U_0/d^2)x^2$ leads to the result

$$\varepsilon_n=\frac{\hbar}{d}\left(\frac{8U_0}{m^*}\right)^{1/2}\left(n+\frac{1}{2}\right),\quad |d_{if}|=\frac{1}{2}ed\hbar i(2m^*U_0)^{1/2}\delta_{f,i-1},$$

where δ_{kl} is the Kronecker symbol. The summation in (11) in the dipole case reduces to a single term, while the equation for the losses of the transverse energy takes the form

$$\frac{d\varepsilon}{dz}=-\frac{16}{3}\frac{e^2}{\hbar c}\frac{\Lambda}{d^2}\frac{U_0}{mc^2}\varepsilon,\tag{16'}$$

where $\Lambda=\hbar/mc$ is the Compton wavelength of the particle.

Besides the radiation that leads to the damping of the angle ψ, there occur also processes that lead to its increase. In a crystal with relatively small number of impurities and dislocations, the principal process of this type is multiple scattering of the particles by the electrons and the vibrating nuclei of the crystal. For positrons or other positively charged particles, which move in the case of channeling far enough from the lattice sites of the crystal, the multiple scattering is mainly by the quasifree valence electrons. For these particles, the parabolic well is a sufficiently approximation to the potential of the planes. The equation for the square of the deflection angle of the particles takes, with allowance for (8) and (16′), the form

$$\frac{d\psi^2}{dz}=-k\psi^2+q,\tag{17}$$

where

$$k=\frac{16}{3}\frac{e^2}{\hbar c}\frac{\Lambda}{d^2}\frac{U_0}{mc^2},\quad q=\frac{2E_s^2}{E^2}\frac{e^2}{\hbar c}n_e r_0^2\ln\frac{\theta_{max}}{\theta_{min}},$$

q is the mean square of the angle of multiple scattering by the valence electrons, $E_s=21$ MeV, $r_0=e^2/mc^2$ is the classical radius of the electron, and n_e is the average density of the valence electrons in the crystal. The maximum angle θ_{max} taken into account in a single scattering act is determined by the critical channeling angle $\theta_{max}\approx(U_0/E)^{1/2}$. The minimal angle is determined by the polarization of the electron gas[14], $\theta_{min}=\hbar\omega_p/E$, where $\omega_p^2=4\pi n_e e^2/m$.

Inasmuch as in the dipole case ($\varepsilon E\ll 1$) the radiative losses of the total energy can be neglected [see (7) and the text that follows], it follows that the mean squared angle of multiple scattering can also be regarded as constant. As a result, the solution of (17) takes the form

$$\psi^2(z)=e^{-kz}\left[\psi^2(0)+\frac{q}{k}(e^{kz}-1)\right].\tag{18}$$

Were it not for the multiple scattering ($q=0$), radiative focusing of the beam would take place over a characteristic length $l_{eff}=1/k$ [see (17)]. At a potential-well depth $U_0=20$ eV and at a width $d=2\times10^{-8}$ cm, typical values in planar channeling, the relaxation length is 2.2 cm for positrons and 7×10^6 cm for protons. This result does not depend on the particle energy so long, of course, as the dipole approximation is valid ($\varepsilon E\ll1$), i.e., up to energies $E\sim10$ GeV for positrons and $E\sim10$ TeV for protons.

Our estimates of the length l_{eff} differ by many orders of magnitude (particulary for protons) from the analogous results of Baryshevskiĭ and Dubovskaya[5]. This difference is due mainly to their incorrect estimate of the matrix element of the dipole moment of the particle in a rectangular potential well [$d_{if}\sim i^{-1}$, cf. Eq. (15) of the present paper]. In addition, according to Eq. (9), the relaxation of the angle ψ is determined in the dipole case by the transverse-energy loss, and not by the lifetime of the initial level, as was previously assumed.[5] Finally, it is well known that a parabolic well is a much better approximation than a square well for the potential of the planes in the case of channeling of positively charged particles.

Whether the beam of particles will actually be focused or whether its angular distribution will broaden because of the scattering and in final analysis because of the channeling, depends on the ratio

$$R=k\psi^2(0)/q.\tag{19}$$

The radiative effects prevail over the effects of multiple scattering by the valence electrons under the condition $R>1$. Estimates show that for positrons and heavier particles this condition cannot be satisfied at energies $\varepsilon E<1$ (see Sec. 1), when radiation decreases rather than increases the deflection angle.

