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Kinetic equation for a system of parametrically excited spin waves

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Kinetic equations describing the magnon system of a parametrically excited ferromagnet are derived. The coherent-state representation is used to find the explicit form of the collision integrals. The stationary distribution is found with allowance for the exchange interactions. It is shown that such a distribution is stable with respect to weak relativistic interactions. The problem of pump-field absorption is investigated. It is shown that the absorbed power is dependent only on the interactions that do not conserve the magnon number.

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1. INTRODUCTION

The macroscopic characteristics of a parametrically excited ferrodielectric are determined by the pair correlators, $n_k = \langle a_k + a_k\rangle$ and $\sigma_k = \langle a_k a_{-k}\rangle$, of the magnon operators. The temporal evolution of the quantities n_k and σ_k is described by kinetic-type equations containing a dynamical part and a collision-integral analog. The aim of the present paper is to find the explicit form of these integrals, something which has not been done before. Further, we investigate the steady-state distribution of the magnons, and look into the problem of energy absorption for such a distribution.

As is well known, by parametric excitation we mean the situation in which, in a trans-threshold pump field, the magnon system is unstable in some region of k space. This leads to the exponential growth of the correlators n_k and σ_k in time. We assume that, as a result of the interaction between the magnons, there occurs such self-consistent renormalization of the magnon energy and pump-field amplitude as is required to make the instability region disappear.

It turns out that the nature of the steady-state distribution essentially depends on the ratio of the strengths of the interactions conserving and not conserving the magnon number. We proceed from the fact that the strongest of them is the exchange interaction between the magnons. We also take into consideration the exchange interaction between the magnons and the phonon subsystem, which is regarded as a thermostat, i.e., as being in thermodynamic equilibrium. We shall consider the relativistic interactions to be weak as compared to the exchange interactions, and shall treat their effect on the steady-state distribution as a perturbation. We establish below the conditions under which the corresponding correction to the stationary magnon distribution will be small. As will be shown, it is precisely the relativistic interactions that are responsible for the absorption of energy in the steady-state regime.

The Hamiltonian of the system under consideration by us has the following form:

$$H=H_m+H_p+H_{mm}{}^{ex}+H_{mp}{}^{ex}+H_{mm}{}^{r}. \tag{1}$$

Here H_m and H_p are the Hamiltonians of the free magnons and phonons, H_m covering the resonance interaction with the pump field:

$$H_m=\sum_k \{\varepsilon_k a_k{}^+a_k+{}^1/_2(V_k a_k a_{-k} e^{i\omega t}+V_k{}^* a_k{}^+ a_{-k}{}^+ e^{-i\omega t})\}, \tag{2}$$

$$H_p=\sum_k E_k b_k{}^+ b_k, \tag{3}$$

where ε_k is the magnon energy, E_k is the phonon ener-

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gy, and V_k is proportional to the amplitude of the uniform monochromatic pump field of frequency ω.

The last three terms in (1) describe the interaction of the magnons with each other and with the phonons:

$$H_{mm}^{ex}=\sum_{1234}\Phi_{1,2;3,4}a_1^+a_2^+a_3a_4 \tag{4}$$

is the exchange magnon-magnon interaction;

$$H_{mp}^{ex}=\sum_{123}(\Psi_{1;2,3}a_1^+a_2b_3+\Psi^*_{1;2,3}b_3^+a_2^+a_1) \tag{5}$$

is the exchange magnon-phonon interaction;

$$H_{mm}^{r}=\sum_{123}(\Phi_{1;2,3}a_1^+a_2a_3+\Phi^*_{1;2,3}a_3^+a_2^+a_1) \tag{6}$$

is the relativistic interaction between the magnons. The The amplitudes of the interactions $\Phi_{1,2;3,4}$, $\Psi_{1;2,3}$, and $\Phi_{1;2,3}$ contain Kronecker symbols ensuring the fulfillment of the momentum conservation laws in such a way that the sums of the momenta on the left and right of the semicolon are equal. Furthermore, from (4) and (6) follow the following properties of the amplitudes:

$$\Phi_{1,2;3,4}=\Phi^*_{1,2;3,4}=\Phi_{3,4;1,2}=\Phi_{2,1;3,4},\quad \Phi_{1;2,3}=\Phi_{1;3,2}.$$

Explicit expressions for the magnon energy and the interaction amplitudes are given in, for example, the monograph by Akhiezer *et al.*[1]

In the second section of the present paper we derive a system of kinetic equation for the correlators n_k and σ_k in the first approximation that takes account of the strongest (the exchange magnon-magnon and magnon-phonon) interactions. In the third section we find with the aid of the obtained kinetic equation the stationary magnon distribution that takes into account the interaction of the magnons with the phonon thermostat. In the fourth section we evaluate the effect of the relativistic interactions on the steady state distribution. For this purpose, we find the contribution of such interactions to the kinetic equations, and determine the corresponding correction to the distribution function. The problem of pump-field energy absorption is investigated in the same section. An expression is obtained for the power absorbable as a result of the magnon-number-nonconserving relativistic interaction. It is also shown that the magnon-number-conserving interactions—in particular, the exchange interactions—cannot lead to absorption.

2. DERIVATION OF THE SYSTEM OF KINETIC EQUATIONS FOR n_k AND σ_k

1. In accordance with what was said in the Introduction, let us limit ourselves in this section to the consideration of the exchange interactions. We shall proceed from the Liouville equation for the density matrix ρ:

$$i\hbar\frac{\partial\rho}{\partial t}=[H,\rho]. \tag{7}$$

With the aid of the unitary transformation

$$\tilde\rho=U^{-1}\rho U,\quad U=\exp\Big[-\frac{i\omega t}{2}\sum_k a_k^+a_k\Big] \tag{8}$$

we eliminate the explicit dependence on time and obtain

$$i\hbar\frac{\partial\tilde\rho}{\partial t}=[\tilde H,\tilde\rho], \tag{9}$$

$$\tilde H=\sum_k\Big[\Big(\varepsilon_k-\frac{\hbar\omega}{2}\Big)a_k^+a_k+\frac{1}{2}(V_ka_ka_{-k}+V_k^*a_k^+a_{-k}^+)\Big]$$
$$+\sum_k E_kb_k^+b_k+\sum_{1234}\Phi_{1,2;3,4}a_1^+a_2^+a_3a_4+\sum_{123}(\Psi_{1;2,3}a_1^+a_2b_3+\Psi^*_{1;2,3}b_3^+a_2^+a_1). \tag{10}$$

The pair correlators

$$n_k\equiv\langle a_k^+a_k\rangle=\mathrm{Sp}\,\tilde\rho a_k^+a_k=\mathrm{Sp}\,\rho a_k^+a_k, \tag{11}$$

$$\sigma_k\equiv\langle a_ka_{-k}\rangle=\mathrm{Sp}\,\tilde\rho a_ka_{-k}=e^{i\omega t}\,\mathrm{Sp}\,\rho a_ka_{-k} \tag{12}$$

satisfy the following equations, obtained from (9) and (10):

$$i\hbar\dot n_k=V_k^*\sigma_k^*-V_k\sigma_k-\Big\{2\sum_{pqr}\Phi_{p,q;r,k}\langle a_p^+a_q^+a_ra_k\rangle$$
$$+\sum_{pq}(\Psi_{p;k,q}\langle a_p^+a_kb_q\rangle-\Psi_{k;p,q}\langle a_k^+a_pb_q\rangle)-\mathrm{c.c.}\Big\},$$
$$i\hbar\dot\sigma_k=(2\varepsilon_k-\hbar\omega)\sigma_k+V_k^*(2n_k+1)+2\sum_{pqr}(\Phi_{p,q;r,k}\langle a_r^+a_pa_qa_{-k}\rangle$$
$$+\Phi_{p,q;r,-k}\langle a_ka_r^+a_pa_q\rangle)+\sum_{pq}(\Psi_{-k;-p,-q}\langle a_ka_{-p}b_{-q}\rangle+\Psi^*_{-p;-k,q}\langle a_ka_{-p}b_q^+\rangle+\mathrm{s.c.}). \tag{13}$$

The terms denoted by s. c. (sign conjugation) here and below are obtained from the terms written out by changing the sign of each of the momenta. The averaging on the right-hand sides of the Eqs. (13) is performed with the density matrix $\tilde\rho$.

