Relation between the critical spin and angular velocity of a nucleus immediately after backbending

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In nonspherical nuclei at $J = J_c + 0$ the relationship between the angular momentum and angular velocity immediately after backbending is the same as in the limiting case $J - J_c \to \infty$. This indicates that there is a unique type of cancellation of the deviations from a rigid-body moment of inertia in the upper phase $J > J_c$. An integral relationship is found which expresses this cancellation quantitatively. This formula permits J_c to be calculated for the rotational bands of the even-even nuclei studied and the results are in agreement with those obtained by other methods of locating the Curie point. For the ground state band of W^{170} the cancellation of the reciprocals of the true and rigid-body moments of inertia can be verified directly. The condition for the stability of the rotation of a nonspherical nucleus is analyzed in the Appendix in close connection with the problem of a reasonable definition of the concept of a variable moment of inertia.

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1. INTRODUCTION

Even before the discovery in 1971 of the singularity exhibited by the rotational band at a certain critical value of the nuclear spin $J = J_c$, it was gradually becoming clear that we are actually dealing with a situation where the moment of inertia I of a nonspherical nucleus varies most significantly. In choosing a reasonable definition of this concept it is desirable to keep in mind the following considerations, in addition to purely aesthetic ones. First of all, for completeness and internal consistency of the theory it is important that both definitions of the moment of inertia, that via the Lagrangian and that via the Hamiltonian, be equivalent. In addition, in accordance with general physical considerations it is natural to expect that precisely at negative values of the correctly defined moment of inertia the rotation becomes unstable; this is discussed in more detail in the Appendix.

The following definition meets all these requirements:

$$\hbar\Omega = \frac{dE}{dJ}, \quad \frac{\hbar^2}{I} = \frac{d(\hbar\Omega)}{dJ} = \frac{d^2E}{dJ^2} \tag{1}$$

(the notation is the same as in Ref. 1). We emphasize that these prerequisites do not create any practical difficulties in a concrete comparison with experiment since in formulas like (1) it is possible to replace the derivatives with respect to J by the corresponding directly observable finite differences. In fact, the widely known Bohr-Mottelson formula

$$E \cong \frac{\hbar^2}{2I'} J(J+1), \quad J \ll J_c \tag{2}$$

has the property that the finite differences calculated with it agree with the result of the formal differentiation according to (1). On the other hand, for $J \gg 1$ the replacement of the derivatives by finite differences suggests itself automatically and no special problems appear.

In our earlier study[1] this phenomenon was viewed as a smooth, continuous rearrangement of the angular momentum coupling scheme in the nucleus. For adiabatically slow rotation, $J \ll J_0$, the internal state of the system is formed mainly by a "nucleon-nuclear symmetry axis" type of interaction, which is due to the axially symmetric deformation of a nucleus "at rest." However, in reality the nucleus is rotating, so that the nucleon-rotation axis interaction (which here emerges as a certain nonadiabatic correction) is always effective to some degree. In the entire region $J < J_c$ there is a complicated "nucleon-nuclear symmetry axis" plus "nucleon-rotation axis" coupling scheme of relatively low symmetry and the two interactions, generally speaking, are comparable. However, in a sufficiently strong rotation "field" the mechanical angular momenta of the individual quasiparticles are aligned parallel to the $\mathbf{\Omega} \parallel \mathbf{J}$ direction and cease to be oriented along the vector $\mathbf{n}$. It can be said that this corresponds to the simplest, most symmetric nucleon-rotation axis coupling scheme, not directly affecting the direction $\mathbf{n}$ of the nuclear axis. Then the vector $\mathbf{n}$ remains "free," that is, it is actually distributed isotropically for $J \geqslant J_c$. As a consequence of the increased symmetry of the rotational state at the point $J = J_c$ the angular velocity of the rotation falls abruptly by some amount $\Delta(\hbar\Omega)$ and in the isotropic upper phase the moment of inertia displays the seemingly paradoxical limiting behavior

$$I \cong j/(J - J_c), \quad J - J_c \ll j/I_0 \tag{3}$$

(j is some coefficient depending on the particular nucleus and I_0 is the rigid body moment of inertia).

