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The self-screening of classical Yang-Mills fields

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It is proved that any static system of Yang-Mills fields produced by charges without currents has Coulomb solutions. However, as a consequence of the lack of a uniqueness theorem, for given charges and asymptotic behavior at infinity there exists a multiplicity of solutions containing a "magnetic" field. Fields produced by an infinite uniformly charged plane are considered. Solutions containing a "magnetic" field and decaying at a distance of the order $\ell_0 = (gh/\sigma c)^{-1/2}$ have minimal energy.

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1. The intrinsic nonlinearity of Yang-Mills (YM) fields endows them with remarkable properties, leading to the hope that there is a possibility that they exist in reality. However, a serious difficulty is the macroscopic inobservability of the Yang-Mills fields if the theory contains long-range Coulomb solutions. In the present paper we shall show that owing to the nonlinearity of the YM equations, for the same charge distribution, in addition to the Coulomb fields there may exist a series of solutions containing a "magnetic" field (there is no uniqueness theorem). In the example we consider (a charged plane), the solution of minimal energy is found among the latter; this allows one to assume that something similar occurs also in more complicated cases: the fields become localized near the charges.

2. Static YM fields have been investigated by Khriplovich,[1] who considered nonlinear properties of the static fields produced by charges. Since in that paper it was assumed that the cause of the appearance of nonlinear solutions was the isotopic nonparallelism of charges producing the fields, Khriplovich has chosen as his object of investigation a relatively complicated two-particle problem, in which it was difficult to find solutions.

In fact, the nonparallelism of the isopins of the charges has no importance, since the relative orientations of isopin spaces at different points is arbitrary in a YM theory: it is meaningless to speak of parallelism or nonparallelism of sources situated at different points.

In order to illustrate this point we consider a static system of YM fields defined by the matrix equations

$$\operatorname{div}\mathbf{E}+[A_iE_i]=4\pi\rho,\qquad (\operatorname{rot}\mathbf{H})_i+e_{ijk}[A_jH_k]+[A_0E_i]=0. \tag{1}$$

In the absence of currents the source of the "magnetic" field is the commutator $[A_0E_i]$. We shall assume that both the vector and scalar potentials are expanded in terms of the generators of the gauge group:

$$A_0=\Phi^a(\mathbf{r})I^a,\quad A_i=a_i^a(\mathbf{r})I^a,\quad i=1,2,3.$$

In A_0 we separate the factor $\Phi(\mathbf{r})$—the absolute value of the potential and the unit vector $I(\mathbf{r})$ of the direction in isospin space

$$A_0(\mathbf{r})=\Phi(\mathbf{r})I(\mathbf{r}),\quad I(\mathbf{r})=u^{-1}(\mathbf{r})I_0u(\mathbf{r}),\quad \langle I_0^2\rangle=1.$$

Then

$$\partial_iI=[IB_i],$$

where

$$B_i=u^{-1}\partial_iu.$$

The corresponding field strength E_i has the form

$$E_i=-\partial_iA_0-[A_iA_0]=-I\partial_i\Phi-\Phi[(A_i-B_i)I].$$

If the vector potential is chosen as "longitudinal":

$$A_i=B_i=u^{-1}\partial_iu\quad (H=0), \tag{2}$$

we obtain

$$E_i=-(\partial_i\Phi)I$$

and the source of the "magnetic" field in the second equation of (1) is absent: the equation becomes an identity. The first of the equations (1) takes the form

$$\Delta\Phi I=-4\pi\rho.$$

At each point the quantities A_0 and ρ are isotopically parallel, confirming the time-independence of the solution. Thus, when the charges are not isotopically parallel there is necessarily present a vector potential which may be purely longitudinal (2): the system has Coulomb solutions. By means of the gauge transformation with the matrix $u(r)$ the isospins at different points have been parallelized) or in the case of groups more complicated than $SU(2)$, reduced to the Cartan subalgebra), which automatically annihilates the longitudinal vector potential.

This solution sheds light also on the problem of gauge-fixing in the YM theories.[2] By means of local gauge transformations a certain isovector field can be reduced to the Cartan subalgebra; this leaves a commutative group of local gauge transformations with generators in that subalgebra (the H-transformations). We expand the YM potential with respect to generators from the Cartan subalgebra (A_i^h) and those which are isotopically orthogonal to them (A_i^ω). We see that under H-transformations at each point of space the potential A_i^ω undergoes a rotation, and A_i^h is subject to a gauge transformation of the electro-magnetic type. Therefore for ultimate gauge-fixing one must impose a gauge condition on A_i^h (e.g., the Lorentz gauge)

$$\partial_i A_i^h = 0.$$

3. In the YM theory there is no analog of the uniqueness theorem of electrodynamics, on account of the nonlinearity of the equations. Therefore, for specified charges and boundary conditions at infinity there may exist also other solutions containing a "magnetic" field.

In order to investigate this possibility we consider a maximally simple system: a homogeneously charged plane.

We select the potential A_i in the form

$$A_0 = F(z)\tau_3, \quad A_1 = u(z)\tau_1, \quad A_2 = v(z)\tau_2, \quad A_3 = 0$$

(z is the distance from the plane, τ_i are matrices satisfying the commutation relations $[\tau_i \tau_j] = e_{ijk}\tau_k$, $i, j, k, = 1, 2, 3$)

$$E_1 = uF\tau_2, \quad E_2 = -vF\tau_1, \quad E_3 = -F'\tau_3,$$
$$H_1 = -v'\tau_2, \quad H_2 = u'\tau_1, \quad H_3 = uv\tau_3.$$

The YM equations for F, u, v can be obtained by extremizing the Lagrangian:

$$\mathscr{L} = \frac{1}{8\pi}\int_{-\infty}^{\infty} (F'^2 - u'^2 - v'^2 + F^2(u^2+v^2) - u^2v^2)\,dz + \sigma F(z_0),$$
$$F'' - (u^2+v^2)F = -4\pi\sigma\delta(z-z_0),$$
$$u'' + (F^2 - v^2)u = 0, \quad v'' + (F^2 - u^2)v = 0, \tag{3}$$

where σ is the charge density on the plane which is situated at z_0.

First of all, the system (3) has the purely Coulomb solution

$$u = v = 0, \quad F = -2\pi\sigma|z - z_0|.$$

In addition there are solutions with $v = 0$; outside the plane F and u satisfy the equations

$$F'' - u^2F = 0, \quad u'' + F^2u = 0, \quad F'(z_0) = -2\pi\sigma. \tag{4}$$

The condition that there be no currents in the plane leads to the requirement

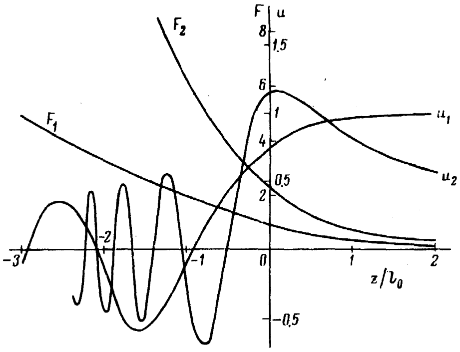

FIG. 1.

$$u'(z_0) = 0. \tag{5}$$

In distinction from the Coulomb solution the field energy per unit area can be finite for fields falling off at infinity. The equations (4) admit the following asymptotic behavior:

$$u|_{z\to\infty} = k, \quad F|_{z\to\infty} = Ce^{-kz}.$$

By means of the transformation

$$kz - \ln C \to z, \quad F \to kF, \quad u \to ku, \tag{6}$$

the boundary conditions can be reduced to the form

$$u|_{z\to\infty} = 1, \quad F|_{z\to\infty} = e^{-z}. \tag{7}$$

It follows from the equations (4) that F increases monotonically as one approaches the plane, whereas the function u oscillates. In Figure 1 the curves with the subscript 1 represent the graphs of the solutions of the system (4) calculated on a computer and showing that there exist a multitude of different solutions (in fact, infinitely many, as can be seen from a qualitative analysis of the system (4)) satisfying the boundary condition (5): the plane can be situated at any of the extrema of the functions u, and the solutions to the left of the plane are obtained by reflection of the solutions to the right of it. Since under the transformation (6) the charge is scaled by a factor k^2, a choice of k transforms the standard solutions with boundary conditions (7) into solutions with a given charge on the plane. The field energy per unit area is defined by the expression

$$\mathscr{E} = \frac{1}{4\pi}\int_{z_0}^{\infty} (\langle E\rangle^2 + \langle H^2\rangle)\,dz = \frac{1}{2\pi}\int_{z_0}^{\infty} (u'^2 + u^2F^2)\,dz,$$

changes proportionally to k^2 under the transformation (6), so that an invariant energetic characteristic is the quantity $\mathscr{E}(2\pi\sigma)^{3/2}$, listed in Table I for the first extrema in the first column. We see that from an energy point of view the most favorable is the first extremum.

