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Diamagnetism of cyclotron waves in plasma

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The nonlinear diamagnetic correction to the magnetic field, needed to account for the high-frequency pressure of a wave packet on the plasma, is obtained for potential cyclotron waves (Bernstein modes) in the case of arbitrary spatial dispersion, with account taken of the nonlinearity of the cyclotron resonance. With allowance for this correction, simplified equations are obtained for the evolution of the cyclotron waves. In the case of ion–cyclotron oscillations on the first harmonic, numerical methods are used to obtain the stationary solutions of these equations. In the one-dimensional case of the solution is a truncated periodic wave, while the cylindrical-symmetry case of the solution is a cyclotron soliton stretched along the external magnetic field. The formation of a magnetic well leads to self-localization, as a result of which the high-frequency pressure of the cyclotron waves can become comparable with the plasma pressure even in the case of weak instability and when the region of existence of the instability is small. This leads to anomalous resistance and to an increase of the particle energy in a direction perpendicular to the magnetic field. Similar effects might be observed in a magnetized semiconductor plasma.

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1. INTRODUCTION

A phenomenon intensively investigated during the last decade in plasma theory is the abrupt increase in the interaction between the Fourier components of the oscillations when the energy density of the latter reaches a certain threshold—the limit of applicability of the weak-turbulence approximation. As indicated in Refs. 1–4 and elsewhere, in the case of Langmuir waves this amplification leads to strongly nonlinear effects, namely to wave collapse or to formation of a set of solutions.

It is of interest to ascertain whether similar effects occur for cyclotron waves, which play very frequently an important role in plasma behavior. This is evidenced, in particular, by the experimental observation of these waves in tokamaks[5,6] and in the auroral region of the earth's magnetosphere,[7] from which intense radiation of electromagnetic waves was observed in the electron cyclotron frequency band (the kilometer radiation of the earth).[8,9] In addition, it was shown theoretically that in an isothermal plasma, in which no ion sound can build up, the principal mechanism of the anomalous resistance can be the interaction of the electrons with the ions via buildup of ion-cyclotron waves.[10]

The first step in the study of strongly nonlinear cyclotron wave is to obtain simplified equations that take into account only the principal linear and nonlinear effects. In Langmuir waves, owing to the decisive dependence of the Langmuir frequency on the plasma density, the main mechanism that leads to the aforementioned strongly nonlinear effects is the formation of density wells in the localization region of the wave packet. The frequency of the cyclotron waves depends mainly on the external magnetic field. Therefore, as shown in Ref. 11 and as will be shown more accurately and in greater detail in the the present paper, the main nonlinear mechanism that determines the behavior of the cyclotron waves is the formation of wells of a constant magnetic field in the region of localization of the cyclotron waves.

The decrease of the magnetic field in the region of localization of the wave packets, called high-frequency (HF) diamagnetism,[11] was discussed in Refs. 12–14. A self-focusing influence of HF diamagnetism on Alfven waves was observed in Ref. 15.

The HF diamagnetism is caused by the diamagnetic current produced in the plasma in a direction perpendicular to the constant magnetic field in the region of

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localization of the wave packet under the influence of the HF pressure of the latter (ponderomotive force) on the plasma. In an earlier[12-14] derivation of an expression for the diamagnetic attenuation of the magnetic field in the packet no account was taken of the influence of the spatial dispersion—the nonlocality of the interaction of the electric field with the medium. This expression is therefore not suitable for cyclotron waves, whose interaction with the plasma has a nonlocal character.

In the present paper we obtain by direct calculation an expression for the diamagnetic field δH, with account taken of spatial dispersion and of a new effect, namely the nonlinearity of the cyclotron resonance for potential cyclotron waves (Bernstein modes). Let the wave-packet frequency be close to $n\omega_j$, where n is an integer and ω_j is the cyclotron frequency of the particles of species j ($j=e$, i denotes respectively electrons and ions), and the characteristic wave number of the packet is much less than the reciprocal Larmor radius ρ_j^{-1}. Then, as follows from the third part of the present paper, we have

$$\delta H=-\frac{r_{Dj}^{-2}}{4n!\,B_0}|\chi|^2, \qquad (1.1)$$

where r_{dj} is the Debye radius of the electrons or ions, B_0 is the external magnetic field and is directed along the Oz axis. If the electric potential in the packet is represented in the form

$$\varphi={}^1/_2[\psi(\mathbf{r},t)\exp(-in\omega_{j0}t)+\text{c.c.}]\ , \qquad (1.2)$$

where ψ is a slow function of the time and ω_{j0} is the cyclotron frequency in the external magnetic field, then the function χ is connected with ψ by the nonlinear equation

$$\left(i\frac{\partial}{\partial\tau}-h\right)\chi=\hat{P}_n\psi,$$

where

$$\tau=n|\omega_{j0}|t,\quad h=\frac{\delta H}{B_0},\quad \hat{P}_n=\rho_j^n\left(\frac{\partial}{\partial x}\pm i\frac{\partial}{\partial y}\right)^n \qquad (1.3)$$

(the plus and minus signs are used for ions and electrons, respectively).

