Propagation of intense laser radiation in an absorbing medium

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(Submitted 25 November 1978)
Zh. Eksp. Teor. Fiz. 76, 1198–1211 (April 1979)

The propagation of high intensity laser pulses in media at rest and in motion in the presence of absorption is studied. The pulses are assumed to be of sufficiently long duration so that the nonlinearity of the medium is mainly due to its heating. The variation of the profile of the intensity and of the phase front of the pulse during its propagation is found. The focusing conditions are obtained.

PACS numbers: 42.60.He, 42.65.Jx

I. INTRODUCTION. BASIC EQUATIONS

As the intensity of electromagnetic waves propagating in a medium increases, their self-action becomes more and more important. Self-action effects are due to the dependence of the dielectric constant of the medium on the intensity of the propagating wave. Electrostriction and the Kerr effect can lead to this dependence. The phenomena due to these mechanisms, which cause the index of refraction to be nonlinear, have been studied in detail (see Refs. 1–3, for example). In addition, variation of the density of the medium and, consequently, of the index of refraction can be due to heating of the medium by absorption of the energy of an intense electromagnetic wave.[4] In the present study we consider the effect of absorption on the propagation of laser pulses of sufficiently long duration.

The propagation of an electromagnetic wave in a medium is described by the wave equation

$$\Delta \mathbf{E} - \operatorname{grad} \operatorname{div} \mathbf{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}[\varepsilon(\mathbf{E}) \times \mathbf{E}] = 0, \qquad (1)$$

where $\varepsilon(\mathbf{E}) = \varepsilon_0 + i\varepsilon_1 + \varepsilon_2(\mathbf{E})$ is the dielectric constant. Let us substitute into this equation the expression

$$\mathbf{E} = \frac{1}{2}\left\{\mathbf{e}E(\mathbf{r})\exp\left[i\int_0^z k\,dz - i\omega t\right] + \text{c.c.}\right\},$$

where $\mathbf{e}$ is the polarization vector, $k = \omega\sqrt{\varepsilon_0}/c$, ω is the frequency of the wave, and the z axis is chosen to lie along the wave propagation direction. Dropping the $\partial^2 E/\partial z^2$ and assuming the polarization to be constant, we find

$$\Delta_\perp E + 2ik\frac{\partial E}{\partial z} + iE\frac{dk}{dz} + \frac{\omega^2}{c^2}(i\varepsilon_1 + \varepsilon_2)E = 0, \qquad (2)$$

where $\Delta_\perp$ is the two-dimensional Laplace operator in the plane perpendicular to the beam, that is, to the z axis. This parabolic equation corresponds to the so-called quasioptical approximation. The conditions under which it is applicable are well known, and we shall not dwell on them here (see Refs. 1 and 5, for example).

Introducing the eikonal S of the complex amplitude $E = E_0 e^{ikS}$ and separating the real and imaginary parts, we get from (2)

$$\frac{\partial I}{\partial z} + \operatorname{div}(I\nabla_\perp S) = -\frac{I}{k}\frac{dk}{dz} - k\frac{\varepsilon_1}{\varepsilon_0}I, \qquad (3)$$

$$\frac{\partial S}{\partial z} + \frac{1}{2}(\nabla_\perp S)^2 = \frac{\varepsilon_2(E)}{2\varepsilon_0} - \frac{S}{k}\frac{dk}{dz} + \frac{1}{2k^2 I^{1/2}}\Delta_\perp I^{1/2}, \qquad (4)$$

where I is the intensity of the electromagnetic wave.

If the duration of the CO_2 laser pulse is greater than 10^{-7} sec (for the atmosphere), it is possible to neglect in the nonlinear increment $\varepsilon_2(E)$ to the dielectric constant the contributions of the Kerr effect and striction, and to take into account only the change of the dielectric constant due to heating of the medium (see Ref. 4):

$$\varepsilon_2 = \frac{d\varepsilon}{dT}\delta T; \qquad (5)$$

here $\delta T = T - T_0$ is the temperature change of the medium. We shall also assume that the pressure manages to become equalized over the diameter of the beam, that is, the pulse duration is $t > a/v_s$, where a is the characteristic transverse dimension of the beam and v_s is the speed of sound. Here

$$\frac{d\varepsilon}{dT} = -\beta\rho\frac{\partial\varepsilon}{\partial\rho}, \quad \beta = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p = \frac{c_p}{v_s^2}\Gamma, \qquad (6)$$

where β is the coefficient of thermal expansion, c_p is the specific heat at constant pressure, ρ is the density of the gas at $T = T_0$, and $\Gamma = c_p/c_V - 1$. The temperature change of the medium is given by the heat-conduction equation

$$\Delta\delta T - \frac{1}{\chi}\frac{\partial\delta T}{\partial t} - \frac{1}{\chi}(\mathbf{v}_0\nabla)\delta T - \frac{1}{\chi}(\mathbf{v}_1\nabla)\delta T = -\frac{\alpha}{\varkappa}I. \qquad (7)$$

Here χ is the coefficient of thermal diffusivity, α is the absorption coefficient, $\varkappa = c_p\rho\chi$ is the coefficient of thermal conductivity, v_1 is the velocity of the convective motion arising from the heating of the medium, and v_0 is the velocity of the medium far from the laser beam (the wind speed).

In the following we shall consider the three cases where the main role is played respectively by the first, second, and third term on the left-hand side of the heat-conductor equation (7). For each of these cases we will find the conditions under which we can neglect the last term on the left-hand side of (7), that is, the convective term.