The analysis of the influence of the multiple scattering on the channeling of negatively charged particles is somewhat more complicated, because both the quasifree electrons of the crystal and the bound electrons and nuclei of the lattice atoms play a role in this process. With decreasing transverse energy of the particles, the effective density of the bound electrons and nuclei increases, therefore the mean squared angle of multiple scattering q depends in this case on the time of motion of the particle in the crystal. The calculation of the dependence of q on the transverse energy can be the subject of a separate investigation. For simplicity we can confine ourselves to some value $\bar{q}$ averaged over all the transverse energies. An equation similar to (17) can then be written for a square well in the form

$$d\psi^2/dz=-\varkappa(\psi^2)^{3/2}+\bar{q}, \tag{20}$$

where

$$\varkappa=\frac{16}{9\pi}\frac{e^2}{\hbar c}\frac{E^2}{(mc^2)^2}\frac{1}{a}.$$

At values $a=10^{-9}$ cm the length of the radiative relaxation $1/\varkappa$ amounts to 0.88 cm for electrons of energy $E=1$ GeV and 3974 cm for π^- mesons of energy 273 GeV.

When account is taken of multiple scattering, the focusing of a particle takes place only under the condition

$$[\varkappa\psi^2(0)]^{3/2}/\bar{q}>1.$$

estimates show that this condition cannot be satisfied at energies $\varepsilon E<1$, for which the corresponding calculation was performed.

In the more general case, when the parameter εE is arbitrary and the radiation has generally speaking a non-dipole character, the expression for the transverse-energy loss takes the form

$$\frac{d\varepsilon}{dt}=e^2\sum_{f<i}\omega_{if}\bar{\omega}_{if}\int_0^1\left|A_{if}\left(\frac{\omega_{if}\xi}{(1-v_z^2)^{1/2}}\right)\right|^2\frac{1-\xi^2}{\xi^2}d\xi, \tag{21}$$

where

$$A_{if}(\zeta)=\int\Psi_i^*(x)e^{-i\zeta x}\Psi_f(x)\,dx$$

is the matrix element of the interaction of the particles with the radiation field. This result is obtained with the aid of the general expression (15) of our preceding paper[2] for the spectral-angular distribution of the radiation probability, in analogy with the derivation of (11). The calculation of the losses of the total energy leads, as expected from simple estimates [see (5)], to the relation

$$\frac{dE}{dt}=\frac{d\varepsilon}{dt}\frac{E^2}{1-E^2\partial\varepsilon/\partial E}.$$

For a parabolic potential and for transitions to the nearest levels $(i-f\ll i)$ with emission of relatively soft photons $(\omega\ll E)$ we get[2]

$$A_{if}(\zeta)=J_{i-f}(\mu\zeta),\quad \omega_{if}=\frac{(i-f)\omega_0}{1+\varepsilon E},\quad \bar{\omega}_{if}=(i-f)\omega_0,$$

where $\omega_0^2=8U_0/d^2E$, $\mu=(2\varepsilon/\omega_0^2E)^{1/2}$, and J_ν is a Bessel function. The remaining calculations similar to those given, for example, in the book[15] (p. 91) for a classical oscillator at rest. The summation over the final states is with the aid of the formula

$$\sum_{\nu=1}^{\infty}\nu^2J_\nu^2(\nu\beta\xi)=\frac{(\beta\xi)^2[4+(\beta\xi)^2]^2}{16[1-(\beta\xi)^2]^{7/2}},$$

where

$$\beta=\left(\frac{2\varepsilon}{E}\right)^{1/2}\frac{(1-v_z^2)^{-1/2}}{1+\varepsilon E}$$

is the velocity of the channeled particle in the coordinate system in which there is no longitudinal motion.

Integrating next in (21) with respect to the variable we obtain

$$\frac{d\varepsilon}{dz}=-\frac{4e^2U_0\varepsilon}{3d^2}[1+(\varepsilon E)^2]^{-5/2}\left[4-\frac{6\varepsilon E}{(1+\varepsilon E)^2}\right],\qquad \frac{dE}{dz}=\frac{2E^2}{2+\varepsilon E}\frac{d\varepsilon}{dz}. \tag{22}$$

At relatively low energies $(\varepsilon E\ll 1)$ expression (22) coincides with (16′). In the opposite case $(\varepsilon E\gg 1)$ we have

$$\frac{d\varepsilon}{dz}=\left(\frac{d\varepsilon}{dz}\right)_{dip}\frac{4}{(\varepsilon E)^5},\quad \frac{dE}{dz}=\left(\frac{d\varepsilon}{dz}\right)_{dip}\frac{8}{\varepsilon^4E^3},$$

where $(d\varepsilon/dz)_{dip}$ are the transverse-energy losses calculated in accordance with the dipole formula (16′).