The theory that we developed earlier[1] does not claim to be able to calculate the specific values of such parameters as J_c, the discontinuity $\Delta(\hbar\Omega)$, or the coefficient j for individual nuclei. In relation to this it should also be noted that the less symmetric lower phase is considerably more difficult to study theoretically. The main result here is the square-root law

$$Q \propto (J_c - J)^{1/2}, \tag{4}$$

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according to which the static quadrupole moment vanishes near the Curie point. It seems almost certain that it is not possible to quantitatively determine the value of the critical angular velocity Ω_{mc}, for example, from only deductive considerations. However, it is remarkable that for a given J_c the value of Ω_{nc} to which the rotational velocity falls after its discontinuous decrease, can be calculated in a closed form.

2. THE ANGULAR MOMENTUM AND ANGULAR VELOCITY IN THE UPPER PHASE $J > J_c$

After reaching the rigid body value of the moment of inertia

$$I=I_0, \quad J-J_c \gg j/I_0 \tag{5}$$

the rotational motion of the two components of nuclear matter can be assumed to be fully concurrent. Let us analyze this rotation in detail, appealing to a physical manifestation of the proton component like the magnetic moment. It is obvious that here

$$g=g_0=Z/A \tag{6}$$

is valid for the gyromagnetic factor.[1)]

However, the nuclear magnetic moment can be calculated in a somewhat different way, using the Larmour theorem (see Ref. 3, for example) and, so to speak, its differential form. If it is favorable for a spherical nucleus to rotate in the above manner, then in an applied magnetic field H it can be viewed as rotating, but with an angular velocity decreased by the Larmour value

$$\Omega_L=g_0eH/2m_pc \tag{7}$$

(m_p is the proton mass). Therefore, the rotational properties of the system that we are interested in here are described by the function $E(\Omega)$, while the change of energy is $-\Omega_L dE/d\Omega$. According to the usual point of view, the nucleus at a level J is a particle having magnetic moment μ and additional energy $-\mu H$ in the field (both the angular momentum vector and the magnetic field vector are assumed to be directed along the z axis).

Let us equate the two expressions for the additional energy of the system in the magnetic field:

$$-\frac{dE}{d\Omega}\Omega_L=-\mu H. \tag{8}$$

Furthermore, transforming the derivatives according to (1) and substituting formula (7), we also take into account the fact that according to the conventional definition of the nuclear g factor, the nuclear magneton $e\hbar/2m_pc$ must serve as the unit of measurement of the magnetic moment:

$$g=g_0I\Omega/\hbar J. \tag{9}$$

Finally, substituting here the values (5) and (6), we find

$$\hbar J=I_0\Omega, \quad J-J_c \gg j/I_0. \tag{10}$$

This seemingly natural recovery of the simple proportionality between the angular momentum and the angular velocity sheds additional light on the physical nature of the upper phase. For $J-J_c \gg j/I_0$ its characteristics cannot depend on such nonuniversal parameters as the coefficient j, for example. Roughly speaking, here we have a fairly clear interpretation of the ordinary rotation of a rigid body. However, if we move down the band in the negative direction of the J axis the stability of this regime deteriorates. In close relation to this the moment of inertia undergoes an odd, nonmonotonic change. First it falls off, then passes through a minimum, and then approaches the pole according to (3). In the region near the transition point $J-J_c \lesssim j/I_0$ of the variable moment of inertia the rotation, of course, can no longer be viewed as simply rigid body rotation.

