plasma microfield is ~10^7 V/cm, i.e., it is comparable with the intensity of the intra-atomic field, thus excluding the possibility of using perturbation theory[25] to calculate the shift of the energy levels.

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Envelope solitons of relativistic strong electromagnetic waves

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We investigate envelope solitons of circularly polarized electromagnetic waves in a cold plasma, due to the joint action of relativistic and striction nonlinearities. It is shown that in the approximation in which the plasma perturbations are quasineutral there exist only soliton small-amplitude solutions in which the oscillatory velocity of the electron is much lower than the velocity of light. Numerical integration with a computer yielded solitons with relativistic amplitudes, for which the plasma charge separation is substantial. These solitons differ in shape from ordinary small-amplitude solitons and have a discrete velocity spectrum. It is demonstrated that the concentration has a limit above which the plasma has no solitons with a specified carrier frequency.

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Recent progress in the development of high-power generators for various frequency ranges make it possible to speak of interaction between electromagnetic radiation and a plasma under conditions when the particle oscillation velocities can be comparable with the speed of light. In this case the relativistic nonlinearity is no longer quadratic and can be rigorously accounted for in only a few particular cases. For circularly polarized radiation, the relativistic nonlinearity was first taken into account rigorously by Akhiezer and Polovin[1] for a wave with constant amplitude.[1)] Gorshkov *et al.*[4] investigated stationary envelope waves of circularly

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polarized radiation in a semiconductor plasma, where the electron effective mass has a strong energy dependence at nonrelativistic energies such that the perturbation of the longitudinal motion under the influence when the ponderomotive force can be neglected. In the present paper we investigate one-dimensional envelope solitons of circularly polarized radiation in an ordinary gas plasma, where longitudinal motion and charge separation can be significant. A similar problem was considered in Ref. 5 in the quasineutrality approximation (i.e., neglecting charge separation in the plasma); it will be shown below that this approximation is valid only for small-amplitude solitions in which the oscillatory velocity is much less than the velocity of light.

It is physically clear that slow solitons of relativistic amplitude, propagating with a velocity on the order of that of sound, can exist only in a strongly post-critical plasma or in a plasma with relativistic temperature. In fact, in a slow soliton the pressure of the RF field is balanced by the excess pressure of the plasma from the outside.[6] In the limit when the entire plasma is forced out of the field region, the condition $nT \approx E^2/4\pi$ that the pressure of the field from the inside be equal to that of the plasma from the outside is equivalent to $\omega_p^2/\omega^2 \approx c^2/v_T^2$, if the oscillatory velocity of the electrons is of the order of the velocity of light. We confine ourselves to fast solitons that can have relativistic amplitudes at more moderate plasma concentrations and temperatures. For simplicity we assume a zero plasma temperature; this approximation is good for solitons with velocities much higher than thermal. The pressure of the RF field is balanced here by the pressure of the plasma incident on the soliton.

1. We describe the transverse field by means of a vector potential **A**

$$\mathbf{H}=\operatorname{rot}\mathbf{A},\quad \mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t},\tag{1}$$

which satisfies the gauge condition

$$\operatorname{div}\mathbf{A}=0.\tag{2}$$

Assuming that all the quantities depend only on a single coordinate z, we can find that the transverse motion of the electron and ion components is determined by the simple relations

$$\mathbf{p}_{\perp e}=\mathbf{C}_1+\frac{e}{c}\mathbf{A},\quad \mathbf{p}_{\perp i}=\mathbf{C}_2-\frac{e}{c}\mathbf{A},\tag{3}$$

and the equation for the vector potential takes the form

$$\frac{\partial^2\mathbf{A}}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}=\frac{4\pi e}{c}(n_e\mathbf{v}_{\perp e}-n_i\mathbf{v}_{\perp i}).\tag{4}$$

In the foregoing relations c is the speed of light in vacuum, e is the electron charge, n, $\mathbf{v}$, and $\mathbf{p}$ are the concentration, velocity, and momentum of the electrons and ions, and are labeled by the subscripts e and i, respectively.

