eigenfrequencies $\omega_{1,2}$ approach one another and for $\varkappa = -0$ they merge ($\omega_{1,2} = -i\gamma$) and then they diverge along the imaginary axis. Before the onset of the instability, when ω_1 goes over to the half-plane Im $\omega > 0$, the other frequency is $\omega_2 = -2i\gamma$. The main contribution to fluctuations comes from the frequency ω_1 closest to the imaginary axis. In calculations of fluctuations at the instability threshold we find from Eq. (19) that the only important part of the trajectory is

$$|\text{Im}\,\omega_1| \lesssim (\gamma/\tau)^{1/2}.$$

Since

$$|-\text{Im}\,\omega_2| \gg (\gamma/\tau)^{1/2},$$

the contribution of the oscillations with the frequency ω_2 is exponentially small and can be ignored. Allowance for just one branch ω_1 gives the results obtained above; all that is necessary is to introduce in Eq. (18) a correction factor ~1 because the fluctuations in the "initial" state at $t = -\infty$ may include comparable contributions from both eigenfrequencies. [It is not possible to reduce the problem to one branch if the merging of the eigenfrequencies occurs at a distance $\lesssim (\gamma/\tau)^{1/2}$ from the instability threshold and, in particular, when it occurs at the threshold itself. This occurs if $\varkappa$ and γ vanish simultaneously.]

We have deliberately ignored limitation of fluctuations by the nonlinear effects because the mechanism of such limitation (and the corresponding criterion) is different for each instability.

[1)] This is pointed out in Ref. 1 for one specific instability. The evolution of noise with time after an abrupt application of a pump field exceeding the parametric instability threshold of a plasma is considered in Ref. 2.

[2)] The opposite case (instantaneous transition to an unstable state) is trivial; it follow from Eq. (13) that the intensity of the fluctuations does not change during the transition time.

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On actuating shock waves in a completely ionized plasma

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The structure of actuating shock waves in a completely ionized plasma with collisions is calculated. Two limiting cases are considered, that of a magnetized and of an unmagnetized plasma. The processes determining the structure of the front of an actuating shock wave in an unmagnetized plasma are Joule dissipation and Hall currents. The width of the shock wave front in this case is determined by Joule dissipation and is equal to the diffusion length of the magnetic field. The magnetic field vector behind the shock wave front may rotate, but (with an accuracy to $\Omega_e\tau_e << 1$) remains in the plane of the initial direction. In the case of a magnetized plasma, the end of the magnetic field vector at the shock front rotates and describes a cone-like helix which expands behind the wave front. The number of revolutions along the helix is proportional to the degree of magnetization of the plasma. For a magnetized plasma, the processes defining the structure of the shock wave front are the electronic thermal conductivity, the electron-ion temperature relaxation and the dispersion due to the Hall terms and to the thermal emf. Correspondingly, the width of the front of an actuating shock wave in a magnetized plasma is equal to the scale of the electronic thermal conductivity. The values of the critical Mach numbers for which isomagnetic discontinuities arise in the shock wave front are found. The structure of the front is investigated in these cases.

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INTRODUCTION

Those shock waves in magnetohydrodynamics in which the magnetic field ahead of the wave front is directed along the normal to the plane of the front, while behind the shock-wave front there is a component of the magnetic field parallel to the plane of the front, are called actuating shock waves. The purpose of the present work is the study of the structure of actuating shock waves in a completely ionized plasma with collisions, within the framework of the hydrodynamic model with classical transport coefficients.[1] A similar problem on a shock wave in a plasma without a magnetic field and for a transverse shock wave was solved in Refs. 2 and 3. Some partial solutions for actuating shock waves were obtained in Refs. 4–8.

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As is well known,[9] the boundary conditions for a normal shock wave (the magnetic field vector is directed along the normal to the plane of the front) permit solutions either in the form of the usual gasdynamic discontinuity, and then the magnetic field no longer has an effect on the properties of the shock wave, completely dropping out of all the equations, or else the magnetic field behind the front of the shock wave changes direction—the transverse component is actuated. In the latter case, the structure of the shock wave front is such that along with the rotation of the magnetic field behind the front, the direction of the gas flow also changes, remaining parallel to the magnetic field. Simultaneously with the rotation of the magnetic field, the transverse component of the field that is generated rotates about the initial direction through an angle that is larger the greater the degree of magnetization of the plasma.

The structure of the actuating shock wave front is different, depending on the degree of magnetization of the plasma. In the case of an unmagnetized plasma, the fundamental dissipative process is Joule dissipation, and the width of the shock wave front is of the order of the diffusion length of the magnetic field. Depending on the intensity of the shock wave, the structure of its front is either continuous or, at sufficiently large values of the Alfven Mach number, has a weak isomagnetic discontinuity, the structure and width of which are determined by processes of electronic thermal conductivity and electron-ion heat exchange. At still higher intensities of the shock wave, a strong isomagnetic (but electronically isothermal) discontinuity appears on the front of the shock wave. The structure and width of this discontinuity are determined by processes of ionic viscosity and thermal conductivity.

In the case of a magnetized plasma, the fundamental dissipative processes which describe the width of the shock-wave front are the electronic thermal conductivity and the electron-ion heat exchange. In comparison with the case of an unmagnetized plasma, dispersion plays a significant role here. The dispersion processes, which lead to polarization rotation (oscillations) of the magnetic field in the plane of the shock wave front, are the Hall currents and the thermal emf.

1. SETUP OF THE PROBLEM. INITIAL EQUATIONS

We consider a plane stationary shock wave propagating with the velocity v_1 along the x axis in a simple ($\gamma_e=\gamma_i=5/3$), completely ionized plasma. The magnetic field ahead of the shock front is normal to the plane of the front, $\mathbf{H}=\{H_1,0,0\}$. Transforming as usual to a set of coordinates moving with the shock front, we have a flow of equilibrium plasma, flowing into the front of the discontinuity with velocity v_1 from $x=-\infty$ (we denote the equilibrium state at $x=-\infty$ by the index 1) and flowing out at $x=+\infty$ (index 2).

As will be shown, the dispersion effects associated with the departure from quasineutrality are insignificant in the considered case. Therefore, we shall consider the plasma to be quasineutral everywhere in what follows, setting $n_i=n_e=n$. In the next approximation in the small ratio of the Debye radius to the width of the shock front, these effects can be taken into account in the same fashion as in Ref. 3.

We assume the problem to be one-dimensional, setting $\nabla=\{d/dx,0,0\}$. Then, from the Maxwell equations,

$$\operatorname{rot}\mathbf{E}=0,\quad \operatorname{rot}\mathbf{H}=\frac{4\pi}{c}j,\quad \operatorname{div}\mathbf{H}=0$$

together with the boundary conditions at $x=-\infty$,

$$\mathbf{v}^e=\mathbf{v}^i=\{v_1,0,0\},\quad \mathbf{H}=\{H_1,0,0\},$$
$$T_e=T_i=T_1,\quad n=n_1$$

it follows that

$$E_y=E_z=\text{const}=0,\quad H_x=H_1=\text{const},\tag{1.1}$$

$$\frac{\partial H_z}{dx}=-\frac{4\pi en}{c}(v_y^i-v_y^e),\tag{1.2}$$

$$\frac{dH_y}{dx}=\frac{4\pi en}{c}(v_z^i-v_z^e).\tag{1.3}$$

As the equations of motion for a medium in a plasma with collisions, we use the equations of two-fluid hydrodynamics for electrons and ions. The complete set of such equations and the values of the kinetic coefficients were obtained by Braginskii.[1] Taking it into account that $v_x^e=v_x^i=v$ follows from the equation of continuity from the condition of quasineutrality, we write down the first integrals of the equations of continuity, momentum flux conservation and energy flux for the entire plasma:

$$nv=C,\tag{1.4}$$

$$(m_e+m_i)vC+n(T_e+T_i)+\pi_{xx}^i+\frac{H_y^2+H_z^2}{8\pi}=P,\tag{1.5}$$

$$(m_ev_y^e+m_iv_y^i)C+\pi_{xy}^i-\frac{H_xH_y}{4\pi}=Q_1,\tag{1.6}$$

$$(m_ev_z^e+m_iv_z^i)C+\pi_{xz}^i-\frac{H_xH_z}{4\pi}=Q_2,\tag{1.7}$$

$$(m_ev^{e2}+m_iv^{i2})\frac{C}{2}+\frac{5}{2}C(T_e+T_i)+\pi_{\alpha\beta}^iv_\beta^i+q_x^e+q_x^i=S.\tag{1.8}$$