Therefore, the system (3)–(7) describes the propagation of an intense pulse of duration $t > a/v_s$ in an iso-

0038-5646/79/040608-08$02.40

tropic medium in the presence of absorption when striction and the Kerr effect are not important. We rewrite these equations in the dimensionless variables

$$\frac{x}{a}=\tilde{x},\quad \frac{y}{a}=\tilde{y},\quad \frac{z}{a}\gamma^{1/2}=\tilde{z},\quad \frac{S}{a\gamma^{1/2}}=\tilde{S},\quad \frac{t}{a^2}\chi=\tilde{t},\quad \frac{\varepsilon_2}{\gamma}=\tilde{\varepsilon}_2,\quad \frac{I}{I_0}=\tilde{I},$$
$$\frac{v_0}{\chi}a=\tilde{v}_0,\quad \frac{\varepsilon_1}{\varepsilon_0}\frac{\omega}{c}\frac{a}{\gamma^{1/2}}=\frac{\alpha a}{\gamma^{1/2}}=\tilde{\alpha},\quad \gamma=\frac{a^2\alpha}{\varkappa}\left|\frac{d\varepsilon}{dT}\right|I_0,\quad \delta=\frac{c^2}{\omega^2a^2}\frac{1}{\gamma}. \tag{8}$$

Here I_0 is the characteristic value of the intensity, and the other quantities were introduced earlier. In these variables (dropping the tildes) the system (3)–(7) becomes

$$\partial I/\partial z+\operatorname{div}(I\mathbf{u})=-\alpha I, \tag{9.1}$$

$$\frac{\partial \mathbf{u}}{\partial z}+(\mathbf{u}\nabla)\mathbf{u}=\frac{1}{2}\nabla_\perp\varepsilon_2+\frac{\delta}{2}\nabla_\perp\frac{1}{I^{1/2}}\Delta_\perp I^{1/2}, \tag{9.2}$$

$$\Delta_\perp\varepsilon_2-\partial\varepsilon_2/\partial t-(\mathbf{v}_0\nabla)\varepsilon_2=I. \tag{9.3}$$

We have assumed that the medium is homogeneous ($d\varepsilon_0/dz=0$) and that $\varepsilon_0=1$, and have operated on (4) with $\nabla_\perp$, introducing as in Ref. 5 a vector $\mathbf{u}\equiv\nabla_\perp S$ that lies in the plane perpendicular to the propagating beam and defines the operator of projection of the propagation direction on the (x, y) plane. In Eq. (9.3) we are neglecting the second derivative with respect to the longitudinal coordinate compared to $\Delta_\perp\varepsilon_2$. This is the same approximation which led us from Eq. (1) to (2).

The system of equations (9) with an arbitrary distribution of the intensity $I(x, y, z=0)=I_0(x, y)$ and phase $\mathbf{u}(x, y, z=0)=\mathbf{u}_0(x, y)$ of the electromagnetic wave at the entrance to the medium can, of course, be solved only numerically. Furthermore, we shall consider beams whose width is large compared to the wavelength. For such beams we can neglect in (9.2) the last term, which describes diffraction effects (see Ref. 5, p. 153). Without this term, Eqs. (9) describe the propagation of electromagnetic waves in an absorbing medium in the geometrical-optics approximation.

2. THE PROPAGATION OF STEADY BEAMS OF ELECTROMAGNETIC WAVES IN A MEDIUM AT REST

We shall begin our study of the propagation of steady beams in a medium at rest with two-dimensional beams. In this case Eqs. (9) take the form

$$(\partial/\partial z+u\partial/\partial x)N=0, \tag{10.1}$$

$$(\partial/\partial z+u\partial/\partial x)u={}^1/_2\partial\varepsilon_2/\partial x, \tag{10.2}$$

$$\partial^2\varepsilon_2/\partial x^2=\partial N/\partial x. \tag{10.3}$$

In (10.1) we have dropped in the right-hand side the term due to linear absorption (the accuracy of this approximation for CO_2 laser beams in the atmosphere is several percent; the inclusion of linear absorption does not affect the subsequent calculations in principle, but only makes them more complicated) and have introduced the power N of the electromagnetic wave

$$N(x, z)=\int_0^x I(x, z)\,dx. \tag{11}$$

The general solution to the system of equations (10) is known (see Ref. 6, for example). For our purposes it is most convenient to write it down introducing the characteristics, that is, the surfaces

$$\varphi(x, z)=\text{const}, \tag{12}$$

which are the solutions of the equation

$$dx/dz=u(x, z). \tag{13}$$

In the two-dimensional case that we are considering the characteristics (12) have the sense of rays, so it is convenient to define the function φ such that $\varphi(x, z=0)=x$. In the variables φ and z the system of equations (10) becomes

$$(\partial N/\partial z)_\varphi=0,\quad (\partial u/\partial z)_\varphi=N/2. \tag{14}$$

From the first equation we find that $N=f(\varphi)$, where f is an arbitrary function. If we take into account the boundary conditions $\varphi(x, z=0)=x$ and $I(x, z=0)=I_0(x)$, that is,

$$N(x, z=0)=\int_0^x I_0(x)\,dx\equiv N_0(x),$$

then we obtain

$$N=N_0(\varphi). \tag{15}$$

From the second of equations (14) we find that $u=N_0(\varphi)z/2+u_0(\varphi)$, where we have used the second boundary condition $u(x, z=0)=u_0(x)$. Substituting this expression into (12), we find the equation for $\varphi(x, z)$:

$$x=\varphi+u_0(\varphi)z+N_0(\varphi)z^2/4. \tag{16}$$

Thus, the problem of the propagation of a two-dimensional steady beam in an absorbing medium has been solved exactly in the geometrical optics approximation for beams with an arbitrary distribution of the intensity $I(z=0)=I_0(x)$ and of the phase $u(z=0)=u_0(x)$ at the entrance to the medium (we recall that the eikonal S is related to $\mathbf{u}$ by the equation $\mathbf{u}=\nabla_\perp S$, that is, $u=\partial S/\partial x$ for the two-dimensional case).

