[1]N. F. Mott and W. D. Twose, Adv. Phys. **10**, 107 (1961).
[2]B. I. Halperin, Adv. Chem. Phys. **13**, 123 (1967); Yu. A. Bychkov, Zh. Eksp. Teor. Fiz. **65**, 427 (1973) [Sov. Phys. JETP **38**, 209 (1974)]; C. T. Papatrantafillou, Phys. Rev. B **7**, 5386 (1972).
[3]V. L. Berezinskiĭ, Zh. Eksp. Teor. Fiz. **65**, 1251 (1973) [Sov. Phys. JETP **38**, 620 (1974)].
[4]A. A. Goglin, Zh. Eksp. Teor. Fiz. **71**, 1912 (1976) [Sov. Phys. JETP **44**, 1003 (1976)].
[5]A. A. Gogolin and V. I. Mel'nikov, Phys. Status Solidi B **88**, 377 (1978).
[6]A. A. Abrikosov and I. A. Ryzhkin, Zh. Eksp. Teor. Fiz. **71**, 1916 (1976) [Sov. Phys. JETP **44**, 1005 (1976)].
[7]A. A. Gogolin, V. I. Mil'nikov, and É. I. Rashba, Zh. Eksp. Teor. Fiz. **69**, 327 (1975) [Sov. Phys. JETP **42**, 168 (1975)].
[8]A. A. Gogolin, S. P. Zolotukhin, V. I. Mel'nikov, E. I. Rashba, and I. F. Shchegolev, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 564 (1975) [JETP Lett. **22** 278 (1975)].
[9]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz, 1962 [Pergamon, 1965]
[10]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhnaika (Quantum Mechanics), Nauka, 1974 [Pergamon].
[11]N. F. Mott, Adv. Phys. **18**, 49 (1967) [Russian translation].
[12]L. P. Gor'kov, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 384 (1977) [JETP Lett. **25**, 358 (1977)].
[13]I. F. Shchegolev, Phys. Status Solidi A **12**, 9 (1972).
[14]D. B. Tanner, C. S.Jacobsen, A. F. Garito, and A. J. Heeger, Phys. Rev. B **13**, 3381 (1976).
[15]M. G.Kaplunov, T. P. Panova, E. B.Yagubskiĭ, and Yu. G. Borod'ko, Zh. Strukt. Khim. **13**, 440 (1972).
[16]R. L. Bush, Phys. Rev. B **13**, 805 (1976).
[17]R. L. Bush, Phys. Rev. B **6**, 1182 (1972).
[18]C. T. Papatriantafillou and E. N. Econonou, Phys. Rev. B 13, 805 (1976).

Translated by J. G. Adashko

Stimulation of superconductivity in an inhomogeneous bridge in a microwave field

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(Submitted 15 November 1978)
Zh. Eksp. Teor. Fiz. **76**, 1775–1780 (May 1979)

Stimulation of superconductivity in a bridge whose neck has a lower critical temperature than the shores is investigated. It is shown that, depending on the microwave-field frequency, the relative contributions made to the stimulation by the "trembling" of the potential well and by the electric field vary, and this leads to different types of phase diagrams of the bridge.

PACS numbers: 85.25. + k, 74.10. + v

Irradiation of a superconductor by a microwave field changes the electron energy distribution, and this disequilibrium can cause a substantial increase of the critical parameters of the superconductor.[1] In superconductors with constrictions (bridges, point contacts, etc.) the electron energy diffusion is caused both by direct acceleration by the electric field,[1,2] and by the "trembling" of the potential well produced as a result of the lowering of the value of the order parameter in the constriction region.[3] The energy of the electrons trapped in the constriction region increases upon reflection from the walls of the trembling well, and the magnitude of the effect depends substantially on the character of the dependence of the order parameter on the coordinates.

For a homogeneous superconductor, the decrease of the order parameter Δ in the constriction region is due to the increase of the density of the superconducting current. At the critical value of the current, Δ has a power-law dependence on the coordinates, and the trembling of the well leads to a substantial increase of the critical current of the bridge in the microwave field.