Using the fact that the interaction is weak, we can decouple the right members of the Eqs. (13) into pair correlators, taking into account in the process the fact that not only the n_k's, but also the σ_k's, are nonzero. As a result, we obtain

$$\dot n_k=\frac{i}{\hbar}(\Delta_k\sigma_k-\Delta_k^*\sigma_k^*)+I_k^n,$$
$$\dot\sigma_k=-\frac{i}{\hbar}(2\xi_k-\hbar\omega)\sigma_k-\frac{i}{\hbar}\Delta_k^*(2n_k+1)+I_k^\sigma, \tag{14}$$

where

$$\xi_k=\varepsilon_k+4\sum_{k'}\Phi_{k,k';k,k'}n_{k'},\quad \Delta_k=V_k+2\sum_{k'}\Phi_{k,-k;k',-k'}\sigma_{k'}^* \tag{15}$$

are the renormalized energy and pump;

$$I_k^n=I^n_{k(mm)}+I^n_{k(mp)},\quad I_k^\sigma=I^\sigma_{k(mm)}+I^\sigma_{k(mp)} \tag{16}$$

are collision-integral analogs:

$$I^n_{k(mm)}=\frac{2i}{\hbar}\sum_{pqr}\Phi_{p,q;r,k}\langle a_p^+a_q^+a_ra_k\rangle+\mathrm{c.c.},$$
$$I^\sigma_{k(mm)}=-\frac{2i}{\hbar}\sum_{pqr}{}'\,\Phi_{p,q;r,k}\langle a_r^+a_pa_qa_{-k}\rangle+\mathrm{s.c.} \tag{16a}$$

for the magnon-magnon correlators and

$$I^n_{k(mp)}=\frac{i}{\hbar}\sum_{pq}(\Psi_{p;k,q}\langle a_p^+a_kb_q\rangle-\Psi_{k;p,q}\langle a_k^+a_pb_q\rangle)+\mathrm{c.c.}$$
$$I^\sigma_{k(mp)}=-\frac{i}{\hbar}\sum_{pq}(\Psi_{k;p,q}\langle a_{-k}a_pb_q\rangle+\Psi^*_{p,-k,q}\langle a_ka_pb_q^+\rangle)+\mathrm{s.c.} \tag{16b}$$

for the magnon-phonon correlators. The summation in (16a) is performed over those values of the quasimomenta for which direct decoupling into pair correlators yields zero. Therefore, the contribution of these sums to Eqs. (14) is quadratic in the small amplitudes Φ and

Ψ.

By constructing equations, similar to (14), for the higher-order correlators, and decoupling their right members into pair correlators, we can, in principle, obtain a closed system of equations. However, since the Hamiltonian (1) contains anomalous quadratic—in the Bose operators—terms ($V_k a_k a_{-k}$ + c. c.), this standard procedure leads in the present case to a system of sixteen equations for the higher-order magnon correlators and to a system of four equations for the magnon-phonon correlators, the direct solution of which is extremely difficult. On the other hand, it is impossible to get rid of the anomalous terms in the Hamiltonian (1), since in the region of parametric magnon excitation $|\varepsilon_k - \hbar\omega/2| < |V_k|$, and the diagonalization of the quadratic—in the Bose operators—part is impossible.

In view of the above-indicated difficulties, to find the higher-order correlators entering into the expressions (16), we shall use the coherent-state distribution function of the magnons. This function is determined by solving a linear, first-order partial differential equation.

2. Let us go over in Eq. (9) to the coherent-magnon-state representation involving those coherent states for which the vector $|z\rangle \equiv |\{z_k\}\rangle$ is a common eigenvector for all the Bose operators a_k, i.e., for which

$$a_k|z\rangle = z_k|z\rangle.$$

For this purpose, let us introduce for consideration the function

$$F(\{z_k\}) = \langle z|\rho_m|z\rangle; \quad \rho_m = \mathrm{Sp}_{ph}\tilde{\rho}, \tag{17}$$

where Sp_{ph} denotes the trace over the complete set of phonon states. The function F gives the probability distribution of the magnons over the set of complex amplitudes z_k.[1)] If we take into account (see Ref. 2) the fact that

$$\langle z|a_k\rho_m|z\rangle = e^{-|z|^2}\frac{\partial}{\partial z_k^*}e^{|z|^2}F = \left(\frac{\partial}{\partial z_k^*} + z_k\right)F,$$
$$\langle z|a_k^+\rho_m|z\rangle = z_k^* F,$$

where

$$e^{|z|^2} = \exp\left(\sum_k |z_k|^2\right),$$

then from (9) we obtain the following equation for F:

$$\partial F/\partial t = \sum_k \hat{L}_k F + M. \tag{18}$$

Here

$$\hat{L}_k = \frac{i}{\hbar}\left[\left(\varepsilon_k - \frac{\hbar\omega}{2}\right) z_k \frac{\partial}{\partial z_k} + V_k^* z_{-k}^* \frac{\partial}{\partial z_k} + \frac{1}{2} V_k^* \frac{\partial^2}{\partial z_k \partial z_{-k}}\right] + \text{c.c.}; \tag{19}$$

$$M = \frac{i}{\hbar}\sum_{1234}\Phi_{1,2;3,4}e^{-|z|^2}z_1 z_2\frac{\partial^2}{\partial z_3 \partial z_4}e^{|z|^2}F - \frac{i}{\hbar}\sum_{12q}(\Psi_{1;2\,q} + \Psi^*_{2;1,q})z_1\frac{\partial F^q}{\partial z_2} + \text{c.c.}; \tag{20}$$

$$F^q = \langle z|\mathrm{Sp}_{ph} b_q \tilde{\rho}|z\rangle. \tag{21}$$

Equation (18) is similar to the Liouville equation for the classical distribution function.[2)]

Let us introduce the distribution function for a pair of magnons with oppositely directed momenta:

$$f_k = \int F \prod_{j\neq\pm k}\left(\frac{1}{\pi}d^2 z_j\right), \quad d^2 z_j = d\,\mathrm{Re}\, z_j d\,\mathrm{Im}\, z_j. \tag{22}$$

The correlators n_k and σ_k can be expressed in terms of it as follows:

$$n_k + 1 = \frac{1}{\pi^2}\int |z_k|^2 f_k d^2 z_k d^2 z_{-k}, \quad \sigma_k = \frac{1}{\pi^2}\int z_k z_{-k} f_k d^2 z_k d^2 z_{-k}. \tag{23}$$

Similarly, we can determine the distribution function for pairs with four different momenta:

$$f_{pqrs} = \int F \prod_{j\neq\pm p,\pm q,\pm r,\pm s}\left(\frac{1}{\pi}d^2 z_j\right) - f_p f_q f_r f_s, \tag{24}$$

in terms of which the corresponding correlators can be expressed, e.g.,

$$\langle a_p^+ a_q^+ a_r a_s\rangle = \int z_p^* z_q^* z_r z_s f_{pqrs} \prod_{j=\pm p,\pm q;\pm r,\pm s}\left(\frac{1}{\pi}d^2 z_j\right). \tag{25}$$

The equation for the function $f_{pq\,rs}$ follows from (18) and (24). It has the following form:

$$\dot{f}_{pqrs} = \sum_{k=\pm p,\pm q,\pm r,\pm s}\hat{L}_k f_{pqrs} + M_{pqrs}, \tag{26}$$

where

$$M_{pqrs} = \int M \prod_{j\neq\pm p,\pm q,\pm r,\pm s}\left(\frac{1}{\pi}d^2 z_j\right) - \sum_{n=p,q,r,s}\int M \prod_{j\neq\pm n} \times \left(\frac{1}{\pi}d^2 z_j\right) f_p f_q f_r f_s f_n^{-1}. \tag{27}$$

Bearing in mind that the distribution function (24) will be used by us only for the computation of correlators of the type (25), we can drop the terms with second derivatives in the operator (19). Indeed, these terms drop out when we go over from Eq. (26) to the equations for such correlators,[3)] and therefore in no way affect the values of the correlators.