The intensity of the electromagnetic wave is

$$I=\partial N/\partial x=I_0(\varphi)\varphi_x'.$$

Substituting φ_x' from (16), we find

$$I=I_0(\varphi)[1+u_0'(\varphi)z+I_0(\varphi)z^2/4]^{-1}; \tag{17}$$

in the dimensional variables we have

$$I=I_0(\varphi)\left[1+u_0'(\varphi)z+I_0(\varphi)\frac{z^2}{4}\frac{\alpha}{\varkappa}\left|\frac{d\varepsilon}{dT}\right|\right]^{-1}.$$

For a vacuum ($\alpha=0$) we have

$$x=\varphi+u_0(\varphi)z,\quad I=I_0(\varphi)[1+u_0'(\varphi)z]^{-1}. \tag{17'}$$

We shall begin our analysis of the solutions that we have obtained with the condition that a point focus exists. At the entrance to the medium let the phase be such that in a vacuum the beam would be focused at a point with the coordinates $x^*=0, z^*=1/b_0$, that is,[1)]

$$u_0=-b_0x. \tag{18}$$

The eikonal in this case is $S(z=0)=-b_0x^2/2$, that is, at the entrance to the medium the phase front of the beam is parabolic.

With the boundary phase (18) the equation of the characteristics (16) becomes

$$x=(1-b_0z)\varphi+{}^1/_4 z^2N_0(\varphi). \tag{19}$$

From this equation we see that the characteristics $\varphi(x,z)=$const intersect at a single point, that is, a focal point exists only when the power N_0 depends linearly on φ; in other words, the beam intensity at the entrance to the medium must be a constant. With accuracy to boundary effects we have

$$I_0(x)=\Theta(1-x^2) \tag{20}$$

$[\Theta(x>0)=1, \Theta(x<0)=0]$. In this case [see (17)]

$$I(\varphi\leqslant 1)=(1-b_0z+z^2/4)^{-1}, \quad I(\varphi>1)=0.$$

The boundary of the beam is determined by the condition $\varphi=1$. For $\varphi\leqslant 1$ the power $N_0(\varphi)=\varphi$ and Eq. (19) can be written as

$$\varphi=(1-b_0z+z^2/4)^{-1}x. \tag{21}$$

The rays $\varphi=$const intersect at the point (the focus)

$$x^*=0, \quad z^*=2[b_0-(b_0^2-1)^{1/2}]. \tag{22}$$

It is interesting that there is a maximum focal length $(z^*)_{max}=2$; in the dimensional variables this is

$$(z^*)_{max}=2\left(\frac{\alpha}{\varkappa}I_0\left|\frac{d\varepsilon}{dT}\right|\right)^{-1/2}.$$

Therefore, when a steady beam propagates an absorbing medium there is a focus only if the boundary distribution of the intensity is of the form (20), when the phase at the boundary is such that

$$|\partial^2S(x,z=0)/\partial x^2|=b_0>1. \tag{23}$$

When this condition is satisfied there is a focus at the point (22), which can differ significantly from the focal point in a vacuum $(x^*=0, z^*=1/b_0)$.

If condition (23) is not satisfied $(b_0<1)$, there is no focus. In this case the width of the beam is a minimum (and the intensity is therefore a maximum) at the point $z_{min}=2b_0$. The behavior of the rays (the characteristics $\varphi=$const) for a steady beam with the intensity distribution (20) and phase distribution (18) at the entrance to the medium is shown in Fig. 1. Beams with an arbitrary intensity and phase profile at the entrance to the medium can be analyzed using expressions (16) and (17) as in the example that we have discussed.

Let us now consider the propagation of steady three-dimensional beams in a medium at rest with absorption. Let the beam be symmetric at the entrance to the medium:

$$I(x,y,z=0)=I_0(r), \quad u_r(x,y,z=0)=u_0(r),$$
$$u_\alpha(x,y,z=0)=0,$$

where $r=(x^2+y^2)^{1/2}$ and α are the cylindrical coordinates in the plane perpendicular to the beam propagation direction z and $\mathbf{u}=\{u_r, u_\alpha\}$. For such beams the system (9) has the following form: the first two equations are the same as (10.1) and (10.2) (with x replaced by r) and the third is

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial\varepsilon_2}{\partial r}=\frac{1}{2\pi r}\frac{\partial N}{\partial r}, \tag{24}$$

where $u\equiv u_r$ and

$$N=2\pi\int_0^r Ir\,dr$$

is the power of the electromagnetic wave.

As in the two-dimensional case we shall introduce the characteristics (12) by means of Eq. (13) [the surfaces $\varphi(r,z)$ now have the sense of tubes of flow]. Then the solution to the system (10.1), (10.2), and (24) with the boundary conditions $I(z=0)=I_0(r)$, $u(z=0)=u_0(r)$ can be written as

$$N=N_0(\varphi)=2\pi\int_0^\varphi I_0(r)r\,dr, \tag{25}$$

$$u^2=u_0^2(\varphi)+\frac{1}{2\pi}N_0(\varphi)\ln\frac{r}{\varphi}. \tag{26}$$

From (13) and (26) we find the equation for $\varphi(r,z)$:

$$z=\frac{\varphi}{u_0(\varphi)}\int_1^{r/\varphi}\left[1+\frac{N_0(\varphi)}{2\pi u_0^2(\varphi)}\ln\xi\right]^{-1/2}d\xi; \quad \varphi(r,z=0)=r. \tag{27}$$

Expressions (25)–(27) solve the problem of the propagation of a steady axially symmetric beam of electromagnetic waves in a medium at rest with weak absorption. The problem was solved for arbitratry boundary profiles of the intensity $I(z=0)=I_0(r)$ and of the phase $u(z=0)=u_0(r)$.

As an example let us consider the propagation of a beam with constant intensity $I_0(r)=I_0\Theta(1-r^2)$ at $z=0$, that is, $N_0(r\leqslant 1)=r^2N_0$ and $N_0(r>1)=N_0$, where N_0 is the total power of the beam in dimensionless units. Let us also assume that the phase at the boundary is such that in a vacuum there would be a point focus: $u_0=-b_0r$, with $b_0=$const [see (18) and footnote 1]. The propagation of this beam is illustrated in Fig. 2. In this case we have

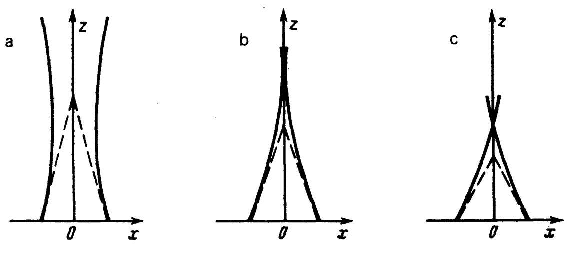

FIG. 1. Propagation of a steady two-dimensional beam with constant intensity at the entrance to the medium (solid line): a) $b_0<1$, b) $b_0=1$, c) $b_0>1$. The dashed lines show the path of the rays in a vacuum.