This paper deals with an inhomogeneous bridge in which the neck has a critical temperature T_c somewhat lower than the critical temperature T_{c0} of the shores of the bridge. The dependence of the order parameter on the coordinates is exponential. In such a bridge the effects of stimulation can lead to a preservation of the superconductivity up to temperatures close to T_{c0}.

1. SUPERCONDUCTIVITY STIMULATION DUE TO THE TREMBLING OF THE POTENTIAL WELL

The change of the electron distribution function in the microwave field depends on the irradiation power. At sufficiently high irradiation powers an equipartition is established of the energies of the electrons trapped in the region of the contact:

$$f(\varepsilon)=\Delta_0/2T,\quad \varepsilon<\Delta_0, \tag{1}$$

where Δ_0 are the values of the order parameter at the shores of the bridge.[3] The electrons with energies $E>\Delta_0$, for bridges that are not too long, can diffuse freely from the contact and therefore have an equilibrium energy distribution (the microwave current density in the shores is negligibly small). As a result, the nonequilibrium term in the Ginzburg-Landau equation for the order parameter[3] takes in the limit of high irradiation power the form

$$\Phi(\Delta)=\Delta\int_{\Delta}^{\infty}\left(f-\operatorname{th}\frac{\varepsilon}{2T}\right)\frac{d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}=\frac{\Delta\Delta_0}{2T}\left[\ln\frac{1+(1-\Delta^2/\Delta_0^2)^{1/2}}{\Delta/\Delta_0}-\left(1-\frac{\Delta^2}{\Delta_0^2}\right)^{1/2}\right]. \tag{2}$$

The superconducting transition temperature T_c^P of the

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bridge irradiated by the microwave field is determined from the Ginzberg-Landau equation

$$\frac{T_c^P-T_c}{T_c}=\max\left\{\frac{\Phi(\Delta)}{\Delta}-\frac{7\zeta(3)}{8\pi^2}\frac{\Delta^2}{T^2}-\frac{T\overline{I_s^2}}{2\pi e^2\rho^2DS^2\Delta^2\Delta_0}\right\},\qquad(3)$$

where $\overline{I_s^2}$ is the mean value of the microwave current in the bridge, ρ is the state density $D=v_F l_{tr}/3$ is the diffusion coefficient, and S is the cross-section area of the contact. As seen from (3), the positive nonequilibrium term $\Phi(\Delta)$ leads to stimulation of the superconductivity, while the negative second and third term lead to its suppression by the nonlinear effects and by the current, respectively. The value $\Delta=\tilde{\Delta}$ corresponding to the maximum of the right-hand side of (3) determines the order parameter produced in the bridge when it becomes superconducting; it is assumed here that the order parameter depends weakly on the coordinates in almost the entire region of the bridge, except at the edges. Near the edges, the order parameter $\tilde{\Delta}$ in the bridge assumes the same value as the parameter Δ_0 in the shores.

To find the superconducting transition temperature T_c^P of the bridge from Eq. (3) it must be borne in mind that the order parameter at the shores Δ_0, which enters the right-hand side of (3), is itself temperature-dependent: at the transition point we have

$$\Delta_0=(8\pi^2/7\zeta(3))^{1/2}(T_{c0}-T_c^P/T_c)^{1/2}.$$

As a result, when solving Eq. (3) with the nonequilibrium term $\Phi(\Delta)$ defined by formula (2), we find that the stimulation can raise the bridge superconducting temperature all the way T_{c0}. In this case in almost the entire temperature range the dependence of T_c^P on the irradiation power is given by

$$T_{c1}^P=T_{c0}(1-0{,}02P^{2/5}),\quad \frac{T_{c0}-T_{c1}^P}{T_c}\gg(\Delta\tau)^2,\qquad(4)$$

where the dimensionless parameter $P=(2\pi e^2\rho^2DS^2T^3)^{-1}\overline{I_s^2}$ is proportional to the irradiation power, and the quantity $\Delta\tau=(T_{c0}-T_c)/T_c$ is assumed small, as it must be for the Ginzburg-Landau theory to be valid. The relation (4) follows from (3), inasmuch as in the corresponding temperature region the maximum is reached at $\tilde{\Delta}=0.64\Delta_0$, and the second term in the right-hand side of (3) is small. At temperatures very close to T_{c0} the dependence of T_c^P on the power becomes slow:

$$T_{c1}^P=T_{c0}\left(1-\frac{63\zeta(3)}{2\pi^2}\frac{(\Delta\tau)^2}{\ln^2[(\Delta\tau)^5/P]}\right),\quad \frac{T_{c0}-T_{c1}^P}{T_c}\ll(\Delta\tau)^2.\qquad(5)$$

The dependence $T_{c1}(P)$ in Figs. 1 and 2 is represented by the curve O'BD.