From Eq. (26) we can derive a closed equation for the function $f_{pq\,rs}$ if, using the fact that the interactions are weak, we decouple the terms in the expression (27) and limit ourselves to the lowest order in the interaction. In this case complete decoupling should be carried out up to the pair functions of the distribution f_k. Furthermore, as in Eqs. (14), the renormalization (15) should be allowed for in (27). As a result, we have

$$\dot{f}_{pq,rs} - \sum_{k=\pm p\pm,q,\pm r,\pm s}\tilde{\hat{L}}_k f_{pq,rs} = J_{pq,rs}, \tag{28}$$

where

$$\tilde{\hat{L}}_k = \frac{i}{\hbar}\left[\left(\xi_k - \frac{\hbar\omega}{2}\right) z_k \frac{\partial}{\partial z_k} + \Delta_k^* z_{-k}^* \frac{\partial}{\partial z_k}\right] + \text{c.c.}, \tag{29}$$

$$J_{pq,rs} \equiv J_{pq,rs}(\{z_k\}) = \left\{\frac{4i}{\hbar}\Phi_{p,q;r,s}e^{-|z|^2}\left(z_p z_q \frac{\partial^2}{\partial z_r \partial z_s} - z_r^* z_s^* \frac{\partial^2}{\partial z_p^* \partial z_q^*}\right) e^{|z|^2} f_p f_q f_r f_s + \text{s.c.}\right\} + \text{c.c.} \tag{30}$$

Here we have taken into consideration the momentum conservation law $\mathbf{p}+\mathbf{q}=\mathbf{r}+\mathbf{s}$, which is contained in the expressions, (16), for the sought collision integrals. Notice that the terms with the magnon-phonon interaction drop out in the indicated decoupling process, since they contain a single phonon operator.

3. Equation (28) is a nonhomogeneous first-order partial differential equation with $J_{pq,rs}$ as its right member. Assuming that the correlators (25) vanish at $t=-\infty$, we shall seek the particular solution to Eq. (28) with the same initial condition. We shall use the meth-

od of characteristics to find the solution.

The characteristics of Eq. (28) are the solution to the system of equations

$$i\hbar\dot{z}_k=\left(\xi_k-\frac{\hbar\omega}{2}\right)z_k+\Delta_k^* z_{-k}^*,\quad -i\hbar\dot{z}_{-k}^*=\left(\xi_k-\frac{\hbar\omega}{2}\right)z_{-k}^*+\Delta_k z_k, \quad (31)$$

which coincide with the equations of motion for the classical amplitudes. As can be seen from (15), into the quantities ξ_k and Δ_k enter the required pair correlators $n_{k'}$ and $\sigma_{k'}$, which are time dependent. However, as is customary in kinetics, this slow dependence can be neglected in Eqs. (31). Then we obtain the following solution:

$$\begin{gathered}z_k=c_k u_k\exp(-i\tilde{\varepsilon}_k t/\hbar)+c_{-k}^* v_k^*\exp(i\tilde{\varepsilon}_k t/\hbar),\\ z_{-k}^*=c_{-k}^* u_k^*\exp(-i\tilde{\varepsilon}_k t/\hbar)+c_k v_k\exp(i\tilde{\varepsilon}_k t/\hbar),\end{gathered} \quad (32)$$

where c_k and c_{-k}^* are constants of the integration,

$$\tilde{\varepsilon}_k=[(\xi_k-\hbar\omega/2)^2-|\Delta_k|^2]^{1/2}, \quad (33)$$

$$u_k=\left[\frac{\xi_k-\hbar\omega/2-\tilde{\varepsilon}_k}{2\tilde{\varepsilon}_k}\right]^{1/2},\quad v_k=-\frac{\Delta_k u_k}{\xi_k-\hbar\omega/2+\tilde{\varepsilon}_k}. \quad (34)$$

In conformity with what was said in the Introduction, we assume that the renormalizations ensure the realness of $\tilde{\varepsilon}_k$ in the entire k space. The sought solution of Eq. (28) then has the following form:

$$f_{pq,rs}=\int_{-\infty}^{t} J_{pq,rs}\left(\left\{c_k u_k\exp\left(-\frac{i}{\hbar}\tilde{\varepsilon}_k\tau\right)+c_{-k}^* v_k^*\exp\left(\frac{i}{\hbar}\tilde{\varepsilon}_k\tau\right)\right\}\right)d\tau,$$

where c_k and c_{-k}^* should be expressed in terms of z_k and z_{-k}^* from (32). As a result, we obtain

$$f_{pq,rs}=\int_{-\infty}^{0} J_{pq,rs}(\{\tilde{z}_k\})\,d\tau, \quad (35)$$

where

$$\tilde{z}_k=\lambda_k^*(\tau)z_k-\mu_k^*(\tau)z_{-k}^*,\quad \tilde{z}_{-k}^*=\lambda_k(\tau)z_{-k}^*-\mu_k(\tau)z_k. \quad (36)$$

Here

$$\begin{gathered}\lambda_k(\tau)=|u_k|^2\exp\left(\frac{i}{\hbar}\tilde{\varepsilon}_k\tau\right)-|v_k|^2\exp\left(-\frac{i}{\hbar}\tilde{\varepsilon}_k\tau\right),\\ \mu_k(\tau)=2iu_k^* v_k\sin\frac{\tilde{\varepsilon}_k\tau}{\hbar}.\end{gathered} \quad (37)$$

4. The found solution, (35), allows us to obtain explicit expressions for the four-magnon correlators of interest to us. From (25) and (35) we have

$$\langle a_p^+ a_q^+ a_r a_s\rangle=\int_{-\infty}^{0} d\tau\int z_p^* z_q^* z_r z_s J_{pq,rs}(\{\tilde{z}_k\})\prod_{j\neq\pm p,\pm q,\pm r,\pm s}\left(\frac{1}{\pi}d^2 z_j\right).$$

It is natural to go over from integration over z to integration over $\tilde{z}$ with the aid of (36). Taking into account the fact that the Jacobian of the transformation is equal to unity, we represent the correlator in the following form:

$$\begin{gathered}\langle a_p^+ a_q^+ a_r a_s\rangle=\frac{4i}{\hbar}\Phi_{pq,rs}\int_{-\infty}^{0} d\tau\{[\lambda_p^*(n_p+1)+\mu_p\sigma_p]\\ \times[\lambda_q^*(n_q+1)+\mu_q\sigma_q](\lambda_r n_r+\mu_r^*\sigma_r^*)(\lambda_s n_s+\mu_s^*\sigma_s^*)-(\lambda_p^* n_p+\mu_p\sigma_p)\\ \times(\lambda_q^* n_q+\mu_q\sigma_q)[\lambda_r(n_r+1)+\mu_r^*\sigma_r^*][\lambda_s(n_s+1)+\mu_s^*\sigma_s^*]\\ -[\lambda_p^*\sigma_p^*+\mu_p(n_p+1)][\lambda_q^*\sigma_q^*+\mu_q(n_q+1)](\lambda_r\sigma_r+\mu_r^* n_r)(\lambda_s\sigma_s+\mu_s^* n_s)\\ +(\lambda_p^*\sigma_p^*+\mu_p n_p)\cdot(\lambda_q^*\sigma_q^*+\mu_q n_q)[\lambda_r\sigma_r+\mu_r^*(n_r+1)][\lambda_s\sigma_s+\mu_s^*(n_s+1)]\}.\end{gathered}$$