Let us return for a while to the region where the moment of inertia is constant $I=I_0$. In our earlier study,[1] in a somewhat formal manner we found

$$\hbar\Omega=\hbar\Omega_{nc}+\hbar^2(J-J_c)/I_0 \tag{11}$$

for the rotational velocity. Now, comparing this to formula (10), we have

$$\hbar J_c=I_0\Omega_{nc}. \tag{12}$$

Therefore, in reaching the pole $J=J_c+0$ the effect of the above-mentioned nonmonotonic change of the moment of inertia on the rotational velocity is as though cancelled. It is easy to find an integral relation expressing this cancellation in a quantitative form. For this according to the second formula in (1) we integrate the inverse value of the moment of inertia over J_c up to some large J; then, performing an even more trivial integration of the analogous expression with the rigid body moment of inertia I_0, we take into account relation (12) and the fact that for $J\to\infty$ (10) is valid. As a result we easily find

$$\int_{J_c}^{\infty}\left(\frac{1}{I_0}-\frac{1}{I}\right)dJ=0. \tag{13}$$

For practical purposes it is convenient to write this relation in a dimensionless form:

$$\int_{J_c}^{\infty}(1-I_0/I)\,dJ=0. \tag{14}$$

In addition to the existing means of determining the location of the phase transition point, formula (12) permits J_c to be calculated according to the observed velocities Ω_{nc}. In Table I we compare the results of processing the experimental data by different methods for the twenty-eight rotational bands of nonspherical even-

TABLE I.

Nucleus	J_c^{exp}	J_c^{extr}	J_c^{theor}	Nucleus	J_c^{exp}	J_c^{extr}	J_c^{theor}
$_{56}Ba^{124}_{68}$	⩾11		⩽13.4	$_{68}Er^{158}_{90}$	13–15		12.6
$_{56}Ba^{126}_{70}$	11–13		12.5		25–27	25.9	23.1
$_{58}Ce^{128}_{70}$	11–13		10.6	$_{68}Er^{160}_{92}$	13–15		14.7
$_{58}Ce^{130}_{72}$	9–11	11.0	9.7	$_{68}Er^{162}_{94}$	13–15	15.7	15.2
$_{58}Ce^{132}_{74}$	11–13		10.3	$_{68}{}^{164}_{96}$	15–17		14.4
$_{58}Ce^{134}_{76}$	9–11		9.5	$_{70}Yb^{164}_{94}$	13–15	13.0	14.0
$_{64}Gd^{154}_{90}$	⩾17		⩽15.9	$_{70}Yb^{166}_{96}$	13–15	15.5	14.5
$_{64}Gd^{154*}_{90}$	11–13	12.4	10.5	$_{70}Yb^{168}_{98}$	15–17		18.3
$_{66}Dy^{154}_{88}$	13–15		14.1	$_{70}Yb^{170}_{100}$	15–17		18.6
$_{66}Dy^{156}_{90}$	15–17		16.0	$_{72}Hf^{168}_{96}$	13–15		13.5
$_{66}Dy^{156*}_{90}$	11–13		9.5	$_{72}Hf^{170}_{98}$	15–17		18.6
$_{66}Dy^{158}_{92}$	15	14.8	15.6	$_{74}W^{170}_{96}$	11–13	13.6	13.1
$_{66}Dy^{160}_{94}$	15–17		16.0	$_{76}Os^{182}_{106}$	13–15	11.9	16.3
$_{68}Er^{156}_{88}$	11–13	13.2	13.8	$_{76}Os^{184}_{108}$	13–15	15.1	18.4
				$_{76}Os^{186}_{110}$	⩾15		⩽17.4

even nuclei that we studied. The β-vibrational bands of gadolinium and dysprosium are starred. In column 2 we indicate, of neccessity tentatively, the location of the discontinuous decrease of the angular velocity of rotation. The value $J_c^{\rm extr}$ is the result of extrapolation according to formula (3) (see Ref. 1 for more details). The last column gives the value of the critical spin calculated according to (12). In calculating the rigid body moment of inertia I_0 we used the value of the nuclear radius obtained in Ref. 1 $r_0 = 1.1 \times 10^{-13}$ cm, which is also in agreement with the data on electron scattering. It corresponds to the working formula