Formulas (4) and (5), which determine the dependence of the bridge transition temperature on the irradiation power, were obtained under the assumption that the electron energy distribution function has the same form (1) as before. This limiting distribution is established at the corresponding power in almost the entire temperature region because of the long time τ_ε of energy relaxation of the electrons. To obtain the necessary estimates we use the kinetic equation for the distribution function $f(\varepsilon)$ (Ref. 3)

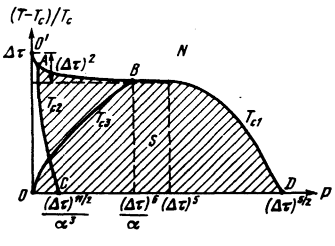

FIG. 1. Plot of $(T_c^P-T_c)/T_c$ against the irradiation power at $\alpha=\omega^2\tau_\varepsilon/T\gg\Delta\tau^{-1/4}$

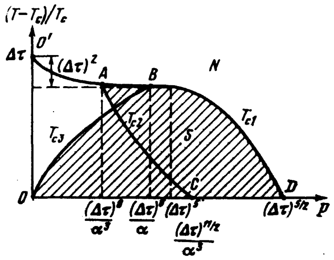

FIG. 2. Plot of $(T_c^P-T_c)/T_c$ against the irradiation power at $\Delta\tau<\alpha<(\Delta\tau)^{-1/4}$

$$\frac{1}{\tau_\varepsilon}\left[f(\varepsilon)-\operatorname{th}\frac{\varepsilon}{2T}\right]\overline{\langle\varepsilon(\varepsilon^2-\Delta^2)^{-1/2}\rangle}=\frac{\partial}{\partial\varepsilon}\left(D_\varepsilon\frac{\partial f}{\partial\varepsilon}\right)\qquad(6)$$

where $\langle\cdots\rangle$ denotes averaging over the region of the contact in which $\varepsilon>\Delta$, and the bar denotes averaging over time. D_ε is the coefficient of the energy diffusion of the electrons.

For the correction to the distribution function $f_1=f(\varepsilon)-\Delta_0/2T$ we get from (6)

$$f_1=-\frac{1}{2T\tau_\varepsilon}\int_\varepsilon^{\Delta_0}\left[\frac{1}{D_\varepsilon}\int_{\tilde{\Delta}}^{\varepsilon}\frac{\varepsilon(\Delta_0-\varepsilon)}{(\varepsilon^2-\tilde{\Delta}^2)^{1/2}}d\varepsilon\right]d\varepsilon.\qquad(7)$$

The coefficient of energy diffusion due to the trembling of the potential well is given by[3,4]

$$D_\varepsilon=-\overline{\left\langle\frac{\partial\theta}{\partial t}\left(D\frac{\partial^2}{\partial r^2}\right)^{-1}\frac{\partial\theta}{\partial t}\right\rangle},\qquad(8)$$

$$\theta=(\varepsilon^2-\Delta^2)^{1/2}-\langle(\varepsilon^2-\Delta^2)^{1/2}\rangle.$$

The dependence of Δ on the coordinates, which is needed to determine D_ε from formula (8) is obtained from the Ginzburg-Landau equation, in which the most important is the nonequilibrium term $\Phi(\Delta)$; this dependence is exponential. As a result, at $\tilde{\Delta}$ close to Δ_0, we get for D_ε

$$D_\varepsilon=\frac{\pi}{144T}\overline{\left[\frac{\partial}{\partial t}\frac{(\varepsilon^2-\tilde{\Delta}^2)^{3/2}}{|\Phi'|^{1/2}\Delta_0(\Delta_0-\tilde{\Delta})}\right]^2}.\qquad(9)$$

