Dynamic damping of a domain wall in a ferromagnet

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A theory is developed to describe the dissipation of the energy of a nonlinear magnetization wave in a ferromagnet through interaction with thermal magnons. The case of a moving domain wall is treated in detail. Dynamic damping of a moving domain wall is investigated. The dependence of the damping force on wall velocity and temperature is calculated. For a wall under the influence of an external magnetic field, the dependence of the velocity of viscous motion on the value of the field is found. It is shown that the damping of a domain wall cannot be described by phenomenological allowance for relaxation in the equations of magnetization dynamics. In particular, at high wall velocities the dependence of the damping force on the velocity becomes nonlinear. At low velocities, the relaxation constant that describes viscous motion of the wall may differ greatly from the relaxation constant that describes, for example, the width of the ferromagnetic resonance line.

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INTRODUCTION

In investigation of dynamic processes in magnetic materials, there is great interest in the problem of the motion of domain walls (DW) and of isolated magnetic domains. Motion of a DW is limited by two qualitatively different phenomena: first, the presence of various crystal defects, whose effect can be described by introduction of a coercive force (a force of static friction)[1,2]; second, the presence of dynamic damping (viscous friction), caused by transfer of the energy of the moving DW to thermal magnons and phonons.[3] With increase of the velocity of the DW, the dynamic damping becomes dominant.

In investigation of dynamic magnon damping of a DW, there arises the problem of describing the interaction of a nonlinear wave of a classical magnetization field with thermal spin waves. A peculiarity of this problem[1)] is that both the nonlinear wave and the magnons are manifestations of the same, essentially nonlinear quantum system, namely the spin system of the ferromagnet (FM). Here the problem arises of choosing the zeroth approximation; that is, of the most natural separation of these subsystems: and also of allowing for their interaction, which leads in particular to dissipation of the energy of the nonlinear wave (in particular of a DW).

This paper investigates the interaction of a nonlinear magnetization wave with magnons. Detailed consideration is given to the case of a moving DW, and a theory is constructed to describe damping of a DW as a result of interaction with thermal magnons. The contribution of various processes to the damping force is analyzed, and the general nature of the variation of the damping force with DW velocity and with temperature is described. On the assumption that the energy of the demagnetizing fields is much smaller than the anisotropy energy (this condition is satisfied, for example, for iron garnet films[2]), an expression is obtained for the damping force over a wide range of values of the DW velocity (up to the limiting value) and of the temperature. For a DW under the action of an external magnetic field, a discussion is given of the effect of interaction of the DW with thermal magnons on the nature of the variation of the velocity of DW motion with the value of the field and with the temperature of the magnet.

1. THE HAMILTONIAN OF SPIN WAVES IN A FERROMAGNET WITH A NONLINEAR MAGNETIZATION WAVE

We consider a uniaxial ferromagnet. The simplest expression for its energy can be written in the form

$$W=\int dr\left\{\frac{\alpha}{2}\left(\frac{\partial\mathbf{M}}{\partial x_i}\right)^2-\frac{\beta}{2}(\mathbf{Mn})^2-\frac{1}{2}(\mathbf{MH}_m)\right\}. \tag{1}$$

Here $\mathbf{M}$ is the magnetization, α the exchange constant, β the anisotropy constant, $\mathbf{n}$ the unit vector along the anisotropy axis (the z axis), and $\mathbf{H}_m$ the magnetic dipole interaction field.

We shall consider small oscillations of the magnetization against the background of a classical nonlinear wave. For this purpose we represent $\mathbf{M}$ in the form $\mathbf{M}_0(\mathbf{r},t)+\mathbf{m}(\mathbf{r},t)$, where $\mathbf{M}_0(\mathbf{r},t)$ is the magnetization distribution in the nonlinear wave, and $\mathbf{m}$ corresponds to the small oscillations of the magnetization against the background of the nonlinear wave.

It is convenient to introduce a new coordinate system in which the axis of quantization $\mathbf{e}_3$ for $\mathbf{m}$ coincides with the equilibrium direction $\mathbf{M}_0(\mathbf{r},t)$ in the nonlinear wave:

$$\begin{aligned}&\mathbf{e}_1=\mathbf{e}_x\cos\varphi+\mathbf{e}_y\sin\varphi,\\&\mathbf{e}_2=\cos\theta(-\mathbf{e}_x\sin\varphi+\mathbf{e}_y\cos\varphi)-\mathbf{e}_z\sin\theta,\\&\mathbf{e}_3=\sin\theta(-\mathbf{e}_x\sin\varphi+\mathbf{e}_y\cos\varphi)+\mathbf{e}_z\cos\theta.\end{aligned} \tag{2}$$

In this system $M_{03}=M_0$, $M_{01}=M_{02}=0$; the nonuniform magnetization distribution corresponding to the moving DW is described by the angles $\theta(\mathbf{r},t)$ and $\varphi(\mathbf{r},t)$. We shall express the components of $\mathbf{m}(\mathbf{r},t)$ in this system in terms of the Holstein-Primakoff operators[6] $a(\mathbf{r})$ and $a^+(\mathbf{r})$:

$$\begin{aligned}&m^{(+)}=m_1+im_2=2(\mu_0M_0)^{1/2}\left(1-\frac{a^+a\mu_0}{M_0}\right)^{1/2}a,\\&m^{(-)}=m_1-im_2=2(\mu_0M_0)^{1/2}a^+\left(1-\frac{a^+a\mu_0}{M_0}\right)^{1/2},\\&m_3=-2\mu_0a^+a,\end{aligned} \tag{3}$$

where μ_0 is the Bohr magneton. The operators $a(\mathbf{r})$ and $a^+(\mathbf{r})$ satisfy the Bose commutation relations