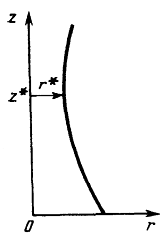

FIG. 2. Thermal defocusing of an axially symmetric beam with constant intensity at the entrance to the medium.

$$I = I_0(\varphi)\varphi\varphi_r'/r\infty\Theta(1-r^2),$$

$$u^2 = [b_0^2 + (N_0/2\pi)\ln(r/\varphi)]\varphi^2, \quad \varphi \leq 1.$$

The minimum radius of the beam r^* is determined by the condition $u=0$ (rays parallel to the z axis); from the preceding equality we have $r^* = \exp\{2\pi b_0^2/N_0\}$. This value of the radius is reached at the point

$$z = z^* = \frac{1}{b_0}\int_{r^*}^{1}\left[1+\frac{N_0}{2\pi b_0^2}\ln\xi\right]^{-1/2} d\xi.$$

In contrast to the two-dimensional case, the radius of the axially symmetric beam does not go to zero anywhere, that is, for such a beam it is impossible to have a focal point in the presence of absorption.

In studying steady beams in an immobile medium we have neglected convection. Let us find the condition for which this approximation is valid (see Ref. 7 for more details). All the above results are valid if the ratio of the discarded convective term in the heat-conduction equation (7) to the first term is $av_1/\chi \ll 1$. From the Navier-Stokes equation

$$\frac{\partial \mathbf{v}_1}{\partial t} + ((\mathbf{v}_0+\mathbf{v}_1)\nabla)\mathbf{v}_1 = -\frac{1}{\rho}\nabla p + \nu\Delta\mathbf{v}_1 + \beta\mathbf{g}\delta T, \tag{28}$$

(ν is the kinematic viscosity and $p = P - (\mathbf{g}\cdot\mathbf{r})\rho$ is the pressure minus the hydrostatic pressure) we get for the convective velocity $v_1 \approx \beta g a^2 \delta T/\nu$. According to (24) $\delta T \approx \alpha N/\pi\varkappa$. Therefore,

$$av_1/\chi \approx \beta g a^3 \delta T/\chi\nu \approx \beta g a^3 \alpha N/\pi c_P \rho \chi^2 \nu.$$

For the atmosphere $\rho\partial\varepsilon/\partial\rho = \varepsilon - 1 = 5.65\times10^{-4}$, the density of air is $\rho_0 = 1.25\times10^{-3}$ g/cm^3, the coefficient of thermal expansion is $\beta = 3.67\times10^{-3}$ deg^{-1}, the specific heat at constant volume is $c_V = 7.143\times10^6$ erg/g·deg, the thermal diffusivity is $\chi = 0.28$ cm^2/sec, the adiabatic exponent is $\gamma = c_p/c_V = 1.4$, the absorption coefficient is $\alpha_{CO_2} = 0.8\times10^{-6}$ cm^{-1}, the speed of sound is $v_s = 3.33\times10^4$ cm/sec, the kinematic viscosity is $\nu = 0.15$ cm^2/sec, and the acceleration of gravity is $g = 0.98\times10^3$ cm/sec^2, so for a beam of radius 25 cm from a CO_2 laser ($\omega = 1.773\times10^{14}$ sec^{-1}, $k = 5.92\times10^3$ cm^{-1}) we have $av_1/\chi \approx 4\times10^6 N$[kW].

3. STEADY PROPAGATION OF ELECTROMAGNETIC WAVES IN A MOVING MEDIUM

In this section we shall consider the propagation of a laser pulse in a medium with absorption in the presence of a wind perpendicular to the beam. Let us begin with the case of two-dimensional beams. If the pulse duration t is such that $t \gg a/v_0$ (here a is the characteristic transverse dimension of the beam and v_0 is the wind speed: for $a = 10$ cm and $v_0 = 10$ m/sec we have $t \gg 10^{-2}$ sec) and the wind speed satisfies the condition $\tilde{v}_0 = av_0/\chi \gg 1$ (for $a = 10$ cm and $\chi = 0.2$ cm^2/sec we have $v_0 \gg 0.02$ cm/sec), in the heat-conduction equation (9.3) we can drop the last term on the left-hand side and for two-dimensional beams the system (9) acquires the following form: the first two equations are the same as (10.1) and (10.2), while the third, dropping the tilde, is

$$\frac{\partial \varepsilon_2}{\partial x} = -\frac{1}{v_0}\frac{\partial N}{\partial x} \tag{29}$$

(we assume that the wind velocity is directed along the x axis). In this approximation the heat is carried away from the wave propagation region by the flow of gas moving in the perpendicular direction, rather than by thermal conduction as in the preceding case. In using the approximation (29) it is necessary to bear in mind the fact that in addition to the condition $\tilde{v}_0 \gg 1$, which (very weakly) limits the wind speed from below, there is another condition limiting it from above: the wind speed must not exceed the speed of sound. If this latter condition does not hold, the pressure in the gas cannot be equalized and the heat-conduction equation alone is insufficient for describing the medium.

As in Sec. 2, using Eq. (13) we introduce the characteristics (12) and the solution of (10.1) (the power N) is of the form (15). The problem is to find the function $\varphi(x,z)$. We find the equation for $\varphi(x,z)$ by substituting (29) into (10.2) and using (13) for u:

$$\left(\frac{\partial x}{\partial \varphi}\right)_z \left(\frac{\partial^2 x}{\partial z^2}\right)_\varphi = -\frac{1}{2v_0} I_0(\varphi). \tag{30}$$

We use next the fact that usually $v_0 \gg 1$. We seek the solution of this equation in the form of a series

$$x = x_0(z,\varphi) + x_1(z,\varphi)/v_0 + x_2(z,\varphi)/v_0^2 + \dots$$

with the boundary conditions

$$(\partial x/\partial z)_\varphi|_{z=0} = u_0(\varphi), \quad x(z=0,\varphi) = \varphi.$$

Substituting this series into (30) and keeping only the first two terms, we find that

$$x = \varphi + u_0(\varphi)z - \frac{I_0(\varphi)}{2b_0^2 v_0}\Phi(b_0 z), \tag{31}$$

where

$$b_0 = -u_0'(\varphi), \quad \Phi(\xi) = \xi + (1-\xi)\ln(1-\xi).$$

It can be shown that

$$\frac{x_2}{v_0 x_1} = \frac{C}{b_0^2 v_0}\ln(b_0^2 v_0), \quad C \approx 1.$$

Expressions (15) and (31) thus solve the problem of the propagation of a beam of electromagnetic waves in a medium with a wind, for a pulse duration $t > a/v_0$ and an arbitrary distribution of the intensity $I(z=0) = I_0(x)$ and phase $u(z=0) = u_0(x)$ at the entrance to the medium, using perturbation theory in the parameter $\zeta = (b_0^2 v_0)^{-1}$. In the dimensional variables

$$\zeta = R_0^2 I_0 \left|\frac{d\varepsilon}{dT}\right| \frac{1}{v_0 c_P \rho}\frac{\alpha}{a}.$$

Here the intensity is $I = \partial N/\partial x = I_0(\varphi)\varphi_x'$. Substituting φ_x' from (31), we obtain

$$I = I_0(\varphi)\left[1 - b_0 z - \frac{I_0'(\varphi)}{2v_0 b_0^2}\Phi(b_0 z)\right]^{-1}. \tag{32}$$

Let us consider beams with a parabolic phase front at the entrance to the medium [see (18), b_0 = const]. For such beams

$$x = (1 - b_0 z)\varphi - \frac{I_0(\varphi)}{2b_0^2 v_0}\Phi(b_0 z). \tag{33}$$

We begin our analysis of our solutions with the case where at the boundary of the medium $z = 0$ the intensity has the form of a step (20). For this boundary profile the intensity (32) looks like $I(\varphi<1)=(1-b_0z)^{-1}$, $I(\varphi>1)\equiv 0$, where we have neglected edge effects. From the equation for the characteristics (33), which in this case is of the form

$$\varphi=(1-b_0z)^{-1}(x+\Phi(b_0z)/2b_0^2v_0),$$

we see that the focus, that is, the point at which the rays $\varphi=$const intersect, has the coordinates $x^*=-1/2b_0^2v_0$ and $z^*=1/b_0$. The location of the focus in the beam propagation direction (along the z axis) is the same as it would be in a vacuum for a given boundary focusing [see (18)]. The wind caused the focus to shift in the transverse direction toward the wind. The path of the beam in this case is illustrated in Fig. 3.

Let us determine the conditions under which a point focus is possible. From the equation for the characteristics (33) we see that all the rays intersect at a single point if the intensity at the boundary $I_0(x)$ is a linear function of x, that is, neglecting edge effects.

$$I_0(x<a)=I_0(x>b)\equiv 0,\quad I_0(a<x<b)=Ax+B \tag{34}$$

(see Fig. 4). For this boundary intensity, Eq. (33) has the form

$$\varphi=\left(x+\Phi(b_0z)\frac{B}{2v_0b_0^2}\right)\left[1-b_0z-\Phi(b_0z)\frac{A}{2v_0b_0^3}\right]^{-1}, \tag{35}$$

$a<\varphi<b$. From this we see that the rays intersect at the point

$$x^*=-\frac{B}{2v_0b_0^2},\quad z^*\approx\frac{1}{b_0}-\frac{A}{2v_0b_0^3}.$$

Let us consider the case where a beam with a plane phase front $u(z=0)\equiv 0$ is incident on a medium with a wind. Letting $b_0\to 0$ in (35), we find

$$\varphi=\left(x+\frac{B}{2v_0}z^2\right)\Big/\left(1-\frac{A}{2v_0}z^2\right).$$

This beam will be focused by the wind at the point $x^*=-B/A$, $z^{*2}=2v_0/A$. As seen from the expression for z^{*2}, the wind focuses only those beams whose intensity increases along the wind: $A=dI_0/dx>0$. Beams whose intensity decreases along the wind ($A<0$) are monotonically defocused.

We can similarly consider the propagation, in a medium with a wind, of a beam such as in (34), which diverges at the entrance to the medium ($b_0<0$). When the intensity of the beam increases in the direction of the wind ($A>0$) the beam is focused by the wind. It can be shown using (35) that in this case the focal point is at $x^*=-B/A$, $z^*\approx|b_0|^{-1}\exp\{2v_0b_0^2/A\}$.

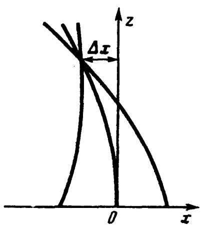

FIG. 3. Propagation of a steady two-dimensional beam with constant intensity at $z=0$ in a medium in motion. The direction of the x axis coincides with the wind direction. The focus is shifted toward the wind by an amount $\Delta x=1/2v_0b_0^2$.

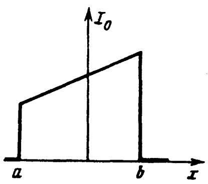

FIG. 4.

Therefore, stationary thermal focusing by a wind is possible for two-dimensional beams with a linear profile at the boundary (and $A=dI_0/dx>0$) propagating in an absorbing medium.

For beams with an arbitrary intensity profile at the entrance to the medium it can be shown using (33) that the point focus that would exist in a vacuum at the point $x^*=0, z^*=1/b_0$ is replaced by a focal region shifted windward. This region is where the rays intersect near the point $x^*=-1/2v_0b_0^2$, $z^*=1/b_0$ and has the dimensions $\Delta x\approx 1/2v_0b_0^2$, $\Delta z\approx 1/2v_0b_0^3$ (Fig. 5).