$$\hbar^2/I_0 = 85\,900/A^{5/3} \text{ [keV]}. \tag{15}$$

On the whole the agreement appears to be satisfactory. It should, however, be noted that the nearness of the magic number $N = 82$ can sometimes manifest itself in very unexpected ways. It can be supposed that the second singularity $J^{(2)}$ found experimentally in the ground state rotational band of Er^{158} (Ref. 4) is due to this; see also Table I. In our opinion the construction of more detailed hypotheses on the nature of the "intermediate phase" $J_c^{(1)} < J < J_c^{(2)}$ is still somewhat premature. Intuition suggests that as the magic nucleus is approached the probability of similar surprises increases. In fact, for Er^{156} we already have circumstantial evidence of this supposition. According to the data of Ref. 5, after the first phase transition the moment of inertia, as usual, passes through a minimum, but then rises very steeply, reaching $I = 1.38 I_0$ for $J = 22$. The available information indicates that Er^{156} will also have a second backbending region.

However, far from the magic number $N = 82$ this type of anomalous behavior of the rotational bands is hardly possible.[2)] It is possible, though, that the effect of the doubly magic lead ($Z = 82$) also serves as a source of some anomalies. It manifests itself mainly via the number of protons in the nucleus and does not extend beyond osmium. Whether or not there is a second singularity in the ground state rotational band of Os^{184} is not yet clear. It was noted earlier[1] that for the preceding isotope Os^{182} the phase transition region is smeared out more than usual.

As an illustration of the agreement between the different methods of finding J_c it is worth noting that the rotional velocity $\hbar\Omega_{nc}$ varies within a fairly wide range: from 216 keV for W^{170} to 340 keV in the ground state rotional band of Ba^{126}. The rigid body moment of inertia I_0 increases by roughly a factor of two throughout the entire table.

For nonspherical nuclei sufficiently far from the possible effect of the magic numbers (see above), experimental data on the upper phase are far from abundant. Therefore, the possibilities of directly verifying the integral relation (13) or (14) at the present time are very limited. Only for W^{170} does the moment of inertia, after passing through the minimum, approach the rigid body value with an accuracy of about 4%. In Fig. 1 we give the graph of the function $1 - I_0/I$ in this case. The accuracy with which the integral of this function becomes zero can be considered satisfactory.

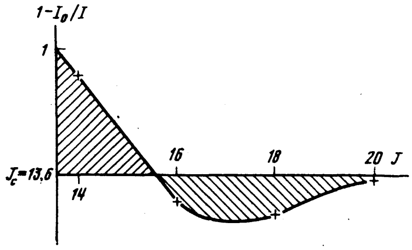

FIG. 1. The integral relation (14) for the W^{170} nucleus. The area under the "anomalous" ($I > I_0$) part of the curve is cancelled with an accuracy of about 10%.

3. CONCLUSIONS

The reason that for tungsten there are few experimental points on a large part of the moment of inertia curve in the upper phase is the following: the ratio j/I_0 is not large, amounting only to 1.88. However, there are nuclei for which the value of this parameter is much larger. For Dy^{158}, for example, the number of neutrons $N = 92$ is sufficiently far from the magic number and $j/I_0 = 6.76$. Here further study of the upper phase makes it possible to construct a more accurate moment of inertia curve using a considerably larger number of experimental points. Beginning, for a rough estimate, from the assumption of similarity to the curve in Fig. 1, we conclude that in the ground state band of Dy^{158} it is of interest to measure the location of the rotational levels up to $J \sim 40$, beyond which the moment of inertia becomes practically the rigid body value.

In addition to the data of Ref. 6 cited in Ref. 1 on the radiation of Ce^{134},[3)] verification of the "area theorem" (13) for different nuclei, including cases with $j/I_0 \gg 1$, would give interesting new material for judging the validity of the theory treating backbending as a macroscopic quantum phenomenon.