$$[a(\mathbf{r}),\ a^+(\mathbf{r}')]=\delta(\mathbf{r}-\mathbf{r}') \tag{4}$$

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and are the creation and annihilation operators of spin waves. We note that the operation of transformation from magnetization operators in the laboratory coordinate system to operators M_i in the coordinate system (2), and as a result to the operators a^+ and a, contains an explicit dependence on time because of the motion of the DW. Because of this fact, when dynamic equations are written for the operators M_i or a, for example in the Hamiltonian form

$$i\hbar\partial a/\partial t=[a, \mathscr{H}], \tag{5}$$

the Hamiltonian operator does not coincide with (1) but must be written in the following form:

$$\mathscr{H}=\int dr\left\{\frac{\hbar}{2\mu_0}\left[(M_3\cos\theta-M_2\sin\theta)\frac{\partial\varphi}{\partial t}-M_1\frac{\partial\theta}{\partial t}\right]\right\}+W\{M_i\}, \tag{6}$$

where $W\{M_i\}$ is the energy (1) of the magnet expressed in terms of M_i. By use of the relations (2) we get

$$\begin{aligned}W\{M_i\}=\int dr\Big\{&\frac{\alpha}{2}(\nabla M_i)^2+\frac{\alpha}{2}(\nabla\theta)^2(M_2^2+M_3^2)\\&+\frac{\alpha}{2}(\nabla\varphi)^2[M_1^2+(M_2\cos\theta+M_3\sin\theta)^2]\\&+\alpha(\nabla\theta)(\nabla\varphi)M_1(M_3\cos\theta-M_2\sin\theta)+\alpha\nabla\theta(M_3\nabla M_2-M_2\nabla M_3)\\&+\alpha(\nabla\varphi)[\sin\theta(M_1\nabla M_3-M_3\nabla M_1)+\cos\theta(M_1\nabla M_2-M_2\nabla M_1)]\\&+\frac{\beta}{2}[M_0^2-(M_3\cos\theta-M_2\sin\theta)^2]-\frac{1}{2}M_iH_{m(i)}\Big\}.\end{aligned} \tag{7}$$

The Hamiltonian (6), (7), written in terms of spin-wave operators, takes the form of a power series in a and a^+:

$$\mathscr{H}=\mathscr{H}_0+\mathscr{H}_1+\mathscr{H}_2+\mathscr{H}_3+\ldots, \tag{8}$$

where $\mathscr{H}_0$ does not contain the operators a and a^+ and describes the dynamics of the classical nonlinear magnetization wave; $\mathscr{H}_1$ is linear in a and a^+; $\mathscr{H}_2$ is quadratic in a and a^+; and so on. In order to write (8) explicitly, it is necessary to express the demagnetizing field $\mathbf{H}_m$ in terms of the components of magnetization by means of the equations of magnetostatics; here the expression for $\mathbf{H}_m$ can also be written in the form of a power series in a and a^+, i.e.,

$$\mathbf{H}_m=\mathbf{H}_m^{(0)}+\mathbf{H}_m^{(1)}+\mathbf{H}_m^{(2)}+\ldots$$

But by use of the relation[6]

$$\int \mathbf{H}_m\delta\mathbf{M}\,dr=\int \mathbf{M}\delta\mathbf{H}_m\,dr,$$

which relates a small change $\delta\mathbf{M}$ of the magnetization to a small change $\delta\mathbf{H}_m$ of the demagnetizing field, one can express $\mathscr{H}_1$ in terms of $\mathbf{H}_m^{(0)}$ alone, namely:

$$\begin{aligned}\mathscr{H}_1=\left(\frac{\mu_0}{M_0}\right)^{1/2}\int dr\Big\{&\frac{(a+a^+)}{\sin\theta}\left[-\frac{\hbar M_0}{2\mu_0}\sin\theta\frac{\partial\theta}{\partial t}-\frac{\delta W_0}{\delta\varphi}\right]\\&+i(a-a^+)\left[\frac{\hbar M_0}{2\mu_0}\sin\theta\frac{\partial\varphi}{\partial t}-\frac{\delta W_0}{\delta\theta}\right]\Big\},\end{aligned} \tag{9}$$

where

$$W_0\equiv W_0\{\theta(\mathbf{r},t),\varphi(\mathbf{r},t)\}$$

is the energy (1) of the ferromagnet expressed as a functional of the angles θ and φ. In our case

$$W_0\{\theta,\varphi\}=\int dr\left\{\frac{\alpha M_0^2}{2}[(\nabla\theta)^2+\sin^2\theta(\nabla\varphi)^2]+\frac{\beta M_0^2}{2}\sin^2\theta+\frac{(\mathbf{H}_m^{(0)})^2}{8\pi}\right\}. \tag{10}$$

To this point we have not specified the form of the functions $\theta(\mathbf{r},t)$ and $\varphi(\mathbf{r},t)$. It is obvious that with a correct choice of the nonlinear wave, i.e., of the ground state for the small oscillations described by a and a^+, the condition $\mathscr{H}_1=0$ must be satisfied; that is, both of the expressions in square brackets in (9) must vanish. This condition determines the well-known dynamic Landau-Lifshitz equations, without damping, for a classical magnetization field, in the angular variables θ and φ.[1]