Before concluding this section let us consider three-dimensional beams. If at the entrance to the medium the beam is symmetric [that is, $I(x,y,z=0)=I_0(r)$, $r=(x^2+y^2)^{1/2}$] and focused at the point $\mathbf{u}(z=0)=-b_0\mathbf{r}$, $b_0=$const, and the wind is directed as before along the x axis, the solution to the system (9) [with Eq. (9.3) in the form (29) and $\delta=0$] has the form

$$N=N_0(\varphi)=2\pi\int_0^\varphi I_0(r)rdr,$$

where

$$N=\int_{-\infty}^{x}dx\int_{-\infty}^{y}dy\,I(x,y,z),$$

while the function $\varphi(r,\alpha,z)$ is determined by the equation

$$r=(1-b_0z)\varphi+\frac{\cos\alpha}{2b_0^2v_0}I_0(\varphi)\Phi_1(b_0z)+O\left(\frac{1}{b_0^4v_0^2}\right),\quad \Phi_1(\xi)=\xi+\ln(1-\xi). \tag{36}$$

In this expression α is the angle between the x axis and the transverse vector $\mathbf{r}$. In Fig. 6 we show the cross sections of the tubes of flow [the surfaces $\varphi(r,\alpha,z)=$const] cut by the planes $z=$const. The shapes of these cross sections are in agreement with the numerical calculations of Wallace and Lilly[8] (Fig. 4 of Ref. 8, the case of constant irradiance).

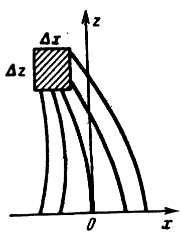

FIG. 5. Thermal defocusing of a two-dimensional beam in a medium in motion. The x axis is chosen to lie along the wind direction and $\Delta x\approx 1/2v_0b_0^2$, $\Delta z\approx 1/2v_0b_0^3$.

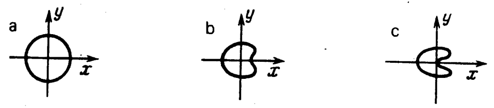

FIG. 6. Cross sections of tubes of flow cut by planes $z = \text{const}$ for a beam which is axially symmetric at $z=0$ and propagates in a medium in motion. The direction of the x axis coincides with the wind direction. The figures are for a) $z=0$, b) $z = z_1$, c) $z = z_2$ ($z_2 > z_1 > 0$).

In the calculations of this section we did not take into account the convective motion of the medium due to heating. We shall show that even for very powerful beams the convective velocity can be neglected compared to the typical velocities of the motion of the medium (the wind speeds). In fact, from the Navier-Stokes equation (28) we have

$$v_1/v_0 \approx \beta g a \delta T / v_0^2.$$

The quantity δT is given by the heat-conduction equation (29)

$$\delta T \approx \alpha a I / c_P \rho v_0,$$

so that

$$v_1/v_0 \approx \alpha a^2 I \beta g / c_P \rho v_0^3.$$

For the atmosphere (see the parameters above) and a CO_2 laser beam we have $v_1/v_0 \lesssim 10^{-6} N$[kW] for $v_0 \gtrsim 1$ m/sec.

4. THE SHORT PULSE APPROXIMATION

If the duration t of the laser pulse is such that $t < a^2/\chi$ and at the same time $t < a/v_0$ (but, of course, $t > a/v_s$, as required by our description of the medium), only the second term remains on the left-hand side of the heat-conduction equation (9.3). For two-dimensional beams in this approximation the system of equations (9) is as follows: the first two equations are the same as (10.1) and (10.2), while the third is

$$\partial \varepsilon_2 / \partial t = -I. \tag{37}$$

As in the two preceding sections, we shall use the method of characteristics to solve the system of equations (10.1), (10.2), and (37). The solution of the first equation with the boundary conditions

$$N(x, z=0, t) = N_0(x) = \int_0^x I_0(x)\, dx$$

has the form (15); the characteristics $\varphi(x, z, t)$ are introduced as the solutions of Eq. (13) [the boundary condition for this equation is $\varphi(x, z=0, t) = x$]. Substituting (37) into (10.2) and using (13) and (15), we find the equation for $\varphi(x, z, t)$:

$$x = \varphi + u_0(\varphi) z + \frac{t}{2b_0^2} I_0'(\varphi) \Phi_1(b_0 z) + O\left(\frac{t^2}{4b_0^4}\right). \tag{38}$$

The quantity $\Phi_1(\xi)$ is defined in (36) and $b_0 = -u_0'(\varphi)$. We have used the boundary condition $u(x, z=0, t) = u_0(x)$. Calculating the ratio of the terms quadratic in the time to those linear in the time in (38), we find for this ratio

$$\gamma \approx \frac{t}{2b_0^2} \frac{1}{1 - b_0 z}. \tag{Z}$$

To estimate the accuracy of our method we compared its predictions for three-dimensional beams [expressions (44) and (45)] to the numerical solution of Eqs. (9) found by computer [Eq. (9.3) was solved using the approximation (37)]. The results are in excellent agreement, even when $\gamma \approx 1$ (see Fig. 7).

Therefore, the problem of the propagation of a two-dimensional beam of electromagnetic waves in a medium with absorption when the pulse duration is $t < \{a^2/\chi, a/v_0\}$ has been solved for an arbitrary distribution of the intensity $I_0(x)$ and phase $u_0(x)$ at the entrance to the medium ($z=0$) using perturbation theory in the parameter γ [expressions (15) and (38)].

For beams with a parabolic phase front at the entrance to the medium [see (18)], $u_0 = -b_0 x$ and $b_0 = \text{const}$. The intensity of these beams is $I = \partial N/\partial x = N_0'(\varphi)\varphi_x' = I_0(\varphi)\varphi_x'$. Substituting φ_x' from (38), we find

$$I = I_0(\varphi)\left[1 - b_0 z + \frac{t}{2b_0^2} I_0''(\varphi)\Phi_1(b_0 z)\right]^{-1}. \tag{39}$$

The function $\varphi(x, z, t)$ is given by Eq. (38).

