A nonlinear theory of excitation of monochromatic waves in a magnetoactive plasma by a relativistic beam of charged particles

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A nonlinear theory of the excitation of monochromatic electromagnetic waves of arbitrary polarization in a magnetoactive plasma by a monoenergetic relativistic beam of low density is presented. It is shown that the beam-plasma interaction is most effective when the component of the phase velocity of the excited waves along the external magnetic field is close to the velocity of light (in vacuo). In this case the maintenance of Cerenkov or cyclotron resonance of the beam particles is enhanced. An energy comparable to the incident kinetic energy of the beam may be transferred to the plasma oscillations.

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1. INTRODUCTION

The interest in the problem of interaction of relativistic particle beams with the electromagnetic waves in a plasma is due to its potential use for the creation of powerful generators and amplifiers in the microwave range. The beam-plasma instability can also be used for heating plasmas to high temperatures (cf. Refs. 1–8).

It is known that interaction of monoenergetic relativistic beams with a plasma situated in an external magnetic field excites fast electromagnetic waves (with a phase velocity of the order of c) under conditions of Cerenkov and cyclotron resonance.[9,10] In the present paper we investigate the nonlinear stage of the interaction of a relativistic beam of low density with the electromagnetic waves in a magnetoactive plasma. It is assumed that the concepts of single-mode amplification regime are valid. The limitation of the exponential growth of the wave amplitude is due to capture of beam particles in the field of the amplified wave. It is proved that the condition of maintenance of synchronism between the amplified wave and the beam particles is improved if the component of the phase velocity of the wave along the external magnetic field is close to the velocity of light c. For Cerenkov resonance this is due in the relativistic case to a decrease of the component of the beam velocity along the magnetic field. Under cyclotron-resonance conditions the wave frequency in a reference frame moving with the beam is close to a harmonic of the gyroscope frequency of the beam particles. If the phase velocity of the wave is close to the velocity of light, the changes in frequency in a reference frame moving with the beam and the harmonics of the gyroscope frequency agree with each other, which leads to an improvement of the synchronism. The energy transfered by the beam to plasma oscillations becomes comparable to the incident kinetic energy of the relativistic beam.[1)]

2. THE BASIC EQUATIONS

We take the distribution function of the beam electrons in momentum space at the initial time $t=0$ in the form

$$f_b(\mathbf{r},\mathbf{p},t=0)=\frac{n_b}{2\pi p_{\perp 0}}\delta(p_z-p_{z0})\delta(p_\perp-p_{\perp 0}),\tag{2.1}$$

where n_b is the beam particle density, $p_\perp$ and p_z are the perpendicular and parallel components of the beam particle momentum, relative to the external magnetic field $\mathbf{B}_0$. The space charge and the electron current in the beam are assumed neutralized in the absence of oscillations. The space charge compensation may be achieved by an ion background and the compensation of the current of the primary beam may be due to a reverse current appearing when the beam is injected into the plasma. As shown in Ref. 11 a pulsed injection of a relativistic electron beam is accompanied by neutralization of the space charge.

We shall assume that at the initial time the amplitude of the wave attains its maximal value near a value $\mathbf{k}_0=(k_{x0},0,k_{z0})$ of the propagation vector. Then the frequency of the excited wave is close to $\omega_0=k_{z0}v_{z0}+n\omega_B(0)$ (n is an integer and $\omega_B(0)$ is the cyclotron frequency of the beam electrons, $\omega_B=eB_0/m_0c\Gamma$, calculated for $p_{z,\perp}=p_{z,\perp 0}$, m_0 is the electron mass and, $\Gamma=[1+(p_z^2+p_\perp^2)]^{1/2}$ is the relativistic factor. The amplitude and the phase of the wave change slowly with time over a characteristic scale γ_L^{-1}, where γ_L is the growth increment of the wave obtained in the linear theory.

In the derivation of the equations which describe the slow changes of the amplitude $\mathscr{E}_\sigma$ and phase α_σ of the wave with time we have used the averaging method (cf. Refs. 12, 13). The Maxwell equations imply

$$i\omega_0\lambda^\sigma(\varkappa_0)\mathscr{E}_\sigma(t)\exp[i\alpha_\sigma(t)]-\frac{\partial}{\partial\omega_0}(\omega\lambda^\sigma)\frac{d}{dt}(\mathscr{E}_\sigma\exp[i\alpha_\sigma])$$
$$=\frac{4en_b}{m_0}\int_{-\pi}^{\pi}d\Phi_{n0}\frac{W_n^\sigma(\varkappa_0)}{\Gamma}\exp(-i\Phi_n)$$
$$+i\frac{4en_b}{m_0}\frac{d}{dt}\int_{-\pi}^{\pi}d\Phi_{n0}\frac{1}{\Gamma}\frac{\partial W_n^\sigma}{\partial\omega_0}\exp(-i\Phi_n).\tag{2.2}$$

Here $\lambda^\sigma(\varkappa)$ is the eigenvalue of the operator[14]

$$\Lambda_{ij}=\frac{c^2k^2}{\omega^2}\left(\frac{k_ik_j}{k^2}-\delta_{ij}\right)+\varepsilon_{ij}(\varkappa)$$

and the letter $\varkappa$ denotes the set of quantities $(\omega,\mathbf{k})$. The electric field of the wave is determined by the following relation:

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$$\mathbf{E}(\mathbf{r},t)=\mathrm{Re}\sum_{\sigma}\exp[i(\mathbf{k}_0\mathbf{r}-\omega_0 t)]\left[\mathbf{f}^{\sigma}(\varkappa_0)+i\frac{\partial\mathbf{f}^{\sigma}}{\partial\omega_0}\frac{d}{dt}\right]\mathscr{E}_{\sigma}(t)\exp[i\alpha_{\sigma}(t)]. \tag{2.3}$$

In the case of a "cold" homogeneous plasma the polarization vector $\mathbf{f}^{\sigma}(\varkappa)$ can be represented in the form (without loss of generality):

$$\mathbf{f}^{\sigma}(\varkappa)=(e_x^{\sigma}(\varkappa),\ ie_y^{\sigma}(\varkappa),\ e_z^{\sigma}(\varkappa)),$$

where the quantities e_x, e_y, e_z are real. For $W_n^{\sigma}(\varkappa)$ we obtain

$$W_n^{\sigma}(\varkappa)=e_x^{\sigma}(\varkappa)\bar{p}_{\perp}\frac{n}{a}J_n(a)-e_y^{\sigma}(\varkappa)\bar{p}_{\perp}J_n'(a)+e_z^{\sigma}(\varkappa)\bar{p}_zJ_n(a), \tag{2.4}$$

where $J_n(a)$ is the Bessel function and $a=k_x\bar{v}_{\perp}/\bar{\omega}_B$.

The integration in the right-hand side of the expression (2.2) is over the initial values of resonance phase $\Phi_n=k_{x0}\xi+k_{z0}\bar{z}-n\bar{\theta}-\omega_0 t$, where ξ is the X-coordinate of the Larmor center of the particle, and θ is the angle in momentum space. The superior bar denotes an average over the rapid oscillations which occur when the particle moves in the wave field in a constant magnetic field. In the sequel we assume that the frequency of the amplified wave is close to the eigenfrequency $\omega(\mathbf{k})$ of plasma oscillations for a polarization $\beta(\lambda^{\beta}(\varkappa_0)\simeq 0)$. In this case the amplitudes of the oscillations corresponding to the two other polarizations are γ_L/ω_0 times smaller than $\mathscr{E}_{\beta}$. We note that the nonlinear character of the motion of the beam particles leads to the appearance of harmonics with propagation vectors $l\mathbf{k}_0$ and frequencies $l\omega_0$ (l an integer). The amplitudes of these harmonics are also γ_L/ω_0 times smaller than the amplitude $\mathscr{E}_{\beta}$ of the main amplified wave.

In order to obtain a complete set of equations it is necessary to add to (2.2) equations which describe the motion of the particle in the constant magnetic field and the field of the amplified electromagnetic wave. Making use of the field equations (2.3) we obtain the equations of motion of the beam particles for resonance condition $\omega_0-k_{z0}v_z\simeq n\omega_B$ ($n=0,\pm1,\ldots$) in the averaged variables $\bar{p}_z$, $\bar{p}_{\perp}$, Φ_n, similar to the way this was done in Ref. 13. The equations in which we are interested have the form:

$$\frac{d\bar{p}_z}{dt}=e\sum_{\sigma}\mathscr{E}_{\sigma}R_{p_z}^{n,\sigma}(\varkappa_0)\cos(\Phi_n+\alpha_{\sigma})-e\frac{d\mathscr{E}_{\beta}}{dt}\frac{\partial}{\partial\omega_0}R_{p_z}^{n,\beta}(\varkappa_0)\sin(\Phi_n+\alpha_{\beta})$$
$$-e\mathscr{E}_{\beta}\frac{d\alpha_{\beta}}{dt}\frac{\partial}{\partial\omega_0}R_{p_z}^{n,\beta}(\varkappa_0)\cos(\Phi_n+\alpha_{\beta})+\varphi_{p_z}+\psi_{p_z},$$
$$\frac{d\bar{p}_{\perp}}{dt}=e\sum_{\sigma}\mathscr{E}_{\sigma}R_{p_{\perp}}^{n,\sigma}(\varkappa_0)\cos(\Phi_n+\alpha_{\sigma})-e\frac{d\mathscr{E}_{\beta}}{dt}\frac{\partial}{\partial\omega_0}R_{p_{\perp}}^{n,\beta}(\varkappa_0)\sin(\Phi_n+\alpha_{\beta})$$
$$-e\mathscr{E}_{\beta}\frac{d\alpha_{\beta}}{dt}\frac{\partial}{\partial\omega_0}R_{p_{\perp}}^{n,\beta}(\varkappa_0)\cos(\Phi_n+\alpha_{\beta})+\varphi_{p_{\perp}}+\psi_{p_{\perp}},$$
$$\frac{d\Phi_n}{dt}=\Delta_n(\varkappa_0)-e\mathscr{E}_{\beta}\frac{k_{z0}}{m_0\omega_{B0}}R_{\Phi}^{n,\beta}(\varkappa_0)\sin(\Phi_n+\alpha_{\beta}), \tag{2.5}$$

where

$$R_{p_z}^{n,\sigma}(\varkappa)=\frac{k_z}{m_0\omega\Gamma}W_n^{\sigma}(\varkappa)-e_z^{\sigma}(\varkappa)\frac{\Delta_n(\varkappa)}{\omega}J_n(a),$$
$$R_{p_{\perp}}^{n,\sigma}(\varkappa)=\left(1-\frac{k_z\bar{v}_z}{\omega}\right)\frac{1}{\bar{p}_{\perp}}W_n^{\sigma}(\varkappa)+e_z^{\sigma}(\varkappa)\frac{\bar{v}_z}{\bar{v}_{\perp}}\frac{\Delta_n(\varkappa)}{\omega}J_n(a), \tag{2.6}$$
$$R_{\Phi}^{n,\sigma}(\varkappa)=\left(1-\frac{k_z\bar{v}_z}{\omega}\right)\frac{1}{a}\frac{\partial}{\partial\bar{p}_{\perp}}W_n^{\sigma}(\varkappa)+e_z^{\sigma}(\varkappa)\frac{\bar{v}_z}{\bar{v}_{\perp}}\frac{\Delta_n(\varkappa)}{\omega}J_n'(a),$$
$$\Delta_n(\varkappa)=n\bar{\omega}_B+k_z\bar{v}_z-\omega,\qquad \omega_{B0}=eB_0/m_0c.$$

The components φ_{pz} and $\varphi_{p\perp}$ describe the next approximation, quadratic in the field amplitude $\mathscr{E}_{\beta}$:

$$\varphi_{p_z,\perp}=-\frac{1}{2}e^2\mathscr{E}_{\beta}^2\sin2(\Phi_n+\alpha_{\beta})\sum_{\substack{r+s=2n\\ r,s\neq n}}\left\{R_{p_z}^{s,\beta}(\varkappa_0)\frac{\partial}{\partial\bar{p}_z}\left(\frac{R_{p_z,\perp}^{r,\beta}(\varkappa_0)}{\Delta_r(\varkappa_0)}\right)\right.$$
$$\left.+R_{p_{\perp}}^{s,\beta}(\varkappa_0)\frac{\partial}{\partial\bar{p}_{\perp}}\left(\frac{R_{p_z,\perp}^{r,\beta}(\varkappa_0)}{\Delta_r(\varkappa_0)}\right)+\frac{1}{\bar{p}_{\perp}}R_{p_z,\perp}^{r,\beta}(\varkappa_0)\frac{aR_{\xi}^{s,\beta}(\varkappa_0)+rR_{\theta}^{s,\beta}(\varkappa_0)}{\Delta_s(\varkappa_0)}\right\}, \tag{2.7}$$

where

$$R_{\theta}^{s,\beta}(\varkappa)=\left[e_x^{\beta}(\varkappa)\left(1-\frac{k_z\bar{v}_z}{\omega}\right)+e_z^{\beta}(\varkappa)\frac{k_x\bar{v}_z}{\omega}\right]J_s'(a)$$
$$-e_y^{\beta}(\varkappa)\left(1-\frac{k_z\bar{v}_z}{\omega}\right)\frac{s}{a}J_s(a)+e_y^{\beta}(\varkappa)\frac{k_z\bar{v}_{\perp}}{\omega}J_s(a), \tag{2.8}$$
$$R_{\xi}^{s,\beta}(\varkappa)=-e_y^{\beta}(\varkappa)\frac{\Delta_s(\varkappa)}{\omega}J_s(a).$$

The terms ψ_{pz} and $\psi_{p\perp}$ take into account the contribution to the equations of motion of the beam particles coming from the alternating fields of the harmonics with propagation vectors $l\mathbf{k}_0$ and frequencies $l\omega_0$:

$$\psi_{p_z,\perp}=e\sum_{\sigma,l>1}\mathscr{E}_{\sigma,l}R_{p_z,\perp}^{ln,\sigma}(l\varkappa_0)\cos(l\Phi_n+\alpha_{\sigma,l}), \tag{2.9}$$

where the quantities $R_{p_z,\perp}^{ln,\sigma}(ln_0)$ are defined by Eqs. (2.6), in which one must set $\varkappa=(l\omega_0,\ l\mathbf{k}_0)$. The amplitudes $\mathscr{E}_{\sigma,l}$ and the phases $\alpha_{\sigma,l}$ of the harmonics satisfy the equation

$$il\omega_0\lambda^{\sigma}(l\varkappa_0)\mathscr{E}_{\sigma,l}(t)\exp[i\alpha_{\sigma,l}(t)]=\frac{4en_b}{m_0}\int_{-\pi}^{\pi}d\Phi_{n0}\frac{W_{ln}^{\sigma}(l\varkappa_0)}{\Gamma}\exp[-il\Phi_n]. \tag{2.10}$$

Let us explain when it is necessary to take into account the terms $\varphi_{p_z,\perp}$ in the equations of motion (cf. Ref. 15). We obtain the following order-of-magnitude estimates for the increments $\delta p_{z,\perp}$ of the momenta and $\delta\Phi_n$ of the phase during the time interval t:

$$\delta p_{z,\perp}/p_{z,\perp}\sim\varepsilon\omega_0 t+\varepsilon^2\omega_0 t, \tag{2.11}$$

$$\delta\Phi_n\sim\left(\frac{\partial\Delta_n}{\partial p_z}\delta p_z+\frac{\partial\Delta_n}{\partial p_{\perp}}\delta p_{\perp}\right)t+\varepsilon\omega_0 t\sim(N_z^2-1)\varepsilon(\omega_0 t)^2+\varepsilon^2(\omega_0 t)^2+\varepsilon\omega_0 t, \tag{2.12}$$

where $N_z=k_{z0}c/\omega_0$, $\varepsilon=e\mathscr{E}_{\beta}k_0/\omega_0 B_0$ is the dimensionless electric field amplitude of the wave of fundamental polarization in the nonlinear stage. In the case of a low-density beam, at the saturation stage $\varepsilon\ll1$. The term in (2.12) which contains the factor N_z^2-1 is due to the first terms in the right-hand sides of (2.11). (In Eqs. (2.5) the corresponding expressions are $e\mathscr{E}_{\beta}R_{p_z,\perp}^{n,\beta}(\varkappa_0)\cos(\Phi_n+\alpha_{\beta})$.) A saturation of beam instability occurs at $\delta\Phi\sim1$. If $|N_z^2-1|\gtrsim1$ the saturation occurs at $t\sim\varepsilon^{-1/2}$. In this case one may neglect the second terms in (2.11) and the second and third terms in (2.12). If however $N_z^2\to1$, it follows from (2.12) that the saturation of the instability occurs at $t\sim\varepsilon^{-1}$. In this case one must retain in the equations of motion the corrections quadratic in the field amplitude $\mathscr{E}_{\beta}$ as well as the terms related to the change of the amplitude $\mathscr{E}_{\beta}$ and phase α_{β} with time, and take into account the fields of harmonics with frequencies which are multiples of ω_0.

Making use of the system (2.5) we obtain the equation for the change of the "detuning" $\Delta_n(\varkappa_0)$:

$$\frac{d\Delta_n}{dt}=e\sum_\sigma \mathscr{E}_\sigma R_\Delta^{n,\sigma}(\varkappa_0)\cos(\Phi_n+\alpha_\sigma)-e\frac{d\mathscr{E}_\beta}{dt}\frac{\partial}{\partial\omega_0}R_\Delta^{n,\beta}(\varkappa_0)\sin(\Phi_n+\alpha_\sigma)$$
$$-e\mathscr{E}_\beta\frac{d\alpha_\beta}{dt}\frac{\partial}{\partial\omega_0}R_\Delta^{n,\beta}(\varkappa_0)\cos(\Phi_n+\alpha_\sigma)+T_\Delta(\varkappa_0)$$
$$+e\sum_{\sigma,l>1}\mathscr{E}_{\sigma,l}R_\Delta^{ln,\sigma}(l\varkappa_0)\cos(l\Phi_n+\alpha_{\sigma,l}),\qquad(2.13)$$

where

$$T_\Delta(\varkappa)=\frac{n}{2}\frac{e^2\mathscr{E}_\beta^2}{m_0^2\Gamma^2}e_z^\beta(\varkappa)e_y^\beta(\varkappa)\frac{k_zk_x}{\omega^2}[J_n^2(a)-J_{n-1}(a)J_{n+1}(a)]\sin 2(\Phi_n+\alpha_\beta),\qquad(2.14)$$

$$R_\Delta^{ln,\sigma}(\varkappa)=\frac{\omega}{c^2}(N_z^2-1)\frac{W_{ln}^\sigma(\varkappa)}{m_0^2\Gamma^2}-\frac{\Delta_{ln}(\varkappa)}{m_0c\Gamma}\left[\frac{W_{ln}^\sigma(\varkappa)}{m_0c\Gamma}+N_ze_z^\sigma(\varkappa)J_{ln}(a)\right].\qquad(2.15)$$

The equation (2.13) allows one to draw conclusions on the conditions under which a resonance is maintained between the beam and the amplified wave.

We note that the quantities $W_n^\sigma(\varkappa)$ which enter in (2.2) and (2.5) characterize the change of the kinetic energy of the resonant particles under the action of the field of the wave, since

$$\frac{d}{dt}m_0c^2\Gamma\approx\frac{e}{m_0\Gamma}\sum_\sigma\mathscr{E}_\sigma W_n^\sigma(\varkappa)\cos(\Phi_n+\alpha_\sigma).\qquad(2.16)$$

Before giving the results of the nonlinear theory of interaction of the beam with the plasma oscillations, we discuss the results of the linear theory.

3. THE RESULTS OF THE LINEAR THEORY

During the linear stage of development of the oscillations the solution of the equations (2.12) and (2.5) can be assumed to have the form

$$\mathscr{E}_\beta(t)=\mathscr{E}_{\beta0}e^{\gamma t},\quad \alpha_\beta(t)=\alpha_{\beta0}+\sigma t.\qquad(3.1)$$

From (2.2) and (2.5) it is not difficult to obtain the dielectric tensor for the beam.[9]

The dispersion relation for the plasma oscillations in the presence of a beam of oscillators has the form

$$\left.\frac{\partial\lambda^\beta}{\partial\omega_0}\right|_{\omega_0=\omega(\mathbf{k}_0)}(\omega_0-\omega(\mathbf{k}_0)+i\gamma-\sigma)=(N_z^2-1)\left(\frac{W_n^\beta(\varkappa_0)}{m_0c\Gamma}\right)^2\frac{\omega_b^2}{(i\gamma-\sigma)^2}$$
$$+\Big[2\left(1-\frac{k_{z0}v_z}{\omega_0}\right)\frac{1}{p_\perp}W_n^\beta(\varkappa_0)\frac{\partial}{\partial p_\perp}W_n^\beta(\varkappa_0)$$
$$+2N_z\frac{W_n^\beta(\varkappa_0)}{m_0c\Gamma}\frac{\partial}{\partial p_z}W_n^\beta(\varkappa_0)-(N_z^2+1)\left(\frac{W_n^\beta(\varkappa_0)}{m_0c\Gamma}\right)^2$$
$$+2(N_z^2-1)\frac{W_n^\beta(\varkappa_0)}{m_0^2c^2\Gamma^2}\omega_0\frac{\partial W_n^\beta}{\partial\omega_0}\Big]\frac{\omega_b^2}{\omega_0(i\gamma-\sigma)},\qquad(3.2)$$

where $\omega_b^2=4\pi e^2n_b/m_0\,\Gamma$. The quantities $W_n^\beta(\varkappa_0)$, Γ and v_z in Eq. (3.2) are computed at $p_z=p_{z0}$ and $p_\perp=p_{\perp0}$.

If N_z^2 is not close to unity, the oscillations grow with the linear increment ($\omega_0\approx\omega(\mathbf{k}_0)$):

$$\gamma_1=\frac{\sqrt{3}}{2}\left[\frac{\omega_b^2}{\partial\lambda^\beta/\partial\omega_0}(N_z^2-1)\left(\frac{W_n^\beta(\varkappa_0)}{m_0c\Gamma}\right)^2\right]^{1/3}_{z,\perp=p_{z,\perp0}},\qquad(3.3)$$

$$\gamma_1/\omega_0\ll|N_z^2-1|\,\omega_0^2a^2/k_0^2c^2.\qquad(3.3a)$$

If however $N_z^2\simeq1$, the linear increment has the following form ($\omega_0\simeq\omega(\mathbf{k}_0)$):

$$\gamma_2=\omega_b\Bigg|\frac{2}{\omega_0\partial\lambda^\beta/\partial\omega_0}\Big[\left(1-\frac{k_{z0}v_z}{\omega_0}\right)\frac{W_n^\beta(\varkappa_0)}{p_\perp}\frac{\partial}{\partial p_\perp}W_n^\beta(\varkappa_0)$$
$$+\frac{W_n^\beta(\varkappa_0)}{m_0c\Gamma}\frac{\partial}{\partial p_z}W_n^\beta(\varkappa_0)-\left(\frac{W_n^\beta(\varkappa_0)}{m_0c\Gamma}\right)^2\Bigg|^{1/2}_{p_{z,\perp}=p_{z,\perp0}},\qquad(3.4)$$

$$\gamma_2/\omega_0\gg|N_z^2-1|\,\omega_0^2a^2/k_0^2c^2.\qquad(3.4a)$$

We note that the inequalities (3.3a) and (3.4) which determine the condition of applicability of the apropriate expressions for the increments of the growth of oscillations have been obtained for $a\lesssim1$, $k_{x0}\gtrsim k_{z0}$, $k_{z0}v_z<\omega_0$, $\omega_0\sim n\omega_B$. An instability arises if the following inequality holds

$$\Big[\left(1-\frac{k_{z0}v_z}{\omega_0}\right)\frac{W_n^\beta(\varkappa_0)}{p_\perp}\frac{\partial}{\partial p_\perp}W_n^\beta(\varkappa_0)+\frac{W_n^\beta(\varkappa_0)}{m_0c\Gamma}\frac{\partial}{\partial p_z}W_n^\beta(\varkappa_0)$$
$$-\left(\frac{W_n^\beta(\varkappa_0)}{m_0c\Gamma}\right)^2\Big]_{p_{z,\perp}=p_{z,\perp0}}<0.\qquad(3.5)$$

In the case $|N_z^2-1|\,\omega_0^2a^2/k_0^2c^2\sim\gamma_2/\omega_0$ and $|\omega_0-\omega(k_0)|\sim\gamma_2$, the magnitude of the linear increment is determined by the estimate $\gamma\sim\gamma_2$.

The fundamental equations (2.2) and (2.5) have been obtained under the assumption that the resonance condition $\omega_0-k_{z0}v_z\simeq n\omega_B$ is satisfied only for a single value of n, i.e., that there is no overlap of neighboring resonance regions. For this one has to require satisfaction of the inequality

$$\omega_B>\gamma_{1(2)}.\qquad(3.6)$$

In the case $|N_z^2-1|\,\omega_0^2=a^2/k_0^2c^2\gg\gamma_1/\omega_0$ we obtain from (3.6) and (3.3) the following estimate:

$$\frac{\omega_{B0}}{\omega_0}\gg\left|\frac{\omega_b^2}{\omega_0^2}\Gamma^3\frac{N_z^2-1}{\omega_0\partial\lambda^\beta/\partial\omega_0}\left(\frac{W_n^\beta(\varkappa_0)}{m_0c\Gamma}\right)^2\right|^{1/3}_{p_{z,\perp}=p_{z,\perp0}},\qquad(3.7)$$

and for the case $|N_z^2-1|\,\omega_0^2a^2/k_0^2c^2\ll\gamma_2/\omega_0$ we have from (3.6) and (3.4)

$$\frac{\omega_{B0}}{\omega_0}\gg\Bigg|\frac{\omega_b^2}{\omega_0^2}\frac{\Gamma^2}{\omega_0\partial\lambda^\beta/\partial\omega_0}\Big[\left(1-\frac{k_{z0}v_z}{\omega_0}\right)\frac{W_n^\beta(\varkappa_0)}{p_\perp}\frac{\partial}{\partial p_\perp}W_n^\beta(\varkappa_0)$$
$$+\frac{W_n^\beta(\varkappa_0)}{m_0c\Gamma}\frac{\partial}{\partial p_z}W_n^\beta(\varkappa_0)-\left(\frac{W_n^\beta(\varkappa_0)}{m_0c\Gamma}\right)^2\Big]\Bigg|^{1/2}_{p_{z,\perp}=p_{z,\perp0}}.\qquad(3.8)$$

4. THE NONLINEAR THEORY OF BEAM INSTABILITY

We now construct a nonlinear theory of beams instability. The system of equations (2.2) and (2.5) describes the capture of beam particles by the field of the amplified wave, which leads to a stabilization of the instability.

If $|N_z^2-1|\,\omega_0^2a^2/k_0^2c^2\gg\gamma_1/\omega_0$ the equations of motion (2.5) imply that the oscillation frequency of the particles captured by the wave near the centers $\bar p_{z,\perp}=p_{z,\perp c}$ in the phase plane $(\bar p_z,\Phi_n)$ or $(\bar p_\perp,\Phi_n)$ equals

$$\Omega_{tr1}=\left|\frac{e\omega_0\mathscr{E}_\beta}{m_0c\Gamma}(N_z^2-1)\frac{W_n^\beta(\varkappa_0)}{m_0c\Gamma}\right|^{1/2}_{\bar p_{z,\perp}=p_{z,\perp c}},\qquad(4.1)$$

and the sizes of the capture region Δp_z and $\Delta p_\perp$ have the order of magnitude for the Cerenkov resonance ($n=0$)

$$\Delta p_z\sim\left|\frac{e\mathscr{E}_\beta m_0}{k_{z0}}\Big[e_z^\beta(\varkappa_0)J_0(a)\right.$$
$$\left.-e_y^\beta(\varkappa_0)\frac{k_{z0}\bar v_\perp}{\omega_0}J_0'(a)\Big]\Gamma\right|^{1/2}\left(1-\frac{\bar v_z^2}{c^2}\right)^{-1/2},\qquad(4.2)$$
$$\Delta p_\perp\sim\frac{e\mathscr{E}_\beta}{\omega_0}e_y^\beta(\varkappa_0)J_0'(a)$$

and for the cyclotron resonances ($n\neq0$)

$$\frac{\Delta p_z}{\bar p_z}\sim\frac{N_z^2}{N_z^2-1}\frac{\Omega_{tr1}}{k_{z0}v_{z0}},$$
$$\frac{\Delta p_\perp}{\bar p_\perp}\sim\frac{N_z^2}{N_z^2-1}\frac{k_{x0}^2}{k_{z0}^2}\frac{n}{a^2}\frac{\Omega_{tr1}}{\bar\omega_B}.\qquad(4.3)$$

One can obtain an estimate for the amplitude of the electric field of the oscillations during the nonlinear stage if one takes into account that saturation of the oscillations occurs when the frequency of oscillations of the captured beam particles becomes equal in order of magnitude to the linear growth increment. Comparing the expressions (3.3) and (4.1) we obtain

$$\mathscr{E}_\beta \sim (4\pi m_0 c^2 n_b \Gamma)^{1/2} \left(\frac{\omega_b}{\omega_0}\right)^{1/3} (N_z^2-1)^{-1/3} \left(\frac{W_n^\beta(\varkappa_0)}{m_0 c\Gamma}\right)^{1/3}. \tag{4.4}$$

As can be seen from the expressions (4.2) and (4.3), in the case $|N_z^2-1|\omega_0^2 a^2/k_0^2c^2 \gg \gamma_1/\omega_0$ the quantities $\bar{p}_z$ and $\bar{p}_\perp$ vary little under the action of the oscillations $\Delta p_{z,\perp} \ll \bar{p}_{z,\perp}$. This circumstance allows one to neglect the changes of $\bar{p}_z$ and $\bar{p}_\perp$ in the right-hand side of (2.2) as well as in the expressions (2.6). Then one can drop from the equations of motion (2.5) the small terms proportional to $d\mathscr{E}_\beta/dt$ and $d\alpha_\beta/dt$, as well as the terms $\varphi_{p_{z,\perp}}$, and $\psi_{p_{z,\perp}}$. In the equation which describes the change of the phase Φ_n one may neglect the last term, which is proportional to the field amplitude.

The system of equations (2.2) and (2.5) reduce to the universal equation[6]:

$$\frac{d\varepsilon}{d\tau} = \chi \int_{-0,5}^{0,5} d\zeta_0 \cos(2\pi\zeta+\alpha),$$

$$\varepsilon\left(\delta - \frac{d\alpha}{d\tau}\right) = \chi \int_{-0,5}^{0,5} d\zeta_0 \sin(2\pi\zeta+\alpha), \tag{4.5}$$

$$\frac{d\nu}{d\tau} = \varepsilon\cos(2\pi\zeta+\alpha), \quad \frac{d\zeta}{d\tau} = \frac{\nu}{2\pi},$$

where we have introduced the dimensionless variables:

$$\zeta = \frac{\Phi_n}{2\pi}, \quad \nu = \gamma_0^{-1}(n\bar{\omega}_B - \omega_0 + k_{z0}\bar{v}_z), \quad \tau = \gamma_0 t,$$

$$\varepsilon = \frac{e\mathscr{E}_\beta(t)}{m_0\Gamma\gamma_0^2}\frac{\omega_0}{c}(N_z^2-1)\frac{W_n^\beta(\varkappa_0)}{m_0 c\Gamma}\bigg|_{\bar{p}_{z,\perp}=p_{z,\perp 0}},$$

$$\gamma_0 = \left|2\frac{\omega_b^2}{\partial\lambda^\beta/\partial\omega_0}(N_z^2-1)\left(\frac{W_n^\beta(\varkappa_0)}{m_0 c\Gamma}\right)^2\right|^{1/3}_{\bar{p}_{z,\perp}=p_{z,\perp 0}}, \tag{4.6}$$

$$\chi = \mathrm{sgn}(N_z^2-1), \quad \delta = \frac{\omega_0 - \omega(\mathbf{k}_0)}{\gamma_0}, \quad \alpha = \alpha_\beta.$$

The energy of the beam is transferred to the plasma oscillations more effectively if the condition $N_z^2 \to 1$ is satisfied. The change of the momenta $\bar{p}_z$ and $\bar{p}_\perp$ under the action of the field of the wave is increased in this case (cf. Eqs. (4.2) and (4.3)) and for $|N_z^2-1| \times\omega_0^2 a^2/k_0^2c^2 \lesssim \gamma_2/\omega_0$ and $\Delta p_{z,\perp} \sim \bar{p}_{z,\perp}$. For the oscillation frequency in the phase plane of the captured particles we obtain from (2.5):

$$\Omega_{tr2}^2 = \bigg| \pm e\mathscr{E}_\beta(N_z^2-1)\frac{\omega_0 W_n^\beta(\varkappa_0)}{m_0^2c^2\Gamma^2}$$
$$+ e^2\mathscr{E}_\beta^2\frac{k_{z0}}{m_0\omega_{B0}}\left\{\frac{1}{m_0\Gamma}\left(\frac{k_{z0}}{\omega_0}\frac{\partial}{\partial\bar{p}_z}W_n^\beta(\varkappa_0)\right.\right.$$
$$\left.+\frac{1}{c^2}\frac{W_n^\beta(\varkappa_0)}{m_0\Gamma}\right) + \frac{W_n^\beta(\varkappa_0)}{\bar{p}_\perp}\left[\frac{k_{z0}\bar{v}_\perp}{\omega_0}\frac{\partial}{\partial\bar{p}_z}\right.$$
$$\left.\left.+\left(1-\frac{k_{z0}\bar{v}_z}{\omega_0}\right)\frac{\partial}{\partial\bar{p}_\perp}\right]\right\}\left(1-\frac{k_{z0}\bar{v}_z}{\omega_0}\right)\frac{1}{a}\frac{\partial}{\partial\bar{p}_\perp}W_n^\beta(\varkappa_0)$$
$$+ e^2\mathscr{E}_\beta^2 n\frac{k_{x0}k_{z0}}{\omega_0^2}\frac{e_z^\beta(\varkappa_0)e_y^\beta(\varkappa_0)}{m_0^2\Gamma^2}[J_{n-1}(a)J_{n+1}(a)-J_n^2(a)]\bigg|_{\substack{\bar{p}_{z,\perp}=p_{z,\perp c}\\ \varkappa=\varkappa_0}}. \tag{4.7}$$

For the amplitude of the electric field of the amplified wave in the saturation stage we obtain from the condition $\gamma_2 \sim \Omega_{tr2}$, with γ_2 and Ω_{tr2} defined respectively by (3.4) and (4.7), the following estimate:

$$\mathscr{E}_\beta \sim (4\pi n_b m_0 c^2\Gamma)^{1/2}. \tag{4.8}$$

The field amplitude $\mathscr{E}_\beta$ determined by Eq. (4.8) can substantially surpass the value (4.4).

As $N_z^2 \to 1$ the initial system of equations (2.2), (2.5) becomes rather complicated. A significant simplification can be achieved if one uses Eq. (2.13) and introduces in place of $\Delta_n(\varkappa_0)$ a new independent variable Δ. One then obtains a closed set of equations for the quantities $\bar{p}_z$, $\bar{p}_\perp$, Φ_n, Δ, $\mathscr{E}_\beta$, and α_β:

$$i\omega_0\lambda^\beta(\varkappa_0)\mathscr{E}_\beta(t)e^{i\alpha_\beta(t)} - \frac{\partial}{\partial\omega_0}(\omega\lambda^\beta)\frac{d}{dt}(\mathscr{E}_\beta e^{i\alpha_\beta})$$
$$= \frac{4en_b}{m_0}\int_{-\pi}^{\pi} d\Phi_{n0}\frac{W_n^\beta(\varkappa_0)}{\Gamma}e^{-i\Phi_n} + i\frac{4en_b}{m_0}\frac{d}{dt}\int_{-\pi}^{\pi} d\Phi_{n0}\frac{\partial W_n^\beta}{\partial\omega_0}\frac{e^{-i\Phi_n}}{\Gamma},$$
$$\frac{d\bar{p}_z}{dt} = e\mathscr{E}_\beta\frac{k_{z0}}{\omega_0 m_0\Gamma}W_n^\beta(\varkappa_0)\cos(\Phi_n+\alpha_\beta),$$
$$\frac{d\bar{p}_\perp}{dt} = e\mathscr{E}_\beta\frac{n\bar{\omega}_B}{\omega_0\bar{p}_\perp}W_n^\beta(\varkappa_0)\cos(\Phi_n+\alpha_\beta), \tag{4.9}$$
$$\frac{d\Delta}{dt} = e\mathscr{E}_\beta R_\Delta^{n,\beta}(\varkappa_0)\cos(\Phi_n+\alpha_\beta)$$
$$- e\mathscr{E}_\beta\frac{d\alpha_\beta}{dt}\frac{\partial}{\partial\omega_0}R_\Delta^{n,\beta}(\varkappa_0)\cos(\Phi_n+\alpha_\beta)$$
$$- e\frac{d\mathscr{E}_\beta}{dt}\frac{\partial}{\partial\omega_0}R_\Delta^{n,\beta}(\varkappa_0)\sin(\Phi_n+\alpha_\beta) + T_\Delta(\varkappa_0),$$
$$\frac{d\Phi_n}{dt} = \Delta - e\mathscr{E}_\beta\frac{n\bar{\omega}_B}{\omega_0\bar{p}_\perp}\frac{\partial}{\partial\bar{p}_\perp}W_n^\beta(\varkappa_0)\sin(\Phi_n+\alpha_\beta).$$

The appearance of the parameter N_z^2-1 in (2.13), in the terms related to the field harmonics with frequency $l\omega_0 (l>1)$, and in the terms corresponding to nonresonant polarizations, makes possible the transition to the system of equations (4.9) which contains the amplitude and phase of only the fundamental amplified wave with polarization β.

5. CONCLUSION

The enhancement of the effectiveness of the interaction between the beam and plasma as $N_z^2 \to 1$ is due to the fact that as $N_z^2 \to 1$ the conditions for maintaining resonance between the excited wave and the beam particles is improved (cf. Eq. (2.13)).

In the case of the Cerenkov resonance ($n=0$) this is due to the fact that as $N_z^2 \to 1$ the variation of the quantity $\bar{v}_z$ is reduced (cf. Ref. 4). This is possible only for ultrarelativistic beams with $\Gamma \gg 1$.

For the cyclotron resonances ($n \neq 0$) as $N_z^2 \to 1$ there occurs a more coordinated change of the quantity $k_{z0}\bar{v}_z$ and $n\bar{\omega}_B$. The improvement of the conditions of maintenance of cyclotron resonances as $N_z^2 \to 1$ leads to the fact that even for moderately relativistic beams ($\Gamma \sim 1$) an energy of the order of the initial kinetic energy of the beam is transferred to the plasma oscillations. This is in agreement with the known fact that for fast waves ($\omega \sim kc$) the relativistic effects turn out to be essential even for nonrelativistic particles.

In spite of the fact that as $N_z^2 \to 1$ the linear increment decreases, the energy transferred to the beam by the plasma oscillations increases, owing to the improved conditions for resonance maintenance. When one considers the spatial amplification of the wave there will also appear an enhancement of the effectiveness of the beam-plasma interaction as $N_z^2 \to 1$.

In the case of a resonance on the first harmonic this phenomenon was noted in Ref. 2 for waves which propagate along an external magnetic field.

We note that the coordinated change of the quantities $k_z v_z$ and ω_B has made it possible earlier to consider the problem of autoresonant particle acceleration in a vacuum ($N_z = 1$) in the field of a circularly polarized wave.[16,17]

It follows from Ref. 12 that a nonrelativistic beam interacts most effectively with potential oscillations of a plasma in the case of transverse propagation ($k_{z0} \ll k_{x0}$). The results of Ref. 12 can be obtained from Eqs. (4.9) in the limit $k_{z0} \ll k_{x0}$, $\omega_0 \ll k_0 c$ (cf. the inequality (3.4a)).

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Influence of a plasma on the interaction of laser radiation with a metal

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An investigation was made of the behavior of a plasma jet and recoil pressure during the action of quasicontinuous millisecond pulses from a neodymium laser on the surface of lead in air. Instability of the laser-beam maintenance of a plasma in the stream of the evaporated target materails was observed near the radiation intensity $I \approx 2$ MW/cm^2. The plasma appearance threshold was less than the intensity needed to maintain it continuously. The plasma jet instability was manifested by periodic detachment of the jet from the target surface and accompanied by oscillations of the recoil pressure $p(t)$. Characteristics of the behavior of $p(t)$ indicated that the effective optical thickness of the jet was $\sigma \lesssim 1$.

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1. Experimental and theoretical investigations of high-temperature characteristics of metals are difficult to carry out. Estimates indicate[1,2] that, for the majority of metals, the critical parameters T_c and p_c lie in the range of temperatures and pressures which are practically inaccessible under static experimental conditions. The use of laser radiation is one of the promising methods of maintaining and investigating high-temperature states of metals but the basic potentialities of this method have not yet been put into practice. However, before this is done, it is necessary to know the relative importance of the various physical properties which occur in the region of interaction of strong radiation with a metal.

Intense evaporation of a metal under the action of laser radiation in air is usually accompanied by the appearance of a plasma jet above the target surface.

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