In the fourth part of the paper, taking into account the diamagnetic field (1.1) that forms a magnetic well, we obtain a simplified nonlinear equation for the amplitude of the potential ψ. In the case of electron cyclotron waves this equation takes the form

$$\Delta_\perp\left(i\frac{\partial\psi}{\partial\tau}-\alpha_e|\hat{P}_{n-1}|^2\psi\right)=\operatorname{div}h\nabla_\perp\psi, \qquad (1.4)$$

where

$$\alpha_e=\frac{(-1)^{n-1}}{2n!}\frac{\omega_{pe}^2}{\omega_{pe}^2+\omega_{e0}^2}$$

ω_{pe} is the electron plasma frequency. We assume that the dependence of ψ on the coordinate z is much weaker than the dependence on the transverse ($\perp$) coordinates relative to the magnetic field. For the ion cyclotron waves we obtain

$$\Delta_\perp\left(i\frac{\partial\psi}{\partial\tau}-\alpha_i|\hat{P}_n|^2\psi\right)=\operatorname{div}h\nabla_\perp\psi, \qquad (1.5)$$

where

$$\alpha_i=\frac{(-1)^n}{n!}\frac{T_{\parallel e}}{T_{\perp i}},$$

$T_{\parallel e}$ and $T_{\perp i}$ are respectively the longitudinal electron and transverse ion frequencies relative to the magnetic field.

The derived equations have solutions in the form of two-dimensional solitons stretched along the magnetic field, as well as periodic waves (see Sec. 5 of the present paper, Figs. 2 and 3).

The ability of cyclotron waves to become self-localized in the form of solitons makes it possible to attain a high wave energy-density under conditions of a small growth rate or a small instability region. This can explain the highly inhomogeneous spatial distribution of cyclotron oscillations observed in the auroral region.[7] The distribution discontinuity is so sharp, that an impression is gained that a shock wave is present with a front along $\mathbf{B}_0$. The relative amplitude of the density oscillations reaches 20–30%.

2. CALCULATION OF THE DIAMAGNETIC FIELD

To calculate the diamagnetic current and hence the diamagnetic field it is necessary first to find the distribution of the plasma particles in the magnetic field in second order in the oscillation amplitude. Solving the Vlasov kinetic equation by successive approximations, we get[16]

$$f_j^{(2)}=\frac{e_j^2}{m_j^2}\sum_{\substack{\mathbf{k}'\mathbf{k}''\\ \omega'\omega''}}\varphi_{\mathbf{k}'\omega'}\varphi_{\mathbf{k}''\omega''}\exp[i(\mathbf{k}'+\mathbf{k}'')\mathbf{r}-i(\omega'+\omega'')t]$$
$$\times\sum_{npqs}\exp[-i(n-p+q-s)\theta+i(q-s)\alpha_{\mathbf{k}'+\mathbf{k}''}+i(n-p)\alpha_{\mathbf{k}''}]$$
$$\times\hat{L}^{npqs}_{j\mathbf{k}'+\mathbf{k}'',\mathbf{k}',\omega'+\omega''}\hat{L}^{np}_{j\mathbf{k}'',\omega''}f_j^{(0)}. \qquad (2.1)$$

The operatores $\hat{L}$ are given here by

$$\hat{L}^{np}_{j\mathbf{k},\omega}=\frac{J_{n\mathbf{k}}J_{p\mathbf{k}}}{\omega-n\omega_j-k_zv_z}\left(\frac{n\omega_j}{v_\perp}\frac{\partial}{\partial v_\perp}+k_z\frac{\partial}{\partial v_z}\right),$$
$$\hat{L}^{npqs}_{j\mathbf{k},\mathbf{k}',\omega}=\frac{J_{s\mathbf{k}}}{\omega-(n-p+q)\omega_j-k_zv_z}\left[J_{q+1\mathbf{k}}\frac{k_\perp'}{2}\left(\frac{\partial}{\partial v_\perp}+\frac{n-p}{v_\perp}\right)\exp[i(\alpha_\mathbf{k}-\alpha_{\mathbf{k}'})]+J_{q-1\mathbf{k}}\frac{k_\perp'}{2}\left(\frac{\partial}{\partial v_\perp}-\frac{n-p}{v_\perp}\right)\right.$$
$$\left.\times\exp[-i(\alpha_\mathbf{k}-\alpha_{\mathbf{k}'})]+J_{q\mathbf{k}}k_z'\frac{\partial}{\partial v_z}\right]. \qquad (2.2)$$

In (2.1) and (2.2), $\varphi_{\mathbf{k}\omega}$ is the Fourier component of the oscillation potential, e_j and m_j are charge and mass of the particles of species j, $f_j^{(0)}$ is the unperturbed distribution function $J_{nk}=J_n(k_\perp v_\perp/\omega_j)$ is a Bessel function of order n, θ and α_k are the azimuthal angles in the space of the velocities and wave vectors in a plane perpendicular to the external magnetic field, and n, p, q, and s are integers.

With the aid of (2.1) we obtain the nonlinear current

$$\mathbf{j}^{(2)}=\sum e_j\int\mathbf{v}f_j^{(2)}d\mathbf{v}. \qquad (2.3)$$

To obtain the diamagnetic field, we average (2.3) over the HF oscillations and assume that $\langle j^{(2)}\rangle$ depends little on the time (the angle brackets $\langle\cdots\rangle$ denote averaging over the HF oscillations). Using next Maxwell's equation

$$\text{rot}\langle \mathbf{H}^{(2)}\rangle = \frac{4\pi}{c}\langle \mathbf{j}^{(2)}\rangle, \tag{2.4}$$

we can determine the diamagnetic field $\langle \mathbf{H}^{(2)}\rangle$.