Thus the natural requirement that terms linear in a and a^+ be absent from the Hamiltonian determines the nonlinear wave of the classical magnetization field (in particular, a moving DW), whereas $\mathscr{H}_2$, $\mathscr{H}_3$, etc. describe quantum and thermal corrections to this state that result from interaction of the nonlinear wave with spin waves. For an arbitrary nonlinear wave, $\mathscr{H}_2$ can be written in the following form:

$$\begin{aligned}\mathscr{H}_2=\mathscr{H}_2^m+2\mu_0M_0\int dr\Big\{&\alpha(\nabla a^+\nabla a)+a^+a\Big[\beta-\frac{\alpha}{2}(\nabla\theta)^2-\frac{3\beta}{2}\sin^2\theta\\&+\frac{\alpha}{2}(\nabla\varphi)^2(1+\cos^2\theta)-\frac{\hbar}{2\mu_0M_0}\cos\theta\frac{\partial\varphi}{\partial t}\Big]\\&+\frac{1}{4}(aa+a^+a^+)[(\alpha(\nabla\varphi)^2+\beta)\sin^2\theta-\alpha(\nabla\theta)^2]\\&+i\frac{\alpha}{2}(aa-a^+a^+)(\nabla\theta\nabla\varphi)+i\frac{\alpha}{2}\nabla\varphi\cos\theta(a\nabla a^+-a^+\nabla a)\Big\},\end{aligned} \tag{11}$$

where $\mathscr{H}_2^m$ is due to dipole-dipole interaction and is expressed in terms of a and a^+ by a complicated integral relation (see Ref. 7). We shall give expressions for $\mathscr{H}_3$ and $\mathscr{H}_4$ for a specific case of a nonlinear wave, a moving DW, in the following section.

In order to describe the spin system of a FM on the basis of our Hamiltonian, it is of course necessary to separate the two sub-systems, the nonlinear wave and the thermal reservoir of magnons, and to describe their interaction. Since the spin-wave Hamiltonian depends explicitly on time, the interaction of the system leads to transfer of energy of the nonlinear wave to the thermal reservoir of magnons, i.e., to dissipation of the energy of the nonlinear wave.

The rate of dissipation of energy, and also the thermal corrections to the values of various physical characteristics of the wave (energy, magnetization, etc.), can be calculated on the basis of the spin-wave Hamiltonian by use of standard methods of many-body theory.[8,9]

2. INTERACTION OF SPIN WAVES WITH A MOVING DOMAIN WALL

We turn to a detailed investigation of the example that is most important in practical respects and simplest in theoretical: motion of a DW. For the case of a plane DW moving along the x axis, we have $\mathbf{M}_0=\mathbf{M}_0(x-Vt)$, and the magnetostatic equations for $\mathbf{H}_m^0$ can be solved in elementary fashion:

$$\mathbf{H}_m^{(0)}=-4\pi M_x\mathbf{e}_x=4\pi M_0\sin\theta\sin\varphi\mathbf{e}_x. \tag{12}$$

From the condition $\mathscr{H}_1=0$, with use of (12), follow the well-known equations that describe the magnetization distribution in a plane nonlinear wave (see, for example, Ref. 1):

$$\alpha(\varphi'\sin^2\theta)'+4\pi\sin^2\theta\sin\varphi\cos\varphi-(\hbar/2\mu_0M_0)\dot{\theta}\sin\theta=0,$$
$$\alpha\theta''-[\beta+4\pi\sin^2\varphi+\alpha(\varphi')^2]\sin\theta\cos\theta+(\hbar/2\mu_0M_0)\dot{\varphi}\sin\theta=0, \quad (13)$$

where $\theta'=\partial\theta/\partial x$, $\dot{\theta}=\partial\theta/\partial t$. The solution of these equations, which describes a DW moving with velocity V, is well known.[10,11] It gives

$$\varphi=\varphi_0(V)=\text{const},\ \sin 2\varphi_0=(V/V_w),$$
$$\cos\theta=\text{th}[(x-Vt)/x_0]\quad\text{for}\quad\beta\gg4\pi. \quad (14)$$

Here $x_0=(\alpha/\beta)^{1/2}$ is the thickness of the DW, and V_w is the well-known Walker limiting value of the DW velocity[10]

$$V_w=(4\pi\mu_0M_0/\hbar)(\alpha/\beta)^{1/2}. \quad (15)$$

But $H_m^{(i)}(i\geq 2)$ are expressed in terms of a and a^+ by complicated integral relations (see, for example, Ref. 7). We shall restrict ourselves, for H_m, to the expression (12). As has already been mentioned above, this approximation corresponds to exact allowance for dipole interaction in $\mathcal{H}_0$ and $\mathcal{H}_1$ and to neglect of the magnetic-dipole terms in $\mathcal{H}_2$ and $\mathcal{H}_3$ in the description of the DW dynamics.[2)] When $\beta\gg4\pi$, this approximation may be expected to be reasonable. Below (see Sec. 4), we shall return to discussion of the contribution of magnetic-dipole interaction to DW damping. By using this simplification, we can write the spin-wave Hamiltonian in explicit form. Using (11) and (14), we get for $\mathcal{H}_2$

$$\mathcal{H}_2=\varepsilon_0\int d\mathbf{r}\{x_0^2(\nabla a^+)(\nabla a)+(1-2/\text{ch}^2\xi)a^+a\}. \quad (16)$$

Here $\varepsilon_0=\hbar\omega_0=2\mu_0\beta M_0$ is the activation energy of spin waves, $\xi=(x-Vt)/x_0$, and $x_0=(\alpha/\beta)^{1/2}$ is the DW thickness. Thus a DW constitutes an effective attractive potential $U(\xi)=-2/\cosh^2\xi$ for spin waves, one that moves with velocity V. The values of $\mathcal{H}_3$ and $\mathcal{H}_4$ are determined by the relations

$$\mathcal{H}_3=i\varepsilon_0x_0\tilde{a}^{3/2}\sqrt{\frac{2}{s}}\int d\mathbf{r}\left\{a^+a\frac{d}{dx}\left[\frac{1}{\text{ch}\,\xi}(a^+-a)\right]\right\}, \quad (17)$$

$$\mathcal{H}_4=-\frac{\varepsilon_0\tilde{a}^3}{2s}\int d\mathbf{r}\,a^+a\{x_0^2(\nabla a^+)(\nabla a)+(1-2/\text{ch}^2\xi)a^+a\}, \quad (18)$$

where $\tilde{a}$ is the interatomic distance and s is the spin of an atom; $2s/\tilde{a}^3=M_0/\mu_0$.