The equations of motion for the electron component and the equation of heat conduction for the ionic component will be

$$m_eC\frac{dv_y^e}{dx}+\frac{en}{c}(v_zH_x-vH_z)=R_y^e,\tag{1.9}$$

$$m_eC\frac{dv_z^e}{dx}+\frac{en}{c}(vH_y-v_yH_x)=R_z^e,\tag{1.10}$$

$$\frac{3}{2}C\frac{dT_i}{dx}+nT_i\frac{dv}{dx}+\frac{dq_i}{dx}-\pi_{\alpha x}^i\frac{dv_\alpha^i}{dx}=3\varepsilon^2\frac{n}{\tau_e}(T_e-T_i).\tag{1.11}$$

Here C,P,Q_1,Q_2,S are constants of integration, determined from the boundary conditions; π_{xx}^i is the viscous stress tensor of the ionic component, $q_x^{e,i}$ are the heat fluxes of electrons and ions, and R^e is the force of friction acting on the electrons. Terms with electron viscosity are small relative to $(m_e/m_i)^{1/2}\equiv\varepsilon\ll1$ and are omitted.

Equations (1.2)–(1.11) represent the complete set of equations for the variables $H_y, H_z, n, v, v_y^{e,i}, T_e, T_i$.

2. DIMENSIONLESS EQUATIONS. BOUNDARY CONDITIONS

Just as in Ref. 3, we transform to dimensionless variables

$$\omega = v/v_k, \quad \lambda_{e,i} = v_y^{e,i}/v_k, \quad \mu_{e,i} = v_z^{e,i}/v_k,$$

$$h_{y,z} = H_{y,z}/H_1, \quad \Theta_{e,i} = T_{e,i}/T_k, \quad \zeta = x/\Delta, \tag{2.1}$$

where the index $k=1$ or 2, depending on whether the quantities are made dimensionless relative to the equilibrium values of the variables ahead of ($k=1$) or behind ($k=2$) the shock front. The Alfven ($M_{ak} = v_k/v_a(k)$) and acoustic ($M_k = v_k/c_s(k)$) Mach numbers are defined as in Ref. 3.

We write down Eqs. (1.2)–(1.11) in dimensionless variables (2.1) with $k=1$ for the equilibrium state 2. It is obvious that in state 2 we have $\lambda_e = \lambda_i = \lambda_2$, $\mu_e = \mu_i$, $\Theta_e = \Theta_i = \Theta_2$. Elminating the dimensionless density and the transverse components of the velocity from the equations, we obtain a set of algebraic equations which connect the values of the equilibrium variables ahead of and behind the shock front, i.e., the boundary conditions

$$\omega_2 - 1 + \frac{3}{5M_1^2}\left(\frac{\Theta_2}{\omega_2} - 1\right) + \frac{h_2^2}{2M_{a1}^2} = 0, \tag{2.2}$$

$$\omega_2^2 - 1 + \frac{3}{M_1^2}(\Theta_2 - 1) + \omega_2^2 h_2^2 = 0, \tag{2.3}$$

$$h_{y2}(\omega_2 - 1/M_{a1}^2) = h_{z2}(\omega_2 - 1/M_{a1}^2) = 0, \tag{2.4}$$

where $h_2^2 = h_{y2}^2 + h_{z2}^2$.

One root $h_{y2} = h_{z2} = 0$ of Eq. (2.4) refers to the gas-dynamic shock wave in which the flow is everywhere parallel to the magnetic field and does not interact with it. The second root of this equation $\omega_2 = 1/M_{a1}^2$ corresponds to the actuating shock wave.

From the necessary condition of stability, which requires that the shock wave be a compression wave with $n_2 > n_1$ and $v_2 < v_1$, i.e., $\omega_2 < 1$ and $h_2^2 > 0$, we obtain the admissible region of change of the Mach numbers: $M_{a1} > 1$ and $M_1 > 1$. It can be shown that the temperature and the entropy here increase through the shock front, i.e., $\Theta_2 > 1$ and $\Theta_2\omega_2 > 1$, where

$$\Theta_2 = 1 + \frac{5M_1^2}{9}\left(1 - \frac{1}{M_{a1}^2}\right)\left(1 - \frac{1}{M_{a1}^2} + \frac{6}{5M_1^2}\right). \tag{2.5}$$

The region of change of the Mach numbers, corresponding to the actuating shock wave, is given by the condition $h^2 > 0$ and $\omega_2 < 1$. Substituting these inequalities in (2.2)–(2.4), we obtain

$$1 < M_{a1}^2 < 4M_1^2/(M_1^2+3). \tag{2.6}$$

Thus the admissible values of the Mach number for the actuating compression shock wave is

$$1 \leqslant M_1 < \infty, \quad 1 \leqslant M_{a1} \leqslant 2.$$

The corresponding region of change of the Mach numbers on the (M_{a1}^2, M_1^2) plane is shown in Fig. 1.

For the Mach numbers in the outgoing flow (behind the shock front), $M_2 = v_2/c_s(2)$, $M_{a2} = v_2/v_a(2)$, we find:

$$M_{a2}^2 = 1, \quad M_2^2 = 9M_1^2/[15M_{a1}^4 - 6M_{a1}^2 + 5(M_{a1}^2 - 1)^2 M_1^2].$$

Then the equation of the curve on the plane (M_{a1}^2, M_1^2) corresponding to the given value of M_2 is

$$M_1^2 = 3M_{a1}^2 M_2^2 (5M_{a1}^2 - 2)/[9 - 5M_2^2(M_{a1}^2 - 1)^2]. \tag{2.7}$$

It follows from (2.7) that the admissible limits of variation of M_2, are $1/5 \leqslant M_2^2 < \infty$. We also note that the maximum value of M_{a1}^2 at a given M_2 is achieved at $M_1 \to \infty$, i.e., at $\beta_1 = 5M_{a1}^2/6M_1^2 = 0$, and is equal to $M_{a1}^2(\beta_1 = 0) = 1 + 3/M_2\sqrt{5}$.