With an aim at investigating envelope solitons of circularly polarized radiation, we introduce the rotating coordinate (see Ref. 4)

$$\mathbf{x}_1=\operatorname{Re}(\mathbf{x}_0+i\mathbf{y}_0)e^{i\omega t-ikz},$$
$$\mathbf{x}_2=\operatorname{Im}(\mathbf{x}_0+i\mathbf{y}_0)e^{i\omega t-ikz}.$$

The projections of Eq. (4) on the new axes are of the form

$$\begin{aligned}\frac{\partial^2A_1}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2A_1}{\partial t^2}-\left(k^2-\frac{\omega^2}{c^2}\right)A_1-2\left(k\frac{\partial A_2}{\partial z}+\frac{\omega}{c^2}\frac{\partial A_2}{\partial t}\right)&=\frac{4\pi e^2}{c^2}A_1\left(\frac{n_e}{m_e}+\frac{n_i}{m_i}\right),\\ \frac{\partial^2A_2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2A_2}{\partial t^2}-\left(k^2-\frac{\omega^2}{c^2}\right)A_2+2\left(k\frac{\partial A_1}{\partial z}+\frac{\omega}{c^2}\frac{\partial A_1}{\partial t}\right)&=\frac{4\pi e^2}{c^2}A_2\left(\frac{n_e}{m_e}+\frac{n_i}{m_i}\right),\end{aligned}\tag{5}$$

where A_1 and A_2 are the projections of the vector potential on the axes $\mathbf{x}_1$ and $\mathbf{x}_2$, while m_e and m_i are the relativistic masses of the electrons and ions expressed in terms of the corresponding rest masses and momenta:

$$m_\alpha=m_{0\alpha}(1+p_\alpha^2/m_{0\alpha}^2c^2)^{1/2}.$$

In Eqs. (5) we have allowed for relations (3) with the integration constants $\mathbf{C}_1$ and $\mathbf{C}_2$ set equal to zero. This corresponds to the case when the electrons and ions are immobile at infinity, where the field is zero.

We are interested in such solutions of the system (5) which depend on the self-similar variable $\xi=z-ut$, and confine ourselves to the case

$$u=c^2k/\omega,\tag{6}$$

in which the system (5) reduces to a single equation[2)]

$$\frac{d^2A}{d\xi^2}+\frac{\omega^2}{c^2}A=\frac{\omega_p^2}{\omega^2}\frac{1}{1-\beta^2}A\left(\frac{n_e}{n_0}\frac{m_{0e}}{m_e}+\frac{n_i}{n_0}\frac{m_{0i}}{m_i}\right),\tag{7}$$

where $\omega_p^2=4\pi e^2n_0/m_{0e}$, n_0 is a certain normalization concentration, and $\beta=u/c$. In the usual coordinates, the solutions that depend on the self-similar variable will contain a carrier component with frequency ω and spatial period $2\pi/k$.

Equation (7) contains two nonlinearities: one expresses the nonlinear dependence of the mass on the RF field (relativistic nonlinearity) and the other takes into account the perturbation of the concentration by the RF field (striction nonlinearity). In the considered "cold" approximation these two nonlinearities have opposite signs, and their competition determines whether solution of the soliton type, for which relativistic nonlinearity is favorable, are possible.

2. To find the solutions of (7) it is necessary also to have expressions for the concentrations and masses in terms of the vector potential. Assuming that the electron and ion concentrations as well as their longitudinal momenta depend only the self-similar variable ξ, we obtain from the continuity equations for the electron and ion components

$$n_\alpha=n_{0\alpha}/(u-v_{\parallel\alpha}),\tag{8}$$

where $n_{0\alpha}$ is the concentration of particles of sort α at the place where their longitudinal velocity $v_{\parallel\alpha}$ is equal to zero. For solutions of the soliton type it follows from the condition that the plasma be neutral at infinity that $n_{0e}=n_{0i}$; we assumed that these values coincide with the normalization n_0 in Eq. (7).

The longitudinal motion of the electrons is determined by the equation

$$(v_{\parallel e}-u)\frac{dp_{\parallel e}}{d\xi}=e\frac{d\varphi}{d\xi}-\frac{e}{c}v_{\perp e}\frac{dA}{d\xi},$$

where φ is the scalar potential of the longitudinal field and is due to the separation of the charges. When account is taken of the relativistic connection between the velocity and the momentum, and also of relation (3), this equation can be rewritten in the form

$$-u\frac{dp_{\parallel e}}{d\xi}=e\frac{d\varphi}{d\xi}-m_0c^2\frac{d}{d\xi}\left[1+\left(\frac{eA}{m_{0e}c^2}\right)^2+\frac{p_{\parallel e}^2}{m_{0e}c^2}\right]^{1/2}. \tag{9}$$

The last term in (9) is a relativistic generalization of the average RF ponderomotive force.[7] Because of the circular character of the polarization, the expression for this force is exact rather than averaged as usual. This expression takes into account both parts of the ponderomotive force, which are connected with the derivatives of the circularly polarized field both with respect to time and with respect to amplitude. In the coordinate system that moves with velocity u, this field is a standing circularly polarized wave, for which we can also write down a general relativistic expression for the ponderomotive force.

Integrating Eq. (9) and recognizing that all the perturbations vanish at infinity, we arrive at the following expression for the longitudinal momentum of the electrons:

$$p_{\parallel e}=\frac{m_{0e}c}{1-\beta^2}\left\{\beta\left(1+\frac{e\varphi}{m_{0e}c^2}\right)-\left[\left(1+\frac{e\varphi}{m_{0e}c^2}\right)^2-(1-\beta^2)\left(1+\frac{e^2A^2}{m_{0e}^2c^4}\right)\right]^{1/2}\right\}. \tag{10}$$

The sign in front of the square root is chosen to ensure that the velocity of the electrons does not exceed u anywhere, and that the concentration does not vanish at infinity.

When account is taken of the connection of the longitudinal velocity with the longitudinal and transverse momenta, as well as of expressions (3), (8), and (10), the electron concentration is

$$n_e=\frac{n_0\beta}{1-\beta^2}\left(1+\frac{e\varphi}{m_{e0}c^2}\right)\left[\left(1+\frac{e\varphi}{m_{0e}c^2}\right)^2-(1-\beta^2)\left(1+\frac{e^2A^2}{m_{0e}^2c^4}\right)\right]^{-1/2}-n_0\frac{\beta^2}{1-\beta^2}. \tag{11}$$

Similar calculations lead to an expression for the ion concentration

$$n_i=\frac{n_0\beta}{1-\beta^2}\left(1-\frac{e\varphi}{m_{0i}c^2}\right)\left[\left(1-\frac{e\varphi}{m_{0i}c^2}\right)^2-(1-\beta^2)\left(1+\frac{e^2A^2}{m_{0i}^2c^4}\right)\right]^{-1/2}-n_0\frac{\beta^2}{1-\beta^2}. \tag{12}$$

Finally, the longitudinal potential is determined by the Poisson equation

$$d^2\varphi/d\xi^2=4\pi e(n_e-n_i). \tag{13}$$

The final system of equations for envelope solitons of circularly polarized radiation, after substituting all the quantities in (7) and (13) and after introducing dimensionless variables, is of the form

$$x''+x\left\{1-\frac{\omega_p^2}{\omega^2}\frac{\beta}{1-\beta^2}\left[\frac{1}{[(1+y)^2-(1-\beta^2)(1+x^2)]^{1/2}}+\frac{\gamma}{[(1-\gamma y)^2-(1-\beta^2)(1+\gamma^2x^2)]^{1/2}}\right]\right\}=0,$$
$$y''-\frac{\omega_p^2}{\omega^2}\frac{\beta}{1-\beta^2}\left[\frac{1+y}{[(1+y)^2-(1-\beta^2)(1+x^2)]^{1/2}}-\frac{1-\gamma y}{[(1-\gamma y)^2-(1-\beta^2)(1+\gamma^2x^2)]^{1/2}}\right]=0, \tag{14}$$

where

$$x=eA/m_{0e}c^2,\quad y=e\varphi/m_{0e}c^2,\quad \tilde{\xi}=\omega\xi/c,\quad \gamma=m_{0e}/m_{0i}.$$

Equations (14) take into account the possible relativistic motion of the ions and the action of the ponderomotive force on the ions. If these effects are neglected, the equations can be simplified somewhat.

The system (14) has a first integral that can be interpreted as a generalized energy integral:

$$\frac{1-\beta^2}{2}(x'^2+x^2)-\frac{1}{2}y'^2-\frac{\omega_p^2}{\omega^2}\frac{\beta}{1-\beta^2}\left\{[(1+y)^2-(1-\beta^2)(1+x^2)]^{1/2}+\frac{1}{\gamma}[(1-\gamma y)^2-(1-\beta^2)(1+\gamma^2x^2)]^{1/2}\right\}=\text{const.} \tag{15}$$

3. A quasineutral plasma corresponds to an approximation in which the derivative y'' in the second equation of the system (14) is small compared with either of the remaining two terms

$$|y''|\ll\frac{\omega_p^2}{\omega^2}\frac{\beta}{1-\beta^2}\left|\frac{1+y}{[(1+y)^2-(1-\beta^2)(1+x^2)]^{1/2}}\right|. \tag{16}$$

Setting y'' equal to zero in this equation, we obtain an algebraic connection between x and y:

$$y=\frac{1}{1-\gamma}\{-(1+\gamma x^2)+[(1+x^2)(1+\gamma^2x^2)]^{1/2}\}, \tag{17}$$

and when this connection is taken into account we obtain for the envelope solitons one nonlinear equation of second order. For simplicity we present this equation for solitons of limited amplitude

$$x\ll 1/\gamma^{1/2}, \tag{18}$$

for which the ion motion is certainly nonrelativistic, and in the equation for the transverse field we can neglect the ion current:

$$x''+x\left[1-\frac{\omega_p^2}{\omega^2}\frac{\beta^2}{1-\beta^2}\frac{1}{(1+x^2)^{1/2}}\frac{1}{[\beta^2+2\gamma(1-(1+x^2)^{1/2})]^{1/2}}\right]=0. \tag{19}$$

Equation (19) is integrated with allowance for the boundary conditions at infinity ($x\to 0, x'\to 0$):

$$x'^2=2\frac{\omega_p^2}{\omega^2}\frac{\beta^2}{1-\beta^2}\left\{1-\left[1+\frac{2\gamma}{\beta}(1-(1+x^2)^{1/2})\right]^{1/2}\right\}-x^2. \tag{20}$$

Differentiating twice the relation (17) and taking into account expressions (19) and (20) for x'' and x', we easily verify that the quasineutrality condition (16) can be satisfied only for small-amplitude solitons:

$$x_{max}^2\ll 1, \tag{21}$$

in which the oscillator velocity is much lower than the speed of light.

For these solitons we can use the weak-relativism approximation and greatly simplify Eqs. (14), by expanding all the radical-containing expression in series [3]:

$$x''+x\left\{1-\frac{\omega_p^2}{\omega^2}\frac{1}{1-\beta^2}\left[1+\frac{1}{\beta^2}\left(\frac{1}{2}x^2-y\right)-\frac{1}{2}x^2\right]\right\}=0,$$
$$y''-\frac{\omega_p^2}{\omega^2}\frac{1}{\beta^2}\left[\left(\frac{1}{2}x^2-y\right)-\gamma y\right]=0. \tag{14a}$$

Equations (14a) were obtained in the approximation

$$x^2\ll 1,\quad y\ll 1,\quad 2\gamma y\ll\beta^2,\quad |y-{}^1\!/_2x^2|\ll\beta^2. \tag{22}$$

The perturbations of the electron and ion concentrations are respectively equal to

$$\frac{\delta n_e}{n_0}=\frac{1}{\beta^2}\left(\frac{1}{2}x^2-y\right),\quad \frac{\delta n_i}{n_0}=\frac{\gamma}{\beta^2}y. \tag{23}$$

The first equation of (14a) thus takes into account the striction nonlinearity due to perturbation of the electron concentration, as well as the relativistic nonlinearity due to the increase of the electron mass.

In the quasineutrality approximation ($y''=0$) we can integrate the system (14a) twice. For soliton solutions, the connection between the amplitude and velocity is determined by the relation [4]

$$x_{max}^2=4\left[1-\frac{\omega^2}{\omega_p^2}(1-\beta^2)\right]\Big/\left(1-\frac{\gamma}{\beta^2}\right). \tag{24}$$

These solitons have the standard form

$$x=x_{max}/\mathrm{ch}\left\{\left[1-\frac{\omega^2}{\omega_p^2}(1-\beta^2)\right]^{1/2}\tilde{\xi}\right\}. \tag{25}$$

It follows from (24) that in the considered "cold" approximation there can propagate in the plasma only envelope solitons with a velocity exceeding a definite critical value

$$u>c(m_{0e}/m_{0i})^{1/2}. \tag{26}$$

Otherwise the striction nonlinearity is stronger than the relativistic nonlinearity, and its sign is such that there are no solutions of the soliton type.

The dependence of the soliton amplitude on the velocity (24) has a nonmonotonic character, the amplitude increases with increasing velocity if $\beta^2 \gg \gamma$; in addition, the amplitude increases when β^2 approaches γ, where the striction nonlinearity assumes a greater role and begins to cancel out the relativistic nonlinearity.

If the finite plasma temperature is taken into account, envelope solitons are possible with velocities of the order of ion-sound velocity.

To verify the applicability of the quasineutrality approximation, we must take into account the fact that $y \approx x^2/2$ and compare the derivative y'' with the remaining terms in the second equation of (14a). A distinction can then be made between two different approximations. The first occurs when the condition

$$|y''|\ll\left|\frac{\omega_p^2}{\omega^2}\frac{\gamma}{\beta^2}y\right|$$

is satisfied. In this case quasineutrality does indeed take place in the sense that the difference between the perturbations of the electron and ion concentration is a small quantity:

$$|\delta n_e-\delta n_i|\ll|\delta n|.$$

It is easy to show that for this purpose it suffices to stipulate the inequality

$$4\frac{\omega^2}{\omega_p^2}\left[1-\frac{\omega^2}{\omega_p^2}(1-\beta^2)\right]\ll\frac{\gamma}{\beta^2}. \tag{27}$$

The other approximation occurs when

$$\left|\frac{\omega_p^2}{\omega^2}\frac{\gamma}{\beta^2}y\right|\ll y''\ll\left|\frac{\omega_p^2}{\omega^2}\frac{1}{\beta^2}y\right|.$$

The first part of this inequality is satisfied under a condition that is the opposite of (17), while the second part is satisfied automatically at small amplitudes ($x_m^2 \ll 1$). In this case the allowance for the perturbations of the ion concentration becomes meaningless, and the physical gist of the approximation is that the difference between the virtual perturbations of the electron concentration, due to the pressure of the RF field and to the longitudinal electric field, is small compared with each of these perturbations. The perturbations of the electron concentration are then much larger than the perturbations of the ion concentration, i.e., there is no quasineutrality in the usual sense of this word.

It should be noted that the electron-concentration perturbations are small in both cases and do not exceed the value

$$\delta n_e/n_0<{}^1/_2x_2x_m^2. \tag{28}$$

Thus, small-amplitude solitons are possible in a transparent plasma, where they constitute a pulse with a high-frequency carrier, propagating with group velocity and having an invariant form, since the dispersion spreading is offset by the nonlinearity. In addition, these solitons can propagate in a plasma with near-critical concentration; their amplitude is also determined by relations (24), and the velocities lie in the range

$$c(m_{0e}/m_{0i})^{1/2}<u\ll c. \tag{29}$$

4. The search for envelope solitons of large amplitude, for which charge separation is essential, was carried out by computer integration of the system (14). At a given ratio ω_p^2/ω^2, the separation of the soliton solutions reduces to finding the eigenvalues β for the eigenfunctions of the system (14), satisfying the boundary condition as $|\tilde{\xi}|\to\infty$

$$x=x'=y=y'=0. \tag{30}$$

This problem is simplified by the following circumstance. On the soliton wings, where the perturbations of all the quantities are small, they are described by a linearized system of equations

$$\begin{aligned} &x''+x\left(1-\frac{\omega_p^2}{\omega^2}\frac{1}{1-\beta^2}\right)=0,\\ &y''+y\frac{\omega_p^2}{\omega^2}\frac{1}{1-\beta^2}=0, \end{aligned} \tag{31}$$

from which it follows that the boundary conditions (30) are satisfied by solutions of the type

$$y=y'=0,\quad x\sim\exp\left\{-\left(\frac{\omega_p^2}{\omega^2}\frac{1}{1-\beta^2}-1\right)^{1/2}|\tilde{\xi}|\right\}.$$

Thus, the problem of finding the eigenvalues and the corresponding eigenfunctions has a single parameter: it is necessary to specify at infinity a sufficiently small perturbation x and integrate the system (14) in the direction of increasing x, separating the soliton solutions by a simple choice of the parameter β.