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APPENDIX

THE CONDITION FOR ROTATIONAL STABILITY OF A NONSPHERICAL NUCLEUS

The considerations discussed below apply equally to either phase $J \gtrless J_c$. Let us view the level J_0 of the ground state rotational band as minimizing the total energy of the nucleus for a given value of the conserved angular momentum of the entire system:

$$E = \min, \quad J = J_0. \tag{A.1}$$

Using the Lagrangian method, we shall drop the auxiliary condition and unconditionally require that

$$E - \lambda J = \min. \tag{A.2}$$

This can be rewritten as

$$\delta(E - \lambda J) > 0. \tag{A.3}$$

Moving now from the minimum along the actually realized rotational band $E(J)$, let us calculate the energy change δE with accuracy to second-order terms inclusive:

$$\left\{\left(\frac{dE}{dJ}\right)_{J=J_0} - \lambda\right\}\delta J + \frac{1}{2}\left(\frac{d^2E}{dJ^2}\right) \qquad (\delta J)^2 > 0. \tag{A.4}$$

The value

$$\lambda=(dE/dJ)_{J=J_0} \quad (A.5)$$

of the Lagrange multiplier ensures the correct location of the extremum and is a minimum for

$$d^2E/dJ^2>0. \quad (A.6)$$

Here we have taken into account the fact that at the beginning the point $J=J_0$ was chosen arbitrarily. Comparison with formula (1) gives

$$I>0. \quad (A.7)$$

Therefore, the requirement that the moment of inertia be positive emerges here as the stability condition.

The question of the rotational stability of a nonspherical nucleus can also be approached from a somewhat different viewpoint. Let us as usual denote the spherical angles giving the orientation of the vector **n** (if convenient it plays the role of the rotational variable, but no separate "rotational Hamiltonian" corresponds to it) in the stationary space by θ and φ. The state of motion in the angle θ is, in general, not "pure." The situation with the motion in the azimuthal angle φ is different: because of the conservation of the z component of the total angular momentum it corresponds to the separate wave function

$$\psi_{rot}=(2\pi)^{-1/2}e^{iM\varphi}, \quad M=J_z. \quad (A.8)$$

For $J_z=J\gg1$ the angular momentum is directed along the z axis and the situation is semiclassical: the fully described state (A.8) of regular precession can in this limit be viewed as changing into classical motion in a cyclic trajectory, corresponding to the variation of the azimuthal angle φ.

In order to remove possible doubts about the validity of using a purely mechanical approach here, we recall that the free rotation of a body in thermodynamic equilibrium is not accompanied by friction (see Ref. 8, for example). We shall proceed directly from the principle of least action

$$\int_{t_1}^{t_2} L dt=\min, \quad \varphi(t_1)=\varphi_1, \quad \varphi(t_2)=\varphi_2 \quad (A.9)$$

(see Ref. 9, for example). The extreme values of the angles φ_1 and φ_2 are viewed as constants which are not varied:

$$\delta\varphi(t_1)=\delta\varphi(t_2)=0.$$

However, let us first consider how the moment of inertia is expressed in terms of the Lagrangian L. We have[4]

$$E=E(M), \quad dE/dM=\Omega, \quad L=L(\Omega).$$

Let us now express the Hamiltonian $E(M)$ in terms of the Lagrangian:

$$E=\Omega dL/d\Omega-L. \quad (A.10)$$

Differentiating (A.10) once with respect to M and cancelling Ω from both sides, after some elementary manipulations we easily find

$$\frac{d^2L}{d\Omega^2}\frac{d^2E}{dM^2}=1 .$$

Comparing this to formula (1), we finally find

$$I=d^2L/d\Omega^2=(d^2E/dM^2)^{-1}. \quad (A.11)$$

This is precisely the relation between the moment of inertia and the Lagrangian that we would naturally expect; see also the preliminary discussion at the beginning of this article.