The Hamiltonian $\mathcal{H}=\mathcal{H}_2+\mathcal{H}_3+\mathcal{H}_4+\ldots$ describes the effect of thermal spin waves on the DW motion. Since for $V\neq0$ the Hamiltonian $\mathcal{H}$ contains terms explicitly dependent on time ($U=U(x-Vt)$), the DW motion leads to the occurrence of inelastic transitions in the spin-wave system. The energy of the magnon system then increases; that is, damping of the domain wall occurs. We note that the time-dependent terms in $\mathcal{H}$ are not small; that is, they cannot be taken into account by perturbation theory.

But at small velocities, we may have a situation in which the potential $U(x-Vt)$ itself is not small, but it varies slowly over characteristic times of the problem. As we shall show below, this condition corresponds to the inequality

$$V\ll2\omega_0x_0, \quad (19)$$

where $2\omega_0x_0$ is the minimum phase velocity of spin waves. Since $V<V_w$ and since $V_w=(2\pi/\beta)\omega_0x_0$ [see (15)], for $\beta\gg4\pi$ the condition (19) is always satisfied.

Thus, the perturbation that produces the inelastic transitions of interest to us in the magnon system is not small but is adiabatic. Consequently, adiabatic perturbation theory[13] can be applied to our problem. In this approach, a state vector of the system is sought in the form of an expansion in eigenstates of a Hamiltonian in which the value of the slowly varying parameter is fixed. In our problem, this corresponds to expansion in eigenstates of a Hamiltonian with the DW at rest at the point x_{DW}, where $Vt\to x_{DW}$. An expansion in these states has the form

$$a(\mathbf{r})=\Omega^{-1/2}\sum_k\Psi_k(\mathbf{r},x_{DW})a_k, \quad (20)$$

where Ω is the volume of the ferromagnetic material, and where $\Psi_k(\mathbf{r},x_{DW})$ is the solution of Schrödinger's equation with potential energy $U=U(x-x_{DW})$:

$$\varepsilon_0\{-x_0^2\Delta+1-2/\text{ch}^2[(x-x_{DW})/x_0]\}\Psi_k=\varepsilon_k\Psi_k. \quad (21)$$

The solutions of this equation, i.e., the energy and the form of the wave functions of magnons in a ferromagnet with a stationary DW, are well known[14]:

$$\Psi_k=\varphi_k(x-x_{DW})e^{i\mathbf{kr}},\ \varepsilon_k=\varepsilon_0(1+x_0^2\mathbf{k}^2),$$
$$\varphi_k(x)=[\text{th}(x/x_0)-ix_0k_x](1+x_0^2k_x^2)^{-1/2}. \quad (22)$$

Equation (21) has still another solution $\Psi_{k\perp}^0(\mathbf{r},x_{DW})$, which corresponds to a spin wave localized near the DW; but this state makes no contribution to the damping of the DW.

It is obvious that the set of functions $\Psi_k(\mathbf{r},x_{DW})$ is a complete orthonormal set; that is,

$$\int d\mathbf{r}\Psi_{k'}^*(\mathbf{r},x_{DW})\Psi_k(\mathbf{r},x_{DW})=\Omega\Delta(\mathbf{k}-\mathbf{k}').$$

Hence it is easily found that

$$a_k=\Omega^{-1/2}\int a(\mathbf{r})\Psi_k^*(\mathbf{r},x_{DW})d\mathbf{r}. \quad (23)$$

In the case of a moving DW, we shall seek $a^+(\mathbf{r})$ and $a(\mathbf{r})$ in the form of an expansion in eigenfunctions of the Hamiltonian for $x_{DW}\to Vt$:

$$a(\mathbf{r})=\Omega^{-1/2}\sum_k\varphi_k(x-Vt)e^{i\mathbf{kr}}a_k. \quad (24)$$

Since the transformation from $a(\mathbf{r})$ to a_k depends explicitly on time, the equation of motion for the operators a_k will have the form

$$i\,\partial a_k/\partial t=[a_k,\tilde{\mathcal{H}}], \quad (25)$$

where $\tilde{\mathcal{H}}$ is the Hamiltonian that describes the evolution in time of the operators a_k. The value of $\tilde{\mathcal{H}}$ is found directly from the equation of motion (5) for the operators $a(\mathbf{r})$ and from the relation (23):

$$i\frac{\partial a_k}{\partial t}=\Omega^{-1/2}\int d\mathbf{r}\left\{i\frac{\partial\Psi_k^*}{\partial t}a(\mathbf{r})+i\psi_k^*\frac{\partial a(\mathbf{r})}{\partial t}\right\}=[a_k,\tilde{\mathcal{H}}]. \quad (26)$$

As before, $\tilde{\mathcal{H}}$ can be represented in the form $\tilde{\mathcal{H}}_2+\tilde{\mathcal{H}}_3+\ldots$. For $\tilde{\mathcal{H}}_2$ it is easy to obtain

$$\tilde{\mathcal{H}}_2=\mathcal{H}_{02}+U_2(t)=\sum_k\varepsilon_ka_k^+a_k+\sum_{1,2}\tilde{U}(1,2)e^{-iq_xVt}a_1^+a_2, \quad (27)$$

where

$$i\equiv\mathbf{k}_i,\ \tilde{U}(1,2)=\Delta(\mathbf{k}_{1\perp}-\mathbf{k}_{2\perp})U(1,2).$$

Here

$$q=k_{1x}-k_{2x},\ \mathbf{k}_\perp=(0,\ k_y,\ k_z),\ \varkappa_i=x_0k_{ix},$$

$$U(1,2)=U(\varkappa_1,\varkappa_2)=-\frac{i\hbar V}{\Omega_x}\int_{-\infty}^{+\infty}dx\left(\varphi_2\frac{d\varphi_1^*}{dx}\right)e^{-iqx}$$
$$=\frac{\hbar V}{\Omega_x}\frac{(\varkappa_1+\varkappa_2)}{[(1+\varkappa_1^2)(1+\varkappa_2^2)]^{1/2}}\frac{\pi q x_0}{2\,\mathrm{sh}(\pi q x_0/2)}. \quad (28)$$

