

Let $V(0) = 0$ and $V(1) = -\beta J$; we then have from (47)

$$1 = C_0 + C_1, \quad e^{-\beta J} = C_0 - \frac{1}{3}C_1;$$

$$\exp\{V(0)\} = 4C_0 = 1 + 3e^{-\beta J}, \quad \exp\{V(1)\} = \frac{1}{3}C_1 = 1 - e^{-\beta J}.$$

Using these expressions we get from (26)

$$\beta^* = -\frac{1}{J} \ln \left(\frac{1 - e^{-\beta J}}{1 + 3e^{-\beta J}} \right). \quad (51)$$

The critical temperature is

$$\beta_c = J^{-1} \ln 3. \quad (52)$$

From (46) and (50) we easily obtain

$$Z(\beta) = [1/2(1 + 3e^{-\beta J})]^{20} Z(\beta^*). \quad (53)$$

We note in conclusion that the models $2DZ_3$ and $2DT$ are equivalent to Potts models of the first type (see Sec. 1) with $N = 3$ and 4 . For these models the phase transition point is known from other transformations of the partition function (see Refs. 6 and 20) and coincides with the results (34) and (52) obtained here as a result of the self-duality of these models.

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Additional localized degrees of freedom in spin glasses

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It is shown that besides the three propagating "acoustic" modes predicted by Halperin and Saslow (*Phys. Rev. B* **16**, 2154, 1977) there exist in spin glasses also localized zero-gap degrees of freedom connected with a system of uniformly distributed disclinations. In this connection, the Poisson-bracket method is used to derive nonlinear equations of motion, which generalize the linearized version presented in a preceding article by the authors [*J. Phys. (Paris)* **39**, 693, 1978]. The cited version of spin-glass description is furthermore extended to other systems and it is shown that it is in fact a variant of the renormalization-group method of Kadanoff and Wilson.

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1. INTRODUCTION

We have recently^[1] constructed a microscopic spin-glass theory wherein the spin glass is represented as consisting of balls of disclinations in a spin system, which are entangled in a complicated and disorderly manner (something recalling a dish of spaghetti). We have arrived at this picture by starting from the microscopic "frustration" concept^[1] developed to apply to spin glasses by Toulouse^[2] and Villain.^[3] We have shown that at the microscopic level such a magnet can be described by specifying at each point of space x a coordinate frame rigidly connected to a disclination ball located at this point, as well as a continuously distributed macroscopic disclination density. The orientation of the coordinate frame secured at the point x_i is specified in natural fashion by its rotation angle

$\varphi(x_i)$ in the "isotopic" space of the spin directions.^[2]

To describe the disclinations we introduce (see Ref. 1), in analogy with dislocation theory (see, e.g., Ref. 4), the quantity b_i . If the macroscopic disclination density is zero, then $b_i = \partial\varphi/\partial x_i$, so that the disclination density is (in the linear approximation)

$$\rho_i = e_{ikl} \partial b_l / \partial x_k, \quad (1)$$

where e_{ikl} is a unit antisymmetrical tensor. In analogy with the theory of plastic flow in elasticity (see, e.g., Ref. 4) we introduce also the disclination flux

$$j_i = \frac{\partial b_i}{\partial t} - \frac{\partial}{\partial x_i} \frac{\partial \varphi}{\partial t}. \quad (2)$$

A characteristic feature of the microscopic spin-glass theory based on the "frustration" concept is the use of local discrete invariance (LDI) of the exchange

forces. The LDI concept was introduced in the theory of spin glasses by Mattis (the so-called "Mattis model"^[5]). Its gist is (for more details see, e.g., Ref. 1) that the spin-system energy remains unchanged if we reverse simultaneously in the Heisenberg Hamiltonian the sign of one of the spins of the system and the signs of all the exchange integrals that connect this spins with all its neighbors. We have shown^[1] that at the macroscopic level the place of the LDI is taken by local exchange invariance (LEI). The LEI is in essence a localization of the continuous SO_3 group—the symmetry group of the exchange interaction. To realize the LEI it is necessary to introduce the so-called Yang-Mills fields^[6] (see also Ref. 1). It was shown that the corresponding Yang-Mills fields are the already mentioned quantities b_i and the field a , which coincides in the linear approximation (see below) with $\partial\varphi/\partial t$.

