Contribution to the theory of normal ionizing shock waves

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A theory of normal ionizing shock waves is developed and their structures are calculated. It is shown that the only stationary solution in a shock tube is a fast switch-on ionization shock wave. The evolution of normal ionizing shock waves as they are produced in an electromagnetic shock tube is considered. A supplementary boundary condition is obtained, wherein the electric field ahead of the front is equal to the gas breakdown field, and it is shown that the Chapman-Jouguet condition used in earlier studies is valid only for the limiting case of the slowest shock wave that does not become detached from the piston. The developed theory is used to calculate the plasma parameters for normal ionizing shock waves in hydrogen and helium, with account taken of the energy lost to ionization and dissociation. A comparison of the calculations with the experimental data shows good agreement between theory and experiment.

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1. INTRODUCTION

Normal shock waves are defined as shock waves propagating along the magnetic field, i.e., the magnetic-field vector ahead of the wave front is normal to the front. If such a shock wave propagates in an ionized gas, then we are dealing with a magnetohydrodynamic (MHD) normal shock wave. If the gas ahead of the shock-wave front is nonconducting, and becomes conducting behind the front as a result of heating, the shock wave is called ionizing. As a result of the ionization of the gas, the shock wave can interact with the magnetic field. The magnetic field and the flow behind the wave front then change their directions relative to the normal to the front (switch-on shock waves).

Whereas magnetohydrodynamic shock waves have been investigated in sufficient detail[1,2] (references to earlier papers are contained in the review of Chu and Gross[3]), no such statement can be made concerning normal ionizing shock waves. As noted in Ref. 4, the only understanding here is that the value of the electric field ahead of the front of the normal ionizing shock wave is an arbitrary quantity. Calculations based on the presently existing theories lead in many cases to substantial discrepancies with the experimental data[5,6] (see Sec. 5 of the present paper).

It was established in many studies that the state of the gas behind the ionizing shock wave front in a magnetic field cannot be uniquely determined by using only the continuity and conservation laws.[3] In other words, the conservation and continuity equations do not suffice to formulate the boundary conditions on the discontinuity surface. A free parameter remains in the problem, for example the electric field in the gas ahead of the front of the shock wave, and its determination calls for a supplementary condition. To obtain this additional condition, a number of workers used the so-called T^* model,[3,7-10] in which it is assumed that the conductivity of the gas is zero at a temperature lower than a certain characteristic value T^*, and becomes finite at $T > T^*$, while the supplementary condition, in principle, is the consequence of the existence of a structure in the wave front. Besides the purely practical inconvenience of using such a model and the limited region of its applicability, the shock-wave structures with gasdynamic discontinuity at the start of the shock-wave front is not realized in installations such as electromagnetic shock tubes (see Sec. 4 of the present paper).

Another approach was proposed by Kunkel and Gross,[3,11] who used a supplementary condition analogous to the Chapman-Jouguet condition in detonation theory, namely that the flow velocity behind the front be equal to the velocity of the small perturbations (see also Ref. 12). Such a supplementary condition is by itself arbitrary and in no way special. The Chapman-Jouguet condition at a given front velocity corresponds to a maximum electric field in the gas ahead of the shock-wave front.[8] If this electric field exceeds the corresponding electric breakdown field of the neutral gas, then the shock-wave front becomes unstable.[13] As shown in Refs. 14, and 15, in the analogous problem for a transverse ionizing shock wave, the front of the shock wave radiates in this case an ionization wave that increases the conductivity of the gas ahead of the front and decreases the electric field.

The purpose of the present paper is to construct a theory of normal ionizing shock waves with account taken of the ionization energy and dissociation energy of the gas. The only requirement of the theory is that the stationary structure by stable, and includes in particular the requirement of ionization stability of the gas ahead of the shock-wave front.[13]

In Secs. 2 and 3 are investigated qualitatively the types of normal ionizing shock waves, their stability, and their possible magnetic structures, particularly as functions of the degree of ionization of the gas ahead of the front. In Sec. 4 is solved the piston problem (thermal explosion or a current sheath playing the role of a piston in an electromagnetic shock tube). The evolution of the produced nonstationary shock wave and the transition to the stationary solution are considered. An additional boundary condition is obtained for the stationary shock wave that becomes detached from the piston.

In Sec. 5 is calculated the structure of a stationary shock wave. The results of the theory are compared with the experimental data on normal ionizing shock waves.

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2. CONSERVATION EQUATIONS AND BOUNDARY CONDITIONS

Let the front of a plane stationary shock wave propagate with velocity v_1 in a neutral gas along the magnetic field H_1. Changing over, as usual, to a system of coordinates that moves with the shock-wave front, and assigning the indices 1 and 2 to the equilibrium values of all the quantities ahead and behind the front, respectively, we have ahead of the shock-wave front a neutral gas flow into the front from $x=-\infty$, where

$$\mathbf{v}=\{v_1,0,0\},\quad \mathbf{H}=\{H_1,0,0\},\quad T_e=T_i=T_a=T_1,\quad N=N_a+n_e=N_a=N_1.$$

Behind the front of the shock wave (at $x=+\infty$) there is an outflow of ionized-gas with a density $N_2=N_{a2}+n_{e2}$ ($n_e=n_i=n$), a temperature T_2, and an ionization degree $\alpha_2=n_2/N_2$. The velocity and magnetic field are respectively

$$\mathbf{v}_2=\{v_{x2},v_{y2},v_{z2}\},\quad \mathbf{H}_2=\{H_{x2},H_{y2},H_{z2}\}.$$

Here e, i, and a pertain respectively to electrons, ions, and neutral atoms. The complete solution of the problem of the structure of the front calls for the use of separate equations of motion for the electron, ion, and neutral components, with account taken of the dissociation and ionization energies of the gas.[16] Such a solution will be obtained below when the structures of the normal ionizing shock waves are calculated and the experimental results are discussed. For the time being, to avoid cumbersome calculations, we confine ourselves in the first part of the paper to a single-fluid model for the investigation of the general character of the possible magnetic structures, for the investigation of the stability, and for the formulation of the boundary conditions.

For the one-dimensional stationary planar problem, Maxwell's equations

$$\operatorname{div}\mathbf{H}=0,\quad \operatorname{rot}\mathbf{E}=0,\quad \operatorname{rot}\mathbf{H}=\frac{4\pi}{c}\mathbf{j}$$

yield

$$H_{x1}=H_{x2}=\text{const},\quad E_{y1}=E_{y2}=\text{const},\quad E_{z1}=E_{z2}=\text{const}. \tag{2.1}$$

From the condition that the current and the electric field in the ionized gas behind the shock-wave front be equal to zero we get

$$\mathbf{j}_2/\sigma=\mathbf{E}_2^*=\mathbf{E}_2+c^{-1}[\mathbf{v}_2\times\mathbf{H}_2]=0.$$

Therefore, taking (2.1) into account, we obtain

$$E_{y1}=E_{y2}=c^{-1}(v_{x2}H_{z2}-v_{z2}H_1)=E_{y1}^*, \tag{2.2}$$
$$E_{z1}=E_{z2}=c^{-1}(v_{y2}H_1-v_{x2}H_{y2})=E_{z1}^*, \tag{2.3}$$

where $\mathbf{E}_1^*$ is the electric field in the gas system ahead of the wave front. We note that, in contrast to a normal shock wave in a plasma, in our case the electric field ahead of the shock-wave front is generally speaking different from zero, and, as can be easily seen, $\mathbf{E}\perp\mathbf{H}$. Without loss of generality, we assume $E_{z2}=0$. We then have also $E_y=0$ and $v_z=0$.

The continuity equation and the equations for the conservation for the momentum and energy flux yield

$$Nv_x=C, \tag{2.4}$$
$$MNv_x^2+nT_e+NT+H_y^2/8\pi=P_x, \tag{2.5}$$
$$MNv_xv_y-H_xH_y/4\pi=P_y, \tag{2.6}$$
$$MNv_x\mathbf{v}^2+5NTv_x+5nT_ev_x-2\frac{c}{4\pi}E_zH_y=S, \tag{2.7}$$

where C, P_x, P_y, and S are constants determined by the boundary conditions.