The variation of the order parameter with time is the result of the oscillations of the superconducting current in the bridge:

$$\dot{\Delta}=\frac{T}{2\pi e^2\rho^2DS^2\Delta_0^3\Phi'}I_s^2.\qquad(10)$$

Calculating the coefficient D_ε from (9) and (10) and then the correction to the limiting distribution function form (7), we can obtain with the aid of the left-hand side of equation (2) the correction Φ_1 for the limiting expression for the nonequilibrium term. As a result, at $\tilde{\Delta}$ close to Δ_0, we have for Φ

$$\Phi=\frac{\sqrt{2}}{3}\frac{\Delta_0^2}{T}\left(1-\frac{\tilde{\Delta}}{\Delta_0}\right)^{3/2}-\frac{9}{5\pi}\left(\frac{\Delta_0}{T}\right)^{12}\frac{\Delta\tau T}{\alpha P^2}\left(1-\frac{\tilde{\Delta}}{\Delta_0}\right)^4,\quad\alpha=\frac{\omega^2\tau_\varepsilon}{T}.\qquad(11)$$

As seen from (11), at powers P and temperatures T_c^P, which are connected by (4) the correction Φ_1 (second term) is small at $\alpha > \Delta\tau$.

Formula (6) enables us also to determine the lower limit of the stimulation effect: at a given temperature, when the power is decreased the value of the nonequilibrium term also decreases, and at a certain power a transition to the normal place takes place again. The transition temperature can be obtained, apart from a numerical coefficient, from Eq. (3), if it is assumed that $\Phi(\Delta)$ is given by formula (11) also at $\Phi_1 \sim \Phi$. The maximum of the right-hand side of (3) is reached at $\tilde{\Delta}$ close to Δ_0, and Eq. (3) takes the form

$$\Delta\tau = \left(\frac{T}{\Delta_0}\right)^5 \left(\frac{\alpha}{\Delta\tau}\right)^{3/5} P^{6/5} - \left(\frac{T}{\Delta_0}\right)^4 P, \tag{12}$$

where the transition temperature T_c^P is expressed in terms of the value of the shore order parameter Δ_0. Solving Eq. (7) for the temperature T_{c2}^P of the second transition, we get

$$\begin{aligned} T_{c2}{}^P &= T_{c0}\left[1 - a\left(\frac{\alpha}{\Delta\tau}\right)^{6/5} P^{2/5}\right], \quad P \ll P_0; \\ T_{c2}{}^P &= T_{c0}[1 - b(\Delta\tau)^{-16/25}\alpha^{6/25}P^{12/25}], \quad P \gg P_0, \end{aligned} \tag{13}$$

where the characteristic power is $P_0 = (\Delta\tau)^{-7}\alpha^{12}$, and a and b are numerical coefficients.

The function $G_{c2}(P)$ at $P \ll P_0$ is shown in Figs. 1 and 2 by the curves AC. It is seen that at $\alpha \gtrsim \Delta\tau$ the plots of $T_{c1}(P)$ and $T_{c2}(P)$ intersect at the temperature

$$\sim T_{c0}[1 - (\Delta\tau)^2/\ln^2(T\Delta\tau/\alpha)],$$

which is very close to T_{c0}, and on the (P, T) diagram there is a region ACD of stimulated superconductivity (Figs. 1 and 2).

At $\alpha < \Delta\tau$ the trembling of the potential well cannot lead to a substantial change of the electron distribution function. In this case perturbation theory is valid, and the stimulation effects, which are proportional to $(I_s^2)^2$, are less than the suppression of the superconductivity by current, the latter being proportional to I_s^2.

2. STIMULATION OF SUPERCONDUCTIVITY BY AN ELECTRIC FIELD

The nonequilibrium effects connected with direct acceleration of electrons by a microwave field were considered for a homogeneous superconductor by Éliashberg and Ivlev.[1,2] In the most essential region of the field intensities, they have obtained the following expression for the nonequilibrium term:

$$\begin{aligned} \Phi(\Delta) &= \frac{5}{4\pi}\frac{e^2DE^2\tau_\varepsilon}{T}\ln\left(\frac{\Delta^2}{e^2DE^2\tau_\varepsilon}\right), \\ 1 &\ll \frac{e^2DE^2\tau_\varepsilon}{\omega^2} \ll \frac{T}{\omega}, \end{aligned} \tag{14}$$

where E is the electric field intensity. This expression can be used to find the superconducting transition temperature of the bridge from formula (3), since the value of the order parameter $\tilde{\Delta}$ in the bridge, which corresponds to the maximum of the right-hand side of (3), is much less in the intensity region of interest to us than the order parameter Δ_0 in the shores (the electron diffusion from the bridge has in this case little effect on the nonequilibrium term).