Since the LEI is the result of localization of the non-Abelian group SO_3 , the simple linear relations (1) and (2) for the density and flux of the disclinations, which are literal replicas of the formulas for the density and flux of dislocations, call for the nonlinear generalization^[1]

$$e_{ik}\rho_i = \frac{\partial b_k}{\partial x_i} - \frac{\partial b_i}{\partial x_k} - [b_i \times b_k], \quad (3)$$

$$j_i = \frac{\partial b_i}{\partial t} - \frac{\partial a}{\partial x_i} - [a \times b_i]. \quad (4)$$

In the absence of disclination density and flux, it follows directly from (3) and (4) that

$$b_i = \partial\varphi/\partial x_i, \quad a = \partial\varphi/\partial t. \quad (5)$$

A description of spin glass in terms of the angle φ in this last case was presented by Halperin and Saslow^[7] and also by Andreev.^[8] They constructed a simple theory of the oscillations in such a system and showed that the excitation spectrum consists of three "acoustic" modes with equal velocities. We, however,^[1] have shown subsequently that the presence of a disclination current in the system transforms these oscillations into dissipative modes at large wavelengths. Moreover, there are weighty grounds for assuming that additional "localized" zero-gap degrees of freedom exist in spin glasses besides the three Halperin-Saslow acoustic modes. This is directly indicated by results of a numerical calculation made by Walker and Walsted.^[9] An indirect confirmation of this same circumstance can be the numerical experiment of Vannimenus and Toulouse.^[10]

In the preceding paper^[1] we considered one of the possible mechanisms that lead to the appearance of additional zero-gap nonlocalized modes—the presence of a "quasi-long-range" order or, as we called it, an "a priori long-range order." We shall show here that the nonlinearity of the equations of motion lead, even in the absence of long-range order, i.e., for "simple" or "genuine" spin glass (see Ref. 1), to the appearance of additional albeit localized zero-gap degrees of freedom.

In the conclusion we mention also one spin-glass model that is described macroscopically in the same terms of disclination density as the "frustration" system. In addition, we discuss the physical meaning of

averaging over the position of the impurities or over the distribution of the exchange integrals, which are used in modern spin-glass theories.

2. DISCLINATION MODEL OF LOW-ENERGY LOCALIZED STATES OF SPIN GLASS

Let us dwell in greater detail on the aforementioned numerical experiments. The numerical experiment of Vannimenus and Toulouse^[10] has shown that the concentration transition from the ferromagnet state to the spin-glass state takes place at that ferromagnetic-bond concentration (more accurately, at that "frustration" concentration) at which the energy of the domain walls vanishes. The experiment is performed on a two-dimensional lattice of Ising spins. Extrapolation of the results of this experiment to a real three-dimensional spin glass (it would be interesting to carry out an analogous numerical experiment on a three-dimensional lattice of Heisenberg spins) leads to the conclusion that in the spin-glass state there is vanishing of not only the energy of the topological domain wall (the singular surface obtained by reversing the spin directions in one of the parts of the system separated by this surface), but also of the energy of a wall of nontopological character. For example, if in one of the system parts separated by the wall all spins are rotated through the same finite angle φ_0 about some axis, then such a nontopological surface can terminate on a singular line—a disclination. The energy per unit length of this disclination is proportional to φ_0^2 and can in principle be arbitrarily small. These disclinations therefore contribute to the low-energy states of the spin glass, and consequently the production and motion of such continuously distributed disclinations must be taken into account in the hydrodynamic equations.

A confirmation of this assumption concerning the low-energy states of spin glass can be discerned in the numerical experiment of Walker and Walsted.^[9] They considered a model of three-dimensional spin glass with a Ruderman-Kittel-Kosuya-Yosida (RKKY) interaction. Starting with an arbitrary initial spin configuration, they gradually decreased the energy of the system in the searches of the ground state, rotating each spin in the direction of the field exerted on the spin by all other spins, and repeating this procedure. It was noted there that the energy-decrease process (or the dissipation) proceeded in two stages. The spin system went over as if to a quasi-equilibrium state, in which each spin was directed in practice along its molecular field. This was followed by further decrease of the energy, whereby each spin together with its own molecular field was inclined by a rather large angle.