The system (3) also has the solutions $u = v$ invariant with respect to all motions of the plane (with the appropiate isospin compensation in the sense of Ref. 3); in distinction from the preceding case which is not invari-

TABLE I.

Number of extremum	Asymmetric case	Symmetric case	Number of extremum	Asymmetric case	Symmetric case
1	0.2486	0.1234	4	0.4656	0.4090
2	0.3385	0.2666	5	0.5158	0.4572
3	0.4078	0.3481			

ant with respect to rotations of the plane, we have

$$F''-2u^2F=0, \quad u''+(F^2-u^2)u=0. \tag{8}$$

The fields which decrease at infinity have the following asymptotic behavior:

$$u=2^{1/2}/z, \quad F=kz^{-s}, \quad s=((17)^{1/2}-1)/2.$$

By means of a scale transformation the quantity k can be reduced to unity. The qualitative behavior of F and u in this case remains the same as for the preceding solution. The corresponding calcualtions carried out on a computer are represented in the figure by curves with the subscript 2. The values of the invariant energies for the first extrema are listed in the table.

Thus, the example we have considered proves the existence of multiple solutions for the YM equations for prescribed charges, solutions which differ from the Coulomb solution and contain a "magnetic" field, and illustrates the localization of the fields near the charges. The characteristic length of decay of the fields can be expressed in terms of the Planck constant and the fundamental YM charge g if one goes over from dimensionless quantities to dimensional ones:

$$l_0=\left(\frac{\hbar c}{g^2}\right)^{1/2}\left(\frac{g}{\sigma}\right)^{1/2}$$

and decreases as the charge density is increased. Such a dependence is essential for an analysis of fields created by a point charge if the latter is considered as the limit of a charged sphere of radius R as $R\to 0$.

Since

$$\sigma=Q/4\pi R^2,$$

the fields will fall off at distances

$$l_0=R(\hbar c/gQ)^{1/2} \tag{9}$$

and in the limit will become unobservable.

The expression (9) bears witness of the strongest self-screening of macroscopic charges (even if one assumes that there is no self-screening of elementary charges). Indeed, for elementary particles

$$\hbar c/g^2\approx 1.$$

When the charge is created by a macroscopic ($Qg\gg 1$) number of charges, the field can be observed in a layer of thickness $l_0 \ll R$ (which in fact justifies the use of the plane results for a macroscopic sphere).

*(*Translator's note*). For the gauge-fixing problem cf. also: I. M. Singer, Some remarks on the Gribov ambituity, Commun. Math. Phys. **60**, 7 (1978). M. F. Atiyah and J. D. S. Jones, Topological aspects of Yang-Mills Theory, Commun. Math. Phys. **61**, 97 (1978).

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Parity breaking effects in diatomic molecules

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It is shown that effects of nonconservation of time and space parities in molecules are considerably enhanced owing to the presence of closely spaced rotational levels of opposite parities. The enhancement factor of the intrinsic electric dipole moment of the electron reaches values of 10^7 to 10^{11}. The degree of circular polarization of the photons from allowed M1 transitions amounts to 10^{-3}, and the optical activity of molecular vapors to 10^{-7} rad/m. In such experiments, now quite feasible, the coupling constant for the weak interaction between the electronic vector current and the nucleonic axial-vector current can be measured. Experiments to measure this constant in heavy atoms are very complicated.

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1. INTRODUCTION

As is well known, diatomic molecules have very closely spaced levels of opposite parities. This is the so called Λ or Ω doubling of rotational levels with given total angular momentum J. In the present paper it is shown that a mechanism based on this close spacing of levels enhances T-odd and P-odd effects. Recently we have seen a paper by Labzovskiĭ[1] on the calculation of P-odd effects in molecules. In the part dealing with enhancement of these effects, our results in principle overlap to a considerable extent with those of Labsovskiĭ. An important difference is that we discuss experiments of a different kind, more promising in our opinion, and also consider the enhancement of the electric dipole moment (EDM) of the electron.

In molecules there is enhancement of the part of the weak interaction caused by the product of the electronic vector and the nucleonic axial-vector currents. This interaction gives a contribution proportional to Z^2 (Ref. 2). The enhancement factor for the EDM of the electron increases in proportion to Z^3 (Refs. 3-5). Therefore we shall consider molecules in which one of

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