Collecting the terms with the same exponential time functions, and integrating over τ, we obtain

$$\langle a_p^+ a_q^+ a_r a_s\rangle=4i\pi\Phi_{p,q;r,s}\{|u_p u_q u_r u_s|^2 R_{pq,rs}\delta_+(\tilde{\varepsilon}_p+\tilde{\varepsilon}_q-\tilde{\varepsilon}_r-\tilde{\varepsilon}_s)+\ldots\}. \quad (38)$$

Here

$$\begin{gathered}R_{pq,rs}=(n_p+1-\gamma_p\sigma_p)(n_q+1-\gamma_q\sigma_q)(n_r-\gamma_r^*\sigma_r^*)(n_s-\gamma_s^*\sigma_s^*)\\ -(n_p-\gamma_p\sigma_p)(n_q-\gamma_q\sigma_q)(n_r+1-\gamma_r^*\sigma_r^*)(n_s+1-\gamma_s^*\sigma_s^*)\\ -[\sigma_p^*-\gamma_p(n_p+1)][\sigma_q^*-\gamma_q(n_q+1)](\sigma_r-\gamma_r^* n_r)(\sigma_s-\gamma_s^* n_s)\\ +(\sigma_p^*-\gamma_p n_p)(\sigma_q^*-\gamma_q n_q)[\sigma_r-\gamma_r^*(n_r+1)][\sigma_s-\gamma_s^*(n_s+1)],\end{gathered} \quad (39)$$

$$\gamma_j=\frac{v_j}{u_j},\quad \delta_+(x)=\delta(x)+\frac{i}{\pi}\mathrm{P}\frac{1}{x}.$$

The terms that have not been explicitly written out in the curly brackets in (38) [and in (40) and (41) below] are obtained by the substitutions

$$\tilde{\varepsilon}_j\to-\tilde{\varepsilon}_j,\quad u_j\rightleftarrows v_j^*\quad (\mathrm{j=p,\ q,\ r,\ s}).$$

The correlator

$$\langle a_r^+ a_p a_q a_{-s}\rangle=-4i\pi\Phi_{p,q;r,s}\{|u_p u_q u_r|^2 u_s v_s^* \overset{*}{R}_{pq,rs}\delta_+(\tilde{\varepsilon}_r+\tilde{\varepsilon}_s-\tilde{\varepsilon}_p-\tilde{\varepsilon}_q)+\ldots\} \quad (40)$$

is computed in entirely similar fashion.

Substituting the expressions (38) and (40) into the formulas (16), we obtain the expressions for the magnon-magnon part of the collision integrals $I^n_{k(mm)}$ and $I^\sigma_{k(mm)}$:

$$\begin{gathered}I^n_{k(mm)}=-\frac{8\pi}{\hbar}\sum_{pqr}{}'\,\Phi^2_{p,q;r,k}\{|u_p u_q u_r u_k|^2 R_{pq,rk}\delta_+(\tilde{\varepsilon}_p+\tilde{\varepsilon}_q-\tilde{\varepsilon}_r-\tilde{\varepsilon}_k)+\ldots\}+\text{c.c.},\\ I^\sigma_{k(mm)}=-\frac{16\pi}{\hbar}\sum_{pqr}{}'\,\Phi^2_{p,q;r,k}\{|u_p u_q u_r|^2 u_k v_k^* \overset{*}{R}_{pq,rk}\delta_+(\tilde{\varepsilon}_r+\tilde{\varepsilon}_k-\tilde{\varepsilon}_p-\tilde{\varepsilon}_q)+\ldots\}.\end{gathered} \quad (41)$$

5. We can use for the computation of the correlators entering into the collision integrals (16b) the function

$$f_{rs}{}^q=\int F^q\prod_{j\neq\pm r,\pm s}\left(\frac{1}{\pi}d^2 z_j\right)-f_r f_s, \quad (42)$$

where the function F^q is given by the formula (21). The equation for F^q is derived from the Liouville equation in exactly the same way as equation (18) was derived for F. Further, performing the integration in (42) and decoupling the terms on the right-hand side, we obtain the equation

$$\dot{f}_{rs}{}^q-\sum_{k=\pm r,\pm s}\hat{L}_k f_{rs}{}^q+\frac{i}{\hbar}E_q f_{rs}{}^q=J_{rs}{}^q, \quad (43)$$

where

$$\begin{gathered}J_{rs}{}^q=\frac{i}{\hbar}e^{-|z|^2}\left\{\overset{*}{\Psi}_{r;s,q}\left[z_r\frac{\partial}{\partial z_s}N_q-\frac{\partial}{\partial z_r^*}z_s^*(N_q+1)\right]\right.\\ \left.+\overset{*}{\Psi}_{-s;-r,q}\left[z_{-s}\frac{\partial}{\partial z_{-r}}N_q-\frac{\partial}{\partial z_{-s}^*}z_{-r}^*(N_q+1)\right]\right\}e^{|z|^2}f_r f_s.\end{gathered} \quad (44)$$

Here $N_q=[\exp(E_q/T)-1]^{-1}$ is the phonon equilibrium Bose function and T is the temperature of the phonon thermostat. Equation (43) is similar to Eq. (28). It has the same characteristics, and therefore its solution can be written in the form

$$f_{rs}{}^q=\int_{-\infty}^{0} J_{rs}{}^q(\{\tilde{z}_k\})\exp\left(\frac{i}{\hbar}E_q\tau\right)d\tau,$$

where the $\tilde{z}_k$ are determined by the formulas (36).

The correlators entering into the expressions (16b) are defined as moments of the function f^q_{rs}, e.g.,

$$\langle a_r^+ a_s b_q\rangle=\int z_r^* z_s f_{rs}{}^q\prod_{j=\pm r,\pm s}\left(\frac{1}{\pi}d^2 z_j\right).$$

As a result, for the magnon-phonon collision integrals, we have

$$I^{n}_{k(mp)}=\frac{\pi}{\hbar}\sum_{pq}\{(|\Psi_{k;p,q}|^2|u_k u_p|^2-\Psi_{p;k,q}\Psi^{*}_{-k;-p,q}$$
$$\times u_k v_k^* u_p^* v_p)R_{kp}{}^{q}\delta_+(\tilde{\varepsilon}_k-\tilde{\varepsilon}_p-E_q)-(|\Psi_{p;k,q}|^2|u_p u_k|^2-\Psi_{k;p,q}\Psi^{*}_{-p;-k,q}$$
$$\times u_p v_p^* u_k^* v_k)R_{pk}{}^{q}\delta_+(\tilde{\varepsilon}_p-\tilde{\varepsilon}_k-E_q)+\ldots\}+\text{c.c.},$$
$$I^{\sigma}_{k(mp)}=\frac{2\pi}{\hbar}\sum_{pq}\{|\Psi_{k;p,q}|^2u_k v_k^*|u_p|^2R_{kp}{}^{q}\delta_+(\tilde{\varepsilon}_k-\tilde{\varepsilon}_p-E_q)\tag{45}$$
$$-|\Psi_{-p;-k,q}|^2u_k v_k^*|u_p|^2R_{kp}{}^{q}\delta_+(\tilde{\varepsilon}_k-\tilde{\varepsilon}_p+E_q)$$
$$+\Psi_{k;p,q}\Psi^{*}_{-p;-k,q}|u_k|^2u_p v_p^*R_{pk}{}^{q}\delta_+(\tilde{\varepsilon}_p-\tilde{\varepsilon}_k-E_q)$$
$$-\Psi_{k;p,q}\Psi^{*}_{-p;-k,q}|u_k|^2u_p v_p^*R_{pk}{}^{q}\delta_+(\tilde{\varepsilon}_p-\tilde{\varepsilon}_k+E_q)+\ldots\},$$

where

$$R_{kp}{}^{q}=(n_k+1-\gamma_k\sigma_k)(n_p-\gamma_p^*\sigma_p^*)N_q-(n_k-\gamma_k\sigma_k)(n_p+1-\gamma_p^*\sigma_p^*)(N_q+1).\tag{46}$$