Since the electric field in the shock wave is different from zero, and reaches a large value ahead of the wave front, the electron temperature T_e, generally speaking, is not equal to the temperature of the atoms and ions. In the region ahead of the front, where the gas is weakly ionized and the electric field is strong, the very concept of electron temperature becomes approximate. Just as for strong shock waves, the hydrodynamic equations cannot be used here to calculate the structure of the wave front, and it is necessary to use the kinetic equation. However, Eqs. (2.4)–(2.7), which are conservation laws, are always valid. The conditions for the applicability of the hydrodynamic equations for the calculation of the wave structure were discussed in Refs. 14–16.

We introduce the dimensionless variables

$$\begin{gathered}\nu=N/N_1,\quad \omega=v_x/v_1,\quad \Theta=T/T_1,\quad \Theta_e=T_e/T_1,\\ \lambda=v_y/v_1,\quad \mu=v_z/v_1,\quad h_{y,z}=H_{y,z}/H_1,\quad \alpha=n/N,\\ s=cE_z/v_1H_1.\end{gathered} \tag{2.8}$$

Changing over in (2.2)–(2.7) to the variables (2.8) and eliminating the density and the transverse velocity components, we obtain[1)]

$$\omega_2-1+\frac{3}{5M_1^2}\left(\frac{\Theta_2}{\omega_2}-1\right)+\frac{h_2^2}{2M_{a1}^2}=0, \tag{2.9}$$

$$\omega_2^2-1+\frac{3}{M_1^2}(\Theta_2-1)+\frac{h_2^2}{M_{a1}^4}+2\frac{h_2s}{M_{a1}^2}=0, \tag{2.10}$$

$$s=-h_2(\omega_2-1/M_{a1}^2). \tag{2.11}$$

Here M_1 and M_{a1} are the acoustic and Alfven Mach numbers.[1] The state of the gas behind the shock-wave front, i.e., the values ω_2, Θ_2, and h_2 (the equilibrium value of the degree of ionization behind the wave front is a function of Θ_2 and ω_2), is not determined uniquely by the conservation laws and is a function of a parameter s which is for the time being arbitrary. Without loss of generality, we can put $s>0$ (it can be shown that by a suitable choice of the coordinate frame it is always possible to attain $s>0$) in the particular case $s=0$, we arrive at the problem of a normal MHD shock wave.[1]

We are eliminating from (2.9)–(2.11) the temperature and the magnetic field, we obtain

$$(1-\omega_2)(\omega_2-\omega_{\mathrm{GD}})\left(\omega_2-\frac{1}{M_{a1}^2}\right)^2=\frac{s^2}{8M_{a1}^2}\left(\omega_2+\frac{2}{M_{a1}^2}\right), \tag{2.12}$$

where

$$\omega_{\mathrm{GD}}=(M_1^2+3)/4M_1^2.$$

At $s=0$, the roots of (2.12) correspond to the following solutions[1]: $\omega_2=1$—trivial solution, $\omega_2=\omega_{\mathrm{GD}}$—pure gasdynamic shock wave, and the double root $\omega_2=1/M_{a1}^2$—switch-on magnetohydrodynamic shock wave. At $s>0$ Eq. (2.12) has four different roots. Whether they can be real solutions, i.e., positive real values of ω_2, depends on the relations between the Mach numbers and on the stability of the corresponding solutions.

Let us list the possible solutions of (2.12) in accordance with those into which they go over at $s=0$. These are the N wave,[8,17] which goes over into the trivial solution $\omega_2 = 1$ at $s = 0$, the gasdynamic shock wave (GD wave), and two switch-on shock waves (SO waves). The last two are fast and slow SO shock waves, depending on whether the flow velocity behind the front of the SO wave is larger or smaller than the velocity of the slow magnetosonic wave.

We note immediately the following important circumstance that follows from (2.12). The solution (2.12) for supersonic shock waves ($M_1 > 1$) is possible only if $\omega_{GD} < \omega < 1$. Thus, at $s > 0$ the compression is less, and the temperature behind the front is generally speaking higher, than in the ordinary pure gas dynamic shock wave with the same value of M_1. From this it follows in turn that all the supersonic ($M_1 > 1$) shock waves are compression waves, and the subsonic waves are rarefaction waves.

The necessary condition for the realization of the solutions of Eq. (2.12) is that they be stable or evolutional. It is known[18] that for the wave to be evolutional it is necessary that the number of waves outgoing from the discontinuity surface be one less than the number of independent boundary conditions. (The stability check is carried out here within the framework of a model which is one-dimensional and quasistationary in the sense of the electromagnetic waves,[19] according to which the discontinuity radiates only acoustic, magnetosonic, and entropy waves.) Recognizing that besides the boundary conditions that follow from the continuity and conservation equations, there is possible one other additional boundary condition (for example, an upper limit on the electric field), we shall list the solutions of (2.12) that are evolutional discontinuity surfaces. A map of these solutions in the (M_1^2, M_{a1}^2) plane is shown in Fig. 1.

In region I ($1 > \omega_{GD} > 1/M_{a1}^2$) only a GD wave is possible subject to a supplementary condition. At $\omega_{GD} < 1/M_{a1}^2 < 1$ (region II) there exist fast SO waves with a supplementary condition and a slow GD wave if there is no supplementary condition. In the region $M_1^2 > 1$, $M_{a1}^2 < 1$ (region III) there are fast N waves, which call for a supplementary condition on the value of the electric field, and slow GD waves, if there is no such condition. At $M_1^2 < 1$ and $M_1^2 < 1$ (region IV), only slow N waves are possible in the presence of a supplementary condition. Finally, at $M_1^2 < 1$ and $M_{a1}^2 > 1$ (region V) there are no stable solutions at all.

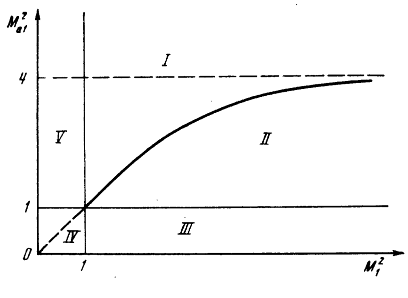

FIG. 1. Regions of possible stable flows on the (M_1^2, M_{a1}^2) plane.

3. QUALITATIVE INVESTIGATION OF MAGNETIC STRUCTURES

We investigate now the possible magnetic structures of normal ionizing shock waves and the ensuing limitations on the value of the electric field or of the parameter s. The only assumption that is made here is that the entropy increases everywhere in the shock-wave front. We regard the gasdynamic jumps as discontinuity surfaces of zero thickness. The shock-wave front constitutes then a region of Joule dissipation, in which the magnetic field changes, and an isomagnetic gasdynamic discontinuity. For the latter it is necessary that the magnetic Reynolds number in the gas dynamic shock wave be small, $\mathrm{Rm} = 4\pi\sigma v\Delta/c^2$, where Δ is a width of the gasdynamic shock wave of the order of the mean free path of the atom.