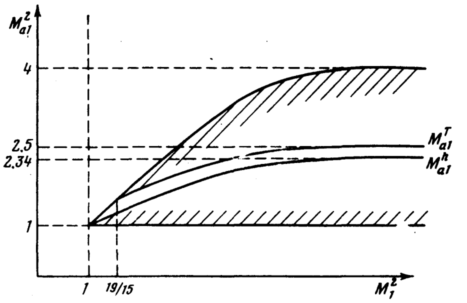

FIG. 1. Region of change of the Mach numbers of the actuating wave, allowed by the boundary conditions $1 \leqslant M_{a1}^2 \leqslant 4$, $1 \leqslant M_1^2 < \infty$. The two lines on the drawing with the asymptotes $M_{a1}^2 = 5/2$ and $M_{a1}^2 = 1 + 3/\sqrt{5}$ at $M_1 \to \infty$ are the levels of the critical Mach numbers M_{a1}^T and M_{a1}^h corresponding to the values $M_2^2 = 1$ and $M_2^2 = 4/5$, respectively.

From Eqs. (2.2)–(2.3), we have

$$h_2^2 = \frac{2}{3}(M_{a1}^2 - 1)\left(4 - \frac{M_1^2+3}{M_1^2} M_{a1}^2\right). \tag{2.8}$$

It then follows that the maximum value of h_2^2 at fixed M_1 in the region (2.6) is

$$h_{2max}^2(M_1^2) = \frac{3}{2}\frac{(M_1^2-1)^2}{M_1^2(M_1^2+3)}. \tag{2.9}$$

Thus h_2 changes from $h_2 = 0$ at $M_1 = 1$ to $h_2^2 = 3/2$ at $M_1 \to \infty$. The values $M_{01}^2 = 5/2$ and $M_2^2 = 4/5$ correspond to the maximum value of h_2.

3. STRUCTURE OF THE SHOCK WAVE IN AN UNMAGNETIZED PLASMA

Since the form of the equations, the kinetic coefficients and, consequently the processes which determine the structure of the shock wave are different in the cases of magnetized and unmagnetized plasma, we consider these two limiting cases separately. We denote the degree of magnetization of the ions by $(\Omega_i \tau_i)^{-1} = \delta$, where Ω_i and τ_i are the cyclotron frequency and the time of Coulomb collisions. For electrons, we have then $(\Omega_e \tau_e)^{-1} = \varepsilon\delta$.

We consider the case of a completely unmagnetized plasma, i.e., we require that the inequalities $\delta \gg 1$ and $\varepsilon\delta \gg 1$ be satisfied over the extent of the entire shock front, both in state 1 and in state 2. We transform in (1.2)–(1.11) to dimensionless variables and, eliminating the density and the transverse components of the electron velocity, we obtain the equations which describe the structure of the shock wave.

Scales that are characteristic for different processes appear in this case in the equations for the dimensionless variables.[3] The scale $\Delta_v = l/M$ corresponds to the ion viscosity, while $\Delta_d = \varepsilon\delta M l/M_a$ and $\Delta_h = M\delta l$ correspond to the dispersion associated with electron inertia and Hall terms, respectively, the scale $\Delta_j = \varepsilon\delta^2 M l/M_a^2$ corresponds to joulean dissipation and $\Delta_{Te} = l/\varepsilon M^3$, $\Delta_{Ti} = l/M^3$, $\Delta_r = Ml/\varepsilon$ corresponds to the electronic and ionic thermal conductivities and the thermal exchange, respectively, where l is the path length.

For an unmagnetized plasma, i.e., at $\varepsilon\delta \gg 1$ and $\delta \gg 1$, the greatest scale will be Δ_j and, consequently, in this case, both the structure and the width of the shock front are determined by the joulean dissipation. It is easy to see that, with accuracy to small quantities of the order of $(\varepsilon\delta)^{-2} \ll 1$, the electron and ion temperatures are equal on the scale Δ_j. Setting $\Theta_e = \Theta_i = \Theta$, and omitting terms that are small in ε, δ^{-1}, $(\varepsilon\delta)^{-1}$, we obtain the following equations for the structure of the shock wave:

$$\omega - 1 + \frac{0.6}{M_k^2}\left(\frac{\Theta}{\omega} - 1\right) + \frac{h^2 - h_k^2}{2M_{ak}^2} = 0, \tag{3.1}$$

$$\omega^2 - 1 + \frac{3}{M_k^2}(\Theta - 1) + \frac{h^2 - h_k^2}{M_{ak}^4} = 0, \tag{3.2}$$

$$h_y\left(\frac{1}{M_{ak}^2} - \omega\right) = -1.2\frac{\Delta_h}{M_{ak}^2}\omega\frac{dh_z}{dx} - 0.52\Theta^{-3/2}\Delta_j\frac{dh_y}{dx}, \tag{3.3}$$

$$h_z\left(\frac{1}{M_{ak}^2} - \omega\right) = 1.2\frac{\Delta_h}{M_{ak}^2}\omega\frac{dh_y}{dx} - 0.52\Theta^{-3/2}\Delta_j\frac{dh_z}{dx}. \tag{3.4}$$

Here the index $k = 1$ or 2 and depends on which dimensionless variables are used. At $k = 1$, we have $h_1 = (\lambda^2 + \mu^2)_1 = 0$ and at $k = 2$, $h_2^2 = (\lambda^2 + \mu^2)_2$.

The Hall terms in the equations of motion for electrons (the corresponding scale is Δ_h) have been left in (3.1)–(3.4). As is well known,[10] the corresponding terms lead to dispersion of the magnetosonic waves propagating at the angle $\Theta \gg \varepsilon$ to the magnetic field. For the actuating shock waves, at $M_a \sim 1$, we have $\Delta_h = M\delta l \approx c/\omega_{pi}$, where ω_{pi} is the ion plasma frequency. Since the scales of the other possible dispersion mechanisms, which correspond to electron inertia c/ω_{pe} and charge separation v_a/ω_{pi}, are considerably smaller, we can neglect them in the considered problem.

In (3.1) and (3.2), we find

$$\omega_\pm(h^2) = \frac{1}{8}\left\{5 + \frac{3}{M_k^2} - \frac{5}{2}\frac{h^2 - h_k^2}{M_{ak}^2} \pm \left[\left(5 + \frac{3}{M_k^2} - \frac{5}{2}\frac{h^2 - h_k^2}{M_{ak}^2}\right)^2 - 16\left(1 + \frac{3}{M_k^2} - \frac{h^2 - h_k^2}{M_{ak}^2}\right)\right]^{1/2}\right\}, \tag{3.5}$$

$$\Theta(h^2) = \frac{M_k^2}{3}\left[1 - \omega^2(h^2) + \frac{h^2 - h_k^2}{M_{ak}^2}\right]. \tag{3.6}$$

We transform in Eqs. (3.3) and (3.4) from the variables h_y, h_z to the variables $h_y = h\sin\varphi$, $h_z = h\cos\varphi$. Then we get the following equations for h^2 and φ.

$$\left(1.38\frac{\omega^2}{\varepsilon^2\delta^2}\Theta^{1/2} + 0.26\Theta^{-3/2}\right)\frac{dh^2}{d\zeta} = \left(\omega - \frac{1}{M_{a1}^2}\right)h^2, \tag{3.7}$$

$$\frac{d\varphi}{d\zeta} = 4.43\frac{\omega}{\varepsilon\delta}\Theta^3\left(\frac{1}{M_{a1}^2} - \omega\right). \tag{3.8}$$

Substituting $\omega(h^2)$ and $\Theta(h^2)$ and integrating, we obtain the solution or the actuating shock wave:

$$\zeta - \zeta_0 = 0.26\int_0^{h^2}\frac{dh^2\Theta^{-3/2}(h^2)}{h^2[\omega(h^2) - M_{a1}^{-2}]}.$$

It is easy to see that the obtained solutions for h^2, $\omega(h^2)$, $\Theta(h^2)$ etc., satisfy all the boundary conditions. It follows from (3.8) that the total angle of rotation of the magnetic field vector in the plane (h_y, h_z) is a small quantity $\Delta\varphi = O(1/\varepsilon\delta)$. Thus, the change in direction of the magnetic field vector in the case of an unmagnetized plasma occurs in one plane, with accuracy to $O(1/\varepsilon\delta)$.