We obtain now the quasistationary distribution function $\langle f_j^{(2)}\rangle$. It is obvious that in this case the largest contribution to (2.1) is made by terms for which the equality $n-p+q=0$ is satisfied. Assuming satisfaction of the condition

$$|\omega - n\omega_j| \gg k_z v_{T\parallel j}, \tag{2.5}$$

where $v_{T\parallel j}$ is the thermal velocity of the particles along the external magnetic field, we neglect in (2.1) the terms connected with the longitudinal electric field E_z. As a result we get

$$\langle f_j^{(2)}\rangle = \frac{e_j^2}{2m_j^2}\sum_{\substack{\mathbf{k}'\mathbf{k}''\\ \omega'\omega''}} \frac{k_\perp'\langle \varphi_{\mathbf{k}'\omega'}\varphi_{\mathbf{k}''\omega''}\rangle \exp[i(\mathbf{k}'+\mathbf{k}'')\mathbf{r} - i(\omega'+\omega'')t]}{[\omega'+\omega''-(k_z'+k_z'')v_z](\omega''-n\omega_j-k_z''v_z)}$$
$$\times \sum_{nsq} \exp[is(\theta - \alpha_{\mathbf{k}'+\mathbf{k}''}) + iq(\alpha_{\mathbf{k}'+\mathbf{k}''} - \alpha_{\mathbf{k}''})]$$
$$\times J_{s\mathbf{k}'+\mathbf{k}''}J_{q\mathbf{k}'+\mathbf{k}''}\Big[J_{q+n+1\mathbf{k}'}\Big(\frac{\partial}{\partial v_\perp} - \frac{n}{v_\perp}\Big)\exp[i(\alpha_{\mathbf{k}'} - \alpha_{\mathbf{k}''})]$$
$$+ J_{q+n-1\mathbf{k}'}\Big(\frac{\partial}{\partial v_\perp} + \frac{n}{v_\perp}\Big)\exp[-i(\alpha_{\mathbf{k}'} - \alpha_{\mathbf{k}''})]$$
$$+ J_{q+n\mathbf{k}'}\frac{k_\perp''}{\omega_j}2i\sin(\alpha_{\mathbf{k}'} - \alpha_{\mathbf{k}''})\Big]J_{n\mathbf{k}''}\frac{n\omega_j}{v_\perp}\frac{\partial f_j^{(0)}}{\partial v_\perp}. \tag{2.6}$$

In the derivation of (2.6) we have used the following relations between the Bessel functions:

$$J_{n\pm1}(x) = \frac{n}{x}J_n(x) \mp \frac{dJ_n(x)}{dx}.$$

Summing further in (2.6) over q with the aid of the formula

$$\sum_q J_{q\mathbf{k}}J_{q+n\mathbf{k}'}\exp[iq(\alpha_{\mathbf{k}} - \alpha_{\mathbf{k}'})] = J_{n\mathbf{k}'-\mathbf{k}}\exp[in(-\alpha_{\mathbf{k}'-\mathbf{k}} + \alpha_{\mathbf{k}'})]$$

and symmetrizing, we get

$$\langle f_j^{(2)}\rangle = -\frac{e_j^2}{2m_j^2}\sum_{\substack{\mathbf{k}'\mathbf{k}''\\ \omega'\omega''}}\langle \varphi_{\mathbf{k}'\omega'}\varphi_{\mathbf{k}''\omega''}\rangle \exp[i(\mathbf{k}'+\mathbf{k}'')\mathbf{r}$$
$$-i(\omega'+\omega'')t]\sum_{ns} J_{s\mathbf{k}'+\mathbf{k}''}\Big[\frac{n\omega_j}{v_\perp}\frac{\partial}{\partial v} - i\frac{k_\perp'k_\perp''}{\omega_j}\sin(\alpha_{\mathbf{k}'}$$
$$-\alpha_{-\mathbf{k}''})\Big]\frac{J_{n\mathbf{k}'}J_{n\mathbf{k}''}}{(\omega'-n\omega_j)(\omega''+n\omega_j)}\frac{n\omega_j}{v_\perp}\frac{\partial f_j^{(0)}}{\partial v_\perp}$$
$$\times \exp[is(\theta - \alpha_{\mathbf{k}'+\mathbf{k}''}) + in(\alpha_{\mathbf{k}'} - \alpha_{-\mathbf{k}''})]. \tag{2.7}$$