On the (h, ω) plane, the equation of the trajectory that describes the structure of the shock-wave front follows from the conservation laws (2.9) and (2.10):

$$F(h,\omega) = 4\omega^2 - \omega\left(5 + \frac{3}{M_1^2} - \frac{5h^2}{2M_{a1}^2}\right) + 1 + \frac{3}{M_1^2} - \frac{h^2}{M_{a1}^4} + \frac{2sh}{M_{a1}^2} = 0. \quad (3.1)$$

It is obvious that the initial state—the point 1 with coordinates $h = 0$ and $\omega = 1$—lies on the curve (3.1). Inasmuch as we have ionized gas behind the shock-wave front, the electric field in the final state (point 2), in its own coordinate system, should be equal to zero. Thus, the point 2 is the intersection of the curve (3.1) with the zero-field hyperbola in the comoving coordinate system

$$s + h(\omega - 1/M_{a1}^2) = 0. \quad (3.2)$$

We consider now the equal-entropy lines on the (h, ω) plane. In terms of the variables ω and Θ the equation for such a line is

$$\omega\Theta^{3/2} = \mathrm{const}.$$

Expressing here the temperature Θ in terms of h and ω from (2.9), we obtain the equal-entropy line equation in the form

$$\frac{h^2}{2M_{a1}^2} = 1 - \omega + \frac{3}{5M_1^2}(1 - \lambda\omega^{-5/3}), \quad (3.3)$$

where λ is a certain constant, in particular $\lambda = 1$ for the line that passes through the point 1. It is convenient to introduce the local (with respect to the flow) value of the acoustic Mach number

$$M^2 = \frac{v^2}{c_s^2} = \frac{M_1^2\omega^2}{\Theta} = 3\omega\left[5 + \frac{3}{M_1^2} - 5\omega - \frac{5h^2}{2M_{a1}^2}\right]^{-1}. \quad (3.4)$$

Differentiating (3.3) and using (3.4), we obtain along the equal-entropy line

$$\left(\frac{\partial h}{\partial\omega}\right)_s = M_{a1}^2\left(\frac{1}{M^2} - 1\right)\frac{1}{h}. \quad (3.5)$$

It can be shown that the entropy increases between the equal-entropy lines towards the "focus" $M = 1$, $h = 0$. Differentiating (3.1) and using (3.4), we get

$$\left(\frac{\partial h}{\partial\omega}\right)_F = \frac{3M_{a1}^2\omega(1/M^2 - 1)}{5h\omega + 2s - 2h/M_{a1}^2}. \quad (3.6)$$

The curve (3.1), which represents the magnetic structure of a normal ionizing shock wave, should begin at the point with coordinates 0 and 1 on the (h, ω) plane and terminate at the point where it crosses the zero-

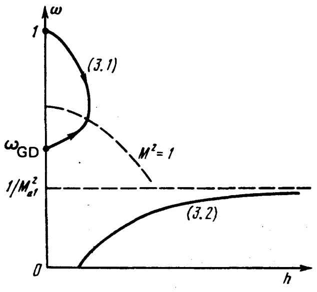

FIG. 2. The curves (3.1) and (3.2) for $1>\omega_{GD}>1/M_{a1}^2$ and $s\neq 0$.

field hyperbola (3.2)—the point 2. From the requirement that the entropy in the shock wave must increase it follows that over the entire motion along the curve (3.1) from point 1 to point 2, which represents the structure of the front, the following inequality must hold

$$(\partial h/\partial\omega)_F<(\partial h/\partial\omega)_J.$$

We consider now different magnetic structures, depending on the relations between the Mach numbers. They are shown in Figs. 2–5 where the arrows indicate the direction of the motion corresponding to increasing entropy. (It follows from the symmetry of the problem that we can confine ourselves only to the quadrant $h>0$, $\omega>0$.)

At $1>\omega_{GD}>1/M_{a1}^2$, see Fig. 2, the trajectory (3.1) crosses the zero-field hyperbola only at $s=0$ and $h=0$, i.e., only a pure gasdynamic shock wave is possible—the discontinuity point $1\to\omega_{GD}$ and the supplementary condition is $s=0$.

At $\omega_{GD}<=/M_{a1}^2<1$, Fig. 3, for each value of s there exists a single structure for the switch-on shock wave, $1\to3\to SO$, and a set of structures of the type $1\to a\to b\to$GD, where $h_{2SO}>h_{2GD}$.

In the region $M_1^2>1$ and $M_{a1}^2<1$, Fig. 4, for each value of s we can have one compression N wave and a set of structures of the type $1\to a\to b\to$GD or $1\to a'\to b'\to$GD.

At $M_1^2<1$ and $M_{a1}^2<1$, Fig. 5, only a rarefaction N wave is possible at each fixed value of s.

Finally, at $M_1^2<1$ and $M_{a1}^2>1$ no shock waves exists at any value of s.[2)]

From among the listed possibilities, the only cases

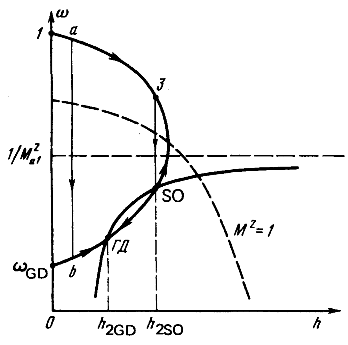

FIG. 3. Possible structures of the shock wave at $1>1/M_{a1}^2>\omega_{GD}$.

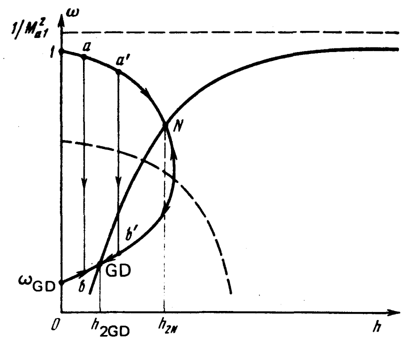

FIG. 4. Possible structures of the shock wave at $M_1^2>1$ and $M_{a1}^2<1$.

realized are those in which the structure of the shock-wave front for a given value of s is uniquely defined.[1] Thus, the condition for the existence of a unique structure selects the same solutions as the evolutionality requirement.

One of the questions that arises in the investigation of the ionizing shock waves is that of the origin of the primary electrons which are necessary to turn on the Joule dissipation. It must be assumed that the primary (or precursor) electrons ahead of the shock-wave front are produced mainly because of photoionization of the cold gas by the radiation, due to the front, of the shock wave proper. The value of the precursor photoionization of the cold gas, α_1, calculated in Ref. 16, is in satisfactory agreement with the experimentally observed one.[20] The gas ahead of the shock-wave front should be regarded as conducting or nonconducting, depending on the magnetic Reynolds number in the incoming flor $Rm_1=4\pi\sigma_1 v_1 L/c^2$, where L is the characteristic dimension in the incoming gas stream. The gas is assumed nonconducting at $Rm_1\ll1$. Obviously, the quantities α_1 and Rm_1 depend on the emissivity, ionization potential, etc. of the concrete gas and are determined mainly by the intensity of the shock wave.

Taking the foregoing into account, we conclude that, depending on the relation between the Mach numbers M_1 and M_{a1} and on the total intensity of the shock wave, the following cases are possible.

If the photoionization is small and the maximum possible value of the electric field ahead of the wave front is less than the electric breakdown field of the gas (see Sec. 4 below), then in region I of Fig. 1 only pure gasdynamic (fast) shock waves are possible with a zero electric field ahead of the front, while in regions II and III *slow* shock waves (GD waves) are possible. Slow

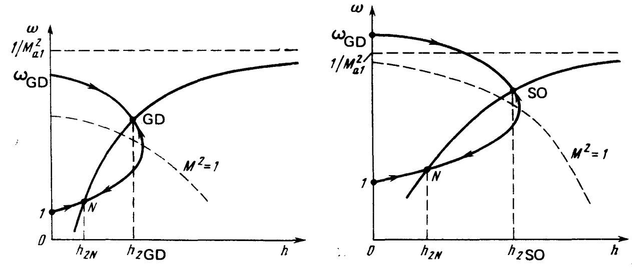

FIG. 5. Structure of shock wave at $M_1^2<1$ and $M_{a1}^2<1$.

shock waves are initiated at the gasdynamic discontinuity, which is followed by a Joule rarefaction region. The characteristic structure of such a wave is illustrated by the transition $1\to\omega_{GD}\to$GD in Fig. 3. It will be shown below (Sec. 4) that in ordinary shock tubes the stable stationary shock waves are always fast. It appears that slow ionizing switch-on shock waves can occur only in the decay of a specially chosen initial discontinuity.

In the case of strong precursor ionization, the electric field ahead of the front should be equal to zero. The problem reduces then to that of a normal shock wave in a plasma.[1] In this case only a pure gas dynamic shock wave is possible in region I of Fig. 1, and only MHD fast switch-on waves are possible in region II. Thus, a sufficiently strong shock wave always has an MHD structure.[21]

We arrive likewise at a magnetohydrodynamic structure in the idealized problem of an ionizing shock wave (infinite plane front; absence of electron losses linear in n_e), inasmuch as in this case the ionization stability requires that the electric field in the gas ahead of the wave front be equal to zero.[13,14,16] In the general case the solution is uniquely determined by the value of the electric field ahead of the wave front, i.e., by the value of the dimensionless parameter s.