We now investigate the obtained solution. We consider (3.5) at the singular points 1 and 2. For $k = 1$ or 2, we have

$$\omega(k) = 1 = \frac{1}{8}\left\{5 + \frac{3}{M_k^2} \pm 3\left|1 - \frac{1}{M_k^2}\right|\right\}. \tag{3.9}$$

It follows from (3.9) that the integral curve, which leaves point 1, is always supersonic (L_+) since $M_1 > 1$ and we must choose the plus sign in (3.9) to satisfy the condition $\omega(1) = 1$. The integral curve ending at point 2 is supersonic and belongs to L_+ at $M_2 > 1$, and is subsonic L_- at $M_2 < 1$. Since the value of ω on the different branches of (3.5) at one and the same h^2 are connected by the conservation laws, a transition is possible in the structure of the shock front from one branch (L_+) to the other (L_-) through the gasdynamic discontinuity, which we can determine by direct calculations similar to those performed in Ref. 11.

The mentioned gas-dynamical discontinuity is an isomagnetic discontinuity, since the magnetic Reynolds number at such a discontinuity is small, in view of smallness of all the characteristic scales of the physical processes in comparison with the scale of Joule dissipation.

We rewrite Eq. (3.7), omitting terms that are small in $1/\varepsilon^2\delta^2$, in the form

$$\frac{dh^2}{d\omega}\frac{d\omega}{d\zeta} = 3.84\Theta^{3/2}\left(\omega - \frac{1}{M_{a1}^2}\right)h^2.$$

Differentiating Eqs. (3.1) and (3.2) once and twice, respectively, and repeating the calculations performed in Ref. 3, we find that the condition for entropy growth, which requires the monotonicity of $d\omega/d\zeta < 0$, leads to the necessity of satisfaction of the inequality

$$dh^2/d\omega \leqslant 0 \tag{3.10}$$

for all ω. Considering (3.10) at the singular points 1 and 2, similar to Ref. 3, we find that (3.10) is always satisfied at point 1, since $M_1 > 1$, and at point 2 at $M_2 > 1$. In the case $M_2 < 1$, the inequality (3.10) is violated and we must introduce the isomagnetic discontinuity in the structure of the shock front.[3] The form of the function $h^2(\omega)$ for the cases $M_2 > 1$ and $M_2 < 1$ is shown in Figs. 2a and 2b.

Linearizing Eq. (3.7) in the vicinity of the singular

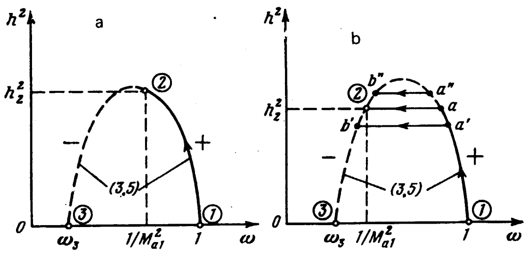

FIG. 2. Form of the function $\omega(h^2)$ for a) purely resistive structure of the actuating shock wave at $M_2 > 1$, and b) structure of the actuating shock wave with isomagnetic jump at $M_2 < 1$; $\omega_3 = (M_1^2 + 3)/4M_1^2$.

point 2, we obtain

$$\frac{d(h^2-h_2^2)}{d\zeta}=\text{const}\cdot(1-M_2^2)(h^2-h_2^2),$$

where const > 0. Since the limit cycle $h^2=h_2^2$ at $M_2<1$ on the plane (h_y, h_z) is unstable as $x\to+\infty$, the integral curve does not approach it. Therefore solutions of the form (Fig. 2b) $1-a'-b'-2$ or $1-a''-b''-2$ are impossible. The only possible solution is $1-a-2$ (see Figs. 2a, b).

To find the structure of the isomagnetic discontinuity, we must rewrite the initial equations in dimensionless variables, referred to the state behind the wave front, setting $h^2=1$. The most important of the dissipative processes determining the structure of the isomagnetic discontinuity are the electronic thermal conductivity and the electron-ion heat exchange, which have a characteristic scale of the order of l_2/ε. It was shown in Ref. 3 that at $4/5<M_2^2<1$ the isomagnetic discontinuity is a weak one with width, according to Prandtl

$$\Delta_s\approx\frac{0.56}{M_2(1-M_2^2)}\frac{l_2}{\varepsilon}.$$

At $M_2^2<4/5$ an internal electronic isometric discontinuity arises inside the isomagnetic discontinuity, the structure of which is determined by the processes of ion viscosity and thermal conductivity. The equations and solutions for the structure of both "weak" and "strong" isomagnetic discontinuities agree with those obtained in Ref. 3 (see Figs. 8 and 9 of Ref. 3).

Setting $M_2^2=1$ and $M_2^2=4/3$ in Eq. (2.6), we obtain the equation for the line of critical values of the Mach numbers on the plane (M_{a1}^2, M_1^2)—see Fig. 1. For $M_{a1}^2<M_{a1}^h(\beta_1)$ the structure of the shock wave is purely resistive, while for $M_{a1}^T>M_{a1}>M_{a1}^h$ there is a weak isomagnetic discontinuity in the structure of the shock front, and at $M_{a1}>M_{a1}^T$ a strong isomagnetic discontinuity. The values of the critical Mach numbers at $\beta_1\to 0$ are the following:

$$M_{a1}^h(\beta_1=0)=(1+3/\sqrt{5})^{1/2}\approx1.530,\quad M_{a1}^T(\beta_1=0)=\sqrt{5/2}.$$

We investigate qualitatively the structure of the shock wave and the behavior of the integral curves. As follows from Eqs. (3.3) and (3.4), there is a singular point (0, 0) on the plane (h_y, h_z) and a limit cycle, determined by the relation $\omega(h^2)=1/M_{a1}^2$—a circle of radius h_2^2 with center at the origin of the coordinates. The characteristic number at the singular point is

$$k_{1,2}=\left(1-\frac{1}{M_{a1}^2}\right)\frac{\Delta_j\pm i\Delta_h}{\Delta_j^2+\Delta_h^2}.$$

Thus, the singular point 1 is an unstable focus. Since $\varepsilon\delta\gg1$ in the considered case of an unmagnetized plasma, i.e., $\Delta_j\gg\Delta_h$, it follows that $|\mathrm{Re}\,k_{1,2}|\gg\mathrm{Im}\,k_{1,2}$. It then follows that the motion in the $(h_y h_z)$ plane is aperiodic–the oscillations of the direction of the transverse component of the magnetic field vector, which arise because of dispersion, are suppressed by the strong Joule dissipation or, as was shown above, the total angle of rotation of the magnetic field vector in the plane of the shock front is a small quantity, of the order of $1/\varepsilon\delta$.

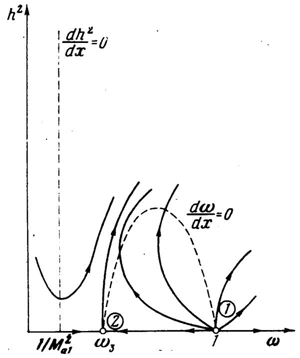

FIG. 3. Field of the integral curves on the (h^2, ω) plane of Eqs. (3.11), (3.14) at $M_{a1}^2<4M_1^2/(M_1^2+3)=1/\omega_3$.