Let us now return to the principle (A.9), rewriting it in the form

$$\delta\int_{t_1}^{t_2} L(\Omega)dt>0. \quad (A.12)$$

Free rotation can occur only as uniform rotation, that is, when the angular velocity is constant:

$$\Omega_0=(\varphi_2-\varphi_1)/(t_2-t_1). \quad (A.13)$$

The deviation $\delta\varphi(t)$ from this simple law of motion gives rise to a time-dependent variation of the velocity

$$\delta\Omega(t)=d\delta\varphi(t)/dt.$$

Let us find the corresponding variation of the action with accuracy to second-order terms inclusive:

$$\left(\frac{dL}{d\Omega}\right)_{\Omega=\Omega_0}\int_{t_1}^{t_2}\delta\Omega(t)dt+\frac{1}{2}\left(\frac{d^2L}{d\Omega^2}\right)_{\Omega=\Omega_0}\int_{t_1}^{t_2}[\delta\Omega(t)]^2dt>0.$$

It is obvious that

$$\int_{t_1}^{t_2}\delta\Omega(t)dt=\delta\varphi(t_2)-\delta\varphi(t_1)=0, \quad \int_{t_1}^{t_2}[\delta\Omega(t)]^2dt>0.$$

Therefore

$$d^2L/d\Omega^2>0. \quad (A.14)$$

Comparing this with formula (A.11), we find the stability condition (A.7). States with a negative moment of inertia as violating the principle of least action are unstable and cannot be realized as stationary rotational energy levels of a nonspherical nucleus.

In conclusion let us clarify the nature of the limiting behavior of the Lagrangian of the upper phase. It can be found explicitly in the region near the transition point, where the moment of inertia has a pole behavior (3). Substituting this formula into (A.11), using formula (30) from Ref. 1 we can express the angular momentum in terms of the angular velocity and then integrate twice with respect to Ω:

$$L_n(\Omega)={}^2/_3(2\hbar j)^{1/2}(\Omega-\Omega_{nc})^{3/2}+I_0\Omega_{nc}\Omega-E_c. \quad (A.15)$$

The constants of integration are here chosen such that the first derivative gives the angular momentum $M=\hbar J$ and the energy (A.10) coincides for $\Omega=\Omega_{nc}$ with the nuclear excitation energy E_c at the Curie point. We see that the Lagrangian of the upper phase has a singularity (a branch point) at the point of the phase transition $\Omega=\Omega_{nc}$.

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Light scattering in cholesteric liquid crystals

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Light scattering in a cholesteric liquid crystal is investigated. Uniaxial fluctuations of the director are considered with a wave vector that has a small component perpendicular to the axis of the cholesteric crystal. It is shown that the spectrum of the scattered light should reveal maxima at $\mathbf{q} = n\mathbf{t}_0$, $n = 0, \pm 1, \ldots$, with intensities that decrease with increasing number n.

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We investigate here the scattering of light in cholesteric liquid crystals. The optical properties of liquid crystals were dealt with in many papers. Kats[1] and Belyakov and Dmitrienko[2] investigated the propagation of light in cholesteric crystals. Kats[1] solved Maxwell's equations for the case of normal incidence of light and for incidence at a small angle to the crystal axis. Belyakov and Dmitrienko[2] considered the passage and selective reflection of light from a cholesteric light crystal with the aid of dynamic diffraction theory, and obtained expressions for the amplitudes and polarizations of the reflected and transmitted waves at an arbitrary angle of incidence of the light on the crystal.

De Gennes[3] and Brazovskiĭ and Dmitriev[4] found the correlation function in the high-temperature phase of a cholesteric crystal. The case of the low-temperature phase, when spontaneous spiral anisotropy is present, was considered in the review of Stephen and Straley,[3] who investigated the scattering of light by long-wave fluctuations. They, however, introduced fluctuating rotation angles of the directors and assumed that these angles vary slowly compared with the period of the spiral. It will be shown, however, that the nonzero harmonics of these functions are also not small.

We consider scattering by fluctuations of arbitrary wavelength. In contrast to Stratonovich,[6] who investigated biaxial fluctuations, we obtain results for wavelengths located directly in the vicinities of the maxima of $\mathbf{q} = \pm n\mathbf{t}_0$, where the terms of the series (3.3) of the Stratonovich paper[6] turn out to be large (since the values of the corresponding τ go through zero) and the expansion turns out to be incorrect.

We investigate uniaxial fluctuations of the director $\delta n_x = -\varphi \sin\varphi_0$, $\delta n_y = \varphi\cos\varphi_0$, δn_z, where $\varphi_0 = t_0 z$, and the z axis is directed along the spiral axis; $n_x = \cos\varphi_0$, $n_y = \sin\varphi_0$, $n_z = 0$.

The assumption that the elastic constants are equal leads to the following expression for the change of the thermodynamic potential:

$$\Delta G = \frac{1}{2} K \int_V d\mathbf{r} \left\{ (\nabla\varphi)^2 + 2t_0 \delta n_z \left[\cos\varphi_0 \left(\frac{\partial\varphi}{\partial x}\right) + \sin\varphi_0 \left(\frac{\partial\varphi}{\partial y}\right) \right] + t_0^2 \delta n_z^2 + (\nabla \delta n_z)^2 \right\}$$

or

$$\Delta G = \frac{1}{2} K \left\{ \sum_q q^2 \varphi_q \varphi_q^* + t_0 \sum_q i q_x \varphi_q (\psi^*_{q+t_0} + \psi^*_{q-t_0}) + t_0 \sum_q q_y \varphi_q (\psi^*_{q+t_0} - \psi^*_{q-t_0}) + t_0^2 \sum_q \psi_q \psi_q^* + \sum_q q^2 \psi_q \psi_q^* \right\},$$

where $\varphi_q \equiv \varphi(\mathbf{q})$, $\psi_q \equiv \delta n_z(\mathbf{q})$ are the Fourier components of the spatial quantities $\varphi(\mathbf{r})$ and $\delta n_z(\mathbf{r})$.

To diagonalize this quadratic form we must solve the system of equations

$$\begin{aligned} \frac{\partial \Delta G}{\partial \varphi_q^*} &= q^2 \varphi_q - i t_0 q_x (\psi_{q+t_0} + \psi_{q-t_0}) + t_0 q_y (\psi_{q+t_0} - \psi_{q-t_0}) = \lambda \varphi_q, \\ \frac{\partial \Delta G}{\partial \psi_q^*} &= (q^2 + t_0^2) \psi_q + i t_0 q_x (\varphi_{q+t_0} + \varphi_{q-t_0}) - t_0 q_y (\varphi_{q+t_0} - \varphi_{q-t_0}) = \lambda \psi_q, \end{aligned} \tag{1}$$

which assumes in the case $q_x = 0$ and $q_y = 0$ the simplest form

$$q^2 \varphi_q = \lambda \varphi_q, \qquad (q^2 + t_0^2) \psi_q = \lambda \psi_q.$$

Then $\lambda_1 = q_z^2$, $\lambda_2 = q_z^2 + t_0^2$,

$$\Delta G = \frac{1}{2} K \sum_q \{ (t_0^2 + q^2) |\psi_q|^2 + q^2 |\varphi_q|^2 \},$$

$$\langle |\varphi_q|^2 \rangle = T/K q_z^2, \qquad \langle |\psi_q|^2 \rangle = T/K (q_z^2 + t_0^2).$$

We shall show that at small $q_\perp$, i.e., $q_\perp / t_0 \ll 1$, the solution can be found in the form of an expansion in this small parameter. The zeroth approximation corresponds to the case $q_\perp = 0$.

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