To find the electric current, as seen from (2.3), it suffices to retain in (2.7) the term with $s=1$, after which we can determine with the aid of (2.3) the complex current $\langle j_x^{(2)} - ij_y^{(2)}\rangle$. Substituting the latter in Maxwell's equation (2.4), we obtain the nonlinear quasistationary magnetic field excited by the HF oscillations:

$$\langle H_z^{(2)}\rangle = -\sum_{\substack{\mathbf{k}'\mathbf{k}''\\ \omega'\omega''}}\langle \varphi_{\mathbf{k}'\omega'}\varphi_{\mathbf{k}''\omega''}\rangle \exp[i(\mathbf{k}'+\mathbf{k}'')\mathbf{r}$$
$$-i(\omega'+\omega'')t]\sum_j \frac{\omega_{pj}^2\omega_j}{2B_0}\sum_n (\mathbf{k}'+\mathbf{k}'')_\perp^{-1}\Big[\frac{\partial}{\partial(\mathbf{k}'+\mathbf{k}'')_\perp}$$
$$\times(\mathbf{k}'+\mathbf{k}'')_\perp S_n^{(1)} + i\frac{k_\perp'k_\perp''}{n\omega_j^2}S_n^{(2)}\sin(\alpha_{\mathbf{k}'} - \alpha_{-\mathbf{k}''})\Big]$$
$$\times \frac{(n\omega_j)^2}{(\omega'-n\omega_j)(\omega''+n\omega_j)}\exp[in(\alpha_{\mathbf{k}'} - \alpha_{-\mathbf{k}''})], \tag{2.8}$$

where ω_{pj} is the Langmuir frequency. The quantities $S_n^{(p)}$, $p=1,2$ are given by

$$S_n^{(p)} = 2\pi\int_0^\infty dv_\perp v_\perp^{2(p-1)}J_{1\mathbf{k}'+\mathbf{k}''}J_{n\mathbf{k}'}J_{n\mathbf{k}''}\frac{1}{v_\perp}\frac{\partial f_j^{(0)}}{\partial v_\perp}. \tag{2.9}$$

The calculation of $S_n^{(p)}$ is given in Appendix 1. Because of the averaging over the HF oscillations, the summation over ω' and ω'' in (2.8) is carried out over the region $|\omega'+\omega''| \ll \omega', \omega''$.

Expression (2.8) obtained by us for the change of the magnetic field in the wave packet is general, since no restrictions whatever were imposed on the transverse wavelength of the oscillations. It is this expression that we must use to obtain the nonlinear correction to the cyclotron frequency when investigating nonlinear cyclotron waves.

3. SOME PARTICULAR EXPRESSIONS FOR THE DIAMAGNETIC FIELD

In this section we consider Eq. (2.8) in some limiting cases. Let the characteristic wavelength of the oscillations be large enough: $k_\perp\rho_j \ll 1$. Taking into account only the first term of the expansion of the Bessel function at small arguments, we obtain

$$\langle H_{zj}^{(2)}\rangle = \frac{\rho_j^{2(n-1)}}{2n!B_0}\sum_{\substack{\mathbf{k}'\mathbf{k}''\\ \omega'\omega''}}(k_\perp'k_\perp'')^n\langle \varphi_{\mathbf{k}'\omega'}\varphi_{\mathbf{k}''\omega''}\rangle \frac{n^2\omega_{pj}^2}{(\omega'-n\omega_j)(\omega''+n\omega_j)}e^{i\phi}, \tag{3.1}$$

where

$$\phi = n(\alpha_{\mathbf{k}'} - \alpha_{-\mathbf{k}''}) + (\mathbf{k}'+\mathbf{k}'')\mathbf{r} - (\omega'+\omega'')t,$$
$$\rho_j = (T_{\perp j}/2m_j\omega_j^2)^{1/2},$$

and the distribution function $f_j^{(0)}$ is assumed Maxwellian. Equation (3.1) determines the diamagnetic field produced by the particles of species j when the oscillation frequency is close to the n-th harmonic of the cyclotron frequency.

It is convenient to introduce in (3.1) the function $\tilde{\chi}_{\mathbf{k}\omega}$ defined by

$$\frac{\omega - n\omega_j}{n\omega_{j0}}\tilde{\chi}_{\mathbf{k}\omega} = 2i^n\rho_j^n k_\perp^n\varphi_{\mathbf{k}\omega}\exp(in\alpha_{\mathbf{k}}). \tag{3.2}$$

Changing over in (3.1) and (3.2) to the coordinate-time representation and taking (1.2) into account, we arrive at Eqs. (1.1) and (1.3)($\delta H \equiv \langle H_{zj}^{(2)}\rangle$, $\chi = \tilde{\chi}e^{-i\tau}$). It is easily seen that the nonlinear correction h in (1.3) is the result of allowance for the diamagnetic decrease of the cyclotron frequency in the resonant denominators in (3.1).

We consider now formula (2.8) for short-wave cyclotron oscillations with $k_\perp\rho_j > 1$. In this case we must first integrate in (2.9) over the velocities and only then can we use the asymptotic Bessel functions. Assume that the Fourier component $\varphi_{k\perp}$ of the potential depends only on the modulus of the wave vector and does not depend on the angle $\alpha_{\mathbf{k}}$. This assumption is equivalent to the expansion

$$\varphi(\mathbf{r}_\perp) = 2\pi\int_0^\infty \frac{dk_\perp k_\perp}{(2\pi)^2}\varphi_{k\perp}J_0(k_\perp r_\perp),$$

i.e., the potential should have cylindrical symmetry in the direction transverse to the external magnetic field, or must have an explicit dependence on the azimuthal angle. Changing over in (2.8) to integration over the wave vectors and using the result (A.27), we get

$$\langle H_{zj}^{(2)}\rangle=\frac{\pi}{2B_0\rho_j^4}\sum_{\omega'\omega''}\frac{n^2\omega_{pj}^2}{(\omega'-n\omega_j)(\omega''+n\omega_j)}$$

$$\times\int_0^\infty\int_0^\infty\frac{dk_\perp'dk_\perp''}{(2\pi)^4}\langle\varphi_{k_\perp'\omega'}\varphi_{k_\perp''\omega''}\rangle J_0(|k_\perp'-k_\perp''|r_\perp)$$

$$\times I_1[2(k_\perp'-k_\perp'')^2\rho_j^2]\exp[-2(k_\perp'-k_\perp'')^2\rho_j^2]. \quad (3.3)$$

Here, just as in (3.1), $n>0$. It must be noted that this equation is valid in the case when the nonlinearity in the resonant denominator can be neglected.

We have thus considered two limiting cases at $k\perp\rho_j\ll1$ and $k\perp\rho_j>1$—of formula (2.8) obtained when the condition (2.5) is satisfied. However, if the electrons have a Boltzmann distribution in the field of the HF oscillations, $\omega<k_z v_{T\parallel e}$ (for example, in the case of ion-cyclotron waves), then Eq. (2.8) does not hold for them. Considering Eqs. (2.1)–(2.4) for hot electrons, we obtain at $k\perp\rho_e\ll1$:

$$\langle H_{ze}^{(2)}\rangle=-\frac{\omega_{pe}^2m_eT_{\perp e}}{4B_0T_{\parallel e}^2}|\psi|^2. \quad (3.4)$$

The contribution of the electrons to the diamagnetic field may turn out to be substantial for ion-cyclotron oscillations at $k\perp\rho_i>1$:

4. EQUATION OF CYCLOTRON WAVES

We now obtain an equation that describes cyclotron waves of finite amplitude. The dielectric constant for the potential cyclotron oscillations takes, when condition (2.5) is satisfied, the form

$$\varepsilon(\mathbf{k},\omega)=1+\frac{g_j}{z_j}\left[\left(\frac{T_{\perp i}}{T_{\parallel e}}\right)_{j=i}+1-I_0(z_j)\exp(-z_j)-I_n(z_j)\exp(-z_j)\frac{\omega}{\omega-n\omega_j}\right], \quad (4.1)$$

where $g_j=\omega_{pj}^2/\omega_j^2$, $z_j 2k\perp^2\rho_j^2$. The term $(T\perp_i/T_{\parallel e})_{j=i}$ need be taken into account only for ion-cyclotron oscillations, and for electrons we assume in this case that the conditions $\omega<k_z v_{T\parallel e}$ and $k\perp\rho_e\ll1$ are satisfied.

The solution of the dispersion equation $\varepsilon(\mathbf{k},\omega)=0$ takes the form

$$\omega=n\omega_j(1+R_j),\ R_j\ll1,$$

$$R_j=\frac{I_n(z_j)\exp(-z_j)}{(T_{\perp i}/T_{\parallel e})_{j=i}+z_j/g_j+1-I_0(z_j)\exp(-z_j)}. \quad (4.2)$$

With the aid of expression (4.2) and of representation (1.2) we easily obtain an equation for the amplitude ψ:

$$\Delta_\perp\left(i\frac{\partial\psi}{\partial\tau}-\hat{R}_j\psi\right)-\operatorname{div}(h\nabla_\perp\psi)=0. \quad (4.3)$$

In the general case we have $h=h_e+h_i$. The operator R_j is defined as follows:

$$\hat{R}_j\psi=\sum_{\mathbf{k}_\perp}R_j(k_\perp)\psi_{\mathbf{k}_\perp}\exp(i\mathbf{k}_\perp\mathbf{r}). \quad (4.4)$$

Concrete expressions for this operator at $k\perp\rho_j\ll1$ are given in the Introduction [see (1.4) and (1.5)].

5. STATIONARY SOLUTIONS

The derived equations (1.1), (1.3) and (1.4) or (1.5) have stationary solutions—either one-dimensional or with radial dependence in a cylindrical frame. We consider the case of ion-cyclotron waves on the first harmonic ($n=1$), since it is the simplest and most important. We seek the solution of Eqs. (1.1), (1.3), and (1.5) in the form

$$\psi=\frac{2^{3/4}}{(g_i)^{1/4}\varkappa}A^2B_0f(\rho)\exp(iA^2\tau),\quad h=-A^2F(\rho), \quad (5.1)$$

where

$$\rho=\varkappa r_\perp,\quad \varkappa=\left(\frac{T_{\perp i}}{T_{\parallel e}}\right)^{1/2}A\rho_i^{-1},\qquad A^2\ll1.$$

Substituting (5.1) in this system, we get

$$\operatorname{div}(1-\Delta)\mathbf{E}=\operatorname{div}F\mathbf{E}, \quad (5.2)$$

$$F(1-F)^2=D^2=|E_x+iE_y|^2, \quad (5.3)$$

here $\mathbf{E}=\nabla f$.

According to (5.1), the function F determines the spatial dependence of the depth of the magnetic well. It follows from (5.3) that in the stationary case the dependence of the well depth on the electric-field amplitude D is not single-valued. Under real conditions a relation with $dF/dD>0$, marked in Fig. 1 by the solid line, is realized. The section aa' cannot be realized, inasmuch as the well depth on it decreases with increasing D. On the section ab, in the vicinity of the exact cyclotron resonance ($F=1$), the depth of the magnetic well can change jumpwise. Thus, the point b is a branch point.

If all the quantities in (5.2) depend only on the coordinate $\xi(\xi=\varkappa x)$ then $\mathbf{E}$ is directed along ξ. In this case the solution of the system (5.2) and (5.3) is shown in Fig. 2. At $\xi<10$ we have a periodic solution in which the plot $a'bc$ on Fig. 1 is realized. At $\xi=10.7$ the depth of the well at the point b breaks away to the point a and subsequently tends to zero with increasing ξ.

In the case when f in (5.1) depends only on the radius $\rho(\rho\varkappa r_\perp)$ in the cylindrical frame, $\mathbf{E}$ is directed along the radius. Then (5.2) and (5.3) have the solution shown in Fig. 3.

It is seen from the figures that on the boundaries of the localized stationary solutions we must have a discontinuity of the magnetic field as well as of the HF pressure (owing to the resonant dependence of the latter on the magnetic field), while the electric field remains continuous. What is realized is a tangential discontinuity, analogous to the known solution for MHD equations,[12] with the magnitude of the magnetic-field discontinuity given by (5.3) and by Fig. 1. The total pressure remains continuous because of the discontinuity of

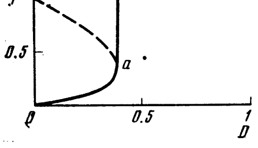

FIG. 1. Dependence of the depth of the magnetic well on the electric-field amplitude in the stationary case (in dimensionless units).

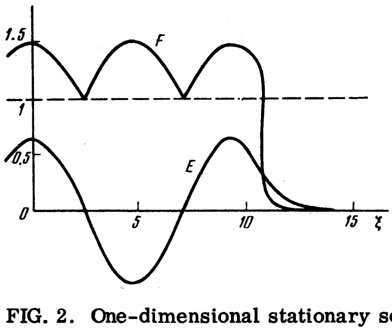

FIG. 2. One-dimensional stationary solution of the equations of ion-cyclotron waves at the first harmonic. It is seen that the cyclotron waves can lead to formation of a periodic structure of the constant magnetic field in the form of domains (curve F, in dimensionless units).

the HF pressure.

The obtained stationary solution can be improved by taking into account the weak dependence on the coordinate z and the presence of a small increment in the next order of perturbation theory, as was done, e.g., in Ref. 3 for a Langmuir soliton.

6. CONCLUSION

Cyclotron waves can build up in plasma for a number of reasons. For example, by particle beams along the magnetic field, as well as in the presence of fast particles transverse to the magnetic field with a loss cone on the distribution function.[17]

Let us discuss briefly the possible kinetic effects produced in a plasma by the appearance of cyclotron solitons, in the case when the plasma pressure is lower than the pressure of the constant magnetic field. Since the HF pressure in the soliton is confined by the magnetic pressure, the HF pressure can approach the value of the plasma pressure or may even exceed it because of the large length of the soliton along the magnetic field, as well as because of the longitudinal inhomogeneity of this field.

The large amplitude of the oscillations of the electric field in the soliton can lead to heating and anomalous resistance of the plasma. The point is that the solitons form along the magnetic field trains of traveling waves or else standing waves. The potential energy of the electrons in such a wave can exceed the kinetic energy, so that in some regimes almost all the electrons turn out to be trapped in the electronic or ionic cyclotron solitons. This can inhibit the growth of the longitudinal current. The electric field energy then goes over, via the cyclotron resonance in the wave, into transverse kinetic energy of the trapped electrons and ions. It is possible that this mechanism can explain the appearance of particles having large transverse energy, which were registered in tokamaks under runaway-electron conditions,[5] as well as the strong drop of the electric potential along the earth's magnetic field in the auroral region, which was observed in satellites.[7]

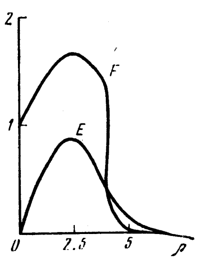

FIG. 3. Solution of the equation of the ion-cyclotron equations in the form of an axisymmetric soliton elongated along the magnetic field (in dimensionless units).

We note in conclusion that the considered effects, which lead to the appearance of cyclotron solitons, can occur in the plasma of semicondictors, where cyclotron waves can propagate.

APPENDIX 1

We integrate with respect to the velocities in the expression

$$S_n^{(p)}=2\pi\int_0^\infty dv_\perp v_\perp^{2(p-1)}\,J_{1\mathbf{k}'+\mathbf{k}''}J_{n\mathbf{k}'}J_{n\mathbf{k}''}\frac{1}{v_\perp}\frac{\partial f_j^{(0)}}{\partial v_\perp},\qquad\text{(A1.1)}$$

where the distribution function $f_j^{(0)}$ is assumed Maxwellian. We apply to (A1.1) the transformation formula 18

$$\int_0^\pi\frac{J_\nu\{(\alpha^2+\beta^2-2\alpha\beta\cos\psi)^{1/2}\}}{(\alpha^2+\beta^2-2\alpha\beta\cos\psi)^{\nu/2}}\sin^{2\nu}\psi\,d\psi=2^\nu\Gamma\left(\nu+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)\frac{J_\nu(\alpha)}{\alpha^\nu}\frac{J_\nu(\beta)}{\beta^\nu},\qquad\text{(A1.2)}$$