For a qualitative investigation of the behavior of the integral curves, we consider a model problem, retaining in Eqs. (3.1)–(3.4) only the dissipative terms with the ion viscosity and Joule losses. We have

$$\Delta_j\frac{dh^2}{dx}=h^2\left(1-\frac{1}{\omega M_{a1}^2}\right),\tag{3.11}$$

$$\omega-1+\frac{0.3}{M_k^2}\left(\frac{\Theta_e+\Theta_i}{\omega}-2\right)+\frac{h^2-h_k^2}{2M_{ak}^2}-\Delta_v\frac{d\omega}{dx}=0,\tag{3.12}$$

$$\omega^2-1+\frac{3}{2M_k^2}(\Theta_e+\Theta_i-2)+\frac{h^2-h_k^2}{M_{ak}^2}-2\Delta_v\omega\frac{d\omega}{dx}=0.\tag{3.13}$$

Eliminating the temperature terms from (3.12) and (3.13), we find

$$3\Delta_v\omega\frac{d\omega}{dx}=(\omega-1)\left(4\omega-1-\frac{3}{M_k^2}\right)+\frac{5(h^2-h_k^2)}{2M_{ak}^2}\left(\omega-\frac{3}{5M_{ak}^2}\right).\tag{3.14}$$

We now investigate the behavior of the integral terms of Eqs. (3.11) and (3.14) on the (h^2, ω) plane. As is seen, in the region of the gasdynamic shock wave, at $M_{a1}^2>4M_1^2(M_1^2+3)$, the isoclines $dh^2/dx=0$ and $d\omega/dx=0$ do not intersect at $h^2>0$ and $\omega>0$ and, consequently, the only possible solution is a gasdynamic shock wave (Fig. 3). Here the point 1 is an unstable focus, while the point 2 is a saddle point $(M_{a2}^2=\omega_2 M_{a1}^2=(M_1+3)M_{a1}^2/4M_1^2>1)$. In the region of Mach numbers corresponding to the actuating shock wave, i.e., at $M_{a1}^2<4M_1^2(M_1^2+3)$, the point 1 is as before an unstable focus (Fig. 4), while the point $2GD$, which corresponds to the gasdynamic shock wave, i.e., $h^2=0$, represents a stable node $(M_{a2}^2<1)$, while the point $2AC$, which corresponds to the actuating shock wave, is a saddle. Obviously, the set of integral curves that enter into the point $2GD$, is

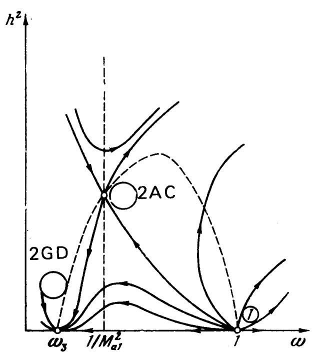

FIG. 4. Field of the integral curves on the (h^2, ω) plane of Eqs. (3.11) and (3.14) for $M_{a1}^2<4M_1^2/(M_1^2+3)$. The point 2AC corresponds to the state behind the front for the actuating shock wave, the point 2GD—to the state behind the front corresponding to the gas-dynamical shock wave with $h^2=0$.

evidence of the ambiguity of the structure of the gas-dynamic shock wave in the region $M_{a1}^2 < 4M_1^2(M_1^2+3)$, i.e., of instability of the solution with the state behind the front, corresponding to $h^2=0$ in this region. It then follows that at $M_{a1}^2 < 4M_1^2/(M_1^2+3)$, only the solution corresponding to the actuating shock wave is realized.

4. STRUCTURE OF THE SHOCK WAVE IN A MAGNETIZED PLASMA

We now consider the case of a strongly magnetized plasma, i.e., we shall assume that $(\Omega_i\tau_i)^{-1}=\delta \ll 1$ everywhere and, correspondingly, $(\Omega_c\tau_c)^{-1}=\delta\varepsilon \ll 1$. In this case, we must take into account the anisotropy of the transport coefficients for the plasma. Here it is convenient to introduce a unit vector directed along the magnetic field:

$$\mathbf{n}=\mathbf{H}/H=(1+h^2)^{-1/2}\{1,h_y,h_z\},$$

and distinguish between the vector components parallel and perpendicular to the magnetic field:

$$\mathbf{a}_\parallel=\frac{\mathbf{an}}{1+h^2}\{1,h_y,h_z\},\quad \mathbf{a}_\perp=\mathbf{a}-\mathbf{a}_\parallel.$$

In particular

$$\nabla_\parallel=\frac{1}{1+h^2}\left\{\frac{d}{dx},\ h_y\frac{d}{dx},\ h_z\frac{d}{dx}\right\}.$$

For example, for the force of fraction, eliminating the currents with the help of Maxwell's equations (1.2) and (1.3), we find

$$R_{uy}=\frac{m_ecH_1}{4\pi e^2\tau_e}\left[-\frac{dh_z}{dx}-0.49\frac{h_y}{1+h^2}\left(h_z\frac{dh_y}{dx}-h_y\frac{dh_z}{dx}\right)\right],$$

$$R_{uz}=\frac{m_ecH_1}{4\pi e^2\tau_e}\left[\frac{dh_y}{dx}-0.49\frac{h_z}{1+h^2}\left(h_z\frac{dh_y}{dx}-h_y\frac{dh_z}{dx}\right)\right].$$

Similarly, for the thermal emf, with accuracy to small terms of the order of $\varepsilon\delta$, we have

$$R_{Ty}=-0.71n\frac{h_y}{1+h^2}\frac{dT_e}{dx},\quad R_{Tz}=-0.71n\frac{h_z}{1+h^2}\frac{dT_e}{dx}.$$

Without writing down the entire set of equations here, we note that since the quantity $1+h^2 \leqslant 5/2$, which appears as a factor in the terms with the electronic thermal conductivity and thermal emf, is of the order of unity, then the basic dissipative process in the case of a magnetized plasma is the electronic thermal conductivity and the electron-ion heat exchange, which have characteristic scales of the order of l_1/ε, while the Joule dissipation, ion viscosity, and thermal conductivity, with scales $\Delta_j=\varepsilon\delta^2M_1/M_a^2$, $\Delta_v\sim\Delta_{Ti}\sim l$ respectively, are small in the small parameters ε and $\varepsilon\delta^2$. Principal among the dispersion mechanisms are the Hall currents and the thermal emf with scales $\Delta_h=M\delta l$ and $\Delta_f=\delta l/M$, respectively.