This can be interpreted in the following manner. Near the ground state of the spin glass there is a set of stationary (or quasi-equilibrium) states; this set is continuous in energy and in spin configuration. Each such quasi-equilibrium state can be represented as a ground state + a disclination system. In fact, if the disclination motion is hindered in the absence of dissipation (and we shall show subsequently that this is indeed the case), then the aggregate of disclination constitutes the stationary state of the system.

3. HYDRODYNAMIC FUNCTIONAL OF THE SPIN-GLASS ENERGY

We arrive thus at the problem of describing the hydrodynamics of a system with continuously distributed disclination, with account taken also of the dynamics of the disclinations themselves. The nondissipative hydrodynamic equations of spin glass can be obtained by using the Poisson brackets for the variables \mathbf{b}_i and \mathbf{a} , which enter in the effective Hamiltonian of the spin glass. Let us examine this Hamiltonian.

We note first that although the variable \mathbf{a} is not the time derivative of the rotation angle φ , since the system contains continuously distributed disclinations, none the less this is locally the case. Therefore \mathbf{a} is the local angular velocity of the spin system. Since the angular velocity is equivalent in its action on the spin to a magnetic field, the system has in the non-equilibrium state a magnetization

$$\mathbf{i} = \chi \mathbf{a} \quad (6)$$

(where χ is the magnetic susceptibility. The system of units is chosen such $\hbar = \gamma = 1$, where γ is the gyromagnetic ratio.) The magnetization makes the following contribution to the hydrodynamic energy of the system:

$$\int d^3r \frac{\mathbf{i}^2}{2\chi} = \int d^3r \frac{\chi \mathbf{a}^2}{2}.$$

Second, it is assumed that spin glass has a spin rigidity, i. e., the energy system depends on the gradients $\Delta_i \varphi$ of the local rotation like $\frac{1}{2} \rho_s (\nabla_i \varphi)^2$. In the presence of a system of continuously distributed disclination, the rotation vector φ can no longer be introduced, in analogy with the impossibility of introducing a displacement vector \mathbf{u}_i in elasticity theory if a dislocation distribution is present (see Ref. 4). In place of $\nabla_i \varphi$ we introduce the quantity \mathbf{b}_i , which plays the role of distortion in a solid. Since locally \mathbf{b}_i can always be represented as a gradient of a rotation angle, the dependence of the hydrodynamic Hamiltonian on \mathbf{b}_i is the same as on $\nabla_i \varphi$ in the absence of disclinations. We thus arrive at the following form of the hydrodynamic functional of the spin-glass energy:

$$H = \int d^3r \left(\frac{\mathbf{i}^2}{2\chi} + \frac{1}{2} \rho_s \mathbf{b}_i^2 \right) \quad (7)$$

(in our earlier paper^[1] we used the symbols λ and μ for the susceptibility χ and the rigidity ρ_s).

It is possible to add to the energy functional (7) higher powers of \mathbf{i} and \mathbf{b}_i , as well as terms that depend on the disclination density ρ_i , but these terms either make a small nonlinear contribution, or are of the next order in the time and space derivatives, and add only small increments of nonhydrodynamic character to the hydrodynamic equations.

4. POISSON BRACKETS

It is now necessary to determine the form of the Poisson brackets for the variables \mathbf{i} and \mathbf{b}_i in the Hamiltonian (7). The Poisson brackets for the components of the local-density vector of the spin angular momentum are well known, viz.,

$$\{i^\alpha(\mathbf{r}), i^\beta(\mathbf{r}')\} = -e^{\alpha\beta\gamma} i^\gamma(\mathbf{r}) \delta(\mathbf{r}-\mathbf{r}') \quad (8)$$

They can be obtained from the commutation relations for the components of the quantum-mechanical angular-momentum operator, which go over in the classical limit into the Poisson brackets. The commutation relations can be used to derive the remaining Poisson brackets.

