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Deformation-potential operator for a screw dislocation

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Geometrical considerations are used to obtain the form of the operator of the deformation potential for a screw dislocation. It is shown that the spectrum of an electron moving along a dislocation in a parallel magnetic field differs from the usual parabolic spectrum.

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In the description of the interaction of an electron with lattice defects one frequently employs the deformation potential, which is a quantity proportional to the divergence of the displacement **u**. In the case of screw dislocations of interest to us, div **u** = 0, whereas the influence of the screw dislocation on the moving electron is subject to no doubt. We shall show how this difficulty can be overcome.

The interaction of an electron with a screw dislocation will be described with the aid of the metric theory. The undeformed medium is assumed to be isotropic. We consider first the situation in the classical approach. The electron spectrum prior to the deformation is then $E=(p_x^2+p_y^2+p_z^2)/2\mu$, μ is the particle mass, and **p** is the momentum. The principal assumption is that the trajectories are frozen into the medium and are deformed together with the medium. Then uniaxial tension by a factor k along the x axis corresponds to a spectrum

$$E=\frac{1}{2\mu}\left(\frac{1}{k^2}p_x^2+p_y^2+p_z^2\right).$$

Any homogeneous deformation can be resolved into similar tensions and rotations. In the case of an inhomogeneous deformation it is possible to introduce a metric ds^2 such that the trajectories coincide with the geodesics. By, the same token, the behavior of a particle in a deformed medium is completely described. The transition to quantum mechanics is in standard fashion—a Laplace operator, followed by a Hamiltonian, is constructed from the metric ds^2.

Let the screw dislocation in an isotropic medium be located along the z axis, and let a positive value of the Burgers vector $b=2\pi a$ correspond to a right-hand screw. We express the metric in the form $ds^2=\omega_1^2+\omega_2^2+\omega_3^2$. In the absence of the dislocation we have $\omega_1=dr$, $\omega_2=rd\varphi$, $\omega_3=dz$. It is almost obvious that when the dislocation is introduced $\omega_3=dz$ is replaced by $\omega_3=-dz-ad\varphi$, while ω_1 and ω_2 remain unchanged. The corresponding Laplace operator can be easily calculated:

$$\Delta=\partial_{xx}+\partial_{yy}+\partial_{zz}+\frac{2a}{r^2}\partial_{z\varphi}+\frac{a^2}{r^2}\partial_{zz}.$$

It corresponds to a Hamiltonian

$$\mathscr{H}=\frac{1}{2\mu}(p_x^2+p_y^2+p_z^2)+V,\quad V=\frac{1}{2\mu}\left(\frac{2a}{r^2}(xp_yp_z-yp_xp_z)+\frac{a^2}{r^2}p_z^2\right),$$

where $p_x=-i\hbar\partial_x$, and the operator V assumes here the role of the deformation potential. The eigenfunctions of this Hamiltonian are of the form $J_{|\nu|}(kr)e^{im\varphi}e^{i\varkappa z}$ where $\nu=m+a\varkappa$. They correspond to energies $\hbar^2(k^2+\varkappa^2)/2m$. If the wave-vector component along the z axis is $\varkappa=0$, then $V=0$ and the dislocation does not influence the motion of the electron.

More meaningful results are obtained if a magnetic field H is directed along the z axis. Then the momentum operators in the Hamiltonian take the form

$$p_x=-i\hbar\partial_x+\frac{e}{2c}Hy,\quad p_y=-i\hbar\partial_y-\frac{e}{2c}Hx,$$
$$p_z=-i\hbar\partial_z.$$

We seek the ψ function in the form $\psi=R(r)e^{im\varphi}e^{i\varkappa z}$. The Schrödinger equation then becomes

$$-\frac{\hbar^2}{2m}\left(R_{rr}+\frac{1}{r}R_r-\frac{\nu^2}{r^2}R-\varkappa^2R\right)+\frac{\hbar\omega\nu}{2}R+\frac{\mu\omega^2}{8}r^2R=ER.$$