In our representation (24), the Hamiltonian $\mathscr{H}_0$ describes a magnon gas whose state at each instant of time is "tuned" to the prescribed position of the DW; $U_2(t)$ describes inelastic transitions between these states. Use of the representation (24) simplifies the problem because the amplitude of the inelastic processes is small ($U_2\sim\varepsilon_0V/\omega_0x_0\ll\varepsilon_0$), and the inelastic processes excited in the magnon system by $U(t)$ can be treated by standard thermodynamic perturbation theory.[8,9] The contribution of $\mathscr{H}_3$ and $\mathscr{H}_4$ is also small, since these operators contain the small parameter $(1/2s)$ and the operators a^+ and a to higher powers. We give the form of $\mathscr{H}_3$:

$$\mathscr{H}_3=\sum_{1,2,3}\{\Phi(1,2;3)\exp(-iq^{(3)}Vt)a_1^+a_2^+a_3+\mathrm{H.c.}\}, \quad (29)$$

where

$$q^{(3)}=k_{1x}+k_{2x}-k_{3x}, \quad (30)$$

$$\Phi(1.2;\ 3)=(\varepsilon_0x_0\tilde{a}^{3/2})(2s\Omega)^{-1/2}\Omega_x^{-1}\Delta(k_{1\perp}+k_{2\perp}-k_{3\perp})\varphi(1.2;\ 3); \quad (31)$$

$\varphi(1.2;3)\sim1$ if $q^{(3)}\leqslant1/x_0$, $\varphi(1.2;3)\sim\exp(-q^{(3)}x_0)$ if $q^{(3)}\gg1/x_0$.

The Hamiltonian $\mathscr{H}$ of the magnons can be represented in the form $\mathscr{H}_0+U(t)$, where $\mathscr{H}_0$ is the Hamiltonian in the absence of the DW, and where $U(t)$ is time-dependent and is due to the presence of the moving DW. The part of $\mathscr{H}_0$ that is quadratic in a^+ and a is determined by (27); $\mathscr{H}_{03}$ and $\mathscr{H}_{04}$ are the same as in a uniformly magnetized ferromagnet and are given, for example, in Ref. 6.

3. DISSIPATION OF ENERGY OF A NONLINEAR WAVE IN A FERROMAGNET (WITH A DW AS AN EXAMPLE)

All the physical characteristics of the spin system of a FM with a DW, i.e., of a nonlinear wave plus magnons system, are determined by the density matrix ρ, which satisfies Liouville's equation

$$i\,\partial\rho/\partial t=[\mathscr{H},\ \rho]. \quad (32)$$

If there is no interaction of the magnons with the DW, that is if $\mathscr{H}=\mathscr{H}_0$, the magnon system is in equilibrium, and its density matrix has the usual Gibbsian form ρ_0. Supposing, as usual, that the operator $U(t)$ is "turned on" adiabatically, one can write (32) in the form of an integral equation and solve it by perturbation theory as regards $U(t)$.[9] Then

$$\rho=\rho_0+\rho_1+\rho_2+\ldots, \quad (33)$$

where ρ_1 corresponds to the approximation of a linear response to $U(t)$, and where the ρ_n are determined by formula (4.1.5) of Ref. 9. By use of (33), one can express the mean value of any physical quantity describing the motion of a DW in a FM in the form

$$\langle A\rangle=A_0+A_1+A_2+\ldots, \quad (34)$$

where $A_n=\mathrm{Sp}(A\rho_n)$. It is easy to show that the calculation of A_n reduces to calculation of many-time Green's functions of the magnons. In particular, in the linear-response approximation

$$A_1=-i\int_{-\infty}^{t}dt_1\langle[\tilde{A}(t),\tilde{U}(t_1)]\rangle_0=\int_{-\infty}^{+\infty}dt_1\,G^{(+)}_{A,V}(t), \quad (35)$$

where the symbol $\langle\ldots\rangle$ represents averaging with weight ρ_0, and where

$$G^{(+)}_{A,U}(t,t_1)=-i\theta(t-t_1)\langle[\tilde{A}(t),\tilde{U}(t_1)]\rangle_0$$

is the retarded equal-time Green's function.[8,9] We note note that, strictly speaking, we are concerned with calculation of the Green's functions of a system of interacting magnons, since $\mathscr{H}_0$ includes the interaction of magnons with each other (and in general with phonons, etc.) in the absence of the DW. But in each specific case, the result, in the zeroth approximation with respect to magnon interaction, can be expressed through "dressed" single-magnon Green's functions.[8]

We turn to investigation of the character of the dissipation of energy of a nonlinear wave (in particular a DW). The dissipation is determined by transfer of the energy of the nonlinear wave to thermal magnons.

The damping force F that acts on unit area of the DW can be expressed in the form

$$F=-\frac{\dot{Q}}{SV}=-\frac{1}{SV}\mathrm{Sp}\left(\rho\frac{\partial U}{\partial t}\right), \quad (36)$$

where $\dot{Q}$ is the change of energy of the magnon system in unit time, S is the area of the DW, and V is, as before, the velocity of the DW.

The operator $U(t)$ has the form $U_2(t)+U_3(t)+\ldots$, where U_n contains n operators a_k^+, a_k. Therefore in the linear-response approximation, F can be expressed as $F^{(2)}_{\mathrm{lin}}+F^{(3)}_{\mathrm{lin}}=\ldots$, where $F^{(n)}_{\mathrm{lin}}$ is determined solely by U_n (schematically, $\langle U_nU_n\rangle$). In subsequent approximations with respect to U there appear, for example, terms of the type $\langle U_2U_2U_2\rangle$, $\langle U_2U_2U_4\rangle$, etc. We shall discuss the contributions of U_2 to F. For $F^{(2)}_{\mathrm{lin}}$ it is easy to derive[3] [see (28)]

$$F^{(2)}_{\mathrm{lin}}=\frac{2\pi}{S}\sum_{k,q>0}q|U(k,k+q)|^2(n_k-n_{k+q})\delta(\varepsilon_{k+q}-\varepsilon_k-qV). \quad (37)$$

Here $n_k=[\exp(\varepsilon_k/T)-1]^{-1}$ is the Bose equilibrium distribution function.