It is easily seen from Figs. 3–5 that s has an upper bound. This upper bound $s=s_{max}$ is determined by the condition of existence of real roots of Eq. (2.12) or, as follows from Figs. 2–5, by the condition that there exists a shock-wave front structure, i.e., by the condition that the hyperbola of the zero field be tangent to the curve (3.1). We shall show that at $s=s_{max}$ the value of the flow velocity v_2 behind the shock-wave front is equal to the local magnetic sound velocity.[8] Indeed, at the point of tangency of curves (3.1) and (3.2) we have

$$(\partial h/\partial\omega)_F=(\partial h/\partial\omega)_J,$$

Substituting here h from (3.1) and (3.2), we obtain the following relation between the Mach numbers and the value of h at the point 2:

$$(1-1/M_2^2)(1-1/M_{a2}^2)=h_2^2/M_{a2}^2. \qquad (3.7)$$

for the velocity of the magnetic sound at the point 2 we have

$$c_{\pm2}^2={}^1/_2\{c_{s2}^2+c_{a2}^2\pm[(c_{s2}^2+c_{a2}^2)^2-4c_{ax}^2c_{s2}^2]^{1/2}\}, \qquad (3.8)$$

where

$$c_{a2}^2=\mathbf{H}_2^2/4\pi\rho_2=c_{ax}^2(1+h_2^2).$$

Equating $c_{\pm2}$ from (3.8) to the value v_2 we obtain (3.7) after simple transformations.

Formally, the equality of the flow velocity behind the shock-wave front to the local velocity of the small perturbations coincides with the known Chapman-Jouguet condition of detonation theory. It was precisely the last condition which was proposed by Kunkel and Gross[11] as the supplementary boundary condition for the normal ionizing shock waves. In a number of papers[12,22] the analogous condition was chosen in the form $v_2=c_{s2}$. Generally speaking, there are no grounds for such conditions. It is known that the Chapman-Jouguet condition in detonation theory follows from the requirement that the flow be stable, a requirement that reduces, in fact, for a plane detonation shock wave, to equality of the flow velocity behind the front to the velocity of the sound in the combustion products.[23]

In the case of a stationary ionizing shock wave in a magnetic field, a stability condition is also necessary. Such a stability requirement for ionizing shock waves in magnetic fields was proposed in the general case by the author in Ref. 13, and for transverse shock waves in Refs. 14 and 15. It includes, besides the hydrodynamics flow stability, i.e., evolutionality of the shock wave, also the condition of ionization stability of the gas ahead of the wave front. The last requirement is connected with the possibility of electric breakdown of the gas ahead of the front, since the gas ahead of the front of the shock wave is in an induced electric field generated by the front of the wave when the transverse magnetic-field component is turned on. In the case of electric breakdown, the shock-wave front radiates an ionization wave that propagates as a result of photoionization (or thermionic conduction), so that the conductivity of the gas increases and the initial electric field decreases. A stationary solution is reached when the electric field decreases to the value of the gas-breakdown threshold field. To realize the described evolution of the shock wave it is necessary that when the initially nonstationary shock wave is produced the value of the electric field of the front be maximal and larger than the breakdown field of the gas.

4. ELEMENTARY THEORY OF ELECTROMAGNETIC SHOCK TUBE

We consider now within the framework of a simple model the formation of a shock wave in en electromagnetic shock tube. The problem of self-similar motion of a conducting gas in an electromagnetic coaxial shock tube was solved numerically for MHD shock waves[24] and for ionizing shock waves.[25] Here we consider an analytically solvable one-dimensional model of a shock tube. Such a model, which is a modification of the well-known snowplow model, makes it possible to obtain in explicit form all the relations that characterize the flow in the self-similar problem. In the case when the magnetic field lies in the plane of the front (transverse shock wave), this solution is exact, and in the remaining cases it is approximate, but agrees well with the results of the exact numerical calculations.

Planar self-similar gas flow is produced when a constant field is instantaneously turned on and its magnetic field pushes the shock wave (magnetic piston). The immobile unperturbed gas (the flow region I) in the shock-wave front is heated, becomes compressed, and acquires a mass velocity in the direction of the wave propagation.

The boundary condition on the rigid conducting rear wall of the shock tube requires either that the gas velocity in the axial direction be zero ($v=0$) or that there be no gas at the well ($\rho=0$). Therefore the region of constant flow at the rear wall of the tube (region 4) cannot coincide with the region of the flow behind the shock-wave front (region 2). These regions are connected with each other through the rarefaction wave (region 3), in which either the density or the axial velocity of the gas decreases to zero on going from region 2 to region 4. Stable flow is possible in this case, obviously, only when the "head" of the rarefaction wave (the boundaries of regions 2 and 3) moves through the gas not faster than the shock wave (the boundary of regions 1 and 2).

The shock-wave velocity relative to the gas that flows out into region 2 is always smaller than the velocity of the fast magnetosonic wave in region 2 and larger (smaller) than the velocity of the slow magnetosonic wave for the fast (slow) shock waves. Since the velocity of the rarefaction wave relative to the gas is one of the characteristic velocities (of the fast or slow magnetosonic waves respectively for the fast or slow rarefaction waves), the only stable configuration in the problem with the piston is a fast shock wave followed by a slow rarefaction wave. The limiting case is a Chapman-Jouguet shock wave, in which the velocity of the outgoing flow and the velocity of the slow rarefaction wave are equal. In the latter case, the distance between the shock-wave front (boundary of regions 1 and 2) and the rarefaction wave front (boundary 2–3), which is equal to zero at the initial instant, remains unchanged, i.e., the Chapman-Jouguet shock wave does not become detached from the piston.

An important conclusion that follows from an analysis of self-similar flows containing rarefaction waves (any problem involving the motion of a gas with a piston, whether it be an electromagnetic shock tube, a planar explosion in a magnetic field, etc.), is that the slow shock waves are in this case unstable and consequently do not start out as stationary flows. The question of setting up an experiment for the observation of slow shock waves is not yet fully clear and is of independent interest.

In the case when the initial magnetic field H_1 is perpendicular to the propagation direction of the shock wave, the velocity of the slow rarefaction wave vanishes and region 3 represents in this case an infinitesimally thin current sheath in which the density drops to zero and the magnetic field increases to its boundary value at the wall H_4. Equating the pressures on both sides of the current sheath we have

$$H_4^2/8\pi = H_2^2/8\pi + p_2. \tag{4.1}$$

Using the continuity equations and the momentum conservation equations

$$\rho_1 v_1 = \rho_2 v_2,$$
$$\rho_1 v_1^2 + p_1 + H_1^2/8\pi = \rho_2 v_2^2 + p_2 + H_2^2/8\pi$$

and neglecting the initial gas pressure p_1, we obtain from (4.1)

$$H_4^2/H_1^2 - 1 = 2M_{a1}^2(1-\omega_2). \tag{4.2}$$

In the case when the magnetic field H_1 does not lie in the plane of the wave front, Eq. (4.2) does not hold. Numerical calculations[24,25] show, however, that even in this case the gas density at the wall and the trapping of the mass by the rarefaction wave are relatively small and have little effect on the flow. Therefore, for fast shock waves it is meaningful to consider the simple model of an electromagnetic shock tube on the basis of Eq. (4.2), which makes it possible to obtain all the relations in explicit form.

Since $H_x = H_1 = \text{const}$ in the entire flow region, we have

$$H_4^2/H_1^2 - 1 = H_{y4}^2/H_1^2.$$

For an MHD switch-on shock wave $\omega_2 = 1/M_{a1}^2$, whence

$$M_{a1}^2 = 1 + H_{y4}^2/2H_1^2. \tag{4.3}$$

In the case of a pure gasdynamic shock wave in an idealized problem (without allowance for the ionization energy) $\omega_2 \approx 1/4$ at $M_1 \gg 1$, and then

$$M_{a1}^2 = 2H_{y4}^2/3H_1^2. \tag{4.4}$$

Calculations on the basis of (4.3) and (4.4) agree with high accuracy with the numerical calculations[25] for normal shock waves in an infinitely conducting gas.

In the case of ionizing shock waves, as discussed above, relation (4.2) does not determine uniquely the velocity of the shock wave from the magnitude of the "pushing magnetic field of the piston," i.e., M_{a1} is no longer a unique function of H_{y4}. The parameter s connected with the electric field ahead of the wave front can take on values in the interval $0 \leq s \leq s_{max}$, where $s_{max} \leq 0.75$ in the idealized problem. (When account is taken of the ionization energy, this value can be also larger.) From (4.2), with account taken of Eq. (2.12), we obtain in the limit of the strong shock wave (omitting terms of order of smallness $1/M_1^2$)

$$s^2 = \frac{H_{y4}^2}{H_1^2}\frac{(3M_{a1}^2 - 2H_{y4}^2/H_1^2)(2M_{a1}^2 - 2 - H_{y4}^2/H_1^2)}{M_{a1}^4(2M_{a1}^2 + 4 - H_{y4}^2/H_1^2)}. \tag{4.5}$$

For fast shock waves, the only ones we consider, the condition for the rarefaction of the current sheath of the piston and of the front of the shock wave leads to the following limitation: The stationary flow, in which the front of the shock wave becomes separated from the piston (or in the limit moves together with it) will be such that its velocity, given by (4.2), is larger than (or equal to) the Chapman-Jouguet shock-wave velocity, i.e., the velocity corresponding to the maximum s from (4.5) at the corresponding Mach numbers.

It is obvious that $\partial s^2/\partial\omega < 0$ for a fast wave. It can be shown that $(\partial s^2/\partial\omega)_{s=s_{max}} < 0$ for the given magnetic piston, i.e., at

$$H_{y4}^2/H_1^2 = 2M_{a1}^2(1-\omega_2) = \text{const}.$$

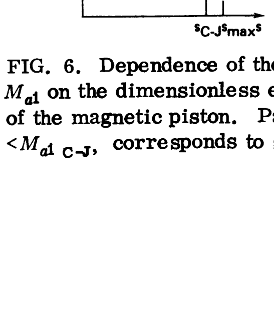

FIG. 6. Dependence of the dimensionless shock wave velocity M_{a1} on the dimensionless electric field s at constant pressure of the magnetic piston. Part of the curve in the figure, for $M_{a1} < M_{a1\,C\text{-}J}$, corresponds to slow waves.

It follows therefore that the electric field in a stationary shock wave, particularly in a Chapman-Jouguet shock wave ($s_{C\text{-}J}$) is always smaller than s_{max}. Figure 6 shows the dependence of the dimensionless velocity M_{a1}^2 on the electric field s for a fixed piston, i.e., at constant H_{y4}/H_1. The part of the $M_{a1}(s)$ curve corresponding to the slow shock waves is shown by a thinner line. The maximum shock wave velocity occurs at $s=0$, i.e., for a magnetohydronamic shock wave (formula (4.3)), and the minimum M_{a1} GD is for a gasdynamic shock wave—formula (4.4). However, the smallest velocity for the stationary shock wave is that of the Chapman-Jouguet shock shock wave: $M_{a1\ C\text{-}J}$ at $s=s_{C\text{-}J}$.

For small values of the transverse component of the switched-on magnetic field, i.e., at $h_2 \ll 1$, the Chapman-Jouguet condition $M_{2-}=1$ is close to the condition $M_2=1$. Indeed, at $M_2=1$ we have, accurate to terms linear in h_2^2

$$M_{2-}=1+\frac{h_2^2}{2(1+h_2^2)(1-{}^5/_6\beta_2)}, \qquad (4.6)$$

where β_2 is the ratio of the gas pressure to the magnetic-field pressure. Recognizing that

$$M_2^2=M_1^2\omega_2^2/\Theta_2=1,$$

and substituting the value of Θ_2 from (2.9), we obtain accurate to small terms of water $1/M_1^2 \ll 1$:

$$h_2^2=2M_{a1}^2(1-{}^8/_5\omega_2).$$

Substituting this expression in (4.6) and using (4.2), we obtain at $H_{y4}/H_1 \ll 1$:

$$M_{2-}=1+\frac{1}{2}\left(\frac{H_{y4}}{H_1}\right)^2\approx M_2=1,\quad M_{a1}\approx\frac{2}{\sqrt{3}}\frac{H_{y4}}{H_1}. \qquad (4.7)$$

It should be noted that in a number of papers[9,22] the condition $M_2=1$ was taken to be the supplementary condition without any limitations whatever, while in Ref. 9 the conditions for the applicability of the indicated approximation (4.7) are not always satisfied. The supplementary condition formulated in Ref. 26 in the form $M_{2+}=1$ has no physical meaning at all, as follows from the foregoing.

It was shown in the preceding section that the stationary structures for fast ionizing shock waves (including the limiting case of a Chapman-Jouguet shock wave) start from the Joule compression zone of the magnetic field. It follows[1] from (4.7) that at $H_{y4}/H_1 \ll 1$ practically the entire structure of the shock wave consists of a Joule zone, and the presence of a viscous hydrodynamic discontinuity in it can be neglected.

Thus, preliminary ionization of the gas ahead of the shock-wave front (for example, precursor photoionization) is a necessary condition for the existence of a stationary structure for a switch-on shock wave. We call attention to the substantial difference between the switch-on shock wave and the transverse ionizing shock wave. In the latter case a stationary solution is always possible, even in the presence of preionization—this is a purely gasdynamic shock wave and is a fast shock wave in this case. From general considerations one should expect at low shock-wave velocities (low temperature and small current in the current sheath of the magnetic piston) the gas conductivity ahead of the current sheath to be insufficient for the formation of a stationary structure. The shock-wave front will in this case not be detached from the current sheath of the piston, i.e., the dynamics of the shock wave does not differ from the case of the Chapman-Jouguet wave. However, measurements of the quantities that characterize the structure of the wave front—the electric and magnetic field, the density and temperature discontinuities—should offer evidence of deviation from the Chapman-Jouguet regime. With increasing discharge current, the slowest of the stationary shock waves is initially formed—the Chapman-Jouguet shock wave, whose structure corresponds to its velocity. At still larger discharge current and still higher velocity, the Chapman-Jouguet induction electric field (the maximum electric field corresponding to the stationary structure at the given piston pressure) exceeds the breakdown threshold of the gas ahead of the shock-wave front. As a result, the front of the shock wave becomes detached from the current sheath of the piston and with further increase of the magnetic field H_1 and of the discharge current a transition should be observed from the Chapman-Jouguet shock-wave structure to a magnetohydrodynamic shock-wave structure, a transition similar to that from the gasdynamic shock wave to the magnetohydrodynamic shock wave for transverse ionizing shock waves.[13-15]

Figure 7 shows the dimensionless wave velocities M_{a1} as functions of the magnetic-piston pressure H_{y4}/H_1 for various regimes: magnetohydrodynamic (MHD), Chapman-Jouguet (C-J), and gas dynamic (GD), calculated without allowance for the energy lost to ionization, so that it is possible to present on a single figure the experimental data pertaining to one experimental setup,[6,27] but for different gases (helium and hydrogen), and for different values of the initial pressure and of the axial magnetic field. (When the ionization energy is taken into account, the lines GD and C-J on Fig. 7 lie somewhat lower). As seen from Fig. 7, the exper-

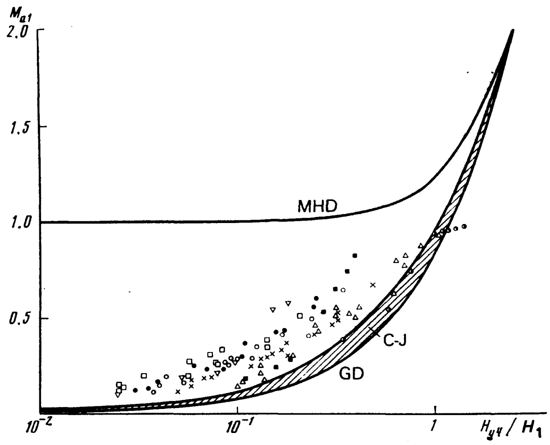

FIG. 7. Dependence of the dimensionless velocity of the shock wave on the magnetic pressure of the piston. Theoretical curves–formulas (4.3), (4.4), and (4.5) at $s=s_{C\text{-}J}$. The experimental points were taken from Refs. 6 and 27. Helium gas, $p_1 = 0.05$, 0.1, and 0.2 Torr: □ – $H_1=12$ kOe, ● – $H_1=10$ kOe, ○ – $H_1=8$ kOe, × – $H_1=6$ kOe, △ – $H_1=3$ kOe. Hydrogen gas, $p_1=0.1$ Torr: ▽ – $H_1=12.5$ kOe, ■ – $H_1=6.3$ kOe, ◑ – $H_1=2$ kOe.

imental points lie between the MHD curves and the C-J curves, and in accordance with the presented theory at large values of H_1, i.e., at large values of the velocity at the given M_{a1}, the experimental points lie closer to the MHD curve.

5. STRUCTURE OF STATIONARY NORMAL SHOCK WAVE. COMPARISON WITH EXPERIMENT

To calculate the structure of the front of a normal ionizing switch-on shock wave we use separate equations of motion for the electrons, ions, and atoms. We assume the plasma to be quasineutral, putting $n_e = n_i = n$. Neglecting the difference between the masses of the ions and the atoms and taking into account the large cross section of the ion-atom collisions, due to the charge exchange, we conclude that $T_i = T_a = T$, and the slippage of the atoms relative to the ions is small: $|\mathbf{v}^i - \mathbf{v}^a| \ll |\mathbf{v}^a|$. From the condition that the x-component of the electric field in the one-dimensional stationary problem be equal to zero, we have

$$v_x^e = v_x^i = v_x^a = v_x.$$

We direct the z axis along the electric field ahead of the wave front:

$$\mathbf{E} = \{0, 0, E_z\}.$$

From Maxwell's equations it follows that

$$\frac{dH_z}{dx} = \frac{4\pi en}{c}(v_y^e - v_y^i), \tag{5.1}$$

$$\frac{dH_y}{dx} = \frac{4\pi en}{c}(v_z^i - v_z^e). \tag{5.2}$$

Neglecting the electron inertia[1] and disregarding in the present approximation the difference between the longitudinal and transverse transport coefficients (inasmuch as the role of the principal dissipation is played here by Joule losses, which are weakly affected by the magnetic field), we write down the equations of motion of the electrons along the axis y and z:

$$v_z^e H_1 - v_x H_z = \frac{c}{en}(R_y^{ei} + R_y^{ea}), \tag{5.3}$$

$$cE_z - v_y^e H_1 + v_x H_y = \frac{c}{en}(R_z^{ei} + R_z^{ea}). \tag{5.4}$$

Adding the equations of motion for the electrons and ions and for all three components, we get

$$\frac{d}{dx}\left[nv_x(mv_y^e + Mv_y^i) - \frac{H_1 H_y}{4\pi}\right] = R_y^{ea} + R_y^{ia}, \tag{5.5}$$

$$\frac{d}{dx}\left[nv_x(mv_z^e + Mv_z^i) - \frac{H_1 H_z}{4\pi}\right] = R_z^{ea} + R_z^{ia}, \tag{5.6}$$

$$nv_x(mv_y^e + Mv_y^i) + N_a v_x M v_y^a - \frac{H_1 H_y}{4\pi} = 0, \tag{5.7}$$

$$nv_x(mv_z^e + Mv_z^i) + N_a v_x M v_z^a - \frac{H_1 H_z}{4\pi} = 0. \tag{5.8}$$

Here $\mathbf{R}^{ea}$, $\mathbf{R}^{ia}$, $\mathbf{R}^{ei}$ are the mutual friction forces between the plasma components; the collision frequencies and kinetic coefficients are defined in the same manner as in Ref. 16.

Changing over in (5.1)–(5.8) to the dimensionless variables (2.8) and leaving out the small terms of order $(m/M)^{1-2}$, we rewrite (5.5)–(5.8) in the form

$$\frac{d}{dx}\left(\alpha\lambda_i - \frac{h_y}{M_{a1}^2}\right) = \frac{\alpha}{\omega}\frac{\lambda_a - \lambda_i}{\tilde{\Delta}_j^{ai}}, \tag{5.9}$$

$$\frac{d}{dx}\left(\alpha\mu_i - \frac{h_z}{M_{a1}^2}\right) = \frac{\alpha}{\omega}\frac{\mu_a - \mu_i}{\tilde{\Delta}_j^{ai}}, \tag{5.10}$$

$$\alpha\lambda_i + (1-\alpha)\lambda_a = h_y/M_{a1}^2, \tag{5.11}$$

$$\alpha\mu_i + (1-\alpha)\mu_a = h_z/M_{a1}^2. \tag{5.12}$$

Here $\tilde{\Delta}_j^{ai} = \frac{3}{4}\sqrt{2}v_1/\nu_{ia}$, where ν_{ia} is the frequency of the ion-atom collisions. Neglecting in first-order approximation the slippage of the ions relative to the atoms, i.e., assuming the scale of the friction between the ions and the atoms to be the smallest in the problem, we obtain

$$\lambda_i = \lambda_a = h_y/M_{a1}^2, \quad \mu_i = \mu_a = h_z/M_{a1}^2. \tag{5.13}$$

From (5.1)–(5.4), using (5.13), we get

$$\Delta_h\omega\frac{dh_y}{dx} - \Delta_j\omega\frac{dh_z}{dx} + h_z\left(\omega - \frac{1}{M_{a1}^2}\right) = 0, \tag{5.14}$$

$$\Delta_h\omega\frac{dh_z}{dx} + \Delta_j\omega\frac{dh_y}{dx} - h_y\left(\omega - \frac{1}{M_{a1}^2}\right) = s, \tag{5.15}$$

where

$$\Delta_h = \tilde{\Delta}_h/\alpha = cH_1/4\pi e\alpha N_1 v_1,$$

$$\Delta_j = \frac{\tilde{\Delta}_j}{\alpha} = \frac{4}{3}\frac{mc^2(\nu_{ei} + \nu_{ea})}{4\pi e^2 \alpha N_1 v_1}.$$

We change over in (5.14) and (5.15) to the Lagrangian coordinate ζ by means of the substitution $\omega d/dx = d/d\zeta$ and introduce new variables h and φ:

$$h_y = h\sin\varphi, \quad h_z = h\cos\varphi.$$

The boundary conditions at $x = +\infty$ yield $h = h_2$ and $\varphi = \pi/2$. After simple transformations we obtain the following equations for h and φ:

$$\frac{dh}{d\zeta} = \alpha\left[\frac{\tilde{\Delta}_j}{\tilde{\Delta}_j^2 + \tilde{\Delta}_h^2}h\left(\omega - \frac{1}{M_{a1}^2}\right) + s\frac{\cos(\varphi - \psi)}{(\tilde{\Delta}_j^2 + \tilde{\Delta}_h^2)^{1/2}}\right], \tag{5.16}$$

$$h\frac{d\varphi}{d\zeta} = -\alpha\left[\frac{\tilde{\Delta}_h}{\tilde{\Delta}_j^2 + \tilde{\Delta}_h^2}h\left(\omega - \frac{1}{M_{a1}^2}\right) + s\frac{\sin(\varphi - \psi)}{(\tilde{\Delta}_j^2 + \tilde{\Delta}_h^2)^{1/2}}\right], \tag{5.17}$$

where $\psi = \tan^{-1}(\tilde{\Delta}_j/\tilde{\Delta}_h) = \tan^{-1}(\Omega_e\tau_e)^{-1}$.

It follows from (5.16) and (5.17) that as $x \to \infty$, when $h \to 0$, we get $\varphi \to \psi$. This circumstance distinguishes our problem with finite s (at $x \to -\infty$ the incoming flow has a preferred direction determined by the electric field) from the magnetohydrodynamic problem considered in Ref. 1, where $s = 0$ and cylindrical symmetry exists relative to the x axis as $x \to \infty$, and therefore all the structures calculated there are determined accurate to the transformation $\varphi \to \varphi + \text{const}$. Thus, in the case of an unmagnetized plasma, when $\Omega_e\tau_e \ll 1$ and $\psi \approx \pi/2$, the transverse component of the magnetic field increases from zero to a finite value, remaining in one plane. In the opposite limiting case $\Omega_e\tau_e \gg 1$ we have $\psi \ll 1$, i.e., the smallest angle of rotation of the magnetic field in the plane of the front is $\pi/2$. The corresponding oscillations of the magnetic field in the forward part of the front were observed in Ref. 26.

If we determine in the second approximation at $\alpha \ll 1$ the relative velocity of the ions and atoms with the aid of (5.9) and (5.10):

$$\lambda_a - \lambda_i = -\frac{\tilde{\Delta}_j^{ai}}{\alpha M_{a1}^2}\frac{dh_y}{d\zeta}, \quad \mu_a - \mu_i = -\frac{\tilde{\Delta}_j^{ai}}{\alpha M_{a1}^2}\frac{dh_z}{d\zeta},$$

then substitution of the obtained expression in the heat conduction equation for the heavy component of the plasma determines its ohmic heating:

$$\Theta-1=\frac{10M_1^2}{9M_{a1}^2}\frac{h^2}{2M_{a1}^2}. \quad (5.18)$$

To compare the theory with experiment it is necessary, as a rule, to take into account the energy lost to ionization and dissociation (for a diatomic gas); the corresponding term should be added to Eq. (2.10). We present the final formulas for a monatomic gas ($\gamma=5/3$). Putting $\Theta_1^{ion}=I/T_1$, where I is the ionization potential of the gas, we have

$$\frac{h_2^2}{2M_{a1}^2}=\left[(1-\omega_2)(4\omega_2-1)+\frac{6\Theta_1^{ion}(\alpha_2-\alpha_1)}{5M_1^2(1+\alpha_1)}\right]\left(\omega_2+\frac{2}{M_{a1}^2}\right)^{-1}, \quad (5.19)$$

$$\Theta_2=\frac{5M_1^2(1+\alpha_1)}{3(1+\alpha_2)}\omega_2\left(1+\frac{3}{5M_1^2}-\omega_2-\frac{h_2^2}{2M_{a1}^2}\right). \quad (5.20)$$

Here α_2 is the equilibrium degree of ionization, determined by the Saha equation with temperature $T_2=\Theta_2 T_1$, and α_1 is the degree of ionization ahead of the shock-wave front. For a diatomic gas that dissociates fully in an ionizing shock wave, it is necessary to take into account the energy lost to dissociation and the smallest values of γ from $\frac{7}{5}$ in the state 1 to $\frac{5}{3}$ in the state 2. This is done by making in (5.19) and (5.20) the substitutions

$$\frac{5M_1^2}{3}\to\frac{7M_1^2}{10},\quad \Theta_1^{ion}(\alpha_2-\alpha_1)\to\Theta_1^{ion}(\alpha_2-\alpha_1)+\frac{\Theta_1^{diss}-1}{2},$$

where $\Theta_1^{diss}=E_{diss}/T_1$ is the dimensionless dissociation energy.

Equations (5.19) and (5.20) together with the Saha equation and the relation $s=h_2(1/M_{a1}^2-\omega_2)$ determine completely the state behind of the shock-wave front. The value of s should be chosen to correspond either to the Chapman-Jouguet condition at small front velocities, before it becomes detached from the magnetic piston but when the flow is already stationary, or else to the dimensionless threshold breakdown field of the gas—at higher velocity when the electric field correponds to the Chapman-Jouguet condition exceeds the breakdown threshold.

Detailed measurements of the parameters of the normal ionizing shock waves were performed by Levine[28] on hydrogen at a pressure 0.1–0.2 Torr in a magnetic field 1–2.5 kOe. The experiments were performed on a coaxial shock tube. His measured values of the electric field ahead of the wave-front greatly exceed the Chapman-Jouguet field at small front velocities ($v_1 \leq 5\times10^6$ cm/sec) and become equal to $E_{1\,C\text{-}J}$ for higher velocities. Levine[28] gives results up to $v_1\approx 8\times10^6$ cm/sec. Inasmuch as for stationary waves we always have $E_1 \leq E_{1\,C\text{-}J}$, the fact that the electric field exceeds $E_{1\,C\text{-}J}$ is evidence that the structures observed at $v_1 < 5\times10^6$ cm/sec are nonstationary. In the velocity interval $(5-8)\times10^6$ cm/sec, a stationary structure corresponding to the Chapman-Jouguet condition is reached, as is confirmed by measurements of the electric magnetic fields at these velocities. The values of the temperature and density behind the front, measured in Ref. 28 at $v_1=7.18\times10^6$ cm/sec, agree with the Chapman-Jouguet hypothesis within the limits of experimental error. For higher front velocities one should expect discrepancies between the result of the experiment and calculations based on the Chapman-Jouguet hypothesis. Unfortunately, there are no corresponding data for an installation with a coaxial magnetic shock tube at $v_1>8\times10^6$ cm/sec. At the same time, for the same pressure, in experiments on a cylindrical shock tube, which are reported in Ref. 27, at $v_1 \gtrsim 10^7$ cm/sec the shock-wave velocities greatly exceed the values calculated with the aid of the Chapman-Jouguet hypothesis from the value of the pushing magnetic field H_{y4}, i.e., the corresponding shock-wave parameters are closer to a magnetohydrodynamic wave, in agreement with our theory.

A substantial deviation from the Chapman-Jouguet condition at high velocities can be seen also in other measurements. Inasmuch as in the Chapman-Jouguet shock wave the state behind the front is subsonic we have in it $\omega_2<\frac{5}{8}$, i.e., when the gas behind the front is fully ionized we should have in any case $n_{e2}>1.6N_1$. At the same time, in a switch-on ionizing wave, which was observed in Ref. 5 at $M_{a1}=0.6$ and $v_1=1.1\times10^7$ cm/sec, no compression of the gas in the front was observed at all. This shock wave is of the N-wave type (see Sec. 3), for which the compression is small, $1-\omega_2\ll1$, but of course $\omega_2\neq1$. Estimates with the aid of (5.19) and (5.20) for this experiment yield $\omega_2\approx0.91-0.05$.

A rigorous calculation of the pre-breakdown field in the gas ahead of the shock-wave front, which determines the value of s, is extremely complicated. The reason is that here, as a rule, it is necessary to consider nonstationary breakdown, $E_1=E_1^*(v_1)$ [see Ref. 15, formula (2.11)]. In addition, under conditions of a real experiment it is necessary to take into account the inhomogeneity of the electric field,[3)] the presence of a strong magnetic field perpendicular to the electric field, the influence of the photoionization of the gas by radiation as a result of the front, secondary processes on the walls of the shock tube, and others. It should also be noted that in cylindrical shock tubes, which are extensively used in recent years for the study of normal ionizing shock waves with high front velocity,[5,6] a stationary electric field is possible at all in the plane normal to the axis of the conducting tube. (In this sense it seems that a coaxial electromagnetic shock tube is a

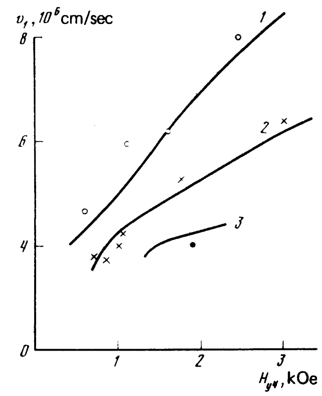

FIG. 8. Dependence of the front velocity of an ionizing shock wave in helium on the magnetic-piston field, formula (4.5): $1-p_1=0.05$ Torr, $2-p_1=0.1$ Torr, $3-p_1=0.2$ Torr; the experimental values for v_1 were taken from Ref. 6 for the same pressures.

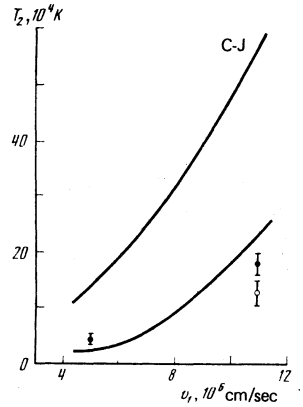

FIG. 9. Temperature behind the shock-wave front in helium as a function of the front velocity, calculated from (5.1)–(5.4) with the pre-breakdown field value (5.5) (lower curve), and calcalculated for a shock wave with the supplemental Chapman-Jouguet condition (upper curve). The experimental points are from Refs. 5 and 6.

more adequate installation for the investigation of normal shock waves.)

We assume for the value of the threshold electric field corresponding to gas breakdown in a transverse magnetic field the semi-empirical formula

$$E_1{}^* = \text{const}\cdot H_1 v_1^{1/2},$$

where the constant must be chosen in accordance with the concrete experimental setup. Under the experimental conditions of Refs. 6 and 26 we can assume

$$E_1{}^* = (150\ \text{V/cm})\frac{H_1}{10\ \text{kOe}}(v_1,\ \text{cm}/\mu\text{sec})^{1/2}\ . \qquad (5.21)$$

Figure 8 shows the shock-wave front velocities in helium, calculated with the aid of (4.5) and (5.21) and measured in Ref. 6, as functions of the propelling magnetic field of the piston H_{y4}. The axial magnetic field in this case is $H_1 = 10^4$ Oe, and the initial pressures are 0.05, 0.1, and 0.2 Torr.

Figures 9 and 10 show the temperature and density, calculated with the aid of (5.19)–(5.21), behind the shock-wave front in helium as a function of the front velocity, and the same obtained on the basis of the Chapman-Jouguet hypothesis. The experimental data are taken from Refs. 5 and 6. As seen from Figs. 9 and 10, the use of the Chapman-Jouguet condition greatly overestimates the density and the temperature.

The electric field ahead of the shock-wave front, measured for helium in Ref. 5 for $p_1 = 0.1$ Torr and $v_1 = 1.1\times 10^7$ cm/sec, is 300 V/cm. The electric field corresponding to the Chapman-Jouguet condition in this case is approximately 700 V/cm, and the value calculated from (5.21) is approximately 500 V/cm. It appears it is precisely the inaccuracy in the pre-breakdown field given by (5.21) which is the cause of the difference between the results of the theory and the experimental data at this shock-wave velocity (see Figs. 9 and 10). If we calculate the compression and the temperature from (5.19) and (5.20) and assume for E_1 the value 300 V/cm, then the results of the calculations agree with the measured quantities within the limits of the experimental error.

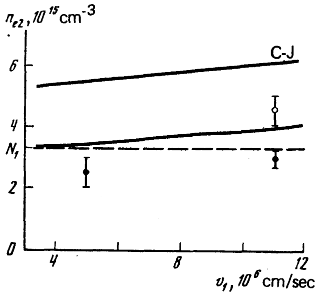

FIG. 10. Electron density behind the shock-wave front in helium, calculated on the basis of the theory developed in this paper and for a shock wave with the Chapman-Jouguet condition. The experimental points are from Refs. 5 and 6.

The Joule overheating of the heavy component of the plasma on account of the small ion-atom slippage can be estimated from formula (5.18), using (5.19) and (5.20). For the conditions given in Ref. 5, different methods of estimating the right-hand side of (5.18) yield $T_{max} \approx 35$–70 eV, while the observed value is $T^e_{max} = 50$ eV. For the ionizing shock wave observed in Ref. 25, a similar calculation yields $T_{max} \approx 30$ eV as against the measured $T^e_{max} = 40$ eV.

It should be noted in conclusion that some discrepancy between the results of the calculations of the theory presented here and the experimental data are due not only to the inaccuracy in the value of the pre-breakdown field. It must also be borne in mind that the representation of normal ionizing shock waves in cylindrical shock tubes as plane, one-dimensional, and stationary is by far not always justified. Thus, for example, in Ref. 26, where switch-on shock waves were investigated in a partially pre-ionized plasma ($\alpha_1 = 0.25$ and 0.5), the measured values of the electron density differ greatly from the value $n_{e2} = M^2_{a1} N_1$ that follows from the magnetohydrodynamic theory. Even greater is the discrepancy between the temperature values measured in Ref. 26 and calculated in accordance with the magnetohydrodynamic theory.

For a critical check on the theory developed here, it is desirable to investigate experimentally the dynamics from the transition from the Chapman-Jouguet regime at small front velocities to their magnetohydrodynamic regime at large velocities. For hydrogen or helium this is a transition from a front moving with $v_1 \approx 5\times 10^6$ cm/sec to a front with velocity $v_1 = 5\times 10^7$ cm/sec.

In conclusion, I am sincerely grateful to A. L. Velikovich for useful discussions, and to Ya. B. Zel'dovich and P. L. Kapitza for helpful remarks.

[1] In (2.9) and (2.10) we have put for brevity $(\Theta + \alpha\Theta_e) \equiv \Theta_2$, but this should not lead to confusion.

[2] The $F(h, \omega)$ curves in Fig. 2–5 are not necessarily closed (we have in mind the half-plane $\omega > 0$). If $F(h, \omega)$ does not cross the parabola $M^2 = 1$, then the trajectory $F(h, \omega)$ is an open line.

[3] Ya. B. Zel'dovich has noted that in a shock tube the field induced ahead of the shock-wave front attenuates over a characteristic distance of the order of the tube diameter, in contrast to an idealized problem with an infinite plane front.

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Thermomagnetic forces in a rarefied gas

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A kinetic theory of the thermomagnetic force (TMF) effect has been constructed and is capable of accounting for the known experimental data. The TMF effect consists of the influence of an external magnetic field on the thermal force acting on a body immersed in a nonuniformly heated gas. To calculate the TMF we use an approach based on the solution of the integral kinetic equation that was previously proposed by the authors for investigating thermomagnetic phenomena in rarefied polyatomic gases. It is shown that in addition to the TMF mechanism associated with the lack of spherical symmetry in collisions between the gas molecules, there is a second TMF mechanism, which is associated with the lack of spherical symmetry in the reflection of polarized molecules from a surface. By taking the second mechanism into account one can explain the observed dependence of the strength of the TMF on the material of the body.

PACS numbers: 51.10. + y, 51.60. + a

1. INTRODUCTION

It was recently found that a body immersed in a nonuniformly heated rarefied polyatomic gas experiences a force when a magnetic field is applied.[1-4] The investigators measured the normal and tangential components of the force that, when the field H is applied, acts on a thin disk (of radius r) of nonmagnetic material suspended in a gas between two surfaces a distance $L\sim 2r$ apart that are maintained at different temperatures. The thermomagnetic force (TMF) is found only in a region of intermediate gas pressures (when $\bar{l}\sim L$, where $\bar{l}$ is the mean free path) and vanishes in the high-pressure limit (as $\bar{l}/L\to 0$). The strength of the TMF depends on the ratio H/p of the field strength to the gas pressure. In some gases (N_2 and CO) the strength of the TMF depends substantially on the material of the disk,[3] but in other gases (O_2 and NO) there is no such dependence.[4]

The earlier theoretical treatment of the TMF[5,6] was limited to the idealized case in which $r\ll\bar{l}\ll L$. The origin of the force was sought in the nonspherical character of the collisions between the gas molecules and in the precession of the molecules in the field. The interaction of the molecules with the walls was assumed to be entirely diffuse. The strength of the force was related phenomenologically to the translational part of the field-dependent[7] heat flux (the Senftleben-Beenakker effect). That theory gave only a qualitative explanation of the effect; it could not account for the observed field dependence (it predicted the onset of the TMF at higher H/p values than were observed), and it could not explain the dependence of the TMF on the material of the disk.

As will be shown below, there is still another contribution to the TMF effect—a contribution due to the nonspherical character of the interaction of the mo-

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