Neglecting the ion viscosity in the scale l/ε, we find from Eqs. (1.6) and (1.7),

$$\lambda_i=h_y/M_{a1}^2;\quad \mu_i=h_z/M_{a1}^2.$$

Eliminating the transverse components of the ion velocity, and omitting terms that are small in the parameters ε and δ, we obtain the following equations for the structure of the front of the actuating shock wave:

$$\omega-1+\frac{3}{5M_k^2}\left(\frac{\Theta_e+\Theta_i}{\omega}-2\right)+\frac{h^2-h_k^2}{2M_{ak}^2}=0, \tag{4.1}$$

$$\omega^2-1+\frac{3}{2M_k^2}(\Theta_e+\Theta_i-2)+\Delta_{Te}\frac{d\Theta_e}{dx}+\frac{h^2-h_k^2}{M_{ak}^4}=0, \tag{4.2}$$

$$\frac{3}{2}\frac{d\Theta_i}{dx}+\frac{\Theta_i}{\omega}\frac{d\omega}{dx}=\frac{\Theta_e-\Theta_i}{\Delta_r}, \tag{4.3}$$

$$\Delta_j\frac{dh_z}{dx}-\Delta_h\frac{dh_y}{dx}+0{,}49\Delta_j\frac{h_y}{1+h^2}\left(h_z\frac{dh_y}{dx}-h_y\frac{dh_z}{dx}\right)+\Delta_f\frac{d\Theta_e}{dx}=h_z\left(1-\frac{1}{\omega M_{a1}^2}\right), \tag{4.4}$$

$$\Delta_j\frac{dh_y}{dx}+\Delta_h\frac{dh_z}{dx}-0.49\Delta_j\frac{h_z}{1+h^2}\left(h_z\frac{dh_y}{dx}-h_y\frac{dh_z}{dx}\right)-\Delta_f\frac{d\Theta_e}{dx}=h_y\left(1-\frac{1}{\omega M_{a1}^2}\right) \tag{4.5}$$

where

$$\Delta_j=\frac{c^2}{4\pi\sigma_1v_1}=\frac{\varepsilon\delta^2M_1l_1}{M_{a1}^2},\quad \Delta_f=\frac{0.71cT_1}{en_1H_1\omega(1+h^2)}=\frac{\delta l_1}{M_1},$$

$$\Delta_{Te}=\frac{6.32T_1T_e\tau_e}{m_em_iv_1^3(1+h^2)}=\frac{l_1}{M_1^3\varepsilon},\quad \Delta_r=\frac{v_1\tau_\varepsilon}{3\varepsilon^2}=\frac{M_1l_1}{\varepsilon}.$$

Transforming in (4.4) and (4.5) from the variables h_y and h_z to the variables h^2 and φ, we obtain

$$\frac{dh^2}{dx}=2h^2\frac{\Delta_f}{\Delta_h}\frac{d\Theta_e}{dx}+2h^2\frac{\Delta_j}{\Delta_h^2}\frac{1+0.51h^2}{1+h^2}\left(1-\frac{1}{\omega M_{a1}^2}\right), \tag{4.6}$$

$$\frac{d\varphi}{dx}=\frac{1}{\Delta_h}\left(\frac{1}{\omega M_{a1}^2}-1\right). \tag{4.7}$$

It follows from (4.6) that since $\Theta_e\sim M^2$, the characteristic scale of change of h^2 is actually the scale of the electronic thermal conductivity $\Delta_{Te}\sim l/\varepsilon$. Thus, the width of the front of the actuating shock wave in a magnetized plasma, i.e., the dimension over which the magnetic field, temperature, velocity, etc., change, is of the order of the electronic thermal conductivity length. The magnetic field vector here in the scale l/ε executes a large number of rotations, of the order of $1/\delta\varepsilon$.

To get the solution, we eliminate Θ_i. As a result, we obtain

$$\Delta_{Te}\frac{d\Theta_e}{dx}=(1-\omega)\left(4\omega-1-\frac{3}{M_k^2}\right)-\frac{h^2-h_k^2}{M_{ak}^2}\left(\frac{5\omega}{2}-\frac{1}{M_{ak}^2}\right), \tag{4.8}$$

$$\frac{3}{2}\frac{d\Theta_e}{dx}+\frac{5M_k^2}{2M_{ak}^2}\omega\frac{dh^2}{dx}+\left\{\frac{5M_k^2}{3}\left[8\omega-3-\frac{3}{M_k^2}-\frac{5(h^2-h_k^2)}{2M_{ak}^2}\right]+\frac{\Theta_e}{\omega}\right\}\frac{d\omega}{dx}+\frac{2}{\Delta_r}\left[\Theta_e+\frac{5M_k^2}{3}\omega^2-\omega\left(\frac{5M_k^2}{3}+1\right)+\frac{5M_k^2}{6M_{ak}^2}\omega(h^2-h_k^2)\right]=0. \tag{4.9}$$

Thus, for a solution of the problem of the structure of the actuating shock wave in a magnetized plasma, there is a set of three equations: (4.6), (4.8), (4.9) for the functions ω, Θ_e, h^2.

In three-dimensional phase space of ω, Θ_e, h^2, these equations have three sinular points. The equilibrium state in the incoming flow at point 1:

$$\omega=1,\quad \Theta_e=1,\quad h^2=0$$

(the dimensionless variables are taken relative to point 1). Point 2, which corresponds to the actuating sound wave, has

$$\omega=1/M_{a1}^2,\quad \Theta_e=\Theta_2,\quad h^2=h_2^2,$$

where Θ_2 and h_2 are defined in (2.5) and (2.8). Point 3, which corresponds to the equilibrium state behind the front for a gas-dynamical shock wave with Mach number M_1 is given by

$$\omega=\omega_3=(M_1^2+3)/4M_1^2,$$

$$\Theta_e=\Theta_3=(5M_1^2-1)(M_1^2+3)/16M_1^2,\quad h^2=0.$$

The sought structure for the actuating shock wave is the integral curve of Eqs. (4.6), (4.8), and (4.9), which emerges from point 1 at $x=-\infty$ and enters into point 2 at $x=+\infty$.

We consider the characteristic equation for (4.6), (4.8) and (4.9) at the singular points. At point 1, we have

$$\left[\frac{\Delta_h^2}{\Delta_j(1)}k-2\left(1-\frac{1}{M_{a1}^2}\right)\right]\Big\{k^2\Delta_{Te}(1)\Delta_r(1)(5M_1^2-4)$$
$$-k\left[\frac{9}{2}\Delta_r(1)\left(1-\frac{1}{M_1^2}\right)-2\Delta_{Te}(1)\left(\frac{5M_1^2}{3}-1\right)\right]-6\left(1-\frac{1}{M_1^2}\right)\Big\}=0. \tag{4.10}$$

The first factor corresponds to a positive root, since $M_{a1}>1$, while one of the other two roots is positive and the other is negative. The two positive roots of Eq. (4.10) correspond to the two-dimensional surface $S_2^-(1)$ of the integral curves emerging from point 1. Since the point 1 is a saddle point on the plane $h^2=0$, only one integral curve emerges from it in the direction of growth of Θ_e and of decrease of ω.

The characteristic equation at the singular point 3 will be

$$\left[\frac{\Delta_h^2}{\Delta_j(1)}k-2\left(1-\frac{4M_1^2/M_{a1}^2}{M_1^2+3}\right)\right]\Big\{k^2\Delta_{Te}(3)\Delta_r(3)(5M_3^2-4)$$
$$-k\left[\frac{9}{2}\Delta_r(3)\left(1-\frac{1}{M_3^2}\right)-2\Delta_{Te}(3)\left(\frac{5M_3^2}{3}-1\right)\right]-6\left(1-\frac{1}{M_3^2}\right)\Big\}=0. \tag{4.11}$$

The dimensionless variables in the first factor here refer to state 1, and those in the second to state 3. The Mach number in the outgoing stream for the gas-dynamic shock wave is

$$M_3^2=\frac{M_1^2+3}{5M_1^2-1}<1\quad\text{at}\quad M_1>1.$$

The root of Eq. (4.11) corresponding to the first fact is negative in the region (2.6), which corresponds to the actuating shock wave. The other two roots are negative at $M_3^2>4/5$, ($M_1^2<19/15$) and have different signs at $M_3^2<4/5$ ($M_1^2>19/15$). Correspondingly, the integral curves entering into point 3 fill the three-dimensional region $S_3^+(3)$ at $M_1^2<19/15$ or the two-dimensional surface $S_2^+(3)$ at $M_1^2>19/15$.