To this end, we connected the variable \mathbf{b}_i with the spin flux density Π_i . The spin flux density Π_i is obtained in standard fashion. A coordinate-dependence transformation of rotation through an angle $\delta\varphi$ is carried out in the Hamiltonian of the system, and then

$$\Pi_i = \frac{\delta H}{\delta \nabla_i \delta \varphi} \quad (9)$$

Under this transformation, the variables that enter in H are transformed in the following manner:

$$\mathbf{i} \rightarrow \mathbf{i} + [\delta\varphi, \mathbf{i}], \quad \mathbf{b}_i \rightarrow \mathbf{b}_i + \nabla_i \delta\varphi + [\delta\varphi, \mathbf{b}_i], \quad (10)$$

therefore

$$\Pi_i = \rho_s \mathbf{b}_i \quad (11)$$

The commutation relations for the components of the spin flux density operator are easy to obtain. As a result we obtain in the classical limit the remaining Poisson brackets

$$\{i^\alpha(\mathbf{r}), b_i^\beta(\mathbf{r}')\} = (\delta^{\alpha\beta} \nabla_i - e^{\alpha\beta\gamma} b_i^\gamma(\mathbf{r})) \delta(\mathbf{r}-\mathbf{r}'), \quad (12)$$

$$\{b_i^\alpha(\mathbf{r}), b_i^\beta(\mathbf{r}')\} = 0. \quad (13)$$

In the derivation of the last relation we have neglected terms of the type

$$e^{\alpha\beta\gamma} \nabla_i \nabla_\lambda i^\gamma(\mathbf{r}) \delta(\mathbf{r}-\mathbf{r}'),$$

which are of high order in the gradients and introduce into the hydrodynamic equations small nonhydrodynamic increments of the same type as the increments due to the inclusion of terms with disclination density (of the type $\text{const} \cdot \rho_i^2$) in the hydrodynamic functional. The Poisson brackets (12) can be derived also without introducing the spin-flux operator Π_i . In fact, we know the law that governs the transformation of the variable \mathbf{b}_i upon rotation of the spin system of coordinates (see (10)), and consequently we know also the action of the rotation operator \mathbf{i} on this variable.

We note that although we used in the derivation of the Poisson brackets (12) and (13) the concrete form of the Hamiltonian (7), actually they do not depend on the form of the Hamiltonian.

5. EQUATIONS OF MOTION

Using the Poisson brackets (8), (12), and (13), we obtain the following hydrodynamic equations in the absence of dissipation:

$$\frac{\partial \mathbf{i}}{\partial t} = \{H, \mathbf{i}\} = \nabla_i \rho_s \mathbf{b}_i, \quad (14)$$

$$\frac{\partial \mathbf{b}_i}{\partial t} = \{H, \mathbf{b}_i\} = \nabla_i \frac{\mathbf{i}}{\chi} + \left[\frac{\mathbf{i}}{\chi} \times \mathbf{b}_i \right]. \quad (15)$$

As seen from (11), Eq. (15) represents the vanishing of the disclination flux density (4), i. e., the disclination flux can be only dissipative. Thus, in the absence of dissipation Eqs. (14) and (15) have stationary solu-

tions with arbitrary $\rho_i(\mathbf{r})$, i. e., with arbitrary distribution of the disclinations. This means that there exists a set, continuous in energy and in spin configuration, of stationary states near the ground state, where $\rho_i = 0$, as is indeed confirmed by the numerical calculation of Walker and Walsted.^[9] In addition to this degree of freedom, for each disclination distribution $\rho_i(\mathbf{r})$ there are the three acoustic spin-wave modes obtained in Refs. 7 and 8. The spectrum of these waves

$$\omega_{1,2,3}^2 = \rho_s q^2 / \chi \quad (16)$$

does not depend on $\rho_i(\mathbf{r})$ (if ρ_s is independent of ρ_i).