Equations (38) and (39) are easily generalized to the case where the intensity profile at the boundary $z=0$ is time-dependent: $I(x, z=0, t) = f_1(t) f_2(x)$ (a modulated signal at the boundary). For such beams it is sufficient to replace the time t in (38) and (39) by

$$\int_0^t f_1(t)\, dt.$$

It is also of interest to consider the case where a beam with a plane phase front is incident on the medium. Letting $b_0 \to 0$ in the equation for the characteristics (38), we find

$$x = \varphi - {}^1/_4 t z^2 I_0'(\varphi). \tag{40}$$

In this case the intensity is

$$I = I_0(\varphi)\varphi_x' = I_0(\varphi)[1 + {}^1/_4 t z^2 I_0''(\varphi)]^{-1}.$$

As the first example let us consider the propagation of a beam with intensity of the form (20) at the boundary. For this profile, expressions (38) and (39) coincide, apart from edge effects, with the corresponding expressions (17') for a vacuum, that is, for such a beam the medium does not have any effect (neglecting edge effects).

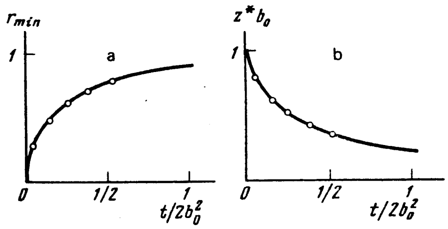

FIG. 7. Minimum radius of a parabolic axially symmetric beam (a) and the distance at which the radius of an axially symmetric beam attains the minimum (b). The solid lines show expressions (46) and (45) and the points show the result of the numerical computer calculation.

As the next example let us consider a beam with the boundary phase (18) $u(z=0)=u_0=b_0x$, $b_0=\text{const}$ and the boundary intensity profile $I(z=0, x, t)=I_0(x) = (1-x^2)\Theta(1-x^2)$. In this case the equation for the characteristics (38) has the form

$$x=[1-b_0z-(t/b_0^2)\Phi_1(b_0z)]\varphi.$$

The minimum width of the beam is determined by the condition $\partial x/\partial z=0$; this condition is satisfied at the point $z=z^*=(1+t/b_0^2)^{-1}b_0^{-1}$, where the width of the beam is

$$x(z^*)=x^*=(t/b_0^2)\ln(1+b_0^2/t).$$

The behavior of the rays for a beam with a parabolic boundary profile is illustrated in Fig. 2.

Let us look at yet another example. Let us consider the propagation of a parabolic beam with a minimum on the axis $I_0(x)=x^2\Theta(1-x^2)$ and a plane phase front at the entrance to the medium ($u_0\equiv 0$). The equation for the characteristics (40) takes the form $\varphi=(1-tz^2/2)^{-1}x$, from which we see that the rays (the curves $\varphi=\text{const}$) intersect at the point $x^*=0$, $z^*=(2/t)^{1/2}$. In the dimensional units

$$z^*=a(2c_p\rho)^{1/2}(\alpha I_0|d\varepsilon/dT|t)^{-1/2},$$

that is, this beam is focused into a point. This is the familiar phenomenon of the thermal self-focusing of a beam with an intensity that decreases toward the center (see Ref. 9, for example).

We shall not consider examples of beams with more complicated boundary intensities and phases. Each of these can be studied using the equation for the characteristics (38) and the expressions for the intensity (39). Let us now look at three-dimensional beams.

If at the entrance to the medium the beam is symmetric, that is, the intensity and phase are of the form

$$I(r, z=0, t)=I_0(r), \quad u_r(r, z=0, t)=u_0(r), \quad u_\alpha(r, z=0, t)\equiv 0$$

[here $r=(x^2+y^2)^{1/2}$ and α are the cylindrical coordinates in the plane perpendicular to the beam propagation direction z], the solution of the system (9) [where the third equation is in the approximation (37)] is such that the power N of the beam [introduced in (24)] is

$$N(r, z, t)=N_0(\varphi)=2\pi\int_0^\varphi I_0(r)r\,dr. \qquad (41)$$

The characteristics $\varphi(r, z, t)$ are given by the equation

$$r=\varphi+u_0(\varphi)z-(t/4b_0^2)I_0'(\varphi)\Phi_2(b_0z)+O(t^2/b_0^4), \qquad (42)$$

where

$$\Phi_2(\xi)=(1-\xi)^{-1}-\xi-1, \quad b_0\equiv -u_0'(\varphi).$$

The ratio of the terms in (42) which are quadratic in the time to those linear in the time is

$$\gamma\approx(t/2b_0^2)(1-b_0z)^{-2}.$$

Expressions (41) and (42) solve the problem of the propagation of an axially symmetric pulse of duration $t<\{a^2/\chi, a/v_0\}$ in a medium with absorption in perturbation theory in the parameter γ for an arbitrary distribution of the intensity $I_0(r)$ and phase $u_0(r)$ at the entrance to the medium (it should be kept in mind that $\mathbf{u}=\nabla_\perp S$, where S is the eikonal).

Since it differs from the two-dimensional case only by details, as an example let us consider only the case of parabolic beams. At the entrance to the medium let

$$I_0(r)=(1-r^2)\Theta(1-r^2); \quad u_0=-b_0r, \quad b_0=\text{const}. \qquad (43)$$

The equation for the characteristics (42) becomes

$$r=[1-b_0z+(t/2b_0^2)\Phi_2(b_0z)]\varphi, \quad \varphi\leqslant 1 \qquad (44)$$

The minimum radius of the beam is given by the condition $(\partial r/\partial z)_\varphi=0$. From (44) we find that here

$$1-b_0z^*=\left[\frac{t}{2b_0^2}\left(1+\frac{t}{2b_0^2}\right)^{-1}\right]^{1/2} \qquad (45)$$

and the radius of the beam is

$$r(z^*)=r^*=2\left[\frac{t}{2b_0^2}\left(1+\frac{t}{2b_0^2}\right)\right]^{1/2}-\frac{t}{b_0^2}. \qquad (46)$$

From expressions (41), (44), and (24) we find the intensity

$$I=\frac{1}{2\pi r}\frac{\partial N}{\partial r}=I_0(\varphi)\frac{\partial\varphi}{\partial r}\frac{\varphi}{r},$$

that is,

$$I=\frac{1}{f^2}\left(1-\frac{r^2}{f^2}\right); \quad f=1-b_0z+\frac{t}{2b_0^2}\Phi_2(b_0z), \quad f\leqslant r.$$

The propagation of this beam is illustrated in Fig. 2. In Fig. 7 we have compared the predictions of our approximate theory [expressions (45) and (46)] to the exact solution of the system of equations (9) found by computer [the third equation is in the approximation (37)] with the boundary conditions (43). This comparison shows that the theory gives a good description of the behavior of the beam for even $\gamma\approx 1$.

The results of this section are valid as long as convection can be neglected, that is, $tv_1/a\ll 1$ [see (7)]. From the Navier-Stokes equation (28) we have $v_1\approx t\beta g\delta T$; according to (37) the temperature change is $\delta T\approx t\alpha I/c_p\rho$, i.e.,

$$tv_1/a\approx\beta g t^2\alpha A/\pi a^3 c_p\rho$$

(here A is the energy in the pulse). For the atmosphere (see above) and a beam of radius 5 cm from a CO_2 laser we have $tv_1/a\approx 10^{-5}t^2A[\text{J}]$.

[1])More accurately, $\tilde{u}_0=-b_0\tilde{x}$; in the dimensional variables the beam is focused at the point $x^*=0$, $z^*=R_0$, so that $u_0=-x/R_0$ and

$$b_0^2=\frac{a^2}{R_0^2}\frac{1}{\gamma}=\frac{1}{R_0^2}\frac{\varkappa}{\alpha I_0}\left|\frac{d\varepsilon}{dT}\right|^{-1}.$$

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Translated by P. Millard

Effect of depolarizing collisions on the photon echo in a magnetic field

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(Submitted 2 October 1978)
Zh. Eksp. Teor. Fiz. 76, 1212–1225 (April 1979)

A connection between the different relaxation characteristics, which describe the change of the optical-coherence matrix under the influence of elastic depolarizing collisions, is established for the first time ever. The calculation is performed for a Van der Waals interaction on a transition with level angular momenta $j_a = j_b = 1$. It is found that the relative difference between the relaxation characteristics ranges from 5 to 20% for the considered values of the interaction parameters. The possibility is demonstrated of experimentally measuring the characteristics that describe the relaxation of the optical-coherence matrix on the transitions $j \to j\, (j > 1)$ and $j \rightleftarrows j+1\, (j \geq 1/2)$ by the photon-echo method in a gas situated in a longitudinal magnetic field. It is found for the $1 \to 1$ and $1/2 \rightleftarrows 3/2$ transitions that the magnetic field intensity can be chosen such that the polarization echo vector component perpendicular to the polarization plane of the exciting pulses is due entirely only to depolarizing collisions.

PACS numbers: 42.65.Gb, 42.50. + q

Recent years have seen progress in nonlinear laser spectroscopy, which permits the study of quantum-transition structures obscured by the Doppler broadening of spectral lines. An extensive bibliography on this topic is contained in the monograph of Letokhov and Chebotaev.[1] In a gas medium, the resonance levels of moving atoms (molecules) are usually degenerate. A consistent calculation of the elastic collisions shows therefore that the relaxation of that density-matrix component which describes optical coherence (transitions between the considered levels) is determined not by a single quantity but by an aggregate $\mathscr{T}^{(\varkappa)}$, where $|j_a - j_b| \leq \varkappa \leq j_a + j_b$, and j_a and j_b are the angular momenta of the levels of the transition. The aggregate of the quantities $\mathscr{T}^{(\varkappa)}$ determines in essence the properties of the resonant electromagnetic radiation that passes through a gas medium. In particular, the gain of a weak probing wave on a transition with level angular momenta $j_a = 1$ and $j_b = 2$, in a medium saturated by a strong field, depends essentially on the ratio of the relaxation characteristics of the dipole polarization $\mathscr{T}^{(1)}$ and of the quadrupole polarization $\mathscr{T}^{(2)}$ of the medium. So far, however, there are no concrete theoretical or experimental results concerning the ratios of the different $\mathscr{T}^{(\varkappa)}$.

In the first part of this paper we calculate all the relaxation characteristics $\mathscr{T}^{(\varkappa)}$ for the transition $j_a = j_b = 1$ in the case of a Van der Waals interaction of the colliding atoms. The connection between the relaxation characteristics having different values of $\varkappa$ are established here for the first time. The results allow us to check the correctness the assumption, used in many papers, that all the $\mathscr{T}^{(\varkappa)}$ are approximately equal.

In the second part of the paper it is shown on the basis of a theoretical calculation that the differences $\mathscr{T}^{(\varkappa)} - \mathscr{T}^{(1)} (\varkappa \neq 1)$ can be determined by direct experiment using photon echo in a gas medium placed in a longitudinal magnetic field.

The photon echo method has been coming into ever increasing use for the study of gas media.[2-16] It is employed to determine successfully the relaxation characteristics of resonant transitions, as well as to identify atomic and molecular transitions. In particular, Wang[6] has obtained theoretically the influence of the difference $\mathscr{T}^{(\varkappa)} - \mathscr{T}^{(1)} (\varkappa \neq 1)$ on the polarization of photon echo on the transitions $j \to j (j > 1)$ and $j \rightleftarrows j+1 (j \geq 1)$. On the remaining transitions, the echo amplitude depended only on the quantity $\mathscr{T}^{(1)}$, and its polarization was not affected by the depolarizing collisions.

Application of a longitudinal magnetic field on a gas medium in which a photon echo is produced extends substantially the capabilities of the photon echo. Thus, for example, a specific rotation of the photon-echo polarization vector is observed, different from the Faraday rotation. This effect was predicted by Aleks-

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