where $\Gamma(\nu)$ is the gamma function. We express the n-th order Bessel function in the obtained expression in terms of a first-order Bessel function in accord with the formula

$$J_n(x)=(-1)^{n-1}x^{n-1}\left(\frac{\partial}{\partial x}\frac{1}{x}\right)^{n-1}J_1(x),\qquad\text{(A1.3)}$$

after which we apply (A1.2) again. As a result we get

$$S_n^{(p)}=-\frac{m_j^2}{T_{\perp j}^2\omega_j^2}\frac{(\mathbf{k}'+\mathbf{k}'')_\perp(k_\perp'k_\perp'')^n}{2^{n+1}\Gamma(n+1/2)\Gamma^2(1/2)\Gamma(3/2)}\left(-\frac{1}{q}\frac{\partial}{\partial q}\right)^{n-1}\times\int_0^\pi d\psi\sin^{2n}\psi\int_0^\pi d\psi_1\sin^2\psi_1 Q^{-1}\int_0^\infty dv_\perp v_\perp^{2p}J_{1Q}\exp\left(-\frac{m_jv_\perp^2}{2T_{\perp j}}\right).\qquad\text{(A1.4)}$$

Here

$$q^2=k_\perp'^2+k_\perp''^2-2k_\perp'k_\perp''\cos\psi,$$
$$Q^2=(\mathbf{k}'+\mathbf{k}'')_\perp^2+q^2-2(\mathbf{k}'+\mathbf{k}'')_\perp q\cos\psi_1.$$

We integrate in (A1.4) with respect to the velocities with the aid of the formula[18]

$$\int_0^\infty dx\,J_\nu(\beta x)e^{-\alpha x^2}x^{\mu+\nu-1}=\frac{\beta^\nu\Gamma(\nu+\mu/2)\exp(-\beta^2/4\alpha)}{2^{\nu+1}\alpha^{\nu+\mu/2}\Gamma(\nu+1)}\,{}_1F_1\left(1-\frac{\mu}{2},\nu+1,\frac{\beta^2}{4\alpha}\right),\qquad\text{(A1.5)}$$

where ${}_1F_1$ is a confluent hypergeometric function. Then the expressions for $S_n^{(p)}$ take the form

$$S_n^{(1)}=-\frac{(\mathbf{k}'+\mathbf{k}'')_\perp(k_\perp'k_\perp'')^n}{2^{n+1}\Gamma(n+1/2)\Gamma^2(1/2)\Gamma(3/2)\omega_j^3}\left(-\frac{1}{q}\frac{\partial}{\partial q}\right)^{n-1}\times\int_0^\pi d\psi\sin^{2n}\psi\int_0^\pi d\psi_1\sin^2\psi_1\exp(-\lambda_1Q^2\rho_j^2),$$

$$S_n^{(2)}=4\frac{T_{\perp j}}{m_j}\left(1+\frac{1}{2}\frac{\partial}{\partial\lambda_1}\right)S_n^{(1)},\qquad\text{(A1.6)}$$

where $\lambda_1=1$.

We integrate next in (A1.6) with respect to the angle ψ_1, using the formula[18]

$$\int_0^\pi d\psi\,e^{\beta\cos\psi}\sin^{2\nu}\psi=\pi^{1/2}\Gamma\left(\nu+\frac{1}{2}\right)\left(\frac{2}{\beta}\right)^\nu I_\nu(\beta),\qquad\text{(A1.7)}$$

where $I_\nu(\beta)$ is a modified Bessel function. We integrate in the obtained expression with respect to q. We integrate next with respect to the angle ψ. As a result we get

$$S_n^{(1)}=-\frac{1}{4\omega_j^3\rho_j^3}\lambda_1^{-3/2}\exp[-\lambda_1(\mathbf{k}'+\mathbf{k}'')_\perp^2\rho_j^2]\left(\frac{\partial}{\partial\lambda_2}\right)^{-1/2}$$
$$\times J_1\left[2(\mathbf{k}'+\mathbf{k}'')_\perp\rho_j\left(\lambda_1\frac{\partial}{\partial\lambda_2}\right)^{1/2}\right]\lambda_2^{-1}I_n(2\lambda_1\lambda_2k_\perp'k_\perp''\rho_j^2)$$
$$\times\exp[-\lambda_1\lambda_2(k_\perp'^2+k_\perp''^2)\rho_j^2]. \qquad (A1.8)$$

In (A1.8) we have introduced formally the operator $(\partial/\partial\lambda_2)^{1/2}$, in order to be able to express the series in the form J_1; $\lambda_2=1$.