We consider now the influence of dissipation. As already shown,^[1] the main contribution to dissipation is made by the dissipative disclination flux $-2\nu\mathbf{b}_i$. We write out Eqs. (14) and (15) and take into account simultaneously this term and the external magnetic field. For this purpose we must add to the energy functional the term

$$- \int d^3r i\mathbf{H}$$

We obtain

$$\frac{\partial \mathbf{i}}{\partial t} + [\mathbf{H} \times \mathbf{i}] = \rho_s \nabla \cdot \mathbf{b}_i, \quad (17)$$

$$\frac{\partial \mathbf{b}_i}{\partial t} = \nabla_i \frac{\mathbf{i}}{\chi} + \left[\frac{\mathbf{i}}{\chi} - \mathbf{H} \right] \times \mathbf{b}_i - 2\nu \mathbf{b}_i. \quad (18)$$

We note that in a magnetic field the spin density \mathbf{i} is connected with the vector \mathbf{a} by the relation

$$\mathbf{i} = \chi(\mathbf{a} + \mathbf{H}) \quad (19)$$

rather than by the relation (6).

Because of the dissipative disclination flux $-2\nu\mathbf{b}_i$, the states with disclinations relax in accord with the law

$$\partial \rho_i / \partial t = -2\nu \rho_i, \quad (20)$$

and the acoustic modes are transformed in the long-wave limit (in the absence of a magnetic field) into diffusion modes (see Ref. 1)

$$\omega_{1,2,3} = -i\rho_s q^2 / 2\chi\nu. \quad (21)$$

In a magnetic field we have

$$\omega_{\parallel} = -\frac{i\rho_s}{2\chi\nu} q^2, \quad \omega_{\perp} = |\mathbf{H}| - \frac{i\rho_s}{\chi(2\nu - i|\mathbf{H}|)} q^2.$$

6. MODEL OF "ANTIFERROMAGNETIC" SPIN GLASS

In our preceding paper^[1] we considered two spin-glass models that could be macroscopically described in terms of Yang-Mills fields \mathbf{b}_i and \mathbf{a} —a Heisenberg lattice with random exchange integrals (lattice with "frustration") and an alloy with RKKY interaction between the magnetic impurities.

There is one other example of this kind, viz., a classical two-sublattice antiferromagnet in which "frustration" is reached not by introducing into the lattice randomly disposed bonds with ferromagnetic sign, but by purely mechanical disturbance of the lattice, i. e., by introducing dislocations into the lattice. The point

is, as shown by one of us,^[11] any dislocation in a two-sublattice³⁾ antiferromagnet is automatically a disclination for its spins. If the number of dislocations and of the disclinations generated by them is macroscopically large, then, averaging as before over the volumes that contain a sufficiently large number of disclinations, we again arrive at a description of matter in terms of the fields \mathbf{b}_i and \mathbf{a} , and perhaps in terms in antiferromagnetic quasi-long-range order (cf. Ref. 1).

This mechanism is remarkable also because it constitutes a well defined and controllable continuous (as a function of the dislocation density) transition from a perfect atomic and magnetic crystal into atomic and magnetic glass (an amorphous magnet).

7. CONCLUSION

One final remark. The universally accepted method of solving spin-glass problems⁴⁾ is to perform the calculations in two steps. The standard statistical-mechanics problem is first solved at a fixed realization of the disposition of the magnetic impurities or of the signs and magnitudes of the exchange integrals. Only then is the free energy averaged over the different realizations (see e. g., the principal papers on spin-glass theory^[13]). A duality of this kind was always somewhat unsatisfactory, since, on the one hand, we always expect "self-averaging" to take place in one manner or another in a correctly formulated theory; on the other hand, both theory and experiment deal with a sample in which one definite realization is actually significant.

Here and in our earlier paper^[1] we have adhered to another method, which is in fact the renormalization-group method in the Kadanoff-Wilson form. The point is that the physically conceivable realizations of the disposition of the magnetic impurities in the alloy and of the signs of the exchange integrals, or the dislocation distribution, are always such that the sample as a whole remains macroscopically homogeneous. Therefore, by starting with some definite realization and carrying out the Kadanoff-Wilson renormalization procedure, i. e., integrating over ever increasing volumes, we obtain at some stage a spatially homogeneous system whose energy is described by a Ginzburg-Landau-Wilson functional that depends on a certain number of relevant and irrelevant fields.

The choice of the relevant fields depends, of course, on the author's intuition and understanding of the physical and microscopic essence of the phenomenon. In particular, in choosing as the relevant variables the Yang-Mills fields \mathbf{b}_i and \mathbf{a} , we started with the microscopic "frustration" concept discovered by Toulouse and Villain.^[2,3] In addition, we included in consideration the ferromagnetic or antiferromagnetic types of the long-range order.