The formulas (41) and (45), together with (16) and (14), solve the problem entailing the derivation of a system of kinetic equations with allowance for the strongest (exchange) interactions. These equations are valid for arbitrary amplitudes of the pump field. If the pump field is weak, the equations can be linearized around the equilibrium distribution, which will lead to previously obtained results.[4] In the case of a strong pump the equations are essentially nonlinear in both the dynamical and collision parts.

3. THE STATIONARY MAGNON DISTRIBUTION

The stationary magnon distribution is determined from the kinetic equations (14), with the collision integrals given by the formulas (16), (41), and (45). Setting $\dot{n}_k=0$, $\dot{\sigma}_k=0$ in (14), we obtain

$$\frac{i}{\hbar}(\Delta_k^*\sigma_k^*-\Delta_k\sigma_k)=I_k{}^{n},$$
$$\frac{2i}{\hbar}\left[\Delta_k^*\left(n_k+\frac{1}{2}\right)+\left(\xi_k-\frac{\hbar\omega}{2}\right)\sigma_k\right]=I_k{}^{\sigma},\tag{47}$$
$$-\frac{2i}{\hbar}\left[\Delta_k\left(n_k+\frac{1}{2}\right)+\left(\xi_k-\frac{\hbar\omega}{2}\right)\sigma_k^*\right]=I_k{}^{\sigma*}.$$

The condition for the consistency of this system follows immediately:

$$(2\xi_k-\hbar\omega)I_k{}^{n}+\Delta_k I_k{}^{\sigma}+\Delta_k^* I_k{}^{\sigma*}=0.\tag{48}$$

Using the fact that the magnon-magnon and magnon-phonon interactions are weak, we shall seek the solution of the system (41) in the form

$$n_k=n_k^{(0)}+n_k^{(1)}+\ldots,\quad \sigma_k=\sigma_k^{(0)}+\sigma_k^{(1)}+\ldots,$$

where $n_k^{(0)}$, $\sigma_k^{(0)}$, and $\sigma_k^{(0)*}$ satisfy Eqs. (47) without the right-hand sides:

$$\Delta_k^{(0)}\sigma_k^{(0)}-\Delta_k^{(0)*}\sigma_k^{(0)*}=0,\quad (\xi_k^{(0)}-\hbar\omega/2)\sigma_k^{(0)}+\Delta_k^{(0)*}(n_k^{(0)}+{}^1/_2)=0,$$
$$(\xi_k^{(0)}-\hbar\omega/2)\sigma_k^{(0)*}+\Delta_k^{(0)}(n_k^{(0)}+{}^1/_2)=0,\tag{49}$$

with

$$\xi_k^{(0)}=\varepsilon_k+4\sum_{k'}\Phi_{k,k';k,k'}n_{k'}^{(0)},$$
$$\Delta_k^{(0)}=V_k+2\sum_{k'}\Phi_{k,-k;k',-k'}\sigma_{k'}^{(0)},\tag{50}$$

$n_k^{(1)}$, $\sigma_k^{(1)}$, ... being small—in the interaction—corrections. The system (49) has a null determinant, and its solution can be expressed in terms of a single quantity, n_k^B, as follows:

$$n_k^{(0)}+\frac{1}{2}=\frac{\xi_k^{(0)}-\hbar\omega/2}{\tilde{\varepsilon}_k^{(0)}}\left(n_k^B+\frac{1}{2}\right),\quad \sigma_k^{(0)}=-\frac{\Delta_k^{(0)*}}{\tilde{\varepsilon}_k^{(0)}}\left(n_k^B+\frac{1}{2}\right).\tag{51}$$

When (51) is substituted into the solvability condition (48), also written in the zeroth approximation, the dominant parts cancel each other out, and we arrive at an equation for the function n_k^B having the form

$$\sum_{pqr}{}'\{|W^{(mm)}_{pq,rk}|^2[n_p^Bn_q^B(n_r^B+1)(n_k^B+1)-(n_p^B+1)(n_q^B+1)n_r^Bn_k^B]\cdot$$
$$\times\delta(\tilde{\varepsilon}_p+\tilde{\varepsilon}_q-\tilde{\varepsilon}_r-\tilde{\varepsilon}_k)+|W^{(mm)}_{pqr,k}|^2[n_p^Bn_q^Bn_r^B(n_k^B+1)$$
$$-(n_p^B+1)(n_q^B+1)(n_r^B+1)n_k^B]\delta(\tilde{\varepsilon}_p+\tilde{\varepsilon}_q+\tilde{\varepsilon}_r-\tilde{\varepsilon}_k)+\ldots\}$$
$$+\sum_{pq}\{|W^{(mp)}_{pqr}|^2[(n_k^B+1)n_p^BN_q-n_k^B(n_p^B+1)(N_q+1)]\cdot$$
$$\delta(\tilde{\varepsilon}_k-\tilde{\varepsilon}_p+E_q)+\ldots\}=0,\tag{52}$$

where

$$W^{(mm)}_{pq,rk}=2\sqrt{2}\,\Phi_{p,q;r,k}(u_p^*u_q^*u_ru_k+v_pv_qv_r^*v_k^*),$$
$$W^{(mm)}_{pqr,k}=2\sqrt{2}\,\Phi_{p,q;r,k}(u_p^*u_q^*v_r^*u_k+v_pv_qu_rv_k^*),$$
$$W^{(mp)}_{pqr}=\Psi_{kpq}u_k^*u_p+\Psi^*_{pkq}v_kv_p^*.$$

The terms that have not been written out pertain to all-possible elementary processes both with conservation, and without conservation, of the "new" quasiparticles (with the corresponding energy conservation laws). Equation (52) has the structure of a normal stationary kinetic equation, and its solution is an equilibrium Bose function with zero chemical potential and a temperature, T, equal to the temperature of the thermostat:

$$n_k{}^B=[\exp(\tilde{\varepsilon}_k/T)-1]^{-1}.\tag{53}$$

The stationary magnon distribution is found by substituting (53) into (51), the functions $\Delta_k^{(0)}$ and $\xi_k^{(0)}$ being determined by the self-consistency conditions.[5] These conditions are nonlinear integral equations, which are obtained from (50) when $n_k^{(0)}$ and $\sigma_k^{(0)}$ are replaced by the found stationary values. The obtained stationary distribution differs essentially from the thermodynamic-equilibrium distribution in that region of **k** space where $|\xi_k-\hbar\omega/2|\sim|\Delta_k|$. The dimensions of the region and its location are determined by the magnitudes of the frequency and pump renormalizations, and are found from the solution to the self-consistency equations. Far from this region the renormalizations become unimportant, and the distribution (51) is close to the equilibrium distribution with a chemical potential equal to $\hbar\omega/2$:

$$n_k^{(0)}=[\exp[(\varepsilon_k-\hbar\omega/2)/T]-1]^{-1},\quad \sigma_k^{(0)}=0.$$

4. EFFECT OF THE RELATIVISTIC INTERACTIONS

1. Let us now find out how the above-found stationary distribution [see (51) and (53)] changes when the small relativistic correction, (6), in the Hamiltonian of the magnon system is taken into consideration.

Since the interaction (6) does not conserve the magnon number, on being transformed with the aid of the unitary transformation (8), the Hamiltonian H^r_{mm} becomes explicitly dependent on time:

$$H_{mm}{}^{r}=U^{-1}H_{mm}{}^{r}U=\sum_{123}(\Phi_{1;2,3}a_1{}^{+}a_2a_3e^{-i\omega t/2}+\text{H.c.}).\tag{6'}$$

As a result, there appear on the right-hand sides of the Eqs. (14) for n_k and σ_k time-dependent relativistic corrections to the collision integrals:

$$I_k^{nr} = -\frac{i}{\hbar} e^{-i\omega t/2} \sum_{pq} (\Phi_{k;\,p,q}\langle a_k^+ a_p a_q\rangle - 2\Phi_{p;\,k,q}\langle a_p^+ a_q a_k\rangle) + \text{c.c.}, \tag{54}$$

$$I_k^{\sigma r} = -\frac{i}{\hbar} \sum_{pq} (e^{-i\omega t/2}\Phi_{k;\,p,q}\langle a_{-k} a_p a_q\rangle + 2e^{i\omega t/2}\Phi^*_{p;\,q,k}\langle a_q^+ a_p a_{-k}\rangle) + \text{s.c.}$$

To find the correlators entering into (54), let us, as before, use the coherent-state distribution function $F(\{z_k\})$, in the equation for which we shall allow for the relativistic corrections. Going over in this equation to the function f_{pqr} [it is determined in much the same way as (24)], we obtain after the decoupling process the equation

$$f_{p,qr} - \sum_{k=pqr} \tilde{L}_k f_{p,qr} = J^r_{p,qr}(t), \tag{55}$$

where

$$J^r_{p,qr}(t) = \frac{2i}{\hbar} e^{i\omega t/2} \left\{ \Phi^*_{p,qr} e^{-|z|^2} \left(z_p \frac{\partial^2}{\partial z_q \partial z_r} - z_q^* z_r^* \frac{\partial}{\partial z_p^*} \right) \times e^{|z|^2} f_p f_q f_r + \text{з. с.} \right\} + \text{c.c.} \tag{56}$$

Equation (55) does not contain terms stemming from the exchange interactions, since these terms vanish in the integration leading to the function $f_{p,qr}$ and in the subsequent decoupling of its terms. In its turn, the relativistic interaction does not contribute to the Eq. (28) for the function $f_{pq,rs}$ because of the difference in the momentum-conservation laws.

The solution to Eq. (55), like the solutions to Eqs. (28) and (43), has the form

$$f_{p,qr} = \int_{-\infty}^{0} J^r_{p,qr}(\tau + t, \{\bar{z}_k\})\, d\tau.$$

The correlators entering into the expressions (54) for the relativistic collision integrals are found, as before, from this solution. In performing the integration over τ, we should bear in mind that the functions n_k and σ_k entering into the integrand contain parts varying in time with frequency ω. This is connected with the explicit dependence of the Hamiltonian $\tilde{H}$ on the time, a dependence which stems from the relativistic interactions. But these rapidly varying parts are corrections to the stationary values n_k^0 and σ_k^0, and are, to the extent that the relativistic interactions are weak compared to the exchange interactions, small. Therefore, in computing the correlators in the expressions (54), we can assume that $n_k = n_k^{(0)}$, $\sigma_k = \sigma_k^{(0)}$. Then for the relativistic collision integrals we have

$$I_k^{nr} = A_k^n + B_k^n e^{i\omega t} + B_k^{n*} e^{-i\omega t},$$
$$I_k^{\sigma r} = A_k^\sigma + B_k^\sigma e^{i\omega t} + C_k^{\sigma *} e^{-i\omega t}, \tag{57}$$

where

$$A_k^n = \frac{2\pi}{\hbar} \sum_{pq} (|\Phi_{k;\,p,q}|^2 |u_k u_p u_q|^2 - 2|\Phi_{p;\,q,k}|^2 |v_k v_p u_q|^2) \times R_{k,pq}\delta_+\left(\tilde{\varepsilon}_k - \tilde{\varepsilon}_p - \tilde{\varepsilon}_q - \frac{\hbar\omega}{2}\right) + \text{к. с.} + \ldots,$$

$$B_k^n = \frac{2\pi}{\hbar} \sum_{pq} (\Phi^{*2}_{k;\,pq} u_k v_k^* u_p v_p^* u_q^* v_q - 2\Phi^{*2}_{p;\,q,k} u_k^* v_k u_p v_p^* u_q^* v_q) R_{k,pq}\delta_+\left(\tilde{\varepsilon}_k - \tilde{\varepsilon}_p - \tilde{\varepsilon}_q - \frac{\hbar\omega}{2}\right) + \ldots.$$

$$A_k^\sigma = \frac{4\pi}{\hbar} \sum_{pq} \Big[|\Phi_{k;\,p,q}|^2 u_k v_k^* |u_p u_q|^2 R_{k,pq}\delta_+\left(\tilde{\varepsilon}_k - \tilde{\varepsilon}_p - \tilde{\varepsilon}_q - \frac{\hbar\omega}{2}\right) - 2\Phi_{p;\,q,-k}\Phi^*_{q;\,p,k} u_k v_k^* u_p^* v_p u_q v_q^* R_{k,pq}\delta_-\left(\tilde{\varepsilon}_k - \tilde{\varepsilon}_p - \tilde{\varepsilon}_q - \frac{\hbar\omega}{2}\right)\Big] + \ldots$$

$$B_k^\sigma = \frac{8\pi}{\hbar} \sum_{pq} \Phi^*_{p;\,q,-k}\Phi^*_{q;\,p,k} |v_k v_p v_q|^2 R_{k,pq}\delta_+\left(\tilde{\varepsilon}_k - \tilde{\varepsilon}_p - \tilde{\varepsilon}_q - \frac{\hbar\omega}{2}\right) + \ldots,$$

$$C_k^\sigma = \frac{4\pi}{\hbar} \sum_{pq} \Phi^2_{k;\,p,q} |u_k|^2 u_p v_p^* u_q v_q^* R_{k,pq}\delta_-\left(\tilde{\varepsilon}_k - \tilde{\varepsilon}_p - \tilde{\varepsilon}_q - \frac{\hbar\omega}{2}\right) + \ldots,$$

$$R_{k,pq} = (n_k^B + 1) n_p^B n_q^B - n_k^B (n_p^B + 1)(n_q^B + 1).$$

Here the terms that have not been explicitly written out are obtained by the substitutions

$$u_j \rightleftarrows v_j^*, \quad \tilde{\varepsilon}_j \to -\tilde{\varepsilon}_j, \quad n_j^B + 1 \rightleftarrows -n_j^B \quad (j = k, p, q).$$