Within the framework of Eqs. (14), solitons of two types are possible:

a) with symmetrical profile of x and y

$$x(\tilde{\xi})=x(-\tilde{\xi}),\quad y(\tilde{\xi})=y(-\tilde{\xi}),$$

b) with antisymmetrical profile of x and symmetrical profile of y:

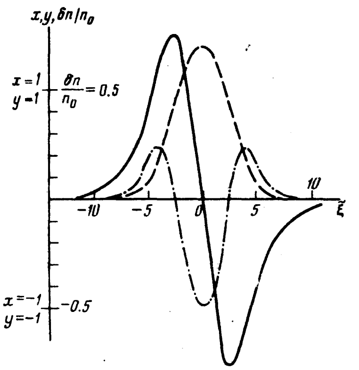

FIG. 1. First discrete-spectrum soliton in a transparent plasma ($\omega_p^2/\omega^2 = 0.5$), having a minimum velocity $\beta \approx 0.764$. Here and in Figs. 2–4 the solid curve is the profile of the potential of the transverse field x, the dashed curve is the profile of the potential of the longitudinal field y, and the dash-dot curve is the profile of the perturbation of the electron density $\delta n_e/n_0$.

$$x(\tilde{\xi}) = -x(-\tilde{\xi}), \quad y(\tilde{\xi}) = y(-\tilde{\xi}).$$

The integration of the system (14) for the soliton solutions must be continued until at some point both x' and y' vanish simultaneously for solitons of type a, or x and y' vanish simultaneously for solitons of type b. This circumstance makes for a more purposeful search for the eigenvalues β. The integration of the equations from infinity in a direction in which x increases exponentially offers additional advantages from the point of view that the form of the solution is not sensitive to the choice of the initial x_∞ and x'_∞ or to the accuracy of the computer calculation.

Numerical integration of Eqs. (14) has confirmed the existence, at $\omega_p^2/\omega^2 \leq 1$, of continuous-spectrum solitons with a continuous dependence of the amplitude on the velocity. These solitons are described with good accuracy by relations (24) and (25), have a low amplitude ($x_m^2 \ll 1$), and the perturbations of the electron and ion concentrations in these solitons illustrate well the premises developed above concerning the quasineutrality approximation.

With increasing velocity β, when the amplitude (24) does not satisfy the condition that (21) be small, the solutions that increase exponentially at infinity are of the soliton type only at certain discrete values of β. At a fixed ratio ω_p^2/ω^2 all the solitons of the discrete spectrum have different shapes; they differ in the number of maxima of the longitudinal-field potential and of the transverse field amplitude.

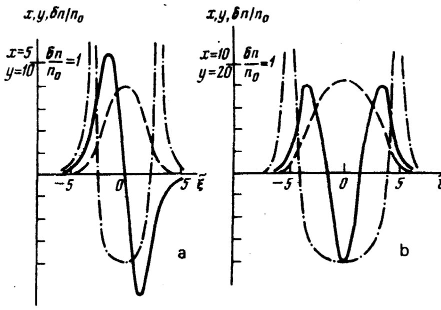

FIG. 3. Discrete-spectrum solitons in a post-critical plasma ($\omega_p^2/\omega^2 = 2.0$); a) $\beta \approx 0.235$, $(\delta n_e/n_0)_{max} \approx 1.7$; b) $\beta \approx 0.356$, $(\delta n_e/n_0)_{max} \approx 2.5$.

Figures 1–4 show some very simple discrete-spectrum solitons at different value the ratio ω_p^2/ω^2. The ratio of the masses of the electrons and ions was assumed to be $m_{0e}/m_{0i} = 1/2000$, corresponding approximately to a hydrogen plasma.

The first figure shows a soliton propagating in a transparent plasma ($\omega_p^2/\omega^2 = \frac{1}{2}$) and having one maximum of the potential and two maxima of the transverse field. The dimensionless soliton velocity is $\beta \approx 0.764$, while the solitons of the continuous spectrum have in such a plasma velocities close to $\beta \approx 0.71$. The amplitudes of both the potential and of the transverse field have relativistic values. The figure shows also the perturbation of the electron concentration, which does not exceed 20% of the unperturbed concentration.

A similar situation takes place also in the case when the concentration of the plasma is close to critical—at small β there exist continuous-spectrum solitons, and at large β solitons with discrete spectra.

The next figures (2 and 3) show solitons of two types for a post-critical plasma ($\omega_p^2/\omega^2 = 1, 2$, and 2), where there are no continuous-spectrum solitons. With increasing post-criticality of the plasma, the velocity decreases and the amplitude increases for solitons of a fixed type, the perturbations of the electron and ion concentrations increase, and the width of the soliton in dimensionless units (i.e., measured in wavelengths of the high-frequency carrier) also decreases somewhat. A particularly strong increase with increasing post-

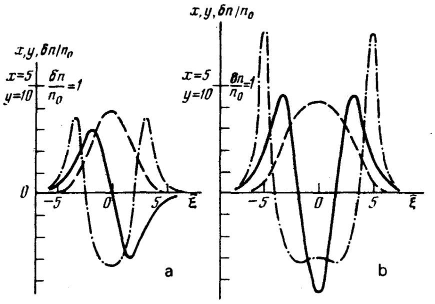

FIG. 2. Discrete-spectrum solitons in a post-critical plasma ($\omega_p^2/\omega^2 = 1.2$): a) $\beta \approx 0.462$; b) $\beta \approx 0.528$.

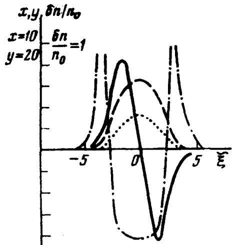

FIG. 4. Example of soliton with noticable perturbations of the ion concentration $\delta n_i/n_0$ (the profile of the ion concentration is dotted); $\omega_p^2/\omega^2 = 2.5$; $\beta \approx 0.174$; $(\delta n_e/n_0)_{max} \approx 3.1$.

criticality is observed in the perturbations of the electron concentration at the edges of the solitons. Figure 4 shows a soliton in a plasma with $\omega_p^2/\omega^2 = 2,5$, in which perturbations of the ion concentration, reaching values of the order of 30%, are quite noticeable. There is a plasma transcriticality limit $(\omega_p^2/\omega^2)_{cr} \sim 5$ above which no solitons solutions are observed. At post-criticality close to the limiting value, there exists only a soliton with one maximum of the potential and two maxima of the transverse field.

The numerical results given an idea of the qualitative picture of the distribution of the envelope solitons of circularly polarized radiation as a function of the velocity and of the ratio of the plasma frequency to the frequency of the RF carrier.[5)]

Envelope solitons exist in the limit of strongly relativistic amplitudes, but their character differs significantly from the character of the soliton solutions with small amplitude. The question of the role of such solitons in the dynamics of nonstationary wave fields still remains open. For a nonstationary wave equation of the Klein–Gordon type it follows from numerical solutions obtained by N. Zabusky (private communication) that in the case of strong nonlinearity the envelop solitons play an equally fundamental role as at small amplitudes.

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1) See also Refs. 2 and 3.

2) Equation (7) describes all the solutions in which the envelope of the RF field, the particle masses, and their concentrations depend only on the self-similar variable. Relation (6) determines only the most successful choice of the coordinate system in which the equation for the envelope take the simplest form. The ratio of ω to k was not fixed in Ref. 5, but a phase shift that depends on the self-similar variable was introduced in the RF carrier. The integration of the equation for the phase shift leads to a result equivalent to (6).

3) In this approximation, the equations are valid for envelopes of waves of arbitrary polarization, and the terms that describe the relativistic and striction nonlinearities are averaged over the period of the RF field. Therefore the coefficient of x^2 changes from $\frac{1}{2}$ for circular polarization of the radiation to $\frac{1}{4}$ for linear polarization.

4) A similar expression for the soliton amplitude can be obtained from the results of Ref. 5 by assuming that the plasma concentration is close to critical and that the soliton velocities are much smaller than the velocity of light.

5) However, the question of the number and shape of the discrete-spectrum soliton as a function of the parameter ω_p^2/ω^2 was investigated within the framework of numerical integration of the system of equations (14).

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Electric field of a plasma produced by optical breakdown in air

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A probe method was used to study electric fields near a plasma resulting from the breakdown of air in the vicinity of a target subjected to CO_2 laser pulses. The probe signal amplitude was determined as a function of the radiation intensity ($10^7 - 10^8$ W/cm^2) and of the distance between the probe and plasma. The appearance of the electric field in the plasma was attributed to the separation of charges in the front of an optical detonation wave. The experimental values of the field potential near the plasma were in agreement with theoretical estimates. The results of the measurements were compared with the experiments in which a plasma was created by the more powerful neodymium laser radiation.

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Explosions of chemical materials are known to create electromagnetic perturbations in the surrounding air. For example, electric signals can be detected by placing a receiving antenna some distance from the center of an explosion.[1,2] However, the origin of such signals is far from clear.

Optical breakdown in air is in many respects similar