Using the connection between the superconducting current and the intensity of the electric field in the bridge[5]

$$I_s = \pi e\rho DS\frac{\tilde{\Delta}^2}{2T}\nabla\varphi = \pi e^2\rho DS\frac{\tilde{\Delta}^2}{\omega T}E, \tag{15}$$

we obtain for the bridge transition temperature T_{c3}^P from Eq. (3)

$$\frac{T_{c3}{}^P - T_c}{T_c} = \frac{3}{\pi^2}P\alpha\left(\frac{T}{\tilde{\Delta}}\right)^5 - P\left(\frac{T}{\tilde{\Delta}}\right)^4, \quad \ln\left(\frac{\tilde{\Delta}^6}{P\alpha T^6}\right) = \frac{6}{5}, \tag{16}$$

where we have discarded the inessential second term in the right-hand side of (3). We note that at $P \gtrsim \alpha^5$ formula (16) is accurate only in order of magnitude, for when the maximum of the right-hand side of (3) was determined only the first term was taken into account. The function $T_{c3}(P)$ corresponding to (16) is plotted in Figs. 1–3 by curve OB.

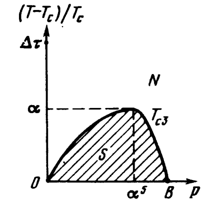

FIG. 3. Plot of $(T - T_c)/T_c$ against the irradiation power at $\alpha < \Delta\tau$

It follows from (16) that when the power is increased the value of the order parameter $\tilde{\Delta}$ in the contact increases, and consequently the suppression of the superconductivity by the current (the second term) becomes ever more important. The maximum increase of the critical temperature by electric-field stimulation, $(\Delta T_c/T_c)_{\max} \sim \alpha$ is reached at a power $P \sim \alpha^5$ and corresponds to the value $\tilde{\Delta} \sim \alpha T$. This statement is valid for both a bridge and a homogeneous superconductor.

In the case of a bridge the superconductivity is stimulated both directly by the electric field and by the trembling of the potential well. The picture of the stimulation depends in this case on the value of the parameter α. At $\alpha > (\Delta\tau)^{-1/4}$ the greatest role is played by the trembling of the potential well; the point B (Fig. 1) of the intersection of the curves T_{c1} and T_{c3} corresponds to a higher power than the point C, and the region of stimulated superconductivity takes the form OAD. In the region of small powers, the bridge superconducting transition temperature decreases with increasing power.

At $\Delta\tau < \alpha < (\Delta\tau)^{-1/4}$ the stimulation of the superconductivity occurs in the main on account of the electric field; the characteristic phase diagram for this case is shown in Fig. 2. At low powers the transition temperature increases with increasing power.

Finally, at $\alpha < \Delta\tau$ the well trembling no longer stimulates the superconductivity. Stimulating with an electric field is still possible in this case (Fig. 3), if the condition $\alpha\tau_\varepsilon > 1$, which is needed for the validity of the Éliashberg theory,[1] is satisfied. It is seen that both conditions can be valid if $\omega < T\Delta\tau$. We note that at $\alpha \sim \Delta\tau$ two individual regions can be produced on the phase diagram, corresponding to stimulation by an electric field (OB) and to trembling of the well (ACD).

Let us discuss also the limitations on the bridge length, which must be satisfied for the theory to hold. First, it was assumed that the bridge is long enough and the order parameter depends little on the coordinates: $a > \xi_0/(\Delta\tau)^{1/2}$, where $\xi_0 = (D/T_c)^{1/2}$ is the pair dimension. On the other hand, the time of spatial diffusion of the electrons in the contact must be small compared with the period of the field, $a < (D/\omega)^{1/2}$, so as to be able to neglect the disequilibrium of the electrons with energies $\omega > \Delta_0$. We see that at $\omega < T\Delta\tau$ both conditions are satisfied for a contact with dimension $\omega < Da^{-2} < T\Delta\tau$.