For spin glasses, unfortunately, this process, while explaining their low-temperature behavior, is not very useful when it comes to phase transition. The point is (see, e. g., Ref. 1 and the bibliography therein) that spin glasses, in the sense of a phase transition, consti-

tute either a case of a lower marginal dimensionality, or else their dimensionality is even lower than the original one. In these situations the Wilson approach yields practically nothing if it is necessary to resort to exactly solvable models.

In conclusion, it is our pleasant duty to thank Walker and Walsted for kindly supplying the results of their numerical experiment.

¹This term was apparently introduced by Anderson for the description of the situation in spin glasses (see, e.g., Ref. 2).

²We use in this article vector symbols ϕ , b_i , ρ_i , and others only for vectors in "isotopic" spin space. For the projections of these vectors we always use Greek superscripts: ϕ^α , b_i^α , etc. Latin subscript always denotes Cartesian coordinate, $x_i \equiv x, y, z$. The fact that in our theory the Latin and Greek indices are never mixed means that the interaction is of the exchange type.

³Dislocations actually generate disclinations in the spin system of not only the simplest two-sublattice antiferromagnets, but also of many-sublattice magnets, such as UO_2 (see, e.g., Ref. 12).

⁴Actually all the statements that follow hold also for any prob-

lem with random distribution of the impurities or with random bonds. For the sake of argument, however, we speak here of spin waves.

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Antiferromagnetic resonance in $\alpha\text{-Fe}_2\text{O}_3$ in the absence of an external magnetic field

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Antiferromagnetic resonance was investigated experimentally in the temperature interval 150-320 K and in the wavelength range 4.5-1.5 mm. It is shown that the experimental results are described by two different formulas for $T < T_M = 260.9$ K and $T > T_M$. The experimental results are used to calculate the temperature dependences of the two uniaxial-anisotropy constants within the framework of the generally accepted premises concerning the magnetic properties of $\alpha\text{-Fe}_2\text{O}_3$.

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Hematite ($\alpha\text{-Fe}_2\text{O}_3$) is an antiferromagnet that can have a weak ferromagnetic moment because of the Dzyaloshinskii interaction.^[1] Even though hematite has attracted the attention of many investigators (see the review^[2]), many of its important properties remain unclear to this day. In particular, we do not know the mechanism whereby anisotropy constant, as a function of temperature, acquires an anomalous behavior that leads to a phase transition from a easy axis state into an easy plane state. Nor can we explain the extremely small anisotropy constants (≈ 0.2 kOe) that follow from the prevailing theoretical premises concerning the magnetic properties of $\alpha\text{-Fe}_2\text{O}_3$.

From the dipole energy calculated by Arman *et al.*^[3] it follows that the dipole field is approximately 9 kOe; according to data on the EPR of Fe^{3+} in $\alpha\text{-Al}_2\text{O}_3$, the

one-ion contribution is about 7 kOe. It must be emphasized that the magnetodipole and one-ion contributions are of the same sign.

According to the accepted premises,^[2] the thermodynamic potential of $\alpha\text{-Fe}_2\text{O}_3$ can be expressed in the form^[1]

$$\Phi = 2M_0 \left[\frac{1}{2} EM^2 - \frac{1}{2} A_1 L_x^2 - \frac{1}{2} A_2 L_x^4 - D(M_x L_y - M_y L_x) \right], \quad (1)$$

where $\mathbf{M} = (\mathbf{M}_1 + \mathbf{M}_2)/2M_0$, $\mathbf{L} = (\mathbf{M}_1 - \mathbf{M}_2)/2M_0$, \mathbf{M}_1 and \mathbf{M}_2 are the sublattice magnetizations, $M_1^2 = M_2^2 = M_0^2$. For homogeneous small oscillations of the magnetic system about the equilibrium value we can calculate the frequencies (see, e.g., Ref. 2) of the antiferromagnetic resonance (AFMR) for the low-temperature ($T < T_M = 261$ K, $L_x \neq 0$) and high-temperature ($T > T_M$, $L_x = 0$) states: