

helix on the concentration.

For example, in the propagation of ultrasound in the CLC mixture, additional relaxation damping should be observed. This is connected with the fact that the period of the mixture (or the inverse quantity  $q_m$ ) is determined by the concentration. Thus, in the propagation of the ultrasound, in addition to the density fluctuations, there is a comparatively slow relaxation (connected with the diffusion) of the pitch of the helix. In the usual fashion,<sup>[7]</sup> we find the following for the complex wave vector  $\kappa$ :

$$\kappa = \frac{\omega}{c_0} + i \frac{\omega^2 \tau}{2c_0^3} (c_\infty^2 - c_0^2), \quad \omega\tau < 1, \quad (38)$$

$$\kappa = \frac{\omega}{c_\infty} + i \frac{c_\infty^2 - c_0^2}{2\tau c_\infty^3}, \quad \omega\tau > 1; \quad (39)$$

here  $\omega$  is the frequency of the ultrasound,  $\tau$  is the characteristic diffusion time of establishing the equilibrium,  $c_0$  and  $c_\infty$  are the velocities of ultrasound at  $\omega\tau \ll 1$  and  $\omega\tau \gg 1$ , respectively.

We note that the diffusion times  $\tau$  differ significantly for directions along and transverse to the axis of the helix, while the natural anisotropy of the elastic properties of pure CLC is very small ( $\sim 10^{-6}$ ). In correspondence with this, the dispersion curves for propagation of ultrasound along and transverse to the axis of the CLC helix should also differ.

The presence of an additional (in relation to pure CLC) hydrodynamic variable (concentration) leads to a change in the spectrum of the collective modes. A more detailed consideration of this question goes beyond the framework of this paper. Here we shall only show the qualitative consequences. Along with ordinary sound

and second sound, which is connected with the compressibility along the axis of the CLC (the modulus  $K_{22}q^2$ ) a damped diffusion mode appears. Depending on the relation of the constant of interaction between the components of the mixture, the character of this latter mode can change from furthermodiffusion of the slipping of one component relative to the other. The structure of the modes also depends on the orientation of the wave vector of the mode relative to the axis of the CLC. All these effects can be observed in principle experimentally, for example, by Brillouin scattering. The real estimate depends on the values of the many parameter of elasticity and viscosity, which are unknown at present for most of the CLC mixtures.

The author expresses his sincere gratitude to I. E. Dzyaloshinskii for discussion of the work and useful criticism.

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## On the possibility of existence of a spin-glass phase in amorphous magnets

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(Submitted 2 February 1978)  
*Zh. Eksp. Teor. Fiz.* **74**, 2331-2341 (June 1978)

It is shown that in amorphous magnets with "rotating" anisotropy of the easy-axis type of a spin-glass phase can exist. The conditions for the existence of such a phase are discussed. The phase diagram for the ferromagnetic, paramagnetic, and spin-glass phases is derived.

PACS numbers: 75.50.Kj, 75.30.Gw

### 1. INTRODUCTION

In this paper we consider the properties of one class of amorphous magnets—alloys of rare-earth (Tb, Dy, Ho) and transition (Fe, Co) metals. These alloys possess a number of unusual magnetic properties; e.g., the magnetization at zero temperature does not

attain the maximum possible value. To explain these phenomena a model was proposed<sup>[1]</sup> according to which each magnetic atom is acted upon by a crystal field that is random in direction but constant in magnitude, leading to anisotropy of the easy-axis type. We shall call this anisotropy rotating anisotropy. Subsequently, Mössbauer-effect experiments<sup>[2]</sup> on these substances

basically confirmed this model. At the same time it was found that the anisotropy constant is large compared with the exchange forces.

In a paper by Harris and Zolner<sup>[3]</sup> it was concluded on the basis of numerical calculations that a spin-glass phase can exist in such magnets. However, it was concluded incorrectly that the existence of this phase depends only on the magnitude of the anisotropy constant and does not depend on the other parameters of the system. Since the problem was solved numerically, the physical mechanism leading to the appearance of such a phase remained unclear. In the present paper we shall consider this problem analytically and shall discuss in detail the physical mechanism leading to the appearance of the spin-glass phase. It will be shown that its existence depends not only on the magnitude of the anisotropy but also on the number of nearest neighbors in the lattice, this dependence being different near the Curie point and at low temperatures. Since these substances are amorphous, it is almost certain that the exchange integral in them also fluctuates. If we take this into account, it turns out that the existence of a spin-glass phase also depends on the size of these fluctuations. The phase diagram of the existence of the paramagnetic, ferromagnetic, and spin-glass phases is also considered in the paper. The properties of the ordered phases, especially near the phase boundaries, are studied.

## 2. MECHANISM OF FORMATION OF THE SPIN GLASS

In order that a spin glass can be formed, it is necessary (but by no means sufficient) that the Hamiltonian of the system be constructed in such a way that in the ordered phase of the system there exist moments directed against the average magnetization. If there are no such moments, there cannot be a spin glass. In ordinary spin glasses such moments arise because of the presence of negative exchange integrals. In our case we have only positive exchange integrals, and, therefore, it would seem that these substances should necessarily be ferromagnets. Nevertheless, as we now show, there is another mechanism, which is connected only with the presence of rotating anisotropy and which, even for ferromagnetic exchange, leads to the appearance of moments directed against the average magnetization.

The interaction Hamiltonian for our system has the form

$$H = - \sum_{ij} J_{ij} s_i s_j - D \sum_i (n_i \cdot s_i)^2, \quad (1)$$

where  $J_{ij}$  is the exchange integral,  $D$  is the anisotropy constant, and  $s_i$  is the spin and  $n_i$  the unit vector of the anisotropy axis for the  $i$ -th atom. We shall assume  $n_i$  to be a random vector, and  $J_{ij}$  to be a random, Gauss-distributed quantity with the parameters

$$\langle J \rangle = J_0, \quad \langle J^2 \rangle - \langle J \rangle^2 = J_1^2. \quad (2)$$

Since in the alloys studied the spins of the rare-earth elements are very large (e.g., we can have  $s = 6, 15/2$ ), we shall assume the spin to be classical.

For simplicity we shall consider first the case of

zero temperature and absence of fluctuations of the exchange integral; i.e., we shall assume that  $T = 0$  and  $J_1 = 0$ . At  $T = 0$  all the spins are completely frozen and are in a position of equilibrium, determined by the condition for the minimum of the energy

$$E = - \sum_{ij} J_{ij} m_i m_j - D \sum_i (n_i \cdot m_i)^2, \quad (3)$$

$$m_i = \langle s_i \rangle_T,$$

where  $m_i$  is the local magnetization. We shall show now that there exist moments for which it is energetically favorable to be directed against the average magnetization  $M$ . For such moments it is obvious that

$$m_i \cdot M < 0. \quad (4)$$

We shall consider the energy of any given moment:

$$E_i = -h_i \cdot m_i - D (n_i \cdot m_i)^2, \quad (5)$$

$$h_i = 2 \sum_j J_{ij} m_j.$$

In order to simplify the problem further, we shall assume that  $D = \infty$ . In this case the moment  $m_i$  will point along the anisotropy axis  $n_i$ , but in such a way that  $h_i \cdot m_i > 0$ . We shall assume that it is energetically favorable for all the nearest neighbors of the  $i$ -th moment to be pointing parallel to  $M$ ; i.e., for all  $j$  the quantity  $m_j \cdot M > 0$ . If, for even just one moment, this is not so, we have already obtained the condition (4) for this moment and our statement is proved. If all the  $m_j \cdot M > 0$ , it is clear that  $h_i \cdot M > 0$  also (see Fig. 1). We note that, because of fluctuations,  $h_i$  is by no means parallel to  $M$ . Since it is the interaction with  $h_i$ , and not with  $M$ , that appears in  $E_i$  in (5), it is obvious that, for all directions of the anisotropy axis  $n_i$  that lie in the shaded sectors formed by the perpendiculars to  $M$  and  $h_i$  in Fig. 1, the condition (4) will be fulfilled for the corresponding moments  $m_i$ .

Thus, we see that even at zero temperature there exist moments directed against the average magnetization. It is entirely clear that all our simplifying assumptions ( $T = 0, J_1 = 0, D = \infty$ ) only make the picture clearer but do not affect the conclusion about the existence of moments directed against  $M$ . It is easy to show that there are always such moments in the ordered phase. It is precisely the presence of such moments that leads to the formation of the spin glass. For the glass to be formed it is necessary, crudely speaking, that there be sufficiently many such moments. The present paper is devoted to elucidating the conditions under which this occurs.

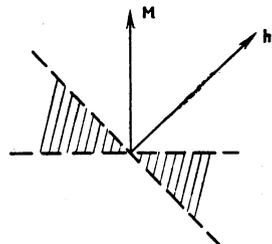


FIG. 1.

### 3. DERIVATION OF THE BASIC EQUATIONS

We shall solve the problems using the method proposed in a paper by the author<sup>[4]</sup> for the derivation of equations for the distribution functions  $f_1(\mathbf{m})$  and  $f(\mathbf{q})$  of the local moments and local molecular fields in the Bethe-Peierls approximation. The solution of these equations will give the answer to the problem posed. First of all it is necessary to make one proviso. Previously,<sup>[4]</sup> the corresponding equations were written for the Ising model. Here we have a Heisenberg interaction. However, it is clear<sup>[4]</sup> that for  $J_0 \ll T$  these equations go over into the usual molecular-field equations. It is more or less clear that in this region we can write the same equations for the Heisenberg interaction too. This is what we shall do.

The equations written below for the local magnetizations and molecular fields are simply the molecular-field equations for these quantities. However, as is well known (see, e.g., Ref. 5), the Bethe-Peierls approximation differs from the molecular-field approximation in this region in that the quantity  $z-1$  appears in place of the number  $z$  of nearest neighbors. Therefore, to simplify the account, we shall proceed as follows. First, we shall obtain all the results in the molecular-field approximation, and, in the section devoted to the discussion of the application of these results to real systems, we shall remember this difference and replace  $z-1$ . We note also that, as can be seen from (3) and (5), our equations are also true for  $T=0$ . Below we shall see that the results for  $T=0$  and for  $J_0 \ll T \ll T_c$  coincide. Therefore, they can also be extrapolated into the region  $T \sim J_0$ .

The molecular-field equations for the local magnetic moments  $\mathbf{m}_i$  and local dimensionless molecular fields  $\mathbf{q}_i$  have the form

$$\mathbf{q}_i = \frac{\mathbf{h}_i}{T_i} = \sum_j \alpha_{ij} \mathbf{m}_j(\mathbf{q}, D, \mathbf{n}_i),$$

$$\mathbf{m}_i(\mathbf{q}, D, \mathbf{n}_i) = \frac{\text{Sp}_s[\mathbf{s} \exp(-H_i/T)]}{\text{Sp}_s[\exp(-H_i/T)]} \quad (6)$$

$$H_i/T = -\mathbf{q}_i \beta - D(\mathbf{n}_i \beta)^2/T, \quad \alpha_{ij} = 2J_{ij}/T.$$

The first formula in (6) gives the equation for the local molecular fields, and the second formula determines the local magnetization in terms of the local molecular field and the local direction of the anisotropy axis.

We now introduce the distribution functions  $f(\mathbf{q})$  and  $f_1(\mathbf{m})$  of the local molecular fields and local magnetizations, respectively, and their Fourier transforms  $F(\mathbf{k})$  and  $F_1(\mathbf{k})$ :

$$f(\mathbf{q}) = \int \frac{d\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\mathbf{q}} F(\mathbf{k}) \quad (7)$$

and analogously for  $f_1(\mathbf{m})$ . Then, by the method developed in our previous paper,<sup>[4]</sup> we obtain from (6) the following equation for  $f(\mathbf{q})$  and  $F(\mathbf{k})$ :

$$F(\mathbf{k}) = \left[ \int \frac{d\mathbf{n}}{4\pi} \int dJ p(J) \int d\mathbf{q} f(\mathbf{q}) \exp\left(-\frac{2i\mathbf{k}J}{T} \mathbf{m}(\mathbf{q}, D, \mathbf{n})\right) \right]^2, \quad (8)$$

$$p(J) = \frac{1}{(2\pi)^{3/2} J_i} \exp\left[-\frac{(J-J_0)^2}{2J_i^2}\right].$$

Here the function  $\mathbf{m}(\mathbf{q}, D, \mathbf{n})$  is defined by the second

formula in (6). Since  $z \gg 1$  always, (8) is easily solved. For  $z \gg 1$ ,  $F(\mathbf{k})$  in (8) is calculated by the method of steepest descents; as a result, we obtain

$$F(\mathbf{k}) = \exp\{-izk_z M \alpha_0 - \frac{1}{2} z [(k_x^2 + k_y^2) \alpha_2^2 Q_1 + k_z^2 (\alpha_2^2 Q_0 - \alpha_0^2 M^2)]\},$$

$$f(\mathbf{q}) = (2\pi z)^{-3/2} [Q_1 \alpha_2^2 (Q_0 \alpha_2^2 - M^2 \alpha_0^2)^{-1/2}]^{-1}$$

$$\times \exp\left\{-\frac{q_x^2 + q_y^2}{2z \alpha_2^2 Q_1} - \frac{(q_z - z \alpha_0 M)^2}{2z (\alpha_2^2 Q_0 - \alpha_0^2 M^2)}\right\},$$

$$\alpha_2^2 = \alpha_0^2 + \alpha_1^2, \quad \alpha_0 = 2J_0/T, \quad \alpha_1 = 2J_1/T, \quad (9)$$

$$M = \int f(\mathbf{q}) d\mathbf{q} \int \frac{d\mathbf{n}}{4\pi} m_z(\mathbf{q}, D, \mathbf{n}),$$

$$Q_0 = \int f(\mathbf{q}) d\mathbf{q} \int \frac{d\mathbf{n}}{4\pi} m_z^2(\mathbf{q}, D, \mathbf{n}),$$

$$Q_1 = \int f(\mathbf{q}) d\mathbf{q} \int \frac{d\mathbf{n}}{4\pi} m_x^2(\mathbf{q}, D, \mathbf{n}).$$

The formulas (9) are written in a system of coordinates in which the  $z$  axis is parallel to the average-magnetization vector  $\mathbf{M}$ . It can be seen from (9) that for  $z \gg 1$  the molecular fields are Gauss-distributed with different variances parallel and perpendicular to  $\mathbf{M}$ . This distribution is characterized by the three parameters

$$M = \langle m_z \rangle, \quad Q_0 = \langle m_z^2 \rangle, \quad Q_1 = \langle m_x^2 \rangle,$$

which are the basic parameters of the theory, to be determined. We shall write equations for them below. If as a result of solving these equations we obtain

$$M=0, \quad Q_0 \neq 0, \quad Q_1 \neq 0; \quad (10)$$

this will imply the existence of a spin-glass phase. In the ferromagnetic phase all three parameters are non-zero, while in the paramagnetic phase all three parameters are equal to zero.

To solve Eq. (9) we need to know the explicit form of the integrals of  $m_x(\mathbf{q}, D, \mathbf{n})$ ,  $m_z^2$ , and  $m_x^2$  over  $\mathbf{n}$ . To calculate these quantities it is convenient to introduce the distribution function of the magnetization in a uniform dimensionless magnetic field  $q$  and a random rotating field:

$$f_1(\mathbf{m}, \mathbf{q}) = \int \frac{d\mathbf{n}}{4\pi} \delta(\mathbf{m} - \mathbf{m}(\mathbf{q}, D, \mathbf{n})). \quad (11)$$

Then, from (9) and (11) we obtain

$$M = \int m_z f_1(\mathbf{m}) d\mathbf{m}, \quad Q_0 = \int m_z^2 f_1(\mathbf{m}) d\mathbf{m}, \quad (12)$$

$$Q_1 = \int m_x^2 f_1(\mathbf{m}) d\mathbf{m}, \quad f_1(\mathbf{m}) = \int f_1(\mathbf{m}, \mathbf{q}) f(\mathbf{q}) d\mathbf{q}.$$

It is clear that the function  $f_1(\mathbf{m})$  appearing in (12) is the distribution function of the local magnetizations. From (9) and (12) it is clear that to solve Eqs. (9) we need to know the function  $f_1(\mathbf{m}, \mathbf{q})$ . We turn now to its calculation.

### 4. CALCULATION OF $f_1(\mathbf{m}, \mathbf{q})$

The calculation of  $f_1(\mathbf{m}, \mathbf{q})$  for arbitrary values of the parameters is difficult. Therefore, we shall calculate it for two particular cases: first, for arbitrary values of the anisotropy constant  $D$  and  $sq \ll 1$ , and, secondly, for  $D = \infty$  ( $y = Ds^2/T \gg qs, y \gg 1$ ) and arbitrary  $q$ . This will be sufficient for our purposes.

First we consider the first case. For  $sq \ll 1$  it is

possible to perform an expansion in  $q$  in the definition (6) of the function  $\mathbf{m}(\mathbf{q}, D, \mathbf{n})$ . It is convenient to perform this expansion first in a coordinate frame in which the  $z$  axis is parallel to the anisotropy axis and then transform to an arbitrary coordinate frame. As a result, from (6) and (11) we obtain

$$f_1(\mathbf{m}, \mathbf{q}) = \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^1 dx \delta\{m_x - m_0 \cos(\psi + \psi_0)\} \\ \times \delta\{m_y - m_0 \sin(\psi + \psi_0)\} \delta\{m_z - 1/3 s^2 q (c \cos \vartheta - b \sin \vartheta \cos \varphi)\}, \\ b(x) = 1/2(3a-1)x(1-x^2)^{1/2}, \quad c(x) = 1 + 1/2(3x^2-1)(3a-1), \\ a(y) = \left[ \int_0^1 \xi^2 d\xi e^{y\xi^2} \right] \left[ \int_0^1 d\xi e^{y\xi^2} \right]^{-1}, \quad y = Ds^2/T, \\ \text{tg } \psi_0(x, \varphi) = \frac{b \sin \varphi}{c \sin \vartheta + b \cos \vartheta \cos \varphi} \\ m_0(q, x, \varphi) = 1/3 s^2 q [b^2 \sin^2 \varphi + (c \sin \vartheta + b \cos \vartheta \cos \varphi)^2]^{1/2}, \\ \mathbf{q} = (q \sin \vartheta \cos \psi, \quad q \sin \vartheta \sin \psi, \quad q \cos \vartheta).$$

In the derivation of (13) we assumed the spin to be classical, i.e., we put  $\mathbf{s} = s\mathbf{p}$ , where  $\mathbf{p}$  is a unit vector. Everywhere below, we also assume the spin to be classical. The formula (13) is written in an arbitrary coordinate frame. The Cartesian coordinates of the vector  $\mathbf{q}$  in this frame are written out for convenience in the last line of (13). We note that  $\varphi$  and  $\arccos x$  are the polar coordinates of the unit vector of the anisotropy axis in a coordinate frame in which the  $z$  axis points along the vector  $\mathbf{q}$ . The integration over  $x$  runs from 0 to 1, and not from  $-1$  to  $+1$ , because the anisotropy axis is not a vector and all  $x < 0$  are equivalent to the corresponding values  $x > 0$ .

We now write  $f_1(\mathbf{m}, \mathbf{q})$  for the important particular case when the  $z$  axis is parallel to the vector  $\mathbf{q}$ ; then  $\vartheta = \psi = 0$  and in (13) we can integrate over  $\varphi$ . As a result we obtain

$$f_1(\mathbf{m}, \mathbf{q}) = \frac{1}{2\pi m_0} \int_0^1 dx \delta\left\{m_x - \frac{s^2}{3} qb(x)\right\} \delta\{m_z - 1/3 s^2 qc(x)\}, \\ m_0^2 = m_x^2 + m_y^2. \quad (14)$$

Below we shall be interested in the function  $f_1(\mathbf{m})$ , which is obtained by averaging  $f_1(\mathbf{m}, \mathbf{q})$  over the distribution function (9), which depends only on  $|\mathbf{q}|$  and the angle  $\vartheta$  between the vectors  $\mathbf{q}$  and  $\mathbf{M}$  but does not depend on the azimuthal angle  $\psi$ . Therefore, it is convenient to average (13) over  $\psi$  immediately; as a result we obtain

$$\int_0^{2\pi} d\psi f_1(\mathbf{m}, \mathbf{q}) = \frac{1}{2\pi m_0} \int_0^{2\pi} d\varphi \int_0^1 dx \delta\{m_x - m_0(q, x, \varphi)\} \\ \times \delta\{m_z - 1/3 s^2 q (c \cos \vartheta - b \sin \vartheta \cos \varphi)\}. \quad (15)$$

If  $\vartheta = 0$ , then, naturally, (15) goes over into (14) (to within a factor  $2\pi$ ).

We now consider the second case:  $D = \infty$  and arbitrary  $\mathbf{q}$ . In this case the problem is simplified by the fact that the local moment is parallel to the local-anisotropy axis but lies at an acute angle to the local molecular field. Then from (6) we have

$$f_1(\mathbf{m}, \mathbf{q}) = \int \frac{d\mathbf{n}}{4\pi} \delta\{\mathbf{m} - s\mathbf{n} \text{ th}(sq\mathbf{n})\}. \quad (16)$$

The magnitude of the moment, as can be seen from

(16), is equal to  $\text{stanh}(sqx)$ , where  $x$  is the cosine of the angle between  $\mathbf{q}$  and  $\mathbf{n}$ . This dependence is easily understood, since the large anisotropy makes the spin "Ising-like" and the spin magnitude depends, naturally, on the projection of the magnetic field on the direction of the moment, which coincides with the direction of the anisotropy axis.

Rewriting (16) in an arbitrary coordinate frame, we obtain a formula analogous to (13) for our case:

$$f_1(\mathbf{m}, \mathbf{q}) = \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^1 dx \delta\{m_x - m_1 \cos(\psi + \psi_1)\} \\ \times \delta\{m_y - m_1 \sin(\psi + \psi_1)\} \delta\{m_z - s \text{ th}(qsx) [x \cos \vartheta - (1-x^2)^{1/2} \sin \vartheta \cos \varphi]\}, \\ \text{tg } \psi_1(x, \varphi) = \frac{(1-x^2)^{1/2} \sin \varphi}{x \sin \vartheta + (1-x^2)^{1/2} \cos \vartheta \cos \varphi} \\ m_1(q, x, \varphi) = s \text{ th}(qsx) \{ (1-x^2) \sin^2 \varphi \\ + [x \sin \vartheta + (1-x^2)^{1/2} \cos \vartheta \cos \varphi]^2 \}^{1/2}.$$

In the case  $sq \ll 1$ , (17) goes over into (13) when  $a = 1$ , which corresponds in (13) to  $D = \infty$ . For  $\vartheta = \psi = 0$ , from (17) we obtain

$$f_1(\mathbf{m}, \mathbf{q}) = \frac{1}{2\pi m_0} \int_0^1 dx \delta\{m_x - s(1-x^2)^{1/2} \text{ th}(qsx)\} \delta\{m_z - s x \text{ th}(qsx)\}. \quad (18)$$

After averaging (17) over  $\psi$  we have

$$\int_0^{2\pi} d\psi f_1(\mathbf{m}, \mathbf{q}) = \frac{1}{2\pi m} \int_0^{2\pi} d\varphi \int_0^1 dx \delta\{m_x - m_1(q, x, \varphi)\} \\ \times \delta\{m_z - s \text{ th}(qsx) [x \cos \vartheta - (1-x^2)^{1/2} \sin \vartheta \cos \varphi]\}. \quad (19)$$

Using the explicit form of  $f(\mathbf{q})$  in (9),  $f_1(\mathbf{m})$  in (12) and  $f_1(\mathbf{m}, \mathbf{q})$  in (15), (19), it is easy to show that  $f_1(\mathbf{m})$  is nonzero for  $m_x < 0$ , even if we assume that  $\alpha_0^2 Q_0 = \alpha_0^2 M^2$ , i.e., if we assume that all  $q_x > 0$ . The characteristic values are

$$(-m_x) \sim |m_{\text{char}}| (Q_0/z)^{1/2} M^{-1},$$

where  $|m_{\text{char}}|$  is the characteristic value of  $|\mathbf{m}|$ , different for (15) and (19). The fact that  $f_1(\mathbf{m})$  is nonzero for  $m_x < 0$  is the mathematical expression of what we talked about in Sec. 2.

To conclude this section we shall consider the moments of the distribution functions (14) and (18) and the relations between them. From (14) and (18), in the case of  $qs \ll 1$  and arbitrary anisotropy, we have

$$\langle m_x \rangle = 1/3 s^2 q, \quad \langle m_x^2 \rangle / \langle m_x \rangle^2 = 1/3 (3a-1)^2, \\ \langle m_x^2 \rangle - \langle m_x \rangle^2 / \langle m_x \rangle^2 = 1/3 (3a-1)^2,$$

while for  $y \gg qs \gg 1$ ,

$$\langle m_x \rangle = 1/s, \quad \langle m_x^2 \rangle = \langle m_x \rangle^2 = 1/s^2. \quad (20)$$

We note that  $s^2 q/3$  is the usual Langevin in magnetization. From (13) it can be seen that as  $D \rightarrow 0$  the quantity  $a - 1/3$  and the variance of the magnetization vanishes. We note that for  $y \gg qs \gg 1$  the quantity  $\langle m_x \rangle$  by no means reaches its maximum possible value  $s$ .  $\langle m_x \rangle$  acquires this value only for  $D = 0$ .

## 5. DERIVATION AND SOLUTION OF THE EQUATIONS FOR $M$ , $Q_0$ , and $Q_1$

We turn now to the derivation of the basic equations for our theory, for the parameters  $\mathbf{M}$ ,  $Q_0$ , and  $Q_1$  of

the distribution function of the molecular fields. For this we make use of the formula (12). We consider first the region near  $T_c$  (by  $T_c$  we shall mean the temperature of the transition from the paramagnetic state to the ordered state, whether this be the ferromagnet or the spin glass). Near  $T_c$  the characteristic values of  $q \ll 1/s$ , and we can make use of the formulas (13) and (15) for  $f_1(\mathbf{m}, \mathbf{q})$ .

From (12), (13), and (15) we obtain the following equations:

$$\begin{aligned} M &= \frac{1}{s} s^2 \langle q_x \rangle, \\ Q_0 &= \frac{1}{s} s^4 \left\{ \left[ 1 + \frac{1}{s} (3a-1)^2 \right] \langle q_x^2 \rangle + \frac{1}{10} (3a-1)^2 \langle q_x^4 \rangle \right\}, \\ Q_1 &= \frac{1}{s} s^4 \left\{ \frac{1}{20} (3a-1)^2 \langle q_x^2 \rangle + \left[ 1 + \frac{1}{20} (3a-1)^2 \right] \langle q_x^4 \rangle \right\}. \end{aligned} \quad (21)$$

The averaging of  $\langle q_x^2 \rangle$ , etc. is performed with the function  $f(\mathbf{q})$ . All the calculations are performed in a coordinate frame in which the  $z$  axis is parallel to the average-magnetization vector  $\mathbf{M}$ . In the derivation of (21) the identities  $\langle m_\rho^2 \rangle = 2 \langle m_x^2 \rangle$ ,  $\langle q_\rho^2 \rangle = 2 \langle q_x^2 \rangle$  have been used. From (9) and (21) we obtain

$$\begin{aligned} M &= \frac{1}{s} s^2 z \alpha_0 M, \\ \left\{ 1 - \frac{1}{s} z (\alpha_2 s^2)^2 \left[ 1 + \frac{1}{s} (3a-1)^2 \right] \right\} Q_0 - \frac{1}{10} (3a-1)^2 z (\alpha_2 s^2)^2 Q_1 & \\ &= \frac{1}{s} s^4 \left[ 1 + \frac{1}{s} (3a-1)^2 \right] z (z-1) \alpha_0^2 M^2, \\ \left\{ 1 - \frac{1}{s} z (\alpha_2 s^2)^2 \left[ 1 + \frac{1}{20} (3a-1)^2 \right] \right\} Q_0 & \\ - \frac{1}{10} z (\alpha_2 s^2)^2 (3a-1)^2 Q_1 & \\ &= \frac{1}{s} s^4 (3a-1)^2 z (z-1) \alpha_0^2 M^2. \end{aligned} \quad (22)$$

Eqs. (21) are written in the linear approximation, and therefore they are valid only near  $T_c$ .

In the ferromagnetic phase the first equation determines  $T_c$  and the other two equations give expressions for  $Q_0$  and  $Q_1$  in terms of  $M^2$ :

$$\begin{aligned} Q_0 &= \Delta_1 / D_0, \quad Q_1 = \Delta_2 / D_0, \\ D_0 &= 1 - \frac{1}{s} z (\alpha_2 s^2)^2 \left[ 2 + \frac{1}{20} (3a-1)^2 \right] \\ &+ \frac{1}{10} z^2 (\alpha_2 s^2)^4 \left[ 1 + \frac{1}{20} (3a-1)^2 + \frac{1}{10} (3a-1)^4 \right], \\ \Delta_1 &= \left\{ 1 + \frac{1}{s} (3a-1)^2 - \frac{\alpha_2^2}{z \alpha_0^2} \left[ 1 + \frac{1}{20} (3a-1)^2 + \frac{1}{10} (3a-1)^4 \right] \right\} (1-1/z) M^2, \\ \Delta_2 &= \frac{1}{20} (3a-1)^2 (1-1/z) M^2. \end{aligned} \quad (23)$$

The expression for  $D_0$  in (23) is written in general form, while the formulas for  $\Delta_1$  and  $\Delta_2$  are written near the point of the phase transition to the ferromagnetic state, where the condition  $1 = s^2 z \alpha_0 / 3$  is fulfilled, since these quantities are of interest only in this region, inasmuch as  $M \neq 0$  there.

From (23) it can be seen that  $D_0$  vanishes when

$$\begin{aligned} z &= \frac{9}{(\alpha_2 s^2)^2} \left[ 1 + \frac{1}{2} (3a-1)^2 \right]^{-1}, \\ z &= \frac{\alpha_2^2}{\alpha_0^2} \left[ 1 + \frac{1}{2} (3a-1)^2 \right], \quad \frac{1}{3} s^2 z \alpha_0 = 1. \end{aligned} \quad (24)$$

The second formula is written in the ferromagnetic phase near  $T_c$ .

From (23) it can be seen that when  $D_0$  vanishes  $Q_0$  and  $Q_1$  can be positive and finite quantities only when  $M = 0$ , as is required of them by their physical meaning. Thus, the condition  $D_0 = 0$  corresponds to the spin-glass phase. If we approach the point  $D_0 = 0$  from the ferromagnetic region, then, as can be seen from (23),  $Q_0$  and  $Q_1$  become large compared with  $M^2$ . Thus, the ferromagnet gradually loses its properties, as it were, inasmuch as  $M$  becomes a small quantity compared with the

standard deviation of the local magnetization, and, finally,  $M$  vanishes at some point. In the spin-glass region,  $T_c$  is now determined not by the condition  $s^2 z \alpha_0 / 3 = 1$  but by the condition  $D_0 = 0$ , i. e., by the first equation (24).

It is easy to show that allowance for the nonlinear terms in Eqs. (22) will change nothing in the elucidation of the conditions for the existence of the ferromagnetic phase and the spin-glass phase and will only make it possible to determine the absolute values of  $M$ ,  $Q_0$ , and  $Q_1$  (Eqs. (22) did not determine them). Allowance for the nonlinearity will also determine for us the critical index  $\beta$ , which in the ferromagnetic phase is, naturally,  $\beta = \frac{1}{2}$ , while in the spin-glass phase  $\beta = 1$ . Therefore, we shall not write out Eqs. (22) with allowance for the nonlinear terms.

We shall consider now the region of low temperatures  $T \ll T_c$ . As we have already said, on the one hand our equations can be used for  $T \gg J_0$ , and, on the other, they are also valid, as is easily shown from (3) and (5), for  $T = 0$ . The results are the same. In fact, we shall calculate the trace over  $\mathbf{s}$  in  $m_j$  in (6) for  $T = 0$  by the method of deepest descents. The expression obtained coincides with the condition for the minimum of the energy  $E_t$  in (5). Clearly, therefore, it is reasonable to assume that these results are also true for all  $T \ll T_c$ .

In this region we consider only the case of strong anisotropy  $y \gg qs$ . Since for  $T \ll T_c$  the local magnetizations are close to saturation, we have  $qs \gg 1$ . Then from (12), (17), and (19), for  $y \gg qs \gg 1$ , we obtain

$$M = \frac{1}{2} s \langle \cos \vartheta \rangle, \quad Q_0 = Q_1 = \frac{1}{3} s^2. \quad (25)$$

In (25)  $\langle \cos \vartheta \rangle$  denotes the average of  $\cos \vartheta$  over the distribution function  $f(\mathbf{q})$  in (9). The modulus of  $\mathbf{q}$  has dropped out in (25), since at low temperatures all the local moments are completely saturated. For the same reason,  $Q_0$  and  $Q_1$  are equal to their values in (20). Thus, (25) is an equation for  $M$  only. Putting

$$\cos \vartheta = q_z / q = q_z / (q_x^2 + q_z^2)^{1/2},$$

from (9) and (25), after straightforward transformations, we obtain

$$\begin{aligned} M &= f(M), \\ f(M) &= \frac{1}{2} s \left[ (2z)^2 \alpha_2^2 Q_1 (\alpha_2^2 Q_0 - \alpha_0^2 M^2) \right]^{-1/2} \int_{-\infty}^{\infty} q_z dq_z \\ &\times \left\{ 1 - \Phi \left( \frac{|q_z|}{(2z \alpha_2^2 Q_1)^{1/2}} \right) \right\} \exp \left\{ \frac{q_z^2}{2z \alpha_2^2 Q_1} - \frac{(q_z - z \alpha_0 M)^2}{2z [\alpha_2^2 Q_0 - \alpha_0^2 M^2]} \right\}, \\ \Phi(z) &= \frac{2}{\pi^{1/2}} \int_0^z e^{-t^2} dt, \end{aligned} \quad (26)$$

$$f(M) = \begin{cases} (2z/3\pi)^{1/2} \alpha_0 M / \alpha_2, & M \rightarrow 0, \\ s/2, & z M^2 \gg s^2. \end{cases}$$

In the last formula of (26) the asymptotic forms of  $f(M)$  are written out; in deriving them we have used the fact that  $Q_0 = Q_1 = s^2/3$ .

It can be seen from (26) that for  $z \gg (\alpha_2/\alpha_0)^2 3\pi/2$  we have the solution  $M = s/2$  which corresponds to the magnetization in (20) (even at  $T = 0$ , the strong anisotropy loosens the system so strongly that the saturation magnetization is equal to  $s/2$  and not  $s$ ). With

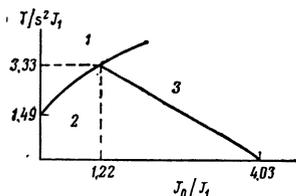


FIG. 2.

decreasing  $z$  the quantity  $M$  decreases and, finally, at

$$z = \frac{3\pi}{2} \left( \frac{\alpha_2}{\alpha_0} \right)^2 \approx 4.71 \left( \frac{\alpha_2}{\alpha_0} \right)^2 \quad (27)$$

the magnetization vanishes. At lower values of  $z$  we have the spin-glass phase. We note that, near the critical values of the parameters, we already have  $M \ll Q_0, Q_1$ , i. e., the situation in analogous to that which obtained near  $T_c$ .

## 6. DISCUSSION OF THE RESULTS. THE PHASE DIAGRAM

We now discuss the results obtained and draw the phase diagram. First of all we remember that, as we have already said in Sec. 3, in the Bethe-Peierls approximation it is necessary to replace  $z$  by  $z-1$ . This means that it is necessary to make this replacement in all the formulas of the preceding sections. For  $z \gg 1$  this is not important. However, for the substances being studied,  $z = 4-6$ ,<sup>[3]</sup> and therefore this replacement is important. We now write out the criterion for the existence of a spin-glass phase with allowance for this replacement. Then, from (24) and (27) we obtain

$$\begin{aligned} \frac{1}{s} (z-1) (\alpha_2 s^2)^2 &= [1 + \frac{1}{2} (3a-1)^2]^{-1}, \\ z-1 &= \frac{\alpha_2^2}{\alpha_0^2} \left[ 1 + \frac{1}{2} (3a-1)^2 \right], \quad \frac{1}{3} s^2 (z-1) \alpha_0 = 1, \\ z-1 &= \frac{3\pi}{2} \frac{\alpha_2^2}{\alpha_0^2} \approx 4.71 \frac{\alpha_2^2}{\alpha_0^2}. \end{aligned} \quad (28)$$

The first two equations are written for  $|T_c - T| \ll 1$ ; the first equation determines  $T_c$  in the spin-glass phase while the second is valid in the ferromagnetic phase and determines the parameter values at which the transition from the ferromagnetic phase to the spin-glass phase occurs. The last equation determines the same parameter values, but for  $T \ll T_c$ . From (28) it can be seen that for large anisotropy ( $a=1$ ) and in the absence of fluctuations of the exchange integral ( $\alpha_2 = \alpha_0$ ) the critical values of  $z$  are equal to

$$\begin{aligned} z=4, \quad |T_c - T| \ll T_c, \\ z=5.71, \quad T \ll T_c. \end{aligned} \quad (29)$$

Since we have  $z = 4-6$ , there is certainly a region of existence of the spin-glass phase. Furthermore, decreasing the anisotropy leads, naturally, to a narrowing of this region (in this case  $a$  becomes  $< 1$  and as  $D \rightarrow 0$  the quantity  $a \rightarrow 1/3$ ). But fluctuations of the exchange integral, naturally, stir up the system and thereby expand the region of existence of the spin-glass phase. For large anisotropy ( $a=1$ ), from (28) and (9) we obtain

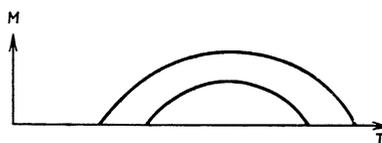


FIG. 3.

$$\begin{aligned} T_{cg} &= \frac{2}{\sqrt{3}} s^2 J_0 (1 + J_1^2/J_0^2)^{1/2} (z-1)^{1/2}, \\ T_{cf} &= \frac{2}{3} s^2 J_0 (z-1), \\ \frac{J_1^2}{J_0^2} &= \frac{z-4}{3}, \quad |T_c - T| \ll T_c, \\ \frac{J_1^2}{J_0^2} &= \frac{z-1}{4.71} - 1, \quad T \ll T_c. \end{aligned} \quad (30)$$

The first two equations determine  $T_{cg}$  for the transition from the paramagnetic to the spin-glass phase  $T_{cf}$  for the transition to the ferromagnetic phase. The third equation determines the point of coexistence of all three phases; when this equation is fulfilled,  $T_{cg} = T_{cf}$ . The last equation determines the line separating the ordered phases at  $T \ll T_c$ . The phase diagram for  $z=6$  and a fixed  $J_1$  is shown schematically in Fig. 2. In Fig. 2 the numbers 1, 2, and 3 correspond to the paramagnetic phase, the glass, and the ferromagnet. We note that for  $z=4$  and large anisotropy there is no ferromagnetic phase; the spin glass or the paramagnet is realized for all values of  $J_1$  and  $J_0$ . For  $z=5$  the ferromagnetic phase exists only at temperatures higher than a certain  $T_0$ .

We consider now, for  $z=6$ , a specific value of  $J_0/J_1$  in the range 1.22-4.03. Then, if we change the temperature, we first have the spin glass, then the ferromagnet, and then the paramagnet. The dependence of the magnetization of the temperature is depicted schematically in Fig. 3. We note that the phase diagrams depicted in Figs. 2 and 3 are very similar to the corresponding phase diagrams of Ref. 6. However, for  $z=4, 5$  the phase diagrams already have a different form.

We note, that since, in reality,  $J_1^2/J_0^2 \approx 1$ , the spin-glass phase can be realized with not very large values of  $z$ . However, as can be seen from (30), even for  $z=8$  at  $T \ll T_c$  we should have  $J_1^2/J_0^2 \approx 0.49$ , which is entirely realistic. For sufficiently large values of  $z$  there will be no spin-glass phase.

In conclusion the author would like to express his gratitude to I. Ya. Korenblit for discussions on the work.

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Translated by P. J. Shepherd