The characteristic equation at point 2 (the dimensionless variables refer to state 2) will be

$$k^3(5M_2^2-4)\frac{\Delta_h^2\Delta_r(2)\Delta_{Te}(2)}{6\Delta_j(2)}+k^2\Big\{\frac{5M_2^2}{6}h_2^2\Delta_r(2)\Delta_{Te}(2)\frac{1+0.51h_2^2}{1+h_2^2}$$
$$+\frac{1}{2}h_2^2\frac{\Delta_r(2)\Delta_f(2)\Delta_h}{\Delta_j(2)}+\left(\frac{5M_2^2}{3}-1\right)\frac{\Delta_{Te}(2)\Delta_h^2}{3\Delta_j(2)}-\frac{3}{4}\left(1-\frac{1}{M_2^2}\right)\frac{\Delta_r(2)\Delta_h^2}{\Delta_j(2)}\Big\}$$
$$+k\Big\{h_2^2\frac{1+0.51h_2^2}{1+h_2^2}\left[\frac{5M_2^2}{9}\Delta_{Te}(2)-\frac{3}{4}\Delta_r(2)\right]$$
$$+\frac{\Delta_h^2}{\Delta_j(2)}\left(\frac{1}{M_2^2}-1\right)+2h_2^2\frac{\Delta_h\Delta_f(2)}{\Delta_j(2)}\Big\}-h_2^2\frac{1+0.51h_2^2}{1+h_2^2}=0. \tag{4.12}$$

It follows from (4.12) that at $M_2^2>4/5$ there is a single positive root and two negative roots, while at $M_2^2<4/5$ there are two positive and one negative root. Thus at $M_2^2>4/5$, the integral curves entering into point 2 fill the two-dimensional surface $S_2^+(2)$, while at $M_2^2>4/5$, only one integral curve $S_1^+(2)$ enters at point 2 in the direction of increase in h^2 and decrease in ω.

The picture of the field of the integral curves for the case $M_1^2<19/15$ (i.e., $M_2^2>4/5$ and $M_3^2>4/5$) is shown schematically in Fig. 5. The two-dimensional surfaces $S_2^-(1)$ and $S_2^+(2)$ intersect along the single integral curve 1-2, which represents the structure of the actuating shock wave. In this case, the shock front is formed entirely by the electronic thermal conductivity, by the electron-ion temperature relaxation, by the Hall effect, and by the frictional forces between the electron and ion components. The variables h^2, Θ_e, ω on the shock front change continuously, while the magnetic field vector (the transverse component) rotates in the plane of the front, completing a large number (of the order of $1/\varepsilon\delta$) of rotations in the shock layer [see Eq. (4.7)], i.e., the end of the magnetic field vector H describes a cone-like helix that expands behind the front and has a pitch of the order of Δ_h. The intersection of the three dimensional regions $S_3^+(3)$ with the two-dimensional surface $S_2^-(1)$ separates the part of this surface filled by integral curves which emerge from point 1 at $x\to-\infty$ and enter point 3 at $x\to+\infty$. The curve 1–3 on the plane $h^2=0$ represents the structure of the shock wave in a plasma without external fields at $M_2^2>4/5$ (or, what amounts to the same thing, the structure of a weak isomagnetic discontinuity, see Ref. 3). As is seen from Fig. 5, such an integral curve is not the only one; there is an infinite set of integral curves which connect the points 1–3 and fill the two-dimensional region $S_2^-(1)\cap S_3^+(3)$. The non-uniqueness of the structure of the gasdynamic shock wave 1–3 in the region of change of the Mach number (2.6) means instability of this wave.[12]

At $M_2^2<4/5$, a single integral curve of the system enters the singular point 2—the curve $S_1^+(2)$ (see Fig. 6). It can be shown that this integral curve does not lie on $S_2^-(1)$. Substituting (4.8) in (4.6) and (4.6) in (4.9), we find that at $M_2^2<4/5$ the factor in the curly brackets before $d\omega/dx$ changes sign in the transition from 1 to 2, i.e., on the corresponding integral curve

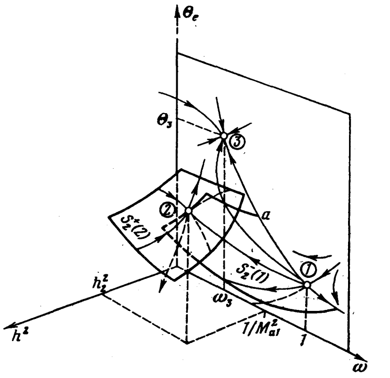

FIG. 5. Field of the integral curves of Eqs. (4.6), (4.8) and (4.9) in the space (ω, h^2, Θ_e) for $M_2^2>4/5$.

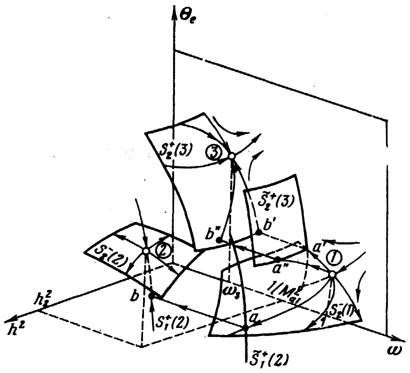

FIG. 6. Field of the integral curves of Eqs. (4.6), (4.8) and (4.9) in the space (ω, h^2, Θ_e) for $M_2^2 < 4/5$.

of the system (4.5), (4.8), (4.9), going from point 1 to point 2, the velocity ω is a double-valued function of the coordinate x, which is physically meaningless. Therefore, it is necessary to introduce an internal discontinuity—the "ion shock wave," which accomplishes the transition from $S_2^-(1)$ to $S_1^+(2)$. Since the scales of the ion viscosity and the ionic thermal conductivity are small in comparison with the scale of electronic thermal conductivity in the parameter ε, such a discontinuity—the isomagnetic discontinuity—is isothermal in the electron temperature and we have in it $\Theta = \text{const}$ accurate to small ε.

For the components of the ion viscous stress tensor that are dimensionless relative to state 1, we have

$$\frac{\pi_{xx}{}^i}{m_i n_1 v_1{}^2} = -\Delta_v \frac{(2-h^2)}{3} G, \quad \frac{\pi_{xy}{}^i}{m_i n_1 v_1{}^2} = -\Delta_v h_y G,$$
$$\frac{\pi_{xz}{}^i}{m_i n_1 v_1{}^2} = -\Delta_v h_z G,$$

where

$$G = \left[(2-h^2)\frac{d\omega}{dx} + 3\left(h_y \frac{d\lambda_i}{dx} + h_z \frac{d\mu_i}{dx} \right) \right] \frac{1}{(1+h^2)^2}.$$

Thus the equations of conservation of momentum of the plasma in the transverse direction will be

$$\Delta_v h_y G = \lambda_i - h_y/M_{a1}{}^2, \quad \Delta_v h_z G = \mu_i - h_z/M_{a1}{}^2,$$

whence we obtain

$$\frac{\lambda_i}{h_y} = \frac{\mu_i}{h_z} = \Delta_v G + \frac{1}{M_{a1}{}^2}. \tag{4.13}$$