Here $\omega=|e|H/\mu c$, $\nu=m+a\varkappa$. The eigenvalues and eigenfunctions are given by[1]

$$E=\frac{\hbar^2}{2\mu}\varkappa^2+\hbar\omega\frac{|\nu|+\nu}{2}+\hbar\omega\left(l+\frac{1}{2}\right),\quad l\geqslant0,$$

$$R=e^{-\gamma r^2/2}r^{|\nu|}F(-l,|\nu|+1;\gamma r^2),\quad \gamma=\mu\omega/2\hbar.$$

Since $a|\varkappa|\ll1$, it follows that at $m<0$ the dependence of the electron energy on its velocity $v=\hbar\varkappa/\mu$ along the dislocation is the usual parabolic one. At $m\geqslant0$ this is

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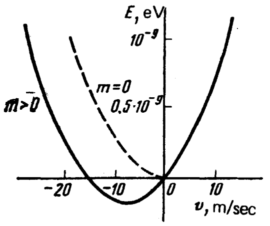

FIG. 1.

no longer the case (see the figure). At $m=0$, the plot of the electron energy against its velocity along the dislocation has a kink at the point $v=0$. On the other hand, if $m>0$ then the minimum of the energy is reached at the drift velocity $-a\omega$, and not at $v=0$. The numbers marked on the figure correspond to $a=0.5\times10^{-8}$ cm, $\omega=1.6\times10^{11}$ rad/sec($H=10^4$ G), and $\mu=9.1\times10^{-28}$ g. The depth of the bound state at $m>0$ is very small ($\sim1.8\times10^{-10}$ eV). If $\varkappa=0$, then the screw dislocation exerts no influence on the electron.

The subject touched upon here is related, for example, to Refs. 2 and 3. In Ref. 2 is considered the dynamics of an electron near a linear defect with axisymmetrical potential $V\sim1/r$. Such a potential offers no advantages to any of the two directions long the defect. In Ref. 3 is considered the case of a screw dislocation. Allowance for the anisotropy of the conductivity tensor leads here to spiral trajectories of the electron when moving along the dislocation, leading to the prediction that a weak magnetic moment appears, parallel to the dislocation when an electron is made to flow along the dislocation. This agrees with our results, since the states of the electron with $m>0$ and $m<0$ are not on a par at $v\neq0$.

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Calculation of critical exponents by quantum field theory methods

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The Gell-Mann–Low function and the anomalous dimensionalities of the quantum-field model $\mathcal{L}_{\text{int}}=-(4\pi)^2g(\varphi^2)^2/4!$ are calculated in a four-loop approximation in the dimensional renormalization formalism. They are used to determine the coefficients of the ϵ expansion for the critical exponents up to the degree ϵ^4 inclusive. To reduce the series of the ϵ expansion, a summation method is used that includes a modified Borel transformation and conformal mapping. The obtained critical exponents are in good agreement with experiment and with results of other theoretical approaches.

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1. INTRODUCTION

The far-reaching analogies between statistical physics and quantum field theory[1] can be used effectively to obtain quantitative predictions concerning the character of the behavior of statistical systems in the vicinity of the phase-transition point.[2] The decisive role in this approach is played by the renormalization-group[3] and ε-expansion[4] methods. On the basis of a calculation of the usual quantum-field Feynman diagram of the φ^4 model in a space of $4-2\varepsilon$ dimensions, and of the solution of the renormalization-group equations, the critical exponents of the phase transitions are presented in the form of series in powers of ε, with the physical (three-dimensional) case corresponding to a value $\varepsilon=1/2$.

The greatest progress in this direction was made by Gurevich and Firsov[5] and by Levinson[6] who succeeded in calculating the contributions of the three-loop and some of the four-loop diagrams. However, in view of the asymptotic character of the obtained series in ε, further progress in this direction presupposes not only inclusion of diagrams of ever increasing order, but also the use of methods for "improving" and summing the asymptotic series. The realization of this program is the purpose of the present paper.

Recently, a number of workers[7,8] have developed simple and quite effective methods of calculating contributions of Feynman diagrams to the renormalization-group functions. The use of this technique has enabled us to

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