Since $U(1,2)$ itself is proportional to V, the damping force varies nonlinearly with the velocity V, and $[F(V)/V]\to0$ as $V\to0$. Turning to the calculation of $F_2(V)$, we note that by virtue of (28)

$$U(k,\ k+q)\propto(2k+q)V. \quad (38)$$

On taking into account the condition [see (37)]

$$\varepsilon(k+q)-\varepsilon(k)=\varepsilon_0x_0^2q(2k+q)=qV, \quad (39)$$

we find that $F^{(2)}=C(T)V^5$.

Thus the contribution of two-magnon processes to DW damping as $V\to0$ is negligibly small. At the same time, two-particle processes are dominant in all known problems of dynamic damping of moving linear defects (see Refs. 4 and 5). This result is due to the special form of the potential energy of interaction of spin waves with a DW in a uniaxial FM; specifically, to the fact that the potential energy of interaction of a magnon with a DW is reflectionless [see (21) and the expression (22) for the wave function]; that is, for an arbitrary wave vector of the incident wave there is no reflected wave.

For a more general model of a ferromagnet, in particular with allowance for magnetic-dipole interaction, the two-magnon terms may lead to a variation of $F^{(2)} \propto V$; then $F^{(2)}$ is expressed by a formula of the type (37) in which $U(k, k+q)$ is replaced by the amplitude of reflection of a magnon from the DW, i.e., by the scattering amplitude $T(\mathbf{k}_\perp, k_x; \mathbf{k}_\perp, -k_x)$.

Reflectionless potentials in the problem of dynamic damping present a great danger, for when perturbation theory is applied to them it turns out that the contribution to the damping force from the first order of perturbation theory with respect to the potential will be proportional to the velocity. In order to obtain the right answer ($F \propto V^5$), it is necessary to sum an infinite perturbation-theory series with respect to the potential. We succeeded in obtaining the right result easily by use of the representation (20); this is equivalent to use of adiabatic perturbation theory.

We turn to the case of a purely uniaxial ferromagnet without allowance for magnetic dipole interaction. Despite the fact that $[F_2(V)/V] \to 0$ as $V \to 0$, two-magnon processes become dominant at a finite velocity (see below). Therefore we turn to a detailed calculation of $F_2(V)$, i.e., of the form of the function $C(T)$. For this purpose, however, linear response theory is inadequate, since a variation $F \propto V^5$ results also from consideration of the subsequent terms in the approximations with respect to $U(1, 2)$; specifically, of terms of the type $\langle U_2 U_2 U_2 \rangle$ and $\langle U_2 U_2 U_2 U_2 \rangle$ along with $\langle U_2 U_2 \rangle$.

It can be shown that inclusion of the subsequent orders of perturbation theory with respect to U leads to replacement of the quantity $U(k, k+q)$ in formula (37) by $T(k, k+q)$, where the "scattering matrix" T is defined by the expression

$$T(k, k+q) = U(k, k+q) + \sum_{\varkappa} \frac{U(k, \varkappa)\, U(\varkappa, k+q)}{\varepsilon_k - \varepsilon_\varkappa} + \ldots \tag{40}$$

It is easily shown that when the condition (39) is taken into account, both terms in (40) are of the same order in the parameter $V/\omega_0 x_0$; that is, $T \propto (V/\omega_0 x_0)^2 + \ldots$.

On calculating $F^{(2)}(V)$ by formula (37) with use of (40), we get

$$F^{(2)} = \frac{T}{2x_0^3} \left(\frac{V}{\omega_0 x_0} \right)^5 \begin{cases} \left(\dfrac{4+\pi^2}{16\pi} \right)^2 \exp\left(-\dfrac{\varepsilon_0}{T} \right), & T \ll \varepsilon_0 \\ \eta & T \gg \varepsilon_0 \end{cases}, \tag{41}$$

where η is a numerical constant:

$$\eta = \frac{1}{8} \int_0^\infty \frac{x\,dx}{(1+x^2)} \left\{ \frac{x}{(1+x^2)\,\mathrm{sh}\,\pi x} + \frac{\pi}{4\,\mathrm{ch}^2\,\pi x/2} \right\}^2 \approx 0.093.$$

We shall discuss the contribution of U_3 and U_4. Restricting ourselves to $F^{(3)}_{\mathrm{lin}}$, we easily obtain

$$F_3 = \frac{4\pi}{S} \sum_{1,2,q^{(3)}} q^{(3)} |\Phi(1.2; 3)|^2 [(n_1+1)(n_2+1) n_3 - n_1 n_2 (n_3+1)]$$
$$\times \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - q^{(3)} V) + \frac{4\pi}{S} \sum_{1,2,q} q \delta(\varepsilon_q - qV) [\Phi(q, 1; 1) \Phi^*(q, 2; 2)$$
$$+ \Phi(q, 2; 2) \Phi^*(q, 1; 1)] n_2 n_3, \tag{42}$$

where $\mathbf{k}_3 = \mathbf{k}_1 + \mathbf{k}_2 - q^{(3)} \mathbf{e}_x$. It is easily seen that the second term in (42) is nonzero only when $\varepsilon(q) = qV$, i.e., when $V \geqslant 2\omega_0 x_0$. Since the DW velocity $V < V_W < 2\omega_0 x_0$ [see (14) and (15)], the second term is identically zero, and the contribution to F_3 comes from the first term alone. After cumbersome but uncomplicated calculations, we get

$$F_3 = \frac{V\hbar}{s x_0^4} \begin{cases} \dfrac{3}{\pi \sqrt{2}\, 2^9\, \mathrm{ch}^2\, \pi/2} \left[\dfrac{T^4}{(sJ_0)^{3/2} \varepsilon_0^{5/2}} \right] e^{(-2\varepsilon_0/T)}, & T \ll \varepsilon_0 \\ \dfrac{1}{9(2\pi)^3} \left[\dfrac{T^2}{(sJ_0)^{3/2} \varepsilon_0^{1/2}} \right] \ln^2 \dfrac{T}{\varepsilon_0}, & T \gg \varepsilon_0 \end{cases}. \tag{43}$$

Here s is the spin of an atom, and $sJ_0 = [2\mu_0 M_0 \alpha/\tilde{a}^2] = \varepsilon_0 (x_0/\tilde{a})^2$ is the value of the exchange integral, of the order of the Curie temperature T_c.

Similar calculations for F_4 lead to the conclusion that F_4 is small in comparison with F_3 over the whole range of temperatures ($T \ll T_c$) and of velocities: $F_4 \lesssim (T/T_c)^{3/2} F_3$. It can also be shown that the succeeding corrections, with respect to U_3, to the linear-response approximation (42) are small according to the parameter $1/2s$.

In concluding this section, we note that the eigenstates of the nonlinear system of a ferromagnet are not exhausted by topological solitons (i.e., DW) and spin waves, but include also nontopological solitons, which cannot be obtained within the framework of linear theory and may be interpreted as coupled states of a large number of magnons.[15] In principle these solitons may make a contribution to DW damping, first because of interaction of the DW with thermal solitons, and second because of Cerenkov radiation of solitons by moving DW.

The contribution of thermal solitons to F is proportional to $\exp\{-E(P, N)/T\}$, where $E(P, N)$ is the energy of a soliton with a given number of magnons N and momentum P. Since in a three-dimensional ferromagnet $E(N, P) \geqslant \varepsilon_0 N_3$, $N_3 \sim (J/\beta)^{3/2} \gg 1$, this contribution may be neglected when $T \ll T_c$.

As regards Cerenkov radiation of a soliton, this is not excluded and is even possible at $T = 0$ if $PV \gg E(P, N)$; that is, radiation of a soliton with a prescribed N is possible if the DW velocity is larger than a certain critical value $V_c(N)$:

$$V_c(N) = \min\{E(P, N)/P\}, \quad N = \text{const.}$$

For a soliton of small amplitude, $V_c(N)$ is close to the minimum phase velocity of spin waves $2\omega_0 x_0$, i.e., larger than the Walker limiting value V_W of (15), and small-amplitude solitons cannot be radiated. With increase of the soliton amplitude, i.e., with increase of N, $V_c(N)$ decreases, but $V_c(N)$ does not vanish for any value of N; that is, $\min V_c(N) = V_c \neq 0$. Cerenkov radiation of solitons is possible only when $V > V_c$, and at small DW velocities it also may be disregarded.

It can be shown that when $4\pi/\beta \to 0$, both V_c and V_W approach zero. It is not excluded that $V_c = V_W$; in this case, the physical meaning of the Walker limit is clarified as a threshold value of the DW velocity, beginning with which stationary motion of a DW is impossible because of Cerenkov radiation of solitons. This, however, is no more than a hypothesis. Investigation of this interesting question is hindered by the fact that at present we do not know an exact solution describing a soliton of arbitrary amplitude with allowance for dipole interaction; that is, we do now know the form of the function $E(N, P)$.

4. GENERAL PICTURE OF DW DAMPING. CONCLUSION

We have reached the conclusion that the main contribution to DW damping in a purely uniaxial FM is made by $F_2(V)$ and $F_3(V)$ [formulas (41) and (43)]. It is easily seen that for $V \to 0$, $F_2 \ll V_3$; but with increase of velocity, $F_2(V)$ increases faster than $F_3(V)$, and at a certain $V \approx V_*$, $F_2 \approx F_3$. For $V \gg V_*$, $F_2 \gg F_3$ and $F \propto V^5$ (see Fig. 1). For V_* one easily gets

$$V_* \approx \omega_0 x_0 \begin{cases} (T^2/T_c\varepsilon_0)^{3/4} \exp(-\varepsilon_0/4T), & T \ll \varepsilon_0 \\ (\varepsilon_0 T^2/T_c^3)^{1/4} \ln^{1/2}(T/\varepsilon_0), & T \gg \varepsilon_0 \end{cases}. \quad (44)$$

It is easily seen that at sufficiently low temperatures, $V_* \ll V_W$. For some FM or ferrites at room temperature, the condition $V_W \lesssim$ may not be satisfied.

Formulas (41) and (43) determine the variation of the damping force on a DW with its velocity. But it is of interest to study the variation of the velocity of viscous DW motion with the value of the external force that produces this motion (usually a magnetic field H_z). This variation is determined by the relation

$$2M_0(H - H_c) = F(V, T), \quad (45)$$

where H_c is the coercive force. It is evident that when $V > V_*$ $(H > H_*)$, the function $V(H)$ is very nonlinear (see Fig. 2). A nonlinear $V(H)$ relation has been observed in ferrites with $\beta/4\pi \approx 30$.[16] At small velocities $(V \ll V_*)$, the $V(H)$ relation is linear, and a DW mobility μ can be introduced in the usual manner:

$$V = \mu(H - H_c) = [2M_0/B(T)](H - H_c), \quad (46)$$

where $B(T) = \lim_{V\to 0}[F(V, T)/V]$ is determined by formula (43).

The following must be mentioned. If one goes outside the framework of our model, i.e., takes into account any other forms of interaction, it may turn out that they produce a nonvanishing amplitude $T(k_x, -k_x)$ of scattering of a magnon by a DW (see above). The contribution of these processes to the damping force is determined in order of magnitude by formula (41) with $(V/\omega_0 x_0)^5$ replaced by $(V/\omega_0 x_0)[T(k_x, -k_x)/\varepsilon_0]^2$. Such discussion of the effect of various types of interaction is beyond the scope of this paper. We note only that consideration of rhombic anisotropy does not lead to reflection of spin waves by a DW and that $F_2^{\text{rhomb}} \propto V^5$. Allowance for dipole scattering, produced by $\mathcal{H}_2^m$, leads to

$$T(k_x, -k_x)/\varepsilon_0 \propto 4\pi/\beta;$$

allowance for cubic anisotropy K or for later terms in the expansion of the uniaxial anisotropy $(\Delta W_a = KM_z^4/M_0^2)$ leads to

$$T(k_x, -k_x)/\varepsilon_0 \propto 2K/\beta M_0^2.$$

Thus these processes lead to $F_2' = VB'$, where $B' \propto [T(k_x, -k_x)/\varepsilon_0]^2$, and contribute to the mobility: $\mu = 2M_0/(B + B')$. Their relative significance is determined by the type of magnetic material. In particular, for films with cylindrical magnetic domains the value of $\beta/4\pi$ varies over a wide range, from several times unity to 10^2 or 10^3.

In analysis of DW mobility, one usually uses a phenomenological description of dissipation on the basis of the equations of motion of the magnetization.[1,2] Then $F = VB_0$, where B_0 is determined by the relaxation constant $\alpha_r = \gamma/\omega_0$ and is given by $B_0 = 2\alpha_r M_0/gx_0$ [see Ref. 2, p. 626 (translation, p. 317)]. In analysis of experimental data on DW mobility, the value is usually taken from data on the width of the ferromagnetic resonance (FMR) line,[17] since in a phenomenological description of the relaxation, γ coincides with the width of the linear FMR line.

We shall discuss the validity of this approach for a description of the relaxation of a nonlinear DW magnetization wave. For this purpose, we compare B of (43) with B_0. By using the theoretical value of the FMR linewidth (see Ref. 6, §31, and Ref. 18), one easily finds that these quantities are different:

$$\frac{B}{B_0} = \begin{cases} (4/3\pi^2)\ln^2(T/\varepsilon_0), & T \gg \varepsilon_0 \\ [3\pi^2\sqrt{2}/64 \operatorname{ch} \pi/2](T/\varepsilon_0)^2 \exp(-\varepsilon_0/T), & T \ll \varepsilon_0 \end{cases}. \quad (47)$$

The difference is especially large at low temperatures but is significant even at room temperature. Supposing at $T \approx 300$ K and $\varepsilon_0 = 2\mu_0 H_A$, and using[17] $H_A \approx 2$ kOe, i.e., $\varepsilon_0 \approx 0.3$ K, we easily find[4)] $B/B_0 \approx 10$.

The difference between the quantities B and B_0 is due in principle to two facts. First, a ferromagnet is a medium with strong spatial and temporal dispersion, and even at small perturbations the dissipation, which

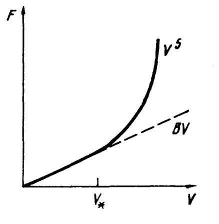

FIG. 1. Variation of damping force with wall velocity.

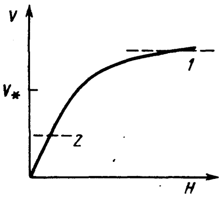

FIG. 2. Variation of the velocity of viscous motion of a wall, under the action of a magnetic field, with the value of the field (schematic). The dashed lines determine the Walker limiting value of the DW velocity, which may be either larger (1) or smaller (2) than the valus of V_*.

is determined by the imaginary part of the magnetic susceptibility, cannot be described by a single phenomenological constant α_r (see Ref. 6, § 31, and Ref. 18). Second, the perturbations of the magnetization field that are due to the nonlinear wave are not small and are not determined solely by the linear susceptibility. Our approach, of course, takes account of both these facts and may prove useful for analysis of the nature of the dissipation of nonlinear waves in any nonlinear media with dispersion.

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[1)] In contrast, for example, to the problem of the damping of a dislocation in a metal[4] or ferromagnet,[5] in which the elastic strain field due to the dislocation may be regarded simply as an external force acting on the electron or magnon subsystem.

[2)] Such "minimal" inclusion of dipole energy is necessary, since without allowance for magnetic dipole energy a DW in a uniaxial FM cannot move at all (see Ref. 12).

[3)] This formula takes no account of the attenuation γ of spin waves. Analysis shows that it is important only at very small DW velocities ($V < x_0\gamma \lesssim$ (1–10) cm/sec).

[4)] The value of (in our notation) B/B_0 at room temperature was measured in Ref. 16. The value obtained was $B/B_0 \approx 6$. Allowance for dipole scattering increases B/B_0, but this is unimportant for analysis of the results of Ref. 17, since in that work ferrite films with $\beta/4\pi \approx 30$ were used.

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Nonlinear theory of the electron temperature superlattice in semiconductors

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A nonlinear theory is developed for the electron temperature superlattice (Bénard phenomenon) in semiconductors with hot electrons. The stability conditions of the superlattice and the amplitude of the spatial oscillations of the electron temperature are determined as functions of the voltage applied to the sample. The asymptotic distribution of the electron temperature T, which is established upon superheating, $(T - T_0)T_0^{-1}$ (T_0 is the lattice temperature), is also obtained when the superheating is suffficiently large but not so great that scattering of the energy by optical phonons is appreciable. The interchange of the energy and momentum scattering mechanisms which occurs at a sufficiently high electron temperature is also taken into account. The asymptotic distribution is found to be one-dimensional and stable, at any rate, on a small scale.

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1. INTRODUCTION AND SETUP OF THE PROBLEM

The electron analog of the hydrodynamic problem of Bénard—the appearance of a spatially inhomogeneous distribution of the electron temperature in a nonuniformly heated electron gas—has been investigated in

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