2. To find the relativistic corrections to the stationary distribution, let us consider the system of equations (14) in which the collision integrals I_k^n and I_k^σ contain the relativistic corrections (57). Setting

$$n_k = n_k^{st} + n_k^r, \quad \sigma_k = \sigma_k^{st} + \sigma_k^r,$$

where n_k^{st} and σ_k^{st} are the stationary distributions satisfying Eqs. (47), while n_k^r and σ_k^r are corrections due to the relativistic interaction, and linearizing the system (14) in the corrections n_k^r and σ_k^r, we obtain a system of linear integro-differential equations. Intending only to estimate the relativistic corrections, we can neglect the integral terms in these equations. As a result, we arrive at the following system:

$$\dot{n}_k^r - \frac{i}{\hbar}(\Delta_k^{st}\sigma_k^r - \Delta_k^{st*}\sigma_k^{r*}) - I_k^{n\,ex}(n_k^r, \sigma_k^r) = I_k^{nr},$$
$$\dot{\sigma}_k^r + \frac{i}{\hbar}(2\xi_k^{st} - \hbar\omega)\sigma_k^r + \frac{i}{\hbar}\Delta_k^{st*}\, 2n_k^r - I_k^{\sigma\,ex}(n_k^r, \sigma_k^r) = I_k^{\sigma r}, \tag{58}$$
$$\dot{\sigma}_k^{r*} - \frac{i}{\hbar}(2\xi_k^{st} - \hbar\omega)\sigma_k^{r*} - \frac{i}{\hbar}\Delta_k^{st} 2n_k^r - I_k^{\sigma ex*}(n_k^r, \sigma_k^r) = I_k^{\sigma r*}.$$

Here $I_k^{ex}(n_k^r\ \sigma_k^r)$ is a function that is linear in n_k^r and σ_k^r, and does not contain the integral terms of the correction to the exchange collision integral.

Let us, in accordance with (57), represent the solution to the system (58) as follows:

$$n_k^r = \tilde{n}_k^r + \alpha_k e^{i\omega t} + \alpha_k^* e^{-i\omega t},$$
$$\sigma_k^r = \tilde{\sigma}_k^r + \lambda_k e^{i\omega t} + \mu_k^* e^{-i\omega t}.$$

For the time-independent part we have

$$-\frac{i}{\hbar}(\Delta_k^{st}\tilde{\sigma}_k^r - \Delta_k^{st*}\tilde{\sigma}_k^{r*}) - I_k^{nex}(\tilde{n}_k^r, \tilde{\sigma}_k^r) = A_k^n,$$
$$\frac{i}{\hbar}(2\xi_k^{st} - \hbar\omega)\tilde{\sigma}_k^r + \frac{i}{\hbar}\Delta_k^{st*}\, 2\tilde{n}_k^r - I_k^{\sigma ex}(\tilde{n}_k^r, \tilde{\sigma}_k^r) = A_k^\sigma, \tag{59}$$
$$-\frac{i}{\hbar}(2\xi_k^{st} - \hbar\omega)\tilde{\sigma}_k^{r*} - \frac{i}{\hbar}\Delta_k^{st} 2\tilde{n}_k^r - I_k^{\sigma ex*}(\tilde{n}_k^r, \tilde{\sigma}_k^r) = A_k^{\sigma*}.$$

Using the fact that the collision part is small compared to the dynamical part, we seek the solution to this system in the form

$$\tilde{n}_k^r = \tilde{n}_{k0}^r + \tilde{n}_{k1}^r + \ldots, \quad \tilde{\sigma}_k^r = \tilde{\sigma}_{k0}^r + \tilde{\sigma}_{k1}^r + \ldots,$$

where $\tilde{n}_{k0}^r$ and $\tilde{\sigma}_{k0}^r$ make the dynamical parts vanish. Therefore, as was done above [see (51)], $\tilde{n}_{k0}^r$ and $\tilde{\sigma}_{k0}^r$ can be expressed in terms of a single quantity, ν_k:

$$\tilde{n}_{k0}^r = \frac{\xi_k^{(0)} - \hbar\omega/2}{\tilde{\varepsilon}_k^{(0)}}\nu_k, \quad \sigma_k^r = -\frac{\Delta_k^{(0)*}}{\tilde{\varepsilon}_k^{(0)}}\nu_k. \tag{60}$$

[We have replaced the quantities ξ_k^{st} and Δ_k^{st} by their values in the zeroth approximation, $\xi_k^{(0)}$ and $\Delta_k^{(0)}$, since the discarded terms give, as can be seen from (60) and

(61), corrections that are of higher order in I_k^{ex}/ε_k.]
The equation for ν_k follows from the solvability condition for the system for $\bar{n}_{k1}^r$, σ_{k1}^r:

$$(2\xi_k^{(0)}-\hbar\omega)I_k^{nex}+\Delta_k^{(0)}I_k^{\sigma ex}+\Delta_k^{(0)*}I_k^{\sigma ex*}=-2\bar{\varepsilon}_k A_k,$$
$$2\bar{\varepsilon}_k A_k=(2\xi_k^{(0)}-\hbar\omega)A_k^n+\Delta_k^{(0)}A_k^\sigma+\Delta_k^{(0)*}A_k^{\sigma*}. \quad (61)$$

From this system we find

$$\nu_k=A_k/D_k, \quad (62)$$

where

$$D_k=\frac{\pi}{2\hbar}\sum_{pqr}\frac{\operatorname{sh}(\bar{\varepsilon}_k/2T)}{\operatorname{sh}(\bar{\varepsilon}_p/2T)\operatorname{sh}(\bar{\varepsilon}_q/2T)\operatorname{sh}(\bar{\varepsilon}_r/2T)}$$
$$\times\{|W_{pq,rk}^{(mm)}|^2\delta(\bar{\varepsilon}_p+\bar{\varepsilon}_q-\bar{\varepsilon}_r-\bar{\varepsilon}_k)+\ldots\}.$$

In the equations determining the coefficients α_k, λ_k, and μ_k, we can neglect the collision terms [we cannot do this in Eqs. (59), since it leads to an inconsistent system]. As a result, we obtain the following solution:

$$\alpha_k=-\frac{i}{\omega}\left(B_k^n+\frac{\Delta_k^{(0)}}{2\xi_k^{(0)}}B_k^\sigma+\frac{\Delta_k^{(0)*}}{2\xi_k^{(0)}}C_k^\sigma\right)$$
$$\lambda_k=\frac{i}{\omega\xi_k^{(0)}}\left[\Delta_k^{(0)*}B_k^n+\frac{1}{2}\left(\frac{|\Delta_k^{(0)}|^2}{\xi_k^{(0)}}-\hbar\omega\right)B_k^\sigma+\frac{(\Delta_k^{(0)*})^2}{2\xi_k^{(0)}}C_k^\sigma\right], \quad (63)$$
$$\mu_k=\frac{i}{\omega\xi_k^{(0)}}\left[\Delta_k^{(0)}B_k^n+\frac{\Delta_k^{(0)2}}{2\xi_k^{(0)}}B_k^\sigma+\frac{1}{2}\left(\frac{|\Delta_k^{(0)}|^2}{\xi_k^{(0)}}-\hbar\omega\right)C_k^\sigma\right].$$

As can be seen from (60), (62), and (63), the time-dependent relativistic corrections are, to the extent that the ratio I_k^{ex}/ε_k is small, small compared to $\bar{n}_k^r$ and $\bar{\sigma}_k^r$. In their turn, the corrections $\bar{n}_k^r$ and $\bar{\sigma}_k^r$ are small compared to the stationary values $n_k^{(0)}$ and $\sigma_k^{(0)}$ for $\nu_k \ll n_k^B=[\exp(\bar{\varepsilon}_k^0/T)-1]^{-1}$. From this and (62) we obtain the inequality

$$A_k/n_k^B \ll D_k.$$

The quantity D_k^{-1} coincides in order of magnitude with the characteristic time of the exchange relaxation to the stationary distribution $n_k^{(0)}$, $\sigma_k^{(0)}$, while the ratio A_k/n_k^B determines the characteristic frequency of transition in the stationary system under the influence of the relativistic interaction (6′). Thus, if the exchange-relaxation frequency is high compared to the relativistic-interaction induced transition rate, then the corrections n_k^r and σ_k^r are small compared to the stationary values $n_k^{(0)}$ and $\sigma_k^{(0)}$. Hence the stationary distribution is stable.

