Inductive interaction of conducting bodies with a magnetized plasma

A. V. Gurevich, A. L. Krylov, and E. N. Fedorov

Institute of Earth Physics, USSR Academy of Sciences
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Magnetohydrodynamic flow of a supersonic stream of plasma around a thin conducting body is considered. A special process of inductive electromagnetic interaction with a plasma is indicated, wherein an electric field that generates a direct electric current is induced on the surface of the body, while magnetohydrodynamic wave perturbations are produced in the medium. The inductive interaction is determined by the conductivity of the body in the stream. Expressions are obtained for the drag, lateral-shifts, and buoyant forces due to the inductive interaction, as well as for the energy flux.

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The flow of a plasma stream with a frozen-in magnetic field **B** around a body is of interest for a number of problems of magnetohydrodynamics.[1,2] We can point out two characteristic processes that determine the interaction of such a body with the plasma. The first is the ordinary mechanical interaction due to the collisions of the particles with the surface of the body. This process leads to hydrodynamic friction and to a charge in the density of the charged particles in the vicinity of the body, owing to the surface recombination of the plasma ions.[3]

The second process is electromagnetic and is connected with the induction of an electric field on the body in the stream. In fact, the magnetic field **B** frozen into the moving plasma induces, in the coordinate frame of the body, an electric field

$$\mathbf{E}=-c^{-1}[\mathbf{v}\times\mathbf{B}]. \qquad (1)$$

Here v is the velocity of the plasma stream relative to the body and c is the speed of light.

The electric field (1) produces an electric current in the conducting body. The current produces polarization perturbations of the electric field and these are transferred to the plasma along the magnetic force lines. As a result, the field (1) should change ($E'\neq E$), meaning that the plasma velocity must also change, $v'\neq v$. Consequently, regardless of the hydrodynamic friction, the induced field (1) alone can cause the body to interact with the incoming plasma stream. We shall call this inductive interaction. Because of this interaction a magnetohydrodynamic wave and an aggregate of other wave perturbations are produced in the plasma if the body moves with supersonic speed. The circuit for the electric current flowing in the body is closed in the plasma by the shock wave. Consequently, a dynamo mechanism is produced and generates a direct electric current on the body.[1)] At the same time, appreciable perturbations are produced in the plasma and lead to the appearance of effective dragging of the body, to lateral shift forces, and to a buoyant force.

We emphasize once more that the considered mechanism of interaction between the body of the plasma is not connected in any way with hydrodynamic friction, and is of pure electromagnetic origin. It is the subject of the present paper. We note that the inductive interaction is apparently of great significance in the case of flow of cosmic plasma around cosmic bodies, particularly the flow of solar wind around the earth. In the latter case, the role of the conducting surface of the body is assumed by the polar ionosphere.

1. FLOW AROUND THE CONDUCTING PLATE. GENERAL RELATIONS

To describe stationary flow of a magnetized plasma around a body we use the equations of ideal magnetohydrodynamics[1,2,4]:

$$\operatorname{div}\bar{\rho}\mathbf{v}=0,\quad \operatorname{div}\mathbf{B}=0,\quad \operatorname{rot}[\mathbf{v}\times\mathbf{B}]=0, \qquad (2)$$

$$\mathbf{v}\nabla\mathbf{v}=-\bar{\rho}^{-1}\operatorname{grad}p+(4\pi\bar{\rho})^{-1}[\operatorname{rot}\mathbf{B}\times\mathbf{B}],\quad \mathbf{v}\operatorname{grad}p\bar{\rho}^{-\gamma}=0. \qquad (3)$$

Here p and $\bar{\rho}$ are the pressure and density of the plasma.

The unperturbed plasma flow is of the form

$$\mathbf{v}=v_0\mathbf{e}_1,\quad \mathbf{B}_0=B_0\mathbf{e}_3,\quad \bar{\rho}=\rho_0,\quad p=p_0. \qquad (4)$$

To separate the effects of inductive electromagnetic interaction we assume that the body in the plasma stream is an infinitesimally thin plate S oriented along the incoming flow, with a normal $\mathbf{n}=n_2\mathbf{e}_2+n_3\mathbf{e}_3$ (see Fig. 1). The plate produces no hydrodynamic perturbations in the plasma and does not change the flow (4) in the absence of

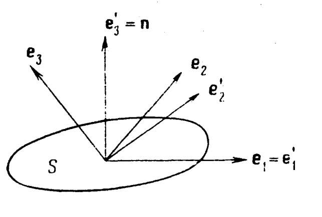

FIG. 1.

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inductive interaction.

Assume that the plate has an anisotropic surface conductivity

$$\hat{\Sigma}=\begin{pmatrix}\Sigma_P & \Sigma_H\\ -\Sigma_H & \Sigma_P\end{pmatrix}.$$

Here Σ_H and Σ_P are the Hall and Pederson integrated surface conductivities. The tangential component of the electric field (1) induced by the plasma stream in the coordinate frame of the plate, produces on the plate surface an electric current

$$\mathbf{I}=\hat{\Sigma}\mathbf{E}_t=-\frac{1}{c}\hat{\Sigma}[\mathbf{v}\times\mathbf{B}]_t. \tag{5}$$

This current leads to perturbation of the plasma flux (4)

$$\mathbf{v}=\mathbf{v}_0+v_0\mathbf{u},\quad \mathbf{B}=\mathbf{B}_0+M_AB_0\mathbf{b},\quad \rho=\rho_0+M\rho_0\rho.$$

Here **u**, **b**, and ρ are dimensionless perturbations, $v_A=B_0(4\pi\rho_0)^{-1/2}$ is the Alfven velocity, $c_s=(dp/d\rho)^{1/2}$ is the speed of sound in the unperturbed stream, and $M=v_0/c_s$ and $M_A=v_0/v_A$ are the hydrodynamic and Alfven Mach numbers. In the case of sufficiently low conductivity Σ, the perturbations are small. We can then linearize Eqs. (2) and (3), and obtain

$$\frac{\partial u_1}{\partial x_1}-\frac{1}{M_A}\left(\frac{\partial b_1}{\partial x_3}-\frac{\partial b_3}{\partial x_1}\right)+\frac{1}{M}\frac{\partial\rho}{\partial x_1}=0,$$
$$\frac{\partial u_2}{\partial x_1}+\frac{1}{M_A}\left(\frac{\partial b_3}{\partial x_2}-\frac{\partial b_2}{\partial x_3}\right)+\frac{1}{M}\frac{\partial\rho}{\partial x_2}=0,$$
$$\frac{\partial u_3}{\partial x_1}+\frac{1}{M}\frac{\partial\rho}{\partial x_3}=0; \tag{6a}$$

$$\frac{\partial b_1}{\partial x_1}-\frac{1}{M_A}\frac{\partial u_1}{\partial x_3}=0,\quad \frac{\partial b_2}{\partial x_1}-\frac{1}{M_A}\frac{\partial u_2}{\partial x_3}=0,$$
$$\frac{\partial b_3}{\partial x_1}-\frac{1}{M_A}\frac{\partial u_3}{\partial x_3}-\frac{M}{M_A}\frac{\partial\rho}{\partial x_1}=0,$$
$$\frac{\partial b_1}{\partial x_1}+\frac{\partial b_2}{\partial x_2}+\frac{\partial b_3}{\partial x_3}=0,\quad \frac{\partial\rho}{\partial x_1}+\frac{1}{M}\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}\right)=0. \tag{6b}$$

The magnetic field is perturbed by the stream (5) flowing on the plate S, the tangential component of the electric field $\mathbf{E}=-c^{-1}[\mathbf{v}\times\mathbf{B}]$ is continuous, and the normal component of the velocity on S vanishes. This leads to the following boundary conditions on S:

$$\{[\mathbf{n}\times\mathbf{b}]\}_S=\mathbf{f},\quad \{[\mathbf{u}\times\mathbf{n}]\}_S=0,\quad (\mathbf{un})|_S=0.$$

Here $\{a\}_s$ denotes the discontinuity of a on going through the plate S, and $\mathbf{f}=4\pi\mathbf{I}/cM_AB_0$.

These boundary conditions together with the requirement that there be no incoming waves are sufficient for a unique determination of the solution (6).

To make the results clearer, we confine ourselves to the hypersonic case

$$M\gg M_A\gg 1. \tag{7}$$

When the condition (7) is satisfied, the system (6) breaks up into three systems that describe the Alfven, fast-magnetosonic, and slow-magnetosonic waves, the last two being coupled by the boundary conditions.

For the Alfven wave we have

$$\frac{\partial u_1}{\partial x_1}-\frac{1}{M_A}\frac{\partial b_1}{\partial x_3}=0,\quad \frac{\partial b_1}{\partial x_1}-\frac{1}{M_A}\frac{\partial u_1}{\partial x_3}=0, \tag{8a}$$
$$\{b_1\}_S=f_P(x_1,x_2),\quad \{u_1\}_S=0. \tag{8b}$$

For the fast sound we have

$$\frac{\partial u_2^+}{\partial x_1}+\frac{1}{M_A}\left(\frac{\partial b_3^+}{\partial x_2}-\frac{\partial b_2^+}{\partial x_3}\right)=0,\quad \frac{\partial b_2^+}{\partial x_1}-\frac{1}{M_A}\frac{\partial u_2^+}{\partial x_3}=0,$$
$$\frac{\partial b_2^+}{\partial x_2}+\frac{\partial b_3^+}{\partial x_3}=0,\quad \rho^+=\frac{M_A}{M}b_3^+, \tag{9}$$
$$\{(n_2b_3^+-n_3b_2^+)\}_S=f_H(x_1,x_2),\quad \{u_2^+\}_S=0.$$

For the slow sound we have

$$\frac{\partial u_3^-}{\partial x_1}+\frac{1}{M}\frac{\partial\rho^-}{\partial x_3}=0,\quad \frac{\partial\rho^-}{\partial x_1}+\frac{1}{M}\frac{\partial u_3^-}{\partial x_3}=0,$$
$$(n_2u_2^++n_3u_3^-)|_S=0,\quad \{u_3^-\}_S=0. \tag{10}$$

Here

$$f_P=4\pi c^{-2}\Sigma_Pv_An_3,\quad f_H=4\pi c^{-2}\Sigma_Hv_An_3.$$

We see that the Alfven wave is excited in this approximation by the Pederson component of the current on the plate, while the magnetosonic waves are excited by the Hall component. We note that in the case $n_2=0(\mathbf{B}_0\perp S)$ no slow sound is excited.

2. ALFVEN WAVE

We consider first the wave excited by the Pederson component of the current. For b_1 we get from (8) the equation

$$M_A^2\frac{\partial^2 b_1}{\partial x_1^2}-\frac{\partial^2 b_1}{\partial x_3^2}=0,\quad \{b_1\}_S=f_P(x_1,x_2),\quad \left\{\frac{\partial b_1}{\partial x_3}\right\}_S=0. \tag{11}$$

The solution of (11) takes the form

$$b_1=\frac{1}{2}\begin{cases} f_P(x_{10}^+,x_{20}), & n_2x_2+n_3x_3>0,\\ -f_P(x_{10}^-,x_{20}), & n_2x_2+n_3x_3<0,\end{cases} \tag{12}$$

where

$$x_{10}^\pm=x_1\mp M_A\frac{n_2x_2+n_3x_3}{n_3},\quad x_{20}=x_2,$$
$$f_P(x_{10},x_{20})=4\pi c^{-2}v_An_3\Sigma_P(x_{10},x_{20}).$$

Thus, as seen from (12), the magnetic-field perturbations due to the Pederson current near the point $(x_{10},x_{20},-n_2x_{20}/n_3)$ on the plate S are transported along the characteristics of Eq. (11) that pass through this point:

$$x_3=-\frac{n_2}{n_3}x_2\pm\frac{1}{M_A}(x_1-x_{10}^\pm),\quad x_2=x_{20}.$$

For the perturbations of the stream velocity we get from (8)

$$u_1=\begin{cases}-b_1, & n_2x_2+n_3x_3>0,\\ b_1, & n_2x_2+n_3x_3<0.\end{cases}$$

We determine now the structure of the current in the plasma:

$$\mathbf{j}=\frac{c}{4\pi}\operatorname{rot}\mathbf{B}=\pm\frac{v_0B_0n_3}{2c}\operatorname{rot}\Sigma_P(x_{10}^\pm,x_{20})\mathbf{e}_1,$$

where the plus and minus signs are taken at $n_2x_2+n_3x_3>0$ and $n_2x_2+n_3x_3<0$, respectively.

The bulk of the current is concentrated on the surface of the discontinuity—on the characteristics that pass through the edge of the plate S. If the plate boundary is defined by the equations

$$F(x_1,x_2)=0,\quad n_2x_2+n_3x_3=0,$$

then the Alfven characteristics that pass through the edge of the plate S form two cylindrical surfaces $\mathbf{F}(x_{10}^\pm,x_{20})=0$, where $x_{10}^\pm$, and x_{20} are defined in (12).

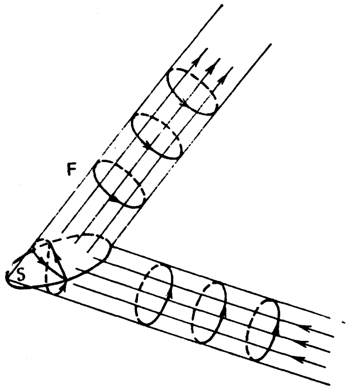

FIG. 2.

The surface current flowing over the surface of the cylinders is

$$\mathbf{j}=\mathbf{I}_S\delta_F, \quad I_S=\frac{v_0|B_0|}{2c}|n_2|\Sigma_P(x_{10}^{\pm}, x_{20}), \tag{13}$$

where δ_F is a delta function concentrated on the surface F.[5,6]

The perturbation picture at a constant conductivity of the plate S, when there is only a surface current (13), is shown in Fig. 2. A discontinuity of low intensity is detached from the plate boundaries and propagates with low velocity v_A. Perturbations are present only inside the regions bounded by the discontinuity. They are produced by the surface current I_s flowing over the surface of the discontinuity. This current completes in the plasma the circuit for the electric current (5) generated on the conducting plate by the induced electric field $\mathbf{E} = -c^{-1}\mathbf{v}\times\mathbf{B}$.

The condition for the applicability of the linear approximation $B_1 \ll B_0$ yields

$$p_1=4\pi c^{-2}\Sigma_P v_0 \ll 1. \tag{14}$$

This condition has a simple physical meaning: the diffusion speed $c^2/4\pi\Sigma_P$ of the magnetic field B_0 in the plate S greatly exceeds the stream speed v_0, so that the magnetic field has time to diffuse through the plate. At finite small values $p_1 < 1$, the picture is similar to that considered above, except that the low-intensity discontinuity is replaced by a shock wave of finite amplitude. At $p_1 \gg 1$ (superconducting case) the magnetic field is pushed out of the plate, and the flow picture is entirely different.

3. FAST MAGNETOSONIC WAVE

We consider now the waves excited by the Hall component of the current on the plate. It is convenient to introduce the scalar potential function $\varphi(x_1, x_2, x_3)$, defining the perturbations b_2^+, b_3^+ and u_2^+ by the relations

$$b_2^+=\frac{\partial\psi}{\partial x_3}, \quad b_3^+=-\frac{\partial\psi}{\partial x_2}, \quad u_2^+=M_A\frac{\partial\psi}{\partial x_1}.$$

It follows then from (9) that

$$M_A^2\frac{\partial^2\psi}{\partial x_1^2}-\frac{\partial^2\psi}{\partial x_2^2}-\frac{\partial^2\psi}{\partial x_3^2}=0,$$

$$\left\{\left(n_2\frac{\partial}{\partial x_2}+n_3\frac{\partial}{\partial x_3}\right)\psi\right\}_{S}=-f_H, \quad \left\{\frac{\partial\psi}{\partial x_1}\right\}_S=0. \tag{15}$$

We represent the solution of (15) in the form[5]

$$\psi=-\frac{2v_A}{c^2}\iint_{S'} d\xi_1 d\xi_2 \Sigma_H(\xi_1,\xi_2)\theta(x_1-\xi_1-M_A[(x_2-\xi_2)^2+(x_3+n_2\xi_2/n_3)^2]^{1/2})$$
$$\times\{(x_1-\xi_1)^2-M_A^2[(x_2-\xi_2)^2+(x_3+n_2\xi_2/n_3)^2]\}^{-1/2}, \tag{16}$$

where S' is the projection of S on the (x_1, x_2) plane, and $\theta(z)$ is the Heaviside function.

Next,

$$\mathbf{b}=\operatorname{rot}\psi\mathbf{e}_1, \quad u_2=M_A\frac{\partial\psi}{\partial x_1},$$
$$\mathbf{j}=\frac{cM_AB}{4\pi}\operatorname{rot}\operatorname{rot}\psi\mathbf{e}_1,$$

or in coordinate notation

$$j_1=-\frac{cM_AB_0}{4\pi}\nabla_\perp^2\psi,$$
$$j_\perp=\frac{cM_AB_0}{4\pi}\nabla_\perp\frac{\partial\psi}{\partial x_1},$$

where

$$\nabla_\perp=\frac{\partial}{\partial x_2}\mathbf{e}_2+\frac{\partial}{\partial x_3}\mathbf{e}_3.$$

We write down formula (16) concretely for the case of a plate strongly elongated along $\mathbf{e}_1$ $[\Sigma_H(x_1, x_2) = G_H\delta(x_2)\theta(x_1)]$:

$$\psi=\frac{2v_AG_H}{c^2}$$
$$\times\ln\left[\frac{x_1}{M_Ar}-\left(\frac{x_1^2}{M_A^2r^2}-1\right)^{1/2}\right]\theta(x_1-M_Ar),$$

where

$$r=(x_2^2+x_3^2)^{1/2}.$$

The electric-current lines inside the Mach cone are described by the expression

$$x_1=\text{const}.$$

These current lines are closed on the surface of the cone along which the surface current flows. Figure 3 shows the lines of the electric current and of the magnetic-field perturbations.

We note that at $n_2 \neq 0$ the velocity component normal to the plate S is $n_2u_2^+ \neq 0$. Therefore a slow magnetosonic wave must be excited in addition to the fast one at $n_2 \neq 0$.

4. SLOW MAGNETOSONIC WAVE

The system is integrated in analogy with (8) with the following boundary condition for u_3^-:

$$u_3^-|_S=-\frac{n_2}{n_3}u_2^+|_S=w(x_1,x_2),$$

which is obtained after the fast wave is determined. The solution takes the form

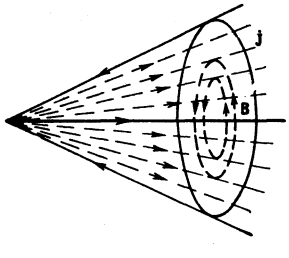

FIG. 3.

$$u_3^- = \begin{cases} w(x_{10}^+, x_{20}), & n_2x_2+n_3x_3>0, \quad x_{10}^+, x_{20}\in S, \\ w(x_{10}^-, x_{20}), & n_2x_2+n_3x_3<0, \quad x_{10}^-, x_{20}\in S, \\ 0, & x_{10}^\pm, x_{20}\notin S, \end{cases}$$

$$0^- = \begin{cases} u_3, & n_2x_2+n_3x_3>0, \\ -u_3, & n_2x_2+n_3x_3<0, \end{cases}$$

where

$$x_{10}^\pm = x_1 \mp M(n_2x_2+n_3x_3)/n_3, \quad x_{20}=x_2$$

and constitutes, in our approximation, a perturbation of purely hydrodynamic quantities. The pressure perturbation, in particular, is balanced by the perturbation of the magnetic field, which is of higher order of smallness compared with b.

The condition for the applicability of the linear approximation for the magnetosonic waves is obtained in analogy with (14) and takes the form

$$p_2 = 4\pi\Sigma_H v_0/c^2 \ll 1.$$

We emphasize that complete separation of the perturbations, i.e., perturbation of only an Alfven wave in the plasma by the Pederson currents and only magnetosonic waves by the Hall currents takes place only at large values of the Mach numbers M_A and M in the first-order approximation in $\varepsilon = 1/M_A$. It is easy to determine the next terms of the expansion in ε. When these are taken into account, the Alfven and magnetosonic perturbations become interlinked. Pederson currents, for example, excite both a strong Alfven wave and weak (of the order of ε) magnetosonic waves that are distributed, as in the case of (16), over and inside the Mach cone.

5. FORCES ACTING ON THE BODY, AND ENERGY DISSIPATION

To calculate the deceleration forces acting on the body and to calculate the absorption of the plasma energy that is converted into Joule heat, we must write down the equations of motion and of energy in the form of the energy and the momentum conservation laws

$$\operatorname{div}\hat{\Pi}=0, \quad \operatorname{div}\mathbf{q}=0, \tag{17}$$

where $\hat{\Pi}$ is the momentum flux density tensor, and $\mathbf{q}$ is the energy flux density vector.[1,4] The force $\mathbf{F}$ exerted on the body by the plasma is equal to the flux of the tensor $\hat{\Pi}$ through an arbitrary closed surface Q that contains the body S. By virtue of the divergent form of (17), the surface Q can be placed directly on the body. In this case the principal part of $\mathbf{F}$, with respect to the parameters p_1 and p_2, is calculated from the characteristics of the unperturbed flux.

For the surface density $\mathbf{f}$ of the force $\mathbf{F}$ we have in a coordinate frame tied to the plate (see Fig. 1)

$$\mathbf{f}=c^{-1}[\mathbf{I}\times\mathbf{B}_0], \quad \mathbf{I}=-c^{-1}\hat{\Sigma}[\mathbf{v}_0\times\mathbf{B}],$$
$$f_1'=c^{-2}v_0B_{03}'^2\Sigma_P, \quad f_2'=-c^{-2}v_0B_{03}'^2\Sigma_H,$$
$$f_3'=c^{-2}v_0B_{03}'(\Sigma_H B_{02}'-\Sigma_P B_{01}').$$

The drag force F_1 is thus due to the magnetic-field component B_3' which is normal to the plate and to the Pederson conductivity Σ_P, the lateral drift force F_2 is due to the Hall conductivity Σ_H and to the same component B_3', while the buoyant force F_3 requires the presence of both a normal and a tangential component of the magnetic field of the stream.

We determine analogously the energy $\mathscr{E}$ absorbed by a unit area of the plate, equal in this approximation to

$$\mathscr{E}=c^{-2}\Sigma_P v_0^2 B_3'^2.$$

It goes over completely into Joule heat released by the current in the plate, and depends only on the Pederson conductivity Σ_P and on the normal component B_3' of the magnetic field.

We note in conclusion that the induction interaction can play a certain role in the flow of cosmic plasma around natural and artifical bodies. Recognizing that the cosmic plasma is highly tenuous we discuss the conditions for the applicability of the magnetohydrodynamic approximation. The possibility of using magnetohydrodynamics for the description of the motion of a rarefied plasma was investigated by a number of workers.[7,8] It is shown that the aggregate of the kinetic equations of a collisionless plasma and of Maxwell's equations, in the description of large-scale motions in the absence of dissipation, reduces to equations (2) under the conditions

$$\frac{D}{R}\to 0, \quad \frac{r_H}{R}\to 0, \quad \frac{m}{M}\to 0, \quad \frac{T_i}{T_e}\to 0. \tag{18}$$

Here R is the characteristic spatial scale of the motion (in our problem R is the dimension of the body), D is the Debye radius, r_H is the Larmor radius of the ions (electrons), and m, M, T_e, and T_i are the masses and temperatures of the electrons and ions. When the first condition of (18) is satisfied, the plasma is quasineutral; the second condition ensures applicability of the drift approximation; owing to the third condition, an equilibrium distribution of the electrons can be assumed; finally, when the last condition is satisfied, the thermal motion of the plasma ions can be neglected. If the electrons have a Maxwell–Boltzmann equilibrium distribution,[2)] then the plasma pressure is

$$p=\bar{\rho}T_e/M, \quad \gamma=1.$$

In this case the magnetohydrodynamics is isothermal, so that Eq. (3) is satisfied identically. Thermal motion of ions in a collisionless plasma cannot be described within the framework of magnetohydrodynamics. However, as shown by comparison with the exact solutions of the kinetic equations, the magnetohydrodynamic approximation leads, even at $T_e \sim T_i$, to a correct qualitative and to a fair quantitative agreement.[9] In addition, in the case of a sufficiently strong magnetic field, when

$$8\pi\bar{\rho}(T_e+T_i)/M_iB^2\to 0, \tag{19}$$

the thermal motion of the plasma is of little significance at all, and magnetohydrodynamics is applicable regard less of the ratio of T_e and T_i; the condition (19) is identical with (7).

[1)] We emphasize that in this case the magnetic flux through the surface of the body remains unchanged, i.e., no current is generated on the body in vacuum (in the absence of plasma).

[2)] In a collisionless plasma the electrons can also have a stationary distribution different from that of Maxwell–Boltzmann, since the electron distribution functions specified at infinity may differ from Maxwellian. In addition, a non-Boltzmann distribution can occur for trapped electrons that execute finite motion (see Ref. 8).

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Nonlinear vibrational excitation waves in a molecular plasma with negative ions

V. G. Dresvyannikov and O. I. Fisun

Institute of Electrodynamics, Ukrainian Academy of Sciences
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The stability of chemically active multi-component plasma with negative ions produced in collisions between electrons and vibrationally excited molecules is investigated theoretically. The conditions are found for which sections with a negative differential conductivity exist on the volt-ampere characteristics (VAC). The characteristic time of development of the instability is derived from the dispersion equation for small perturbations. It is shown that during the nonlinear stage of the discharge a vibrational excitation wave is formed and transforms the system into a stable stationary state. The structure and velocity of the traveling wave front are found. The results explain the experimentally observed onset of instability of a molecular plasma in which nitrous oxide is introduced.

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There is a large class of collective phenomena which are some way or other associated with the presence of an internal structure of the plasma particles. As well known examples, we can mention ionization waves[1] and various thermonuclear instabilities (see, for example, Ref. 2).

The singularity of a low-temperature plasma is that the electron, along with motion in a continuous energy spectrum, can execute transitions in the space of discrete states. Here localization of the electron on a neutral particle frequently turns out to be energetically more favorable than its existence in the free state.[3] A negative charge in such a plasma will be connected both with the electron and with the ion components, and the concentration of the latter can significantly exceed the electron concentration, especially in a plasma of molecular gases.[4] The presence of negative ions leads to the appearance of specific collective processes in the plasma. Thus, for example, in discharges in O_2 and CO_2, and also in CO_2-N_2-He mixtures, instability of dissociative sticking, observed in the form of strata,[5-8] is most effectively excited. The reason for its appearance is the dependence of the rate of dissociative sticking k_a on the electron temperature T_e. However, if the dissociative sticking (DS) reaction $AB+e\xrightarrow{k_a(T_e)}A+B^-$ has the activation energy $E_{th}=D-U\lesssim kT_e$ (D is the energy of dissociation of the molecule AB, and U is the energy of affinity to the complex B), then $\partial \ln k_a/\partial \ln T_e \ll 1$ and the excitation of DS the mode becomes impossible.[5,7,9,10] In this work we shall show that another mechanism of instability of DS is possible in a plasma of molecular gases, and can be realized even at $E_{th} \lesssim kT_e$.

It is known that the relative contribution of the vibrationally excited molecules to the total rate of production of negative ions can be large, in spite of their relative small population. Such an effect is observed in the formation of O^- from O_2 and CO_2; in N_2O it appears more strongly—the quantity k_a increases by four orders of magnitude when the vibrational temperature T_V charges from 30 to 1000 K.[11] The reason for such a strong $k_a(T_V)$ dependence is connected with the impor-