It is seen that upon violation of the dipole approximation $(\varepsilon E\gg 1)$ the law that governs the decrease of the transverse energy changes. Therefore the statement made by Wedell[7] that this law does not depend on the conditions of applicability of the dipole approximation is incorrect. When the multipole expansion of the radiation field becomes inapplicable, the transverse-energy losses in a parabolic well begin to decrease like E^{-3}. This is the result of the faster decrease, that in the case $\varepsilon E\ll 1$, of the frequencies ω_{if} of the transverse motion and of the corresponding faster decrease of the emission probability (for more details see Ref. 4, sec. 3). Therefore the tendency of the transverse-energy loss rate to decrease at high energies $(\varepsilon E\gg 1)$ should be observed also for other types of potentials.

CONCLUSION

The presented analysis shows that the electromagnetic radiation of channeled particles can lead to a substantial decrease of both the transverse and total energy of the particle. There is an optimal total particle energy, determined by the condition $|\varepsilon|E\sim 1$, at which the rate of energy loss is maximal. Under the optimal conditions the total and transverse energies of the positrons decrease to half their values over a path of several centimeters. For heavier particles the radiation losses of the energy are practically always negligibly small. However, even for electrons and positrons with optimal energies, the length over which the particles become dechanneled as a result of multiple scattering by electrons and nuclei of the crystal is shorter by 10–100 times than the effective length of the energy relaxation.

The relaxation of the transverse energy leads to a decrease of the mean squared deflection angle of the particles from the planes or axes of the crystal, and the relaxation of the total energy, on the contrary, leads to its increase. The total effect depends on the particle energy. Radiation leads to a decrease of the deflection angle only at relatively low energies $|\varepsilon|E<C$, where the constant C is determined by the concrete form of the channeling potential. The radiation

focusing of the electrons and positrons could become noticeable in a crystal over a particle path of several centimeters. However, multiple scattering, as already noted above, leads in this case to a relatively strong opposing effect, against the background of which the observation of focusing of even light particles is practically impossible.

Note added in proof (18 September 1979). A more detailed analysis shows that in the general (non-dipole) case under the condition $\varepsilon E \gtrsim 1$ an important role is assumed by the parametric coupling of the transverse and longitudinal motions. Therefore the results (21) and (22) of the present paper, obtained without allowance for this coupling, should be regarded as incorrect.

[1)]The choice of a more realistic form of the potential of the plane leads to the existence of a critical energy for electrons, too.

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Ultracold-neutron storage experiments

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Results of experiments on storage of ultracold neutrons (UCN) with energies in the range $(10\text{–}30)\times 10^{-9}$ eV are presented. The UCN storage time for an aluminum vessel was found to depend on the temperature of the walls. The energy spectra of UCN stored in a copper vessel for different times were measured. It is shown that the time averaged increase in the energy of a UCN in a single collision with the wall does not exceed $\sim 0.7\times 10^{-10}$ eV.

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That the anomalously short ultracold-neutron (UCN) storage times observed experimentally are due to heating of the neutrons to thermal energy in impacts with the walls of the storage vessel is now regarded as established.[1] The interest in this heating is due mainly to two circumstances. First, it has been found in experiments on the storage of UCN in vessels whose walls were coated with various materials[2] that the cross section for the anomalous heating is virtually independent of the material of the wall and amounts to ~10 b on conversion to thermal energy. Second, none of the experiments[3-5] has revealed any temperature dependence of the UCN storage time, although in some experiments[4] the temperature was varied over a fairly wide range (80–600 °K). In the work reported here we investigated the temperature dependence of the storage time in a vessel made of aluminum—a material that has not previously been used for UCN storage.

Another problem considered in this work is the possibility of slightly heating (or cooling) UCN while storing them in vessels. Quasielastic reflections of UCN (in which the energy transfer ΔE per impact is much smaller than E_{lim}) might lead to loss from the vessel of neutrons whose energies rise above the limiting energy E_{lim} of the wall material. We also estimated how much broadening of the spectrum of the stored UCN could be attributed to quasielastic reflections.

Low-energy $[(20\text{–}30)\times 10^{-9}$ eV$]$ UCN were used in both experiments. This was necessary in investigating the temperature dependence of the storage time since the limiting energy of aluminum is small ($E_{\text{lim}} = 52 \times 10^{-9}$ eV). To estimate the possible broadening of the UCN spectrum due to quasielastic reflections we had to compare the spectra of the UCN in the storage vessel at different times. The UCN spectrum is most easily

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