APPENDIX 2

We introduce the quantity [see(2.8)]

$$D_j=-\frac{\omega_j^3}{(\mathbf{k}'+\mathbf{k}'')_\perp}\left[\frac{\partial}{\partial(\mathbf{k}'+\mathbf{k}'')_\perp}(\mathbf{k}'+\mathbf{k}'')_\perp S_n^{(1)}+i\frac{k_\perp'k_\perp''}{n\omega_j^2}\right.$$
$$\left.\times S_n^{(2)}\sin(\alpha_{\mathbf{k}'}-\alpha_{-\mathbf{k}''})\right]\exp[in(\alpha_{\mathbf{k}'}-\alpha_{-\mathbf{k}''})+i(\mathbf{k}'+\mathbf{k}'')\mathbf{r}]. \qquad (A2.1)$$

Substituting here the expressions for $S_n^{(1)}$ and $S_n^{(2)}$ obtained in Appendix 1, we can represent D_j in the form

$$D_j=\frac{1}{2\rho_j^2}\left(\frac{\partial^2}{\partial\lambda_2\partial\lambda_3}\right)^{-1/2}I_1\left[2\left(\frac{\partial^2}{\partial\lambda_2\partial\lambda_3}\right)^{1/2}\right]\left[\frac{\partial}{\partial\lambda_3}\lambda_3\right.$$
$$\left.+\frac{1}{n}k_\perp'k_\perp''\rho_j^2\left(2+\frac{\partial}{\partial\lambda_1}\right)2i\sin(\alpha_{\mathbf{k}'}-\alpha_{-\mathbf{k}''})\right](\lambda_1\lambda_2)^{-1}$$
$$\times I_n(2\lambda_1\lambda_2k_\perp'k_\perp''\rho_j^2)\exp[-\lambda_1\lambda_2(k_\perp'^2+k_\perp''^2)\rho_j^2$$
$$-\lambda_1\lambda_3(\mathbf{k}'+\mathbf{k}'')_\perp^2\rho_j^2]\exp[in(\alpha_{\mathbf{k}'}-\alpha_{-\mathbf{k}''})+i(\mathbf{k}'+\mathbf{k}'')\mathbf{r}], \qquad (A2.2)$$

where $\lambda_3=1$.

We obtain the value of $\langle D_j\rangle$ averaged over the angles $\alpha_{\mathbf{k}'}$ and $\alpha_{\mathbf{k}''}$:

$$\langle D_j\rangle=\frac{1}{(2\pi)^2}\int_0^{2\pi}d\alpha_{\mathbf{k}'}\int_0^{2\pi}d\alpha_{\mathbf{k}''}D_j. \qquad (A2.3)$$

To this end we expand in (A2.2) the exponentials $\exp[-\lambda_1\lambda_3 2k_\perp'k_\perp''\rho_j^2\cos(\alpha_{\mathbf{k}'}-\alpha_{\mathbf{k}''})]$ and $\exp[i(\mathbf{k}'+\mathbf{k}'')\mathbf{r}]$ in Bessel-function series

$$e^{x\cos\alpha}=\sum_n I_n(x)e^{in\alpha},\qquad e^{ix\sin\alpha}=\sum_n J_n(x)e^{in\alpha} \qquad (A2.4)$$

and substitute the obtained expression in (A2.3). Integrating with respect to the angles $\alpha_{\mathbf{k}'}$ and $\alpha_{\mathbf{k}''}$, we get

$$\langle D_j\rangle=\frac{1}{2\rho_j^2}\left(\frac{\partial^2}{\partial\lambda_2\partial\lambda_3}\right)^{-1/2}I_1\left[2\left(\frac{\partial^2}{\partial\lambda_2\partial\lambda_3}\right)^{1/2}\right]\sum_p J_{n+p}(k_\perp'r_\perp)$$
$$\times J_{n+p}(k_\perp''r_\perp)\left[\frac{\partial}{\partial\lambda_3}\lambda_3+\frac{p}{n}\left(2+\frac{\partial}{\partial\lambda_1}\right)(\lambda_1\lambda_3)^{-1}\right]$$
$$\times(\lambda_1\lambda_2)^{-1}I_p(2\lambda_1\lambda_3k_\perp'k_\perp''\rho_j^2)I_n(2\lambda_1\lambda_2k_\perp'k_\perp''\rho_j^2)$$
$$\times\exp[-\lambda_1(\lambda_2+\lambda_3)(k_\perp'^2+k_\perp''^2)\rho_j^2]. \qquad (A2.5)$$

We obtain now the limiting values of (A2.5). With the condition $k_\perp\rho_j\ll1$ satisfied, we retain in (A2.5) the term with $p=0$. Taking into account the first term of the series expansion of the modified Bessel function I_n and differentiating with respect to λ_i, $i=1,2,3$, we obtain

$$\langle D_j\rangle=\frac{1}{2n!\rho_j^2}(k_\perp'k_\perp''\rho_j^2)^nJ_n(k_\perp'r_\perp)J_n(k_\perp''r_\perp). \qquad (A2.6)$$

With the condition $k_\perp\rho_j>1$ satisfied, we use the asymptotic expressions for modified Bessel functions at large arguments. Applying the differentiation operation with respect to λ_i to the resultant exponential $\exp[-\lambda_1(\lambda_2+\lambda_3)(k_\perp'-k_\perp'')^2\rho_j^2]$ we get

$$\langle D_j\rangle=\frac{1}{8\pi k_\perp'k_\perp''\rho_j^4}J_0(|k_\perp'-k_\perp''|r_\perp)I_1[2(k_\perp'-k_\perp'')^2\rho_j^2]$$
$$\times\exp[-2(k_\perp'-k_\perp'')^2\rho_j^2]. \qquad (A2.7)$$

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