Using the equality which follows from (4.13),

$$h_y \frac{d\lambda_i}{dx} + h_z \frac{d\mu_i}{dx} = \Delta_v h^2 \frac{dG}{dx} + O(\varepsilon),$$

we obtain the following equations for the structure of the viscous isomagnetic discontinuity:

$$(2-h^2)\frac{d\omega}{dx} + 3h^2 \frac{d(\Delta_v G)}{dx} = G(1+h^2)^2, \tag{4.14}$$

$$\omega - 1 + \frac{0.3}{M_1{}^2}\left(\frac{\Theta_e + \Theta_i}{\omega} - 2 \right) + \frac{h^2}{2M_{a1}{}^2} = \frac{1}{3}\Delta_v (2-h^2) G, \tag{4.15}$$

$$\frac{3}{2}\frac{d\Theta_i}{dx} + \frac{\Theta_i}{\omega}\frac{d\omega}{dx} = \frac{10}{9} M_1{}^2 G^2 (1+h^2)^2 + \frac{d}{dx}\left(\Delta_{Ti} \frac{d\Theta_i}{dx} \right). \tag{4.16}$$

The first equation here is a consequence of (4.13), the second is the conservation of momentum flux of the plasma (2.2). The last is the equation of ionic thermal conductivity. It is integrated once with the help of (4.14) and (4.15). As a result, we get in place of (4.16)

$$\Delta_{Ti}\frac{d\Theta_i}{dx} = \frac{3}{2}\Theta_i - \frac{5M_1{}^2}{3}\omega^2 + \omega\left(\frac{10M_1{}^2}{3} + 2 - \frac{5M_1{}^2 h^2}{3M_{a1}{}^2} \right) - \frac{5M_1{}^2}{3} h^2 \Delta_v{}^2 G^2 - \Theta_e \ln \omega + F, \tag{4.17}$$

where F is a constant of integration.

The boundary conditions for the ionic shock wave are the vanishing of all the derivatives in Eqs. (4.14), (4.15) and (4.17). We thus obtain

$$G = 0, \tag{4.18}$$

$$\omega - 1 + \frac{0.3}{M_1{}^2}\left(\frac{\Theta_e + \Theta_i}{\omega} - 2 \right) + \frac{h^2}{2M_{a1}{}^2} = 0, \tag{4.19}$$

$$\frac{3}{2}\Theta_i - \frac{5M_1{}^2}{3}\omega^2 + \omega\left(\frac{10M_1{}^2}{3} + 2 - \frac{5M_1{}^2 h^2}{3M_1{}^2} \right) - \Theta_e \ln \omega + F = 0. \tag{4.20}$$

Eliminating Θ_i from (4.19) and (4.20), we find the equation which connects the values of the velocity at the points a and b—the boundary points of the ionic shock wave:

$$\Theta_e \ln \frac{\omega}{\omega_b} = \frac{5M_1{}^2}{3}(\omega - \omega_b)\left[5 + \frac{3}{M_1{}^2} - \frac{5h^2}{2M_{a1}{}^2} - 4(\omega + \omega_b) \right]. \tag{4.21}$$

Equation (4.21) allows us to find for each point b in the space (h^2, Θ_e, ω) a point a from which we can go to the given point b through the ionic shock wave.

Thus a solution is constructed for $M_2^2 < 4/5$ in the following way. For each integral curve $S_1^+(2)$, we find the corresponding point a with the help of (4.21). These points form a certain curve $S_1^+(2)$ (Fig. 6). Let a be the intersection of $S_1^+(2)$ and $S_2^-(1)$, and b the point on S_1^+ corresponding to it. Then the structure of the actuating shock wave will be $1 - a - b - 2$ and will contain the ionic shock wave $a = b$ in it. In a strong shock wave (large M_1), the temperature of the ions in the internal discontinuity reaches values higher than equilibrium corresponding to the point 2; then a relaxational layer follows behind the point 2 (see similar solutions and the drawing for the structures of the shock waves in Ref. 3).

As in the previous case, a set of possible structures corresponds to the transition 1-3, which corresponds to a gasdynamic shock wave. For points of the surface $S_2^+(3)$, in accord with (4.21), we can construct another surface $S_2^+(3)$ the transition to which takes place on $S_2^+(3)$ through the ionic shock wave. The intersection of such surfaces $S_2^+(3)$ and $S_2^-(1)$ is the locus of points a "which appear at the beginning of the ionic shock wave in the structure $1 - a'' - b'' - 3$. In particular, the transition $1 - a' - b' - 3$ in the plane $h^2 = 0$ is always possible; it represents a shock wave in the plasma without external magnetic fields at $M_2^2 < 4/5$.

It can be seen from Fig. 1, [see also (2.7)], that at $M_2^2 < 4/5$ we also have $M_3^2 > 4/5$. If $M_2^2 > 4/5$ and $M_3^2 < 4/5$ (the region below the curve $M_2^2 = 4/5$ at $M_1^2 > 19/15$ on Fig. 1), then everything that has been said about the structure of the actuating shock wave remains without change. The vicinity of point 3 here will have the form shown in Fig. 6.

CONCLUSION

Thus, the structures of shock waves propagating in a plasma, along the magnetic field depend on the values of the Mach numbers M_a and M. In the region outside the "wedge" in Fig. 1, the shock wave is gas-dynamic and the magnetic field plays no role, generally dropping out of the equations. Inside the wedge, the gas-dynamical shock wave is unstable, and the shock wave is magnetohydrodynamic with front width equal either to the diffusion length of the magnetic field, $c^2/4\pi\sigma v_1$ in the case of an unmagnetized plasma, or to a length that is characteristic for the electronic thermal conductivity l/ε for a magnetized plasma. The characteristic size of the oscillations of the magnetic field is connected with the Hall terms and the thermal emf, leading to a dispersion of the magnetosonic waves propagating at an angle to the magnetic field; the corresponding scale is $\Delta \approx M\delta l/M_a^2 = c/\omega_{pi}$.

In the present work, we have not considered the problem of the stability of the actuating sound wave, which is discussed in a series of theoretical works (Refs. 12–14). In particular, it has been shown by Roikhvarger and Syrovatskii[14] that while the actuating wave is evolutionary, i.e., there exists for it a unique solution of the problem of small perturbations, the actuating shock wave is non-evolutionary in the linear approximations, i.e., i is unstable to the spontaneous emission of Alfven waves. The instability of the actuating shock wave is evidently connected with the fact that azimuthal symmetry of the original unperturbed flow is disrupted in it. An arbitrarily small azimuthal asymmetry ahead of the shock front removes such a degeneracy in the intermediate region. The solution of the problem of the stability of the actuating shock wave in such an arrangement is the object of a separate paper.

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Scattering and interaction of sound with sound in a turbulent medium

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The effect of developed hydrodynamic turbulence on sound is studied. The correlation time and length of the acoustic field, the isotropization length and the frequency diffusion coefficient for the acoustic wave packet are calculated. The region of applicability of the kinetic equation for sound with a linear dispersion law are found. The parameter $k_s LM$ (k_s is the sound wave vector, L is the energy-containing scale, M is the Mach number) is of interest in principle for solution of the aforementioned problems. Precisely this parameter determines whether the second-order perturbation theory is sufficient or an infinite set of diagrams must be summed (*i.e.*, transport must be taken into account) in studies of the interaction between sound and hydrodynamic turbulence.

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INTRODUCTION

Various aspects of the problem of the interaction of sound with hydrodynamic turbulence have been studied in a number of researches.[1-5] Thus, the propagation of sound in a turbulent atmosphere was considered in the work of Tatarskii[2] under conditions in which the principal role is played by processes of elastic scattering of a monochromatic sound wave; in our previous work,[3] processes of absorption and emission of sound by homo-

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