3. Let us consider the problem of pump-field-energy absorption by the magnon system in the steady-state regime. By definition, the absorbable energy is

$$Q=\langle\partial H/\partial t\rangle,$$

where H is the Hamiltonian (1) and the averaging is performed with the density matrix ρ determined by Eq. (7). Carrying out the differentiation, and using (12), we obtain

$$Q=\frac{i\omega}{2}\sum_k(V_k\sigma_k-V_k^*\sigma_k^*).$$

On the other hand, the equation for the correlator n_k can be represented in the form

$$-i\hbar\dot{n}_k=V_k\sigma_k-V_k^*\sigma_k^*+\langle[\hat{H}_{int},a_k^+a_k]\rangle,$$
$$\hat{H}_{int}=\hat{H}_{mm}^{ex}+\hat{H}_{mp}^{ex}+\hat{H}_{mm}^r,$$

the averaging being performed with $\hat{\rho}$. Hence for the absorbable power we obtain the following exact expression:

$$Q=\frac{\hbar\omega}{2}\dot{N}-\frac{i\omega}{2}\left\langle\left[\hat{H}_{int},\sum_k a_k^+a_k\right]\right\rangle, \quad (64)$$

where $N=\Sigma_k n_k$ is the total number of magnons.

Since the exchange interactions conserve the total magnon number (the corresponding terms in the Hamiltonian commute with the operator $\Sigma_k a_k+a_k$), only the relativistic terms in the second addend of formula (64) will remain. Performing the averaging over time, and taking into account the boundedness of N as a function of time in the steady-state (not necessarily stationary) regime, we obtain

$$\bar{Q}=-\frac{i\omega}{2}\overline{\left\langle\left[\hat{H}_{mm}^r,\sum_k a_k^+a_k\right]\right\rangle},$$

or

$$\bar{Q}=-\frac{\hbar\omega}{2}\sum_k\overline{I_k^{nr}}. \quad (65)$$

Thus, of all the interactions entering into the Hamiltonian (1), only the relativistic interactions, which do not conserve the magnon number, lead to absorption in the steady-state regime. Let us emphasize that, for a system with the Hamiltonian (1), this statement, like the formula (65), is exact. It follows from it, in particular, that if the distribution is stationary, then there should be no absorption. Indeed, as shown above, the stationary distribution can be established only by the magnon-number-conserving exchange interactions, while the relativistic interactions, which are responsible for the absorption, make the distribution nonstationary. The stationary distribution (51), (53) found by us leads, as it should, to zero absorption, and the absorbable power is determined by the nonstationary relativistic corrections.

Because of the weakness of the relativistic interaction, we can use the approximate expression (57) for the collision integral. The oscillating terms in I_k^{nr} drop out in the averaging over time, and we have

$$\bar{Q}=\frac{\pi\omega}{2}\operatorname{sh}\frac{\hbar\omega}{4T}\sum_{pqr}\frac{|\Phi_{p;qr}|^2}{\operatorname{sh}(\bar{\varepsilon}_p/2T)\operatorname{sh}(\bar{\varepsilon}_q/2T)\operatorname{sh}(\bar{\varepsilon}_r/2T)}$$
$$\times\left\{|u_pu_qu_r|^2\delta\left(\bar{\varepsilon}_p-\bar{\varepsilon}_q-\bar{\varepsilon}_r-\frac{\hbar\omega}{2}\right)+\ldots\right\}. \quad (66)$$

Here, as before, the dotted line in the curly brackets in (66) denotes terms that are obtained from those that have been written out by the substitutions $u_s\to v_s^*$, $\bar{\varepsilon}_s\to-\bar{\varepsilon}_s(s=p,q,r)$. The positive definiteness of the found absorbable power is evident; what is more, the substitution $\omega\to-\omega$ does not destroy this property.

In conclusion, let us note that, for the realization of such a distribution with the corresponding relativistic corrections, besides the need for the self-consistency equations to be compatible, it is necessary that the pump field exceed the threshold value, whose order of magnitude is determined by the linear damping.

[1)]In the classical limit the quantities z_k coincide with the amplitudes of the classical waves.

2) A similar type of equation for a phonon system is considered in Ref. 3.

3) The terms with the second derivatives drop out in going from Eq. (26) to the equations, (25), for the correlators, since there is not a single pair of indices among the indices **p**, **q**, **r**, **s** that differ only in sign.

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The permittivity tensor and increase in the transmittance of the spinel ferrites upon their conversion into single-sublattice structures

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Investigations of the optical properties of substituted spinel ferrites have indicated a significant increase in the transmittance of these compounds upon their conversion into a structure with a single Fe^{3+}-ion magnetic sublattice. The off-diagonal components of the permittivity tensor are determined from the optical and magneto-optical spectra, and some of the transitions that occur in the hexagonal ferrites are identified. It is suggested that the two-ion optical transitions with charge transfer between neighboring magnetically active ions play the decisive role, and a selection rule for such transitions is proposed.

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INTRODUCTION

In ferromagnets with two magnetic iron sublattices intense allowed transitions in the visible and near-ultraviolet regions of the spectrum are observed only in the case when they are due to pair excitation of the Fe^{3+} ions.[1-7] This follows from the fact that, in the indicated spectral region, the electric dipole transitions of the Fe^{3+} ion in the internal crystal field are spin and parity forbidden, as well as from the quadratic dependence of the intensity of these transitions on the iron-ion concentration.[1] The first intense single-ion optical transitions of the type of a charge transfer from the $2p$ orbitals of oxygen to the $3d$ orbitals of iron[8] lie, according to Ref. 4, in the shorter-wavelength region of the spectrum.

The two-exciton mechanism of simultaneous excitation of two Fe^{3+} ions located on different sublattices, and coupled by a strong exchange interaction, has been considered in investigations of the garnet ferrites[5] and substituted spinel ferrites.[6,7] In particular, in Ref. 6, where chromium-substituted spinel ferrites are considered, the conclusion that the two-ion optical-transition mechanism plays the decisive role is based on the fact that the magnitude of the magneto-optical effect has been found in investigations of the magneto-optical spectra of the spinel ferrites of cobalt and nickel to decrease sharply when the iron ions on one of the sublattices are replaced by Cr^{3+} ions. Another possible mechanism for the pair excitation of the iron ions is the transfer of charge between Fe^{3+} ions on different sublattices with the formation of Fe^{4+}- and Fe^{2+}-ion pairs.[2,4] Analysis of the splitting of the energy levels of the Fe^{3+} ion in tetrahedral and octahedral crystal fields and the computation of the energies of the possible transitions both in the case of two-exciton excitation and in the case of charge transfer between sublattices lead to good agreement with the experimental data.[4,6]

Thus, if we consider the pair mechanism of transition excitation in iron ions located on different sublattices to have been reliably established, then we should expect to observe not only the above-indicated decrease in the magnitude of the magneto-optical effect, but also a decrease in the absorption coefficient of such ferromagnets when the iron ions on the tetrahedral or octahedral sublattices are replaced. With the object of verifying this assertion, we carried out optical investigations of the aluminate ferrites and chromite ferrites of cobalt and nickel, in which we replaced in turn the iron ions on the octahedral or tetrahedral sublattice respectively by Al^{3+} and Cr^{3+} ions. On the basis of the optical and the earlier-performed magneto-optical measurements, we computed the permittivity-tensor components, which were used for a more reliable identification of the optical transitions and the determination of the nature of their splitting.

By qualitatively comparing the magneto-optical spectra of the spinel ferrites with those of the hexagonal ferrites with the M, W, Z, Y, and X structures, we have identified some of the optical transitions in the