The authors thanks A. I. Larkin and B. I. Ivlev for valuable discussions of the results.

[1]G. M. Eliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **11**, 186 (1970) [JETP Lett. **11**, 114 (1970)]; Zh. Eksp. Teor. Fiz. **61**, 1254 (1971) [Sov. Phys. JETP **34**, 668 (1972)].
[2]B. I. Ivlev and G. M. Éliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **13**, 464 (1971) [JETP Lett. **13**, 333 (1971)]; B. I. Ivlev, S. G. Lisitsyn, and G. M. Eliashberg, J. Low Temp. Phys. **10**, 449 (1973).
[3]L. G. Aslamazov and A. I. Larkin, Zh. Eksp. Teor. Fiz. **74**, 2184 (1978) [Sov. Phys. JETP **47**, 1136 (1978)].
[4]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **68**, 1915 (1975) [Sov. Phys. JETP **41**, 960 (1975)].
[5]P. De Gennes. Conductivity of Metals and Alloys, Oxford, 1963.

Translated by J. G. Adashko

Fluctuations in layered superconductors in a parallel magnetic field

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The effect of fluctuations on the properties of layered superconductors in a magnetic field parallel to the layers is considered. The fluctuations lead to a phase transition with respect to the field. In strong fields the long-range order is destroyed in both the longitudinal and transverse direction. The pair correlation function falls off in a power-law manner along the layer and exponentially across the layer. In this state the superconductivity is retained along the layers but disappears in the direction perpendicular to the layers.

PACS numbers: 74.40.+k

In certain layered superconductors Josephson interaction of the layers evidently occurs. The intercalation compounds TaS_2 and NbS_2 can serve as examples. The spectrum of the one-electron energies in the normal state of such compounds can be described by the dependence

$$\varepsilon(\mathbf{p}) = \mathbf{p}_\parallel^2/2m - 2W\cos p_z d, \qquad (1)$$

where $\mathbf{p}_\parallel$ is the quasi-momentum along the layers, m is the effective mass, p_z is the quasi-momentum in the direction perpendicular to the layers, and d is the distance between the layers.

Josephson interaction of the layers occurs when the electrons can move over a distance of the order of the size of a Cooper pair without once hopping to neighboring layers. This situation obtains if the condition

$$W \ll T_c \qquad (2)$$

is fulfilled, where T_c is the superconducting-transition temperature, calculated in the BCS approximation.

In a paper by Bulaevskii,[1] Ginzburg-Landau differential-difference equations were derived to describe layered superconductors. These equations go over into the ordinary Ginzburg-Landau equations for anisotropic superconductors if the temperature is close to the critical temperature. In the opposite limiting case

$$(T_c - T)/T_c = \tau \gg W^2/T_c^2 \qquad (3)$$

such a transition is impossible. The most important differences arise in a magnetic field parallel to the layers. The Josephson interaction of the layers leads to the result that the diamagnetic currents are limited in magnitude and cannot destroy the superconducting order parameter. It was shown in Ref. 1 that only a paramagnetic effect can lead to suppression of the superconductivity in a parallel magnetic field. If the magnetic field is not very strong ($\mu H \ll T_c$, where μ is the Bohr magneton), or if the Chandrasekhar-Clogston paramagnetic limit is absent for any of the reasons in Refs. 2–4, the modulus $|\Delta|$ of the order parameter is close to the value obtained in the BCS approximation. In this case all the magnetic properties are described by the changes in the phase.

In a purely two-dimensional superconductor, phase fluctuations are important and lead to destruction of the long-range order.[5] However, even a very small probability of hops from layer to layer leads to restoration of the long-range order.[6] This result is obtained in the absence of a magnetic field. A magnetic field parallel to the layers weakens the interaction of the layers and enhances the fluctuations. In fields $\mu H \gg W/p_0 d$ the layers cease to interact and the fluctuations become purely two-dimensional. In this region of fields the long-range order is destroyed. The pair correlation function within the layers falls off in a power-law manner. The superconductivity is retained within the layers but vanishes in the